FE 5108: Portfolio Theory and Investments

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The following document presents the answer for the FE 5108 Class Project, completed by a team of three students: **Samarth Bahukhandi (A0297746H), Raditya (A0296600E), and Sai Kiran Reddy (A0297745J).** Despite our efforts, we were unable to find additional members, so this project was undertaken by the three of us alone. The project is done using **R, Python & Excel** software **.**

Question 1)

PART A.

- For t-stat, calculate **mean/se** for each factor. Since **return spreads** are calculated, **arithmetic mean** is **preferred** over **geometric mean.** The arithmetic mean shows a period-to-period difference between the returns, and spreads don't compound.
- The t-stat for **SMB, and HML** factors is higher for the 1927-1992 period than for the 1993-2023 period. This indicates that the factors in the **pre-publication period** were **more significant** than after the model was published. SMB and HML don't remain high-conviction investment ideas after the publication compared to before publication.

```
T-Stats for 1927-1992

With Arithmetic Mean
Mkt-RF:  3.2205625120989674
SMB:  2.1271515550073685
HML:  3.335204505925686
```

1.A.1 T-stats for 1927-1992 period

```
T-Stats for 1993-2023

With Arithmetic Mean
Mkt-RF:  3.095629461680783
SMB:  0.6002049027386961
HML:  0.8315327229762526
```

1.A.2 T-stats for 1993-2023 period

PART B.

- The **Investable universe** for the MSCI index consists of all the existing constituents of an underlying **MSCI parent index**. This ensures that the investable universe has

sufficient liquidity and **capacity** by depending on the broader MSCI country or regional Index. **(Section 2.1)**

- Price momentum in the MSCI momentum index is calculated using each security's 6—and 12-month local price performance.

  Price momentum calculation:
  6-month price momentum = $((P_{T-1}/P_{T-7}) - 1)$ – (Local Risk-free rate)
  12-month price momentum = $((P_{T-1}/P_{T-13}) - 1)$ - (Local Risk-free rate)
  Where P_{T-K} = Security local price K months before the rebalancing date (T).

  If **12-month** price momentum is **missing**, only **6-month** price momentum is used for computing momentum value and vice versa if **6 months** is missing. **Local risk-free rates** are the short-term rates in the **local currency** of the country, typically the **3M LIBOR rate** or **short-term deposit rate**. **(Section 2.2)**

- **Price momentum** is standardized across stocks by calculating a risk-adjusted momentum value for each security, converted to **Z-Score**. Risk-adjusted momentum is calculated by **dividing price momentum** by its **annualized standard deviation** of weekly returns over 3 years. This normalizes the impact of volatility across stocks. These are standardized into **Z-Scores** and then **averaged** out with equal weight using **6 months** and **12 months** to produce a single combined **momentum z-score**. To control the extreme values the momentum z-score is **winsorized** at **+/-3**. The values beyond 3 or below -3 are capped at the respective values. **(Section 2.2)**

- Selected stocks are weighted by calculating their weight (**each stock's weight** in the index is the **product** of its **market cap weight** in the parent index and its **momentum score**), and then the momentum weight is **normalized** to 100%.

  Benefits:
  1. By including a market cap, the index **emphasizes larger, more liquid stocks, reducing** the impact of **individual trade** on stock prices.
  2. The market cap's weight also **reduces the index's volatility**.

  Detriments:
  1. **Lower weight** will be given to the **high-momentum stocks** that have a **smaller market cap**. This could dilute the momentum factor.

- The **MSCI Momentum Index** is rebalanced **semi-annually** in **May** and **November** coinciding with the May and November **Semi-Annual Index Review(SAIR)** of the MSCI Global Investable Market Indexes.

- In addition to semi-annual reviews, the MSCI momentum index also undergoes **ad-hoc trigger-based rebalancing** based on trigger conditions checked every month. If the **monthly volatility** crosses the threshold (**95 percentile** of monthly change of parent index) rebalancing happens. During these periods, the **index rebalances** using only the **6-month momentum score**, rather than the typical combination of 6- and 12-month scores, to capture recent **shifts in momentum more accurately**.

  Benefits:

  1. By rebalancing, the index can **adapt** its composition to better reflect the current market, reducing exposure to a **sharp decline of stocks**, and minimizing potential loss.
  2. Ad-hoc can help address cases when stocks **lose their momentum advantage**. Investors can refocus on other stocks or securities that continue to exhibit strong momentum.
  3. Ad-hoc allows investors to **quickly respond to changes** in the market. Investors can **recalibrate** to other stocks with **stronger recent momentum** even when market volatility is high

- The MSCI Momentum Index applies caps on individual security weights to reduce concentration risk.

  Broad Regional and Country Indexes: For these indexes, an individual issuer's weight is capped at 5%. This ensures no single security can dominate the index, promoting a more balanced exposure across multiple stocks.

  Narrow Country or Regional Indexes: is defined as an index for which the maximum market cap weight in the Parent Index is greater than 10%. This is particularly relevant when an index has a few dominant companies, as it prevents these large-cap stocks from skewing the index too heavily.

  <u>PART C.</u>

- Calculate annualized return and Sharpe ratio:
  1. $annualized\ return\ =\ (\Pi(1\ +\ return))^{12/terms}\ -\ 1$

  2. $sharpe\ ratio\ =\ (annualized\ return\ -\ annualized\ risk\ free\ rate)/(std(return)\ *\ \sqrt{12})$

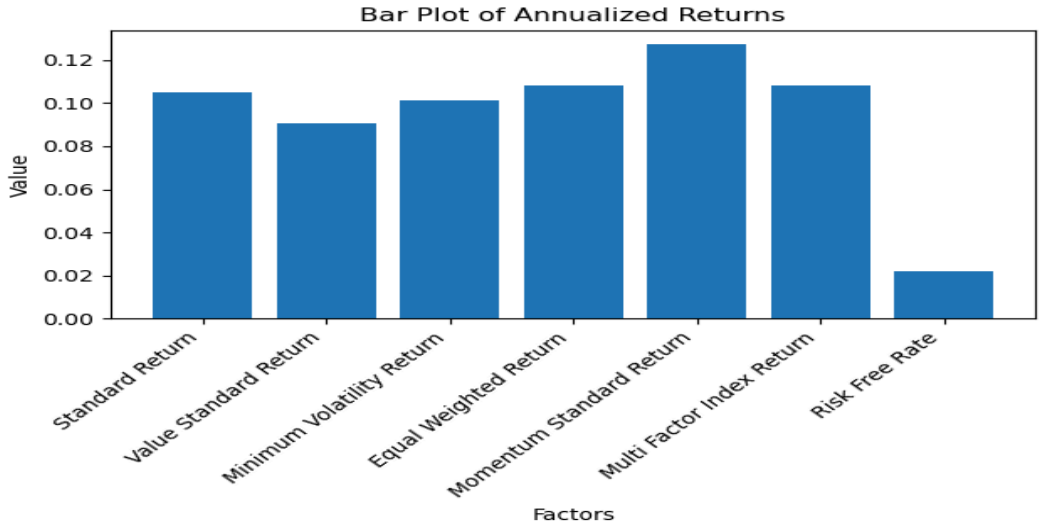
```
Annualized Returns

Standard: 0.1049 = 10.49%
Value Standard: 0.0905 = 9.05%
Minimum Volatility: 0.1013 = 10.13%
Equal Weighted: 0.1082 = 10.82%
Momentum Standard: 0.1272 = 12.72%
Multi Factor Index: 0.1082 = 10.82%
Risk-free rate: 0.0218 = 2.18%
```

