FE 5108 Portfolio Theory and Investments

Assignment 1 due on 19-Sep-2023, 11.59pm

Risk Management Institute

National University of Singapore

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Question 1

Asset 1 and asset 2 both cost \$150. Yields on asset 1 in states 1 and 2 are $(z_{11}, z_{12}) = (100, 200)$ and on asset 2 are $(z_{21}, z_{22}) = (200, 100)$. An individual with an initial wealth of \$150 has a utility function:

$$v(c) = -e^{-c}$$

(A) Show that the state-contingent budget constraint can be expressed as:

$$c_1 + c_2 = 300$$

(B) If the individual believes that state 1 will occur with probability π, show that his optimal consumption in state 1 is:

$$c_1^* = 150 + \frac{1}{2}\ln(\pi/(1-\pi))$$

(C) If q_1 is the number of units of asset 1 purchased show that:

$$c_1^* = 200 - 100q_1^*$$

and hence obtain an expression for q_1^* in terms of π, the probability of state 1.

(D) What values do c_1^* and q_1^* approach as the probability of state 1 becomes very small?

Question 2

In a competitive economy there are I investors, all having the same utility function $U = \mu^{10} e^{-\sigma}$. Each individual is endowed with exactly one unit each of assets 1, 2, and 3 with payoff statistics as shown in the table below, all the payoff distributions being uncorrelated ($\sigma_{ab} = 0$, for all $a \neq b$). Given asset prices are also shown:

	μ_a	σ_a	P_a^A
Asset 1	1	0	1.0
Asset 2	1	3	0.46
Asset 3	1	4	0.04

(A) Sketch the indifference curves on $\mu(c)$, $\sigma(c)$ axes. Locate, for any single individual, the three single-asset portfolios he might hold.

(B) Under the assumptions here, each individual's optimum portfolio H* must evidently be the same as his endowed portfolio. Locate this portfolio, and also the mutual fund portfolio F. What fraction of his wealth does the individual hold in the mutual fund?

(C) Verify that the price of risk reduction is $\Theta = \frac{3}{10}$. What is the equation of the individual's budget line? What is his Marginal Rate of Substitution (the slope of the indifference curve) at H*?

Question 3

An individual with an initial wealth of \$50 must choose a portfolio of two assets, both of which have a price of \$50. The first asset is riskless and pays off \$50 in each of the two possible states. The second returns z_{2s} in state s, for $s = 1, 2$. The probability of state 1 is π.

(A) If the individual splits his wealth equally between the two assets, confirm the correctness of the following table, where the risky asset returns may have the form of α, β, or γ.

(B) Suppose the individual has a utility function:

$$v(c) = -e^{-Ac}$$

where $A = \frac{\ln 4}{30}$ (and hence $e^{30A} = 4$). Confirm that the individual's preference ranking of the three risky assets is $\gamma > \alpha > \beta$.

	Risky asset returns (z_{21}, z_{22})	Probability of state 1	Final consumption (c_1, c_2)	E(c)	$\sigma^2(c)$
α	(20,80)	1/5	(35,65)	59	144
β	(38,98)	1/2	(44,74)	59	225
γ	(30,90)	1/3	(40,70)	60	200

(C) With preferences as given in (B) show that in each case the individual's optimal decision is to spend an equal amount on each of the two assets.

Group Assignment (due on 19-Sep-2023, 11.59pm)

- Each group consists of 4 or 5 people.
- Include a cover letter explaining the contribution of each group member.
- Submit to *sunyifei.econ@gmail.com* by 19-Sep-2023.

Suggestions:

- All the questions are based on the first three lecture notes.