1.C.1 Annualized returns for MSCI Indexes

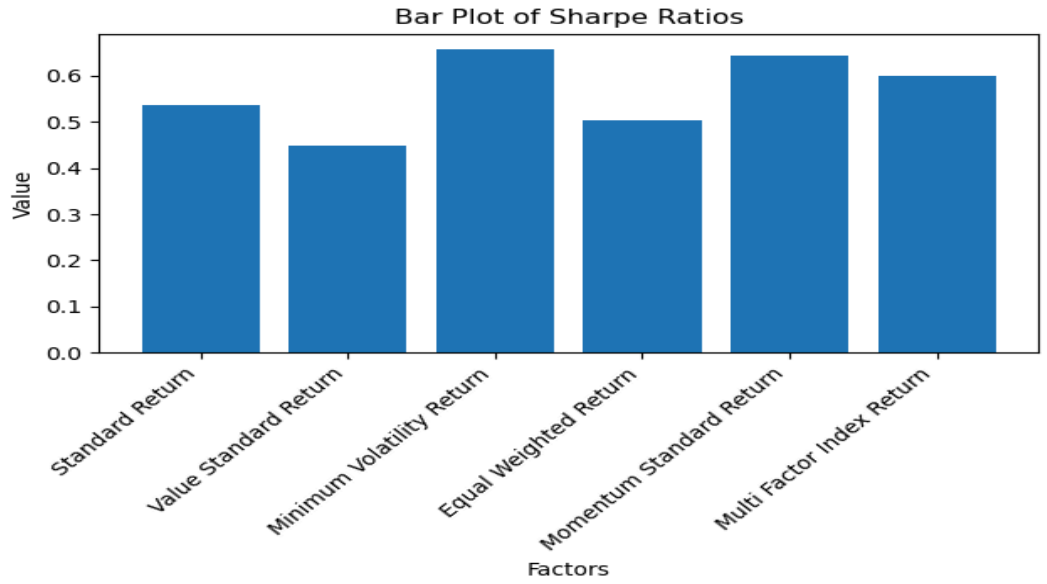
```
Sharpe Ratios

Standard: 0.5379
Value Standard: 0.4497
Minimum Volatility: 0.6569
Equal Weighted: 0.5042
Momentum Standard: 0.6446
Multi Factor Index: 0.6000
```

1.C.2 Sharpe ratio for MSCI Indexes

- **Momentum Standard Return** performed the best with **12.72%** returns and the Value Index performed worst with **9.05%** returns. **Equal weighted** and **Multifactor index** performed the second best with **10.82%**, followed by Standard and Minimum Volatility.
- **Minimum Volatility** has the best Sharpe ratio of **0.6569** and **Value Standard** the least with **0.4497**. **Momentum Standard** is the second best with **0.6446**, followed by **Standard** with **0.5379** and **Equal Weighted** with **0.504**



1.C.3 Bar Plot of Annualized returns on MSCI Indexes



1.C.4 Bar Plot of Sharpe Ratios returns on MSCI Indexes

PART D.

- **MSCI Value Spread index spread** is correlated to **HML** with a correlation of **0.728**. **MSCI Minimum Volatility Spread** is correlated to **Beta Spread** with a correlation of **0.815**.
- **The Momentum Index Spread** is correlated with the calculated **Momentum Spread** with a correlation of **0.642**.

	HML	SMB	MOM	BAB
Value Spread	0.72832	-0.177026	-0.243478	0.414829
Minimum Volatility Spread	0.231505	-0.306972	0.303188	0.814676
Equal Weighted Spread	0.430786	0.321624	-0.448468	-0.199402
Momentum Spread	-0.22251	0.079246	0.642596	0.14025

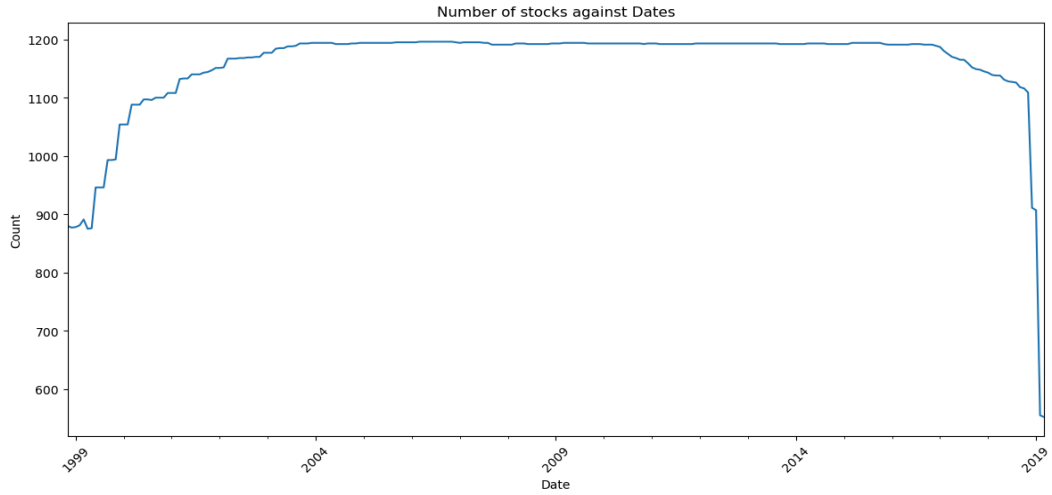
1.D.1 Correlation matrix of MSCI Index Spreads and Factors

Question 2)

PART A.

- The investable universe is **not consistently** defined. The number of stocks picks up from the start (December 1998 Returns) **880 stocks**, to a uniform number of stocks of around **1200** from 2004 to 2017, and drops to **552** at the end (April 2019 Returns). Considering the **2004 to 2017** period would be a consistently defined investable universe.

- **April 2019 returns** and **March 2019** returns are particularly questionable with a very low number of stock data.

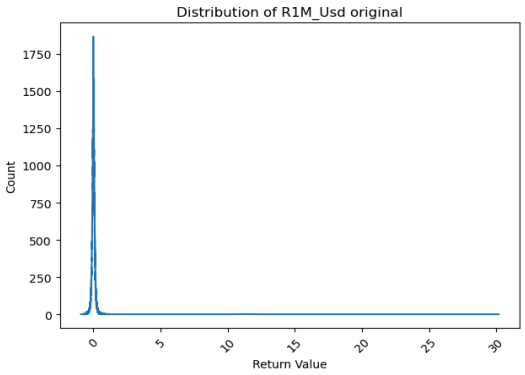


2.A.1 Number of Stocks against Dates

- Returns (R1M_Usd) have more outliers on the **upside** with the maximum being **30.176**. The following shows stats and the original distribution of the returns

count	283380.000000
mean	0.012633
std	0.177877
min	-0.922000
25%	-0.044000
50%	0.011000
75%	0.064000
max	30.176000

2.A.2 stats of R1M_Usd returns

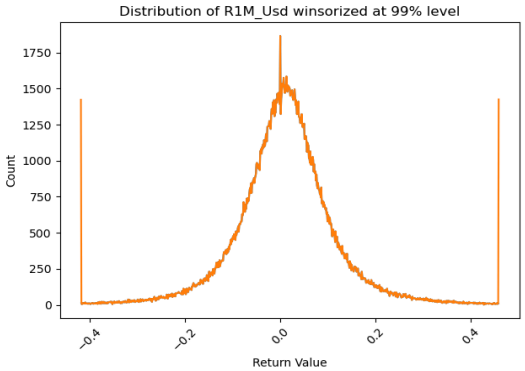


2.A.3 Distribution of Original Returns

- Winsorization is done on the data at 99% and the outliers are set to edge values. The following stats and diagram show the new distribution.

count	283380.000000
mean	0.010709
std	0.110137
min	-0.418000
25%	-0.044000
50%	0.011000
75%	0.064000
max	0.458000

2.A.4 Stats of winsorized R1M_Usd returns

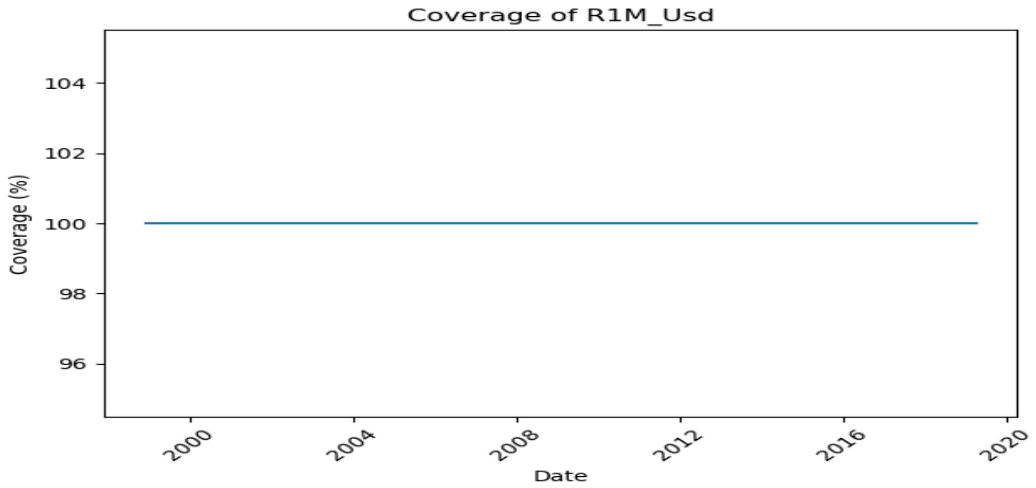


2.A.5 Distribution of winsorized R1M_Usd returns

- The datasheet doesn't contain any missing data. However as observed from the plot above, An unnaturally large number of returns is set as 0. The hypothesis is these fields are missing data and are filled as a substitute.
- The reasons for doing this might be stock has disappeared or the returns for that period are not known.

Options to treat missing data:

1. **Remove stocks with missing data**: This is the simplest approach but may reduce the sample size.
2. **Replace missing values**: Replace missing values with the mean, median, or immediately past value of the stock.
3. **Use models that handle missing data**: Use models to predict and fill missing data.

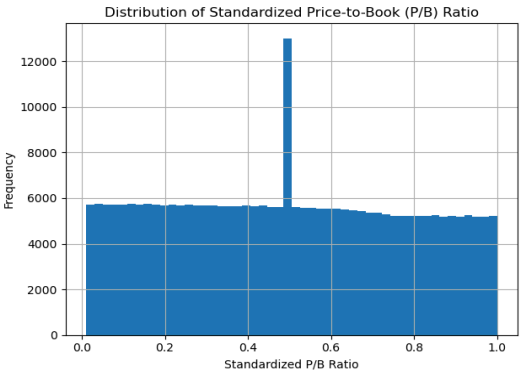


2.A.6 Coverage of R1M_Usd against time

- A **P/B value** of 0.5 stands out from the distribution. The other **P/B** are almost uniform around 6000 frequency, P/B value of 0.5 spikes to almost **13000**.
- Given how the HML factor is constructed the bottom and high Price to Book value will be considered. So the large number of stocks at a P/B value of 0.5 will not be considered in the calculation.

count	283380.000000
mean	0.495121
std	0.283646
min	0.010000
25%	0.250000
50%	0.500000
75%	0.730000
max	1.000000

2.A.7 stats of P/B standardized values

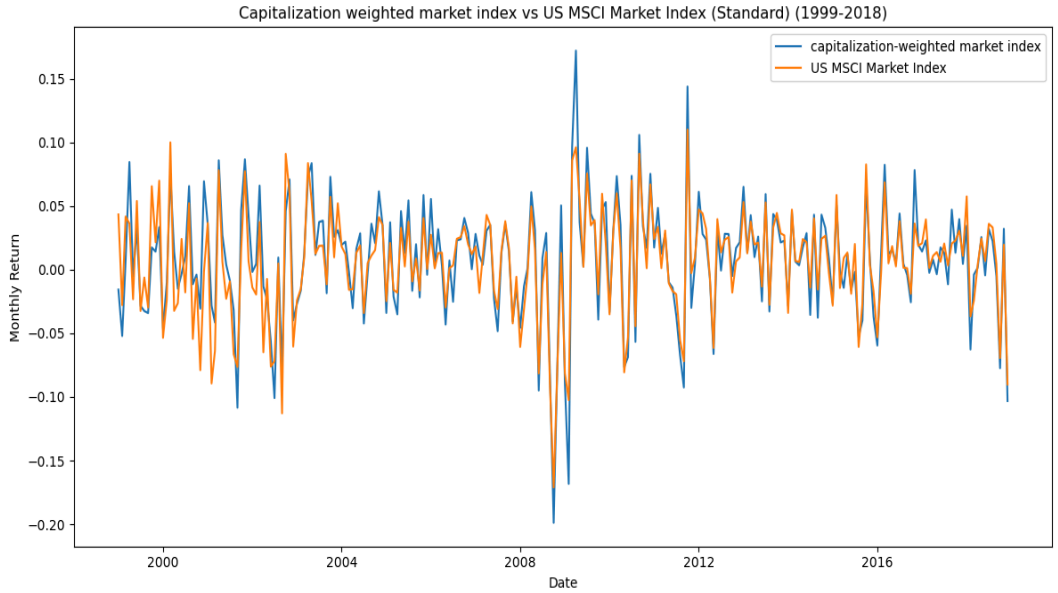


2A.8 distribution of Standardized Price to Book ratio

- Sanity check would be the Correlation between the multiplied value of Standardized Price to Book value and Standardized Book value vs Standardized Market Cap. Multiplication is a better indicator for standardized values than division.
- The **Spearman correlation** from the above operation comes out to be **0.7954**. This indicates that the P/B value is indeed the ratio between market cap and book value.

PART B.

- Comparison between **market cap weighted returns** and the **MSCII standard market index** are visually similar with both the highs and lows. This provides confidence that the **market cap-weighted** returns consider a broad universe of US stocks with an average number of monthly stocks as **1165**.
- The correlation between them comes out to be **0.9095**.



2.B.1 Plot of Market Cap weighted index and US MSCI Market Index

PART C.

- **Note: Winsorized returns are used for calculations and since P/B is standardized, the B/P standardized value is taken as 1-standardized(P/B).**
- The **annualized return** for the equally weighted portfolio is higher than that for the **Book to Price-weighted** portfolio, which is higher than that for the Book to Price and market-weighted portfolio. Stocks with lower prices to book give higher returns as they are undervalued. Stocks with higher market caps don't fluctuate as much, so returns when there is a higher weight to market cap are lower.

```
Annualized return for equally weighted portfolio: 0.1692 = 16.92%
Annualized return for Bp weighted portfolio: 0.1292 = 12.92%
Annualized return for Bp*MktCap weighted portfolio: 0.0958 = 9.58%
```

2.C.1 Annualized returns of portfolios constructed

- The annualized standard deviation for an **equally weighted portfolio** is higher than that for a Book to price-weighted portfolio as they are more volatile. The standard deviation for a **Book to price-weighted** portfolio is **higher** than that for a **Book to Price** and Market Cap-weighted portfolio as the stocks with higher market caps are less volatile.

```
Annualized standard deviation for equally weighted portfolio: 0.2053 = 20.53%
Annualized standard deviation for Pb weighted portfolio: 0.1792 = 17.92%
Annualized standard deviation for Pb*MktCap weighted portfolio: 0.1689 = 16.89%
```

2.C.2 Annualized Standard Deviation of the Portfolios constructed

- The results without winsorization are as follows:

```
Annualized return for equally weighted portfolio (without winsorization): 0.2484 = 24.84%
Annualized return for Bp weighted portfolio (without winsorization): 0.1665 = 16.65%
Annualized return for Bp*MktCap weighted portfolio (without winsorization): 0.1115 = 11.15%
Annualized standard deviation for equally weighted portfolio (without winsorization): 0.2350 = 23.50%
Annualized standard deviation for Pb weighted portfolio (without winsorization): 0.1901 = 19.01%
Annualized standard deviation for Pb*MktCap weighted portfolio (without winsorization): 0.1756 = 17.56%
```

2.C.3 Annualized returns and Standard Deviation for strategies without winsorization

- The portfolio with the least number of stocks is the **equal-weighted portfolio** with **219** stocks, while the portfolio with the **highest single-stock weight** is the **Market Price Book to Price Weight Portfolio**.
- The Market Cap Book to Price Weight Portfolio has the highest single-stock weight of **0.38%** as it is weighted by both the **book-to-price ratio** and the **market capitalization** of the stocks. This is expected as companies with **higher market cap** weight improve the **weight of a high B/P single stock**.

```
Number of stocks in MktCap BP Weight Portfolio: 1118
Number of stocks in BP Weight Portfolio: 1118
Number of stocks in equal weighted portfolio: 219
Highest single-stock weight in BP Weight Portfolio: 0.0018
Highest single-stock weight in MktCap BP Weight Portfolio: 0.0038
Highest single-stock weight in equal weighted portfolio: 0.0009
```

2.C.5 Analysis of Last period for number and Weight of Stocks

PART D.

- **Book-to-Price and Market Cap weighted Index** has the lowest tracking error of **2.65%** as it includes Market Cap as weights making it **closest** to similar **Market Cap weighted** benchmark.

```
Annualized Standard Deviation of the strategies over broad market cap weighted index
Equally Weighted Excess      0.084355
Bp Weighted Excess           0.045201
Bp*MktCap Weighted Excess    0.026588
```

2.D1 Annualized Standard Deviation of Strategies over Broad Market Cap Index

- **Note: Indexes data is moved by a month to calculate correlation with HML. As index data is 1 month forward.**
  Equally weighted Excess has a high correlation of **0.951** with **Book Price Weighted Excess**. The correlations imply portfolios are indeed investing in the same style factor- value.
- HML has a high correlation with book-to-price and market-weighted Excess with 0.856. This is expected as **HML** calculation includes **Market Cap** and **book-to-price weights**.

```
Correlation Matrix                          Equally Weighted Excess  Bp Weighted Excess  \
Equally Weighted Excess                    1.000000                 0.950988
Bp Weighted Excess                         0.950988                 1.000000
Bp*MktCap Weighted Excess                  0.771046                 0.766655
HML                                        0.593781                 0.603273

                           Bp*MktCap Weighted Excess       HML
Equally Weighted Excess                     0.771046  0.593781
Bp Weighted Excess                          0.766655  0.603273
Bp*MktCap Weighted Excess                   1.000000  0.856963
HML                                         0.856963  1.000000
```

2.D2 Correlation matrix of equal weighted

Question 3)

PART A.

- We download the history of monthly industry total returns (including dividend payouts) of 49 industries in value-weighted measure from January 1995.
- We apply data manipulation to get cleaned data from seed year: January 1995 - December 2004. Finally, we calculate the covariance matrix:

	Agric	Food	Soda	Beer	Smoke	Toys	Fun	Books	Hshld	Clths	...	Boxes	Trans	
Agric	0.003302	0.000600	0.000387	0.000573	0.000418	0.001208	0.000956	0.000844	0.000482	0.001324	...	0.001324	0.000916	0.0
Food	0.000600	0.001929	0.001527	0.001595	0.001577	0.000897	0.000790	0.000759	0.000962	0.001224	...	0.001156	0.001070	0.0
Soda	0.000387	0.001527	0.007589	0.002968	0.001380	0.001303	0.001656	0.001437	0.001962	0.001721	...	0.001831	0.001902	0.0
Beer	0.000573	0.001595	0.002968	0.003662	0.001202	0.001144	0.001327	0.000838	0.001878	0.001231	...	0.001365	0.001523	0.0
Smoke	0.000418	0.001577	0.001380	0.001202	0.007729	0.001114	0.000873	0.000353	0.000790	0.000608	...	0.001253	0.000422	0.0
Toys	0.001208	0.000897	0.001303	0.001144	0.001114	0.003947	0.002320	0.000978	0.001059	0.002119	...	0.002404	0.001834	0.0
Fun	0.000956	0.000790	0.001656	0.001327	0.000873	0.002320	0.004887	0.001760	0.001097	0.002574	...	0.002788	0.002180	0.0
Books	0.000844	0.000759	0.001437	0.000838	0.000353	0.000978	0.001760	0.002072	0.000829	0.001808	...	0.001787	0.001530	0.0
Hshld	0.000482	0.000962	0.001962	0.001878	0.000790	0.001059	0.001097	0.000829	0.002162	0.000974	...	0.001432	0.001184	0.0
Clths	0.001324	0.001224	0.001721	0.001231	0.000608	0.002119	0.002574	0.001808	0.000974	0.004044	...	0.002813	0.002575	0.0
Hlth	0.000949	0.000999	0.001979	0.001755	0.001550	0.001283	0.001054	0.001097	0.001146	0.001898	...	0.001635	0.001462	0.0
MedEq	0.000826	0.000540	0.001426	0.000877	0.001044	0.000864	0.001487	0.000849	0.000850	0.001022	...	0.001047	0.000933	0.0
Drugs	0.000319	0.001050	0.001204	0.001497	0.000917	0.000485	0.001316	0.000768	0.001153	0.000618	...	0.000553	0.000794	0.0
Chems	0.001030	0.001061	0.001622	0.001191	0.000659	0.001942	0.001982	0.001412	0.001072	0.002334	...	0.002350	0.002004	0.0
Rubbr	0.000911	0.000559	0.002036	0.001169	0.000951	0.002041	0.002305	0.001303	0.001001	0.001770	...	0.002371	0.001617	0.0
Txtls	0.001023	0.000976	0.001853	0.001304	0.000438	0.001867	0.002374	0.001356	0.001024	0.002479	...	0.002483	0.002052	0.0
BldMt	0.001186	0.001101	0.002123	0.001675	0.001188	0.001950	0.002024	0.001379	0.001506	0.002285	...	0.002521	0.001972	0.0
Cnstr	0.001177	0.000851	0.001722	0.001101	0.001519	0.002297	0.002528	0.001716	0.000944	0.002885	...	0.002295	0.002307	0.0

3.A.1 Covariance Matrix of Average Value Weighted Monthly Industry Total Returns (including Dividends) for 49 industries; January 1995 - December 2004; Source

- Next, calculate the optimal weights from three strategies:

  - **Unconstrained Minimum Variance (long and shorts)**

```
Minimum Variance Portfolio Weights (unconstrained):
[ 0.04627885  0.18769057 -0.05869298 -0.02033556  0.00877537 -0.01191305
  0.00060732  0.44168817  0.23027653  0.04925941 -0.04975295  0.2691647
  0.01401752 -0.26821209  0.09001033 -0.01371139 -0.01473738 -0.18937241
  0.05534781  0.02645966 -0.01256868 -0.07843831  0.06967438 -0.19417598
 -0.02873605  0.12987025 -0.01563558  0.06359535 -0.01817007  0.21535218
  0.20304538  0.00495424 -0.13536246 -0.34444481  0.01645449  0.1062553
  0.01023675 -0.02270744  0.21542414 -0.05839574  0.10740772 -0.07029518
  0.02327959  0.12704142 -0.1867899  -0.05035545  0.19952425 -0.04696174
 -0.02192649]
```

3.A.2 Unconstrained Minimum Variance Portfolio Weights (long and shorts included), January 1995 - December 2004

  - **Constrained Minimum Variance (Long-only)**

```
Minimum Variance Portfolio of Long-only weights:
[ 1.12818560e-01  8.94331683e-02  8.59735524e-21  3.26561802e-21
  1.11076314e-20  6.94025337e-21  5.39364910e-22  1.06955046e-01
  1.23220454e-01  5.12833843e-21  2.40759490e-21  6.08433465e-02
  9.18358424e-02  7.30396855e-21 -1.68944105e-21  2.42550608e-21
  6.92295260e-21  2.34944336e-21  6.79912874e-21  3.35169508e-21
  8.68539729e-21  4.08360614e-21 -8.67518013e-21  6.72071879e-21
  1.14739430e-20  9.22894994e-02  6.87114553e-03  4.91974332e-21
  8.46440465e-21  1.21663291e-02  1.66963620e-01  2.60147592e-02
  5.66151476e-21  2.76904566e-21  1.65871984e-20  1.23350719e-20
  2.38841537e-20  6.16843882e-21  8.86214479e-21  1.55322092e-20
  2.61398098e-21  7.38189848e-21  5.59191377e-21  3.75686970e-21
  5.47082176e-21  4.76013221e-21  1.10588229e-01  2.51733010e-21
 -3.22516530e-21]
```

3.A.3 Constrained Minimum Variance Portfolio Weights (long-only), January 1995 - December 2004
Note: Some weights displayed above have negative signs but they are extremely small, approximating zero, and not actually negative values; they appear as such due to their very low magnitude

- **Inverse Volatility Portfolio**

```
Inverse Volatility Portfolio:
[0.02222129 0.02907317 0.01465717 0.02109895 0.01452356 0.02032462
 0.01826529 0.02805034 0.02746123 0.02007828 0.0177263  0.02850458
 0.02510685 0.02410806 0.02211148 0.02067371 0.02410881 0.01855018
 0.01461823 0.01772885 0.01826057 0.01953055 0.01802023 0.01813243
 0.01935681 0.0171913  0.01002034 0.02005607 0.00999685 0.02513102
 0.0279764  0.02037736 0.02148073 0.02389854 0.01193397 0.0132963
 0.01165467 0.01601064 0.02444983 0.01861411 0.0243701  0.02607215
 0.0227845  0.02462785 0.02246677 0.02448158 0.02697282 0.0159675
 0.01787706]
```

3.A.4 Inverse Volatility Portfolio Weights, January 1995 - December 2004

- Lastly, get all the weights in a table for comparison and sanity check as shown below. *Top 5 in each strategy are highlighted in green while bottom 5 are highlighted in red.*

RESULTS:

Table 1.1 Optimal Weights Allocation across different strategies

Industries	Minimum Variance Portfolio (Unconstrained)	Minimum Variance Portfolio (Constrained)	Inverse Volatility Portfolio
Agric	0.04627885	0.11281856	0.022221288
Food	0.18769057	0.08943317	0.029073166
Soda	-0.05869298	0.00000000	0.014657171
Beer	-0.02033556	0.00000000	0.021098947
Smoke	0.00877537	0.00000000	0.014523557
Toys	-0.01191305	0.00000000	0.020324616
Fun	0.00060732	0.00000000	0.018265287
Books	0.44168817	0.10695505	0.028050344
Hshld	0.23027653	0.12322045	0.027461231
Clths	0.04925941	0.00000000	0.020078279
Hlth	-0.04975295	0.00000000	0.017726304
MedEq	0.26916470	0.06084335	0.028504584
Drugs	0.01401752	0.09183584	0.025106854
Chems	-0.26821209	0.00000000	0.024108062
Rubbr	0.09001033	0.00000000	0.02211148
Txtls	-0.01371139	0.00000000	0.020673709
BldMt	-0.01473738	0.00000000	0.024108807
Cnstr	-0.18937241	0.00000000	0.018550179
Steel	0.05534781	0.00000000	0.014618233
FabPr	0.02645966	0.00000000	0.017728848
Mach	-0.01256868	0.00000000	0.018260571
ElcEq	-0.07843831	0.00000000	0.019530551
Autos	0.06967438	0.00000000	0.018020227
Aero	-0.19417598	0.00000000	0.018132425
Ships	-0.02873605	0.00000000	0.019356805
Guns	0.12987025	0.09228950	0.017191301
Gold	-0.01563558	0.00687115	0.010020335
Mines	0.06359535	0.00000000	0.02005607
Coal	-0.01817007	0.00000000	0.009996848
Oil	0.21535218	0.01216633	0.025131022
Util	0.20304538	0.16696362	0.027976402
Telcm	0.00495424	0.02601476	0.020377358
PerSv	-0.13536246	0.00000000	0.021480733
BusSv	-0.34444481	0.00000000	0.023898544
Hardw	0.01645449	0.00000000	0.011933975
Softw	0.10625530	0.00000000	0.013296296
Chips	0.01023675	0.00000000	0.011654673
LabEq	-0.02270744	0.00000000	0.016010645
Paper	0.21542414	0.00000000	0.024449832
Boxes	-0.05839574	0.00000000	0.018614114
Trans	0.10740772	0.00000000	0.024370099
Whlsl	-0.07029518	0.00000000	0.02607215
Rtail	0.02327959	0.00000000	0.022784503
Meals	0.12704142	0.00000000	0.024627849
Banks	-0.18678990	0.00000000	0.022466767
Insur	-0.05035545	0.00000000	0.024481575
RlEst	0.19952425	0.11058823	0.026972823
Fin	-0.04696174	0.00000000	0.015967498
Other	-0.02192649	0.00000000	0.017877063
Sum of Weights	**1.000000000**	**1.000000000**	**1.000000000**

3.A.5 Comparison of optimal weights across the three strategies within 49 industries, January 1995 - December 2004

IMPLICATIONS:

- The *Books and Household industries* consistently rank within the top 5 optimal weights across all strategies.
- The bottom 5 and other top-weighted industries *show no clear pattern across strategies* due to differing constraints and allocation methods.
- Each strategy's *weights sum to 1***,** confirming a balanced and *fully invested* allocation.

LONG-ONLY MINIMUM VARIANCE PORTFOLIO

- The *long-only minimum-variance problem* seeks to minimise portfolio variance *subject to the constraint* that all weights are non-negative. Here's the formulation:

  Objective:

  Minimise the portfolio variance, which can be expressed as:

  $$min\,\sigma^2 = min_W\, W^T \Sigma\, W$$

  where:

  - W is the vector of portfolio weights,
  - Σ is the covariance matrix of asset returns.

  Constraints:

  - **Fully invested**: The sum of weights must equal 1: $\Sigma_{i=1}^{n} W_i = 1$
  - **Long-only**: No short positions are allowed, so each weight must be non-negative $W_i \geq 0$ for all i = 1, 2, …., n

  Code:

- For this optimization, we use **convex optimization** and quadratic programming with Python's **CVXPY** library.
- We cp.quad_form(weights_long, cov_matrix) to set up the portfolio variance based on the covariance matrix. We define the problem with cp.Problem(objective_long, constraints_long) and solve it with problem_long.solve().
- The output min_weights_constrained gives the optimal weights for each asset in the long-only minimum-variance portfolio

```python
# Minimum Variance Portfolio: Constrained
weights_long = cp.Variable(n)

# Define objective (minimize variance)
portfolio_variance_long = cp.quad_form(weights_long, cov_matrix)
objective_long = cp.Minimize(portfolio_variance_long)

# Constraints: fully invested
constraints_long = [weights_long >=0, cp.sum(weights_long) == 1]

# Solve optimization problem
problem_long = cp.Problem(objective_long, constraints_long)
problem_long.solve()

min_weights_constrained = weights_long.value


# Print results
print("Minimum Variance Portfolio of Long-only weights:")
print(min_weights_constrained)
```

3.A.6 Minimum Variance Portfolio Code(Long-only), January 1995 - December 2004

PART B.

```
IN-SAMPLE ANNUALIZED VOLATILITY:


Unconstrained (Shorts & Long): 7.52392%
Constrained (Long-Only): 10.2887%
Inverse Volatility: 13.97532%
```

3.B.1 In-Sample Annualised Volatility across 3 strategies , January 1995 - December 2004

RESULTS:

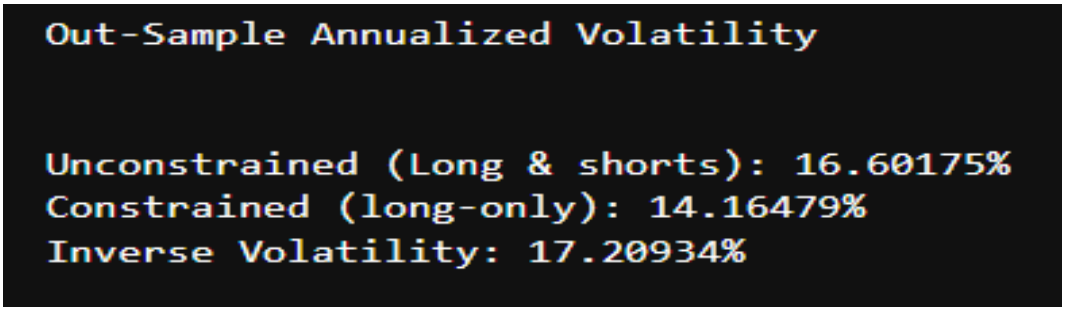
- For the in-sample period from January 1995 to December 2004, the calculated annualised return volatilities for each of the three portfolios are as follows:
    - Unconstrained Portfolio (long & shorts) = 7.52392% **(lowest)**
    - Constrained Portfolio (long-only) = 10.28870%
    - Inverse Volatility Portfolio = 13.97532% **(highest)**

IMPLICATIONS:

- The order of results *is as expected* and corroborate with what we have studied in portfolio theory and discussions:
    - The unconstrained portfolio has the flexibility to short and adjust risk exposures, which can optimise efficiently and thus has lowest volatility

- The constrained portfolio is limited to long positions but can still minimise risk through diversification
- The inverse volatility portfolio, while it might reduce individual asset volatility, doesn't consider the overall risk profile or correlations between assets, leading to higher realised volatility

PART C.



3.C.1 Out-of-Sample Annualised Volatility across 3 strategies , January 2005 - July 2023

RESULTS:

- For the out-of-sample period from January 2005 to July 2023, the calculated annualised return volatilities for each of the three portfolios are as follows:
  - Unconstrained Portfolio (including shorts) = 16.60175%
  - Constrained Portfolio (long-only) = 14.16479% **(lowest)**
  - Inverse Volatility Portfolio = 17.20934% **(highest)**

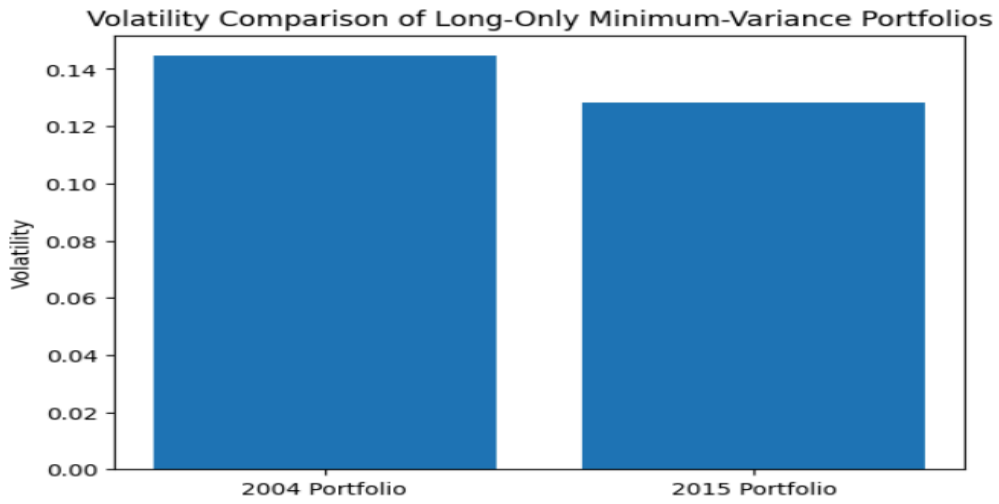
IMPLICATIONS:

- **As seen,** the order of the best performing strategies change:
  - The *long-only portfolio* performs best in terms of low volatility, as expected, due to its diversification and avoidance of short-selling risks
  - The *unconstrained portfolio* has higher volatility than expected, potentially due to market conditions where short positions were less beneficial or too risky
  - The *inverse-volatility portfolio* underperformed in terms of volatility, likely because it didn't consider asset correlations and broader portfolio risk, focusing only on individual asset volatility

- This reflects the limitations of minimum variance optimization, which works well in idealised conditions but can struggle with *out-of-sample performance* due to overfitting, lack of robustness, and failure to account for real-world correlations between assets

PART D.

```
Volatility of the portfolio optimized in 2004: 14.45862%
Volatility of the portfolio optimized in 2015: 12.81076%
```



3.D.1 Long-only Minimum Variance Portfolio in 1995-2004 and 2006 -2015, tested on the out-sample time frame January 2016 - July 2023

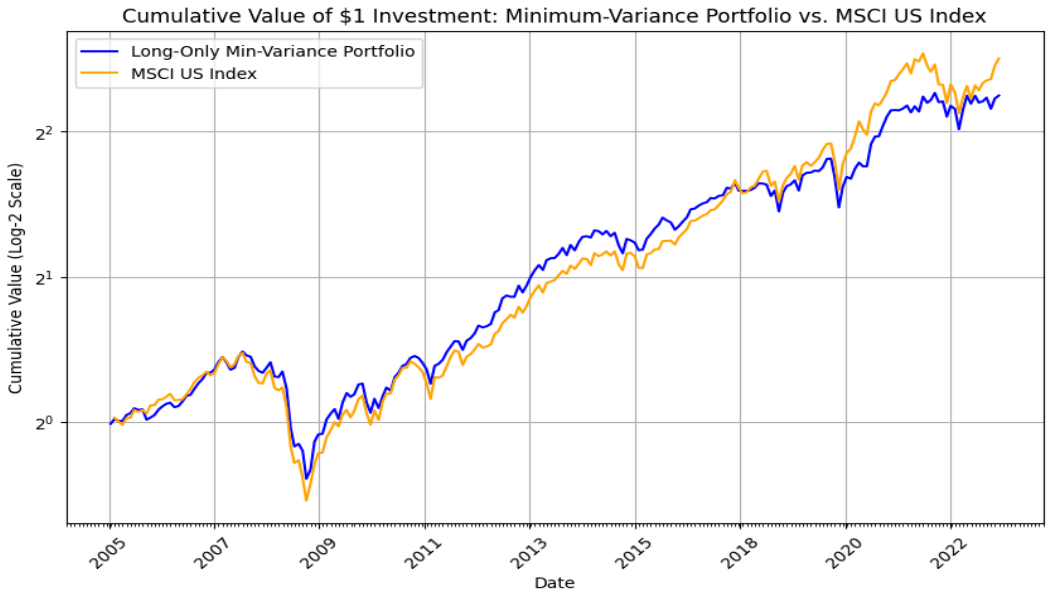
RESULTS:

- The portfolio optimised in 2015 has *a lower volatility* (12.81%) compared to the one optimised in 2004 (14.46%) performed on the time frame of January 2016 to July 2023

IMPLICATIONS:

- *This result is as expected* and demonstrates the *value of re-optimization*, as the updated covariance matrix (2016 - 2023) and weights *reflects changes in market conditions*.
- Re-optimization is crucial because it adapts the portfolio to current market dynamics, potentially reducing risk and enhancing stability
- In essence, the results show that *the closer the optimization to the current period, the better it is likely to suit* the present market environment

PART E.



3.E.1 Cumulative Value of $1 investment: Minimum-Variance Portfolio vs MSCI US Index, January 2005 - July 2023

RESULTS:

- The graph shows that our long-only minimum variance portfolio *closely mirrored* the broader MSCI Index.
- As seen in the graph, the long-only minimum variance portfolio performed better up until 2018 but has underperformed since then, likely due to changes in market conditions.
- During the Global Financial Crisis (September 2008 - February 2009), the defensive nature of our minimum variance strategy, built with historical 10-year weights from December 2004, *led to better performance, with smaller losses* compared to the MSCI index.
- This supports the theory that optimal portfolio weights *can perform well in closer* out-of-sample periods.
- However, during the COVID-19 market meltdown (February 2020 - March 2020), the *defensive strategy underperformed*, resulting in worse returns than the MSCI index.

```
Top 10 industries during the 2008 Financial Crisis:
Gold    -0.116976
Beer    -0.217268
Drugs   -0.232648
PerSv   -0.240713
Food    -0.248909
Meals   -0.260257
Soda    -0.294691
Smoke   -0.299143
Rtail   -0.300527
Util    -0.327138
Name: 991, dtype: float64
```

```
Top 10 industries during the COVID-19 Market Crisis:
Gold    -0.031706
Drugs   -0.070460
Rtail   -0.102399
Smoke   -0.130164
Food    -0.131457
Softw   -0.145720
Hshld   -0.146390
LabEq   -0.158744
Boxes   -0.162104
Paper   -0.164095
Name: 1124, dtype: float64
```

3.E.2 Top 10 industries based on cumulative returns during Global Financial Crisis and Covid Meltdown

IMPLICATIONS:

2008 Financial Crisis:

- **Defensive Sectors:** Gold, Beer, Drugs, Food, Tobacco, and Utilities performed well, as they are considered essential goods that people continue consuming during recessions (*Exhibit 3.E.2)*
- **Unexpected Sectors:** Personal Services and Retail were less typical defensive sectors but may have benefited from stable demand for basic services and essential retail during the crisis.
- **Summary:** Traditional defensive sectors outperformed, aligning with the theory that essential, low-volatility industries fare better in crises

COVID-19 Market Crisis:

- **Defensive Sectors:** Gold, Drugs, and Tobacco performed well, similar to the 2008 crisis, due to their safe-haven and essential nature.
- **Unexpected Sectors:** Retail (e-commerce), Software, and Lab Equipment benefited from pandemic-driven shifts, like increased online shopping, remote work, and healthcare needs (*Exhibit 3.E.2)*
- **Summary:** The top industries included more from *unexpected sectors like Retail (especially e-commerce), Software, and Lab Equipment* showing that growth-oriented industries can perform well during crises when their services become essential.

Question 4)

PART A.

```
NVDA - Standard Deviation: 3.746% ; VaR 95%: -6.121% ; CVaR 95%: -7.0725%

NVDL - Standard Deviation: 7.4795% ; VaR 95%: -12.0288% ; CVaR 95%: -14.1617%
```

4.A.1 NVIDIA (NVDA - stock) and GraniteShares 2x Long NVDA Daily (NVDL - ETF) Standard Deviation, Value at Risk and Conditional Value at Risk; January 2024 - August 2024

IMPLICATIONS:

- The results above corroborate the notion that *leverage increases the risk of an investment*. NVDL, which uses leverage to magnify the returns of NVDA, shows higher volatility (Standard Deviation), greater potential losses (VaR), and more severe losses in the tail risk (CVaR).
- All risk measures increase roughly by the same factor, which in this case is the leverage ratio (2x) illustrating that *leverage is a double-edged sword*

PART B.

```
Intercept (beta_0): -0.000612, Slope (beta_1): 1.995975
Confidence Intervals:
              0         1
const -0.000935 -0.000289
NVDA   1.987407  2.004543
```

4.B.1 NVIDIA (NVDA - stock) and GraniteShares 2x Long NVDA Daily (NVDL - ETF) Standard Deviation, Value at Risk and Conditional Value at Risk; January 2024 - August 2024

IMPLICATIONS:

- The *intercept of -0.000612* is near zero, indicating minimal offset between NVDL and NVDA returns aside from the expected leverage
- The *slope of 1.995975 (close to 2)* suggests NVDL's daily returns are roughly twice NVDA's, aligning with the ETF's goal
- The *confidence interval for the slope (1.987407 to 2.004543)* includes 2, reinforcing that the multiplier is likely around 2
- In conclusion, the regression results provide strong evidence that NVDL is fulfilling its stated objective of delivering *2 times the daily return* of NVDA

PART C.

```
Estimated Annual Performance Drag:-0.154134%
```

4.C.1 Annual Performance Drag of the ETF, January 2024 - August 2024

IMPLICATIONS:

- The estimated daily intercept from the regression is *-0.000612*. This represents the daily performance drag, which suggests that the ETF's returns lag the underlying NVDA stock by *0.0612% per day.*
- The annualised performance drag is approximately *-0.154%* which is is considerably smaller than the expense ratio *1.15% p.a.*, indicating that the lag in performance is not solely due to the ETF's operating costs
- Two key contributors in costs inherent in the ETF's operations that could account for the remaining drag are:
    - **Transaction Costs**: The ETF trades NVDA stock on a daily basis to maintain the desired exposure which incurs *transaction costs* such as bid-ask spreads and brokerage fees
    - **Leverage Costs**: Since the ETF uses leverage to amplify returns, borrowing costs to maintain that leverage (e.g., interest paid on borrowed funds) will be an additional expense that could contribute to performance drag

PART D.

```
Monthly NVDA Returns %:
 Date
2024-02-29    28.5970
2024-03-31    14.2099
2024-04-30    -4.3724
2024-05-31    26.8781
2024-06-30    12.6996
2024-07-31    -5.2781
2024-08-31     2.0084
Freq: ME, Name: NVDA, dtype: float64%
```

```
Monthly NVDL Returns %:
 Date
2024-02-29    59.1128
2024-03-31    25.7576
2024-04-30   -12.2169
2024-05-31    55.7233
2024-06-30    22.5983
2024-07-31   -14.6801
2024-08-31    -1.6009
Freq: ME, Name: NVDL, dtype: float64%
```

4.D.1 Monthly Returns (in %) of NVDA and NVDL, January 2024 - August 2024

```
Total Period Return NVDA: 94.0497%
```

```
Total Period Return NVDL: 181.5333%
```

4.D.2 Total Return (in %) of NVDA and NVDL, January 2024 - August 2024

IMPLICATIONS:

- The results corroborate with the fund's warning: *NVDL seeks 2x the daily return of NVDA, not cumulative returns over extended periods*
- **Monthly Return Variations**: NVDL's monthly returns approximate but do not exactly double NVDA due to daily compounding (*Exhibit 4.D.1)*
- **Cumulative Return Divergence**: Across February to August, NVDA's cumulative return was *94.05%*, while NVDL's was *181.53%* reflecting the shortfall from the 2x goal due to compounding
- These results confirm that *NVDL closely tracks 2x daily returns but diverges from 2x cumulative returns over longer periods*, aligning with the factsheet's caution about using leveraged ETFs for multi-day holding

PART E.

```
Two-Day Return Scenario:
Two-day return NVDA: 0.10000000000001119%
Two-day return NVDL: -1.6000000000000014%
```

4.E.1 Two Day Return Simulation on NVIDIA stock

IMPLICATIONS:

- While NVDA ends with a small net gain over two days (**+0.1%**), NVDL's cumulative return is **-1.6%** due to the effects of compounding with leverage.
- **Illustrating the Fund's Warning**: This example aligns with the fund's cautionary note that "the Fund will lose money if the underlying stock's performance is flat over time" and even that losses can occur when the underlying stock sees moderate gains.
- Leveraged ETFs like NVDL multiply not only daily returns but also daily fluctuations, which can lead to erosion of value in volatile markets, even if the underlying asset shows a slight positive trend

PART F.

IMPLICATIONS:

- **Scenario 1: Bullish on NVDA on the chips release day**
  - We would choose the **ETF (NVDL)**. Given the ETF's 2x leverage on daily returns, a strong positive movement in NVDA's price could result in amplified gains for NVDL on that day.
  - For short-term bets on single-day events, this leverage can maximise potential returns
- **Scenario 2: Bullish on NVDA for the next decade but stock would see price volatility**
  - We would choose the **stock (NVDA).** ETF NVDL would not perform well in a volatile market as we have seen before due to its volatility drag.
  - For a long-term investment where you expect fluctuations, owning NVDA directly would avoid the compounding effect and provide more reliable exposure to the stock's performance

---

Question 5)

PART A

- **The MSCI methodology consist of:**

1. Value investment style using:
- Book Value to Price Ratio (BV/P)
- 12-Month Forward Earnings to Price Ratio (E fwd/P)
- Dividend Yield (D/P)

2. Growth investment style using:
- Long-Term Forward Earnings Per Share (EPS) Growth Rate (LT fwd EPS G)
- Short-Term Forward EPS Growth Rate (ST fwd EPS G)
- Current Internal Growth Rate (g)
- Long-Term Historical EPS Growth Trend (LT his EPS G)
- Long-Term Historical Sales Per Share (SPS) Growth Trend (LT his SPS G)

- The MSCI Global Investable Market Value and Growth methodology is implemented for the Standard and Small Cap Indexes on a **market-by-market basis**.
- In Developed Market (DM) Europe, however, MSCI uses a **regional approach instead of focusing on specific countries**. Furthermore the Small Cap Global Value and Growth Indexes do not incorporate the long-term forward EPS growth rate.

- Contrasting this with Fama-French (FF), their "value" (HML) metric relies solely on the size and Book-to-Market ratio, categorizing stocks as "value" or "growth". Therefore, MSCI's multifactor approach is **broader than Fama-French's indicator**.

PART B

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.07128    0.07869  -0.906   0.3657
`MKT-RF`     0.94803    0.01231  76.986   <2e-16 ***
SMB         -0.17437    0.01761  -9.905   <2e-16 ***
HML          0.32164    0.01634  19.682   <2e-16 ***
RF           0.86464    0.31008   2.788   0.0056 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.007 on 338 degrees of freedom
Multiple R-squared:  0.9484,    Adjusted R-squared:  0.9478
F-statistic:  1554 on 4 and 338 DF,  p-value: < 2.2e-16
```

Figure 5.B.1 Regression of the independent variables (Mkt-Rf, SMB, HML, and the Risk-Free rate) and the dependent variable (MSCI Value Index Return) monthly from January 1995 to July 2023.

IMPLICATIONS :

- From the figure, the MSCI US Standard (Large+Mid Cap) Value Equity Index and Fama-French factors appear to have the stated exposure, or are "as-advertised", because the Adjusted R-squared for its return is relatively high at 0.9478, which is close to 1.
- The regression coefficients are significant in 99% confidence level except for the intercept term that will be explained in the next part.

PART C

- The lack of an intercept term with 99% confidence level (P-value = 0.367 >0.01) which we fail to reject null hypothesis equal to zero) indicates that the MSCI Value Index's monthly average return is not statistically distinct from the replication return generated by the Fama-French factors.
- This suggests that MSCI's refined "value" definition does not yield a performance advantage over the Fama-French academic definition in this context.
- The absence of a significant intercept implies that the Fama-French factors sufficiently capture the returns of the MSCI Value Index, meaning MSCI's modifications to "value" do not provide a statistically meaningful improvement in average returns over its replication.

PART D

- Replication portfolio is constructed using coefficients from the regression results between January 1995 to July 2023, excluding the intercept term to focus on a specific investment area with direct investability.
- The intercept term was omitted as it only represents the average monthly excess return and does not correspond to an actual investment instrument. The model that we use to replicate the portfolio is

$$Replicating\ Portfolio = 0.948\ (Mkt - Rf) - 0.1743\ (SMB) + 0.3216(HML) + 0.8645(RF)$$

- Furthermore since the sum of the regression coefficients exceeds 1, this portfolio construction involves the use of leverage. The replication portfolio monthly return is in replicating_portfolio.txt file

```
[1] "Correlation between Replicating Portfolio and MSCI Value Index: 0.973865609174585"
```

Figure 5.D.1 Correlation Result

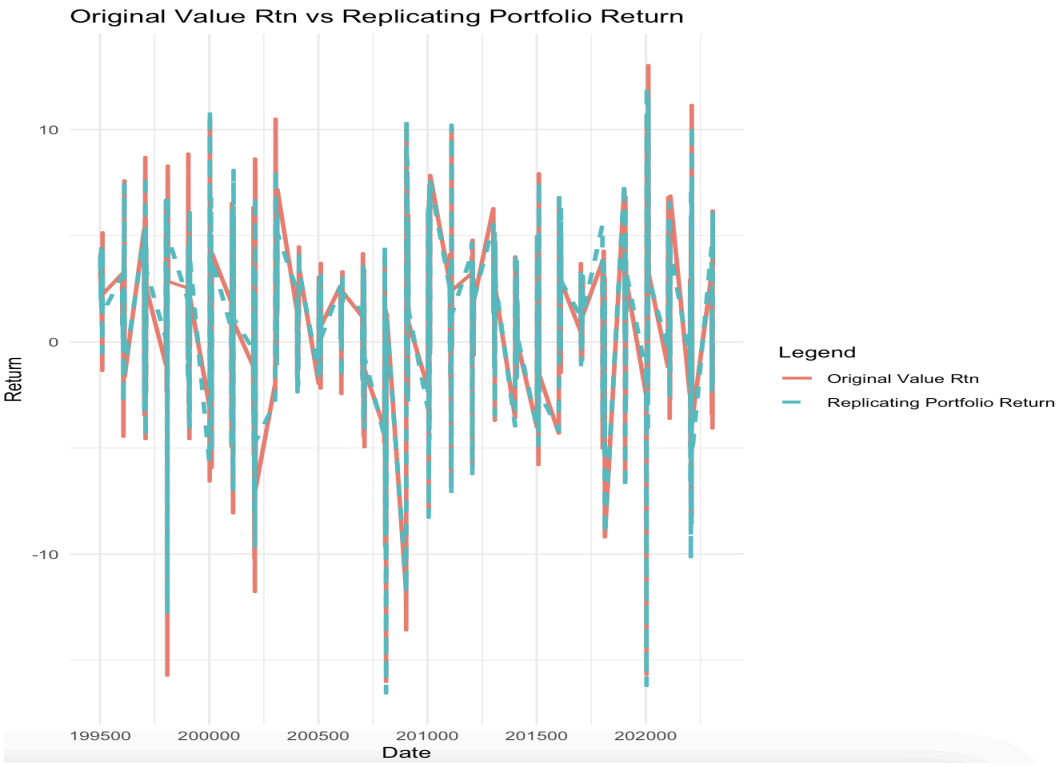


Figure 5.D.2 Plot between MSCI USA Value Index Return vs Replicating Portfolio Return in monthly

IMPLICATIONS :

- The replicating portfolio is effective to reflect the MSCI Value Index because the correlation between replicating portfolios is 0.9738 which is almost 1.
- Furthermore, based on the plot we draw, the original MSCI value Index return to the replicating portfolio between January 1995 to July 2023 is also quite similar.

	Metric	MSCI.Value.Index	Replicating.Portfolio
1	Annual Return	9.0494701	10.0458839
2	Annual Volatility	15.2667028	14.8677168
3	Sharpe Ratio	0.4497084	0.5287952

Figure 5.D.3 Annualized return, annualized volatility, and Sharpe Ratio between MSCI Value Index and Replicating Portfolio

IMPLICATIONS :

- Annualized Return, Annualized volatility, and Sharpe Ratio of replicating portfolio return perform better than MSCI Value Index return.
- Sharpe ratio higher is better, volatility higher is worse, and annual return higher is better. However as there are portfolio cost:
    1. **Market price impact for high volume trade** as replication portfolio invest in more than 1 area other than MSCI therefore if high volume trade happen, there will be cost because of price rocketing when buying or plummeted when selling
    2. **Transaction Cost/Management/Agent Fee cost per trade** will also effect as the trader/broker will require more fees as the number of trade increase
    3. **Bid-Ask Spread** which will happened per asset traded

- Which in summary, as replicating portfolios invest in a lot of markets rather than a single MSCI index, the cost of the transaction of the replicating portfolio will be higher than MSCI index even though the return gives more benefit to the replicating portfolio.
- Furthermore our weight coefficient in the replicating portfolio is using leverage which cause higher risk than investing in MSCI

PART E

- We calculate monthly active return to the benchmark rate by:

$$Monthly\ Active\ Return = Multi\ Factor\ monthly\ return - MSCI\ USA\ standard\ monthly\ return$$

- Next we compute annualized active return, tracking error, and information ratio:

$$Annualized\ Active\ Return = Annualized\ Multi\ Factor\ return - Annualized\ MSCI\ Standard\ return$$

$$Tracking\ Error = STDV\ (Monthly\ Active\ Return)\ x\ \sqrt{12}$$

$$Information\ Ratio = \frac{Annualized\ Active\ Return}{Tracking\ Error}$$

```
[1] "Annualized Active Return: 0.3363"
> print(paste("Annualized Tracking Error:", round(annualized_active_sd, 4)))
[1] "Annualized Tracking Error: 3.3867"
> print(paste("Annualized Information Ratio:", round(information_ratio, 4)))
[1] "Annualized Information Ratio: 0.0993"
```

Figure 5.E.1 Annualized active return, annualized tracking error, and information ratio between Multi Factor Index Return and MSCI standard return

IMPLICATIONS :

- An Information Ratio of 0.1 suggests that the Multi Factor Return is providing only a slight excess return over the benchmark per unit of tracking error.
- This small margin might not be substantial enough to justify an active strategy, especially if transaction costs, management fees, or other expenses are considered.