FE 5108 - Portfolio Theory and Investments

Assignment 2

Factor Models in Portfolio Management

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Factor models are financial tools designed to explain asset returns based on underlying factors that influence stock price movements. These factors help differentiate between price changes driven by market-wide influences and those resulting from company-specific events. Essentially, factor models provide insights into how much of a stock's return can be attributed to broader, factor-driven influences as opposed to unique corporate developments.

> *In finance, factor modelling applies standard multivariate statistical techniques to model returns, variances, and correlations (Rosenberg & McKibben, 1973).*

A core assumption of these models is that the factors are linearly dependent, with stock prices reflecting a weighted combination of these factors based on their respective exposures. The factors can be categorised into two broad groups:

1. **Macroeconomic Factors:** These are systematic risks impacting all companies within an industry, such as inflation, interest rates, or GDP growth.
2. **Firm-Specific Factors:** These are unique to individual companies, such as profitability, size, or earnings growth. These factors are often easier to estimate accurately compared to broader macroeconomic variables.

I. Single Factor Models

Factor models trace their origins to single-factor models, which breaks down a security's anticipated return into two key components: common (macroeconomic) factors and firm-specific events. This decomposition of risks provides insight into overall economic uncertainty, captured by a *systematic market factor denoted as* ***F***, and the uncertainty specific to an individual firm, represented by a *firm-specific random variable* e_i.

In this framework, the single-factor model for any stock i, can be described as follows:

$$R_i = E(R_i) + \beta_i * F + e_i$$

Here, the nonsystematic components of returns, e_i, are assumed to be uncorrelated across stocks and independent of the market factor F. The term *$E(R_i)$ represents the expected excess return on stock i.*

Among the various single-factor models, the most prominent is *William Sharpe's Capital Asset Pricing Model (CAPM)*, developed in the 1960s, which has become a cornerstone of modern finance.

a) Capital Asset Pricing Model (CAPM)

The capital asset pricing model is a set of predictions concerning equilibrium expected returns on risky assets. Harry Markowitz laid down the foundation of modern portfolio management in 1952. The CAPM was published 12 years later in articles by *William Sharpe, John Lintner, and Jan Mossin*. (*Bodie, Kane, and Marcus, 2023)*
The model is based on **two key assumptions** :

- *Individual behaviour*
  1. Investors are rational, mean-variance optimizers
  2. Their common planning horizon is a single period
  3. Investors all use identical input lists, an assumption often termed as homogeneous expectations. Homogeneous expectations are consistent with the assumption that all relevant information is publicly available
- *Market Structure*:
  1. All assets are publicly held and traded on public exchanges
  2. Investors can borrow or lend at a common risk-free rate, and they can take short positions on traded securities
  3. There are no taxes or transaction costs

In a well-diversified portfolio, the volatility of individual stocks has little impact on overall risk, as this risk is primarily determined by the proportion of each stock held. As a result, **the market only prices in systematic risk** — *the risk affecting all assets* — while firm-specific risk is disregarded. In regression models, firm-specific risk is eliminated through diversification, leading to the absence of an intercept term. Instead, *beta coefficients measure a stock's sensitivity to overall market risk*, indicating how its returns move in relation to market fluctuations

Therefore, the Capital Asset Pricing Model (CAPM) formula for a stock can be written as:

$$R_i - R_f = \beta(R_m - R_f)$$

R_i = *Return on a stock*
R_f = *Return on a stock*
B = *Sensitivity of excess market return and excess stock return (excess from risk free rate)*
R_m = *market rate*

With the CAPM formula, investors can evaluate the expected return of risky assets in comparison to a riskless asset as a benchmark by its historical data

i) Implementation

[File Link] In this study, we will examine the correlation between the S&P 500 and the risk-free rate on individual stocks for Apple, Amazon, and JPMorgan using the Capital Asset Pricing Model (CAPM). We will create a stock excess return model using Excel regression. To avoid industry-specific risks influencing the CAPM, we will analyze three different industries: mobile devices, e-commerce, and banking. For our analysis, we will use the risk-free rate based on the 1-month U.S. Treasury Bill rate, with data covering a five-year period.

Here are the steps for conducting an empirical study using the CAPM model for three different stocks:

1. **Data Collection:** We gather stock price, T-bill rates, and market index data from September 24, 2019, to September 24, 2024 (a 5-year timeframe). Stock price data and market index come from investing.com

2. **Return Calculation:** We calculate the daily return of the stocks and the market index using the following formulas:

$$Ri = \frac{Current\ Stock\ Price - Stock\ Price\ yesterday}{Stock\ Price\ yesterday}$$

$$Rm = \frac{Current\ S\&P\ 500\ \ Index - S\&P\ 500\ Index\ yesterday}{S\&P\ 500\ index\ \ yesterday}$$

3.**Risk-Free Rate:** T-bills are taken as a benchmark for risk-free rates. Since the T-bills are yearly rates, dividing by 365 would give an approximate daily rate. We select the shortest tenor (1 month) from the U.S. Treasury website, as we are regressing daily data, and this provides the most relevant rate due to its minimal interest fluctuation compared to other tenors. In cases where data for certain dates in 2019–2021 is unavailable, we substitute the previous day's Treasury Bill rate. The Treasury Bill data is sourced from the U.S. Department of the Treasury website

4.**Regression Analysis:** We use Excel to perform regression analysis, with the independent variable $X = R_m - R_f$ and the dependent variable $Y = R_i — R_f$.

5. **Significance Testing:** We conduct a t-statistic test and calculate the p-value to determine the significance of the beta coefficient and intercept at a 95%

confidence level using a two-tailed distribution. The null hypothesis H0 assumes the coefficient is zero.

ii) Results & Implications

Upon using Excel toolkit for Data Analysis on the three stocks, we get the following results:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.6405
R Square	0.4103
Adjusted R Sq	0.4098
Standard Errc	0.0172
Observations	1258

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.2584	0.2584	873.8059	0.0000
Residual	1256	0.3714	0.0003		
Total	1257	0.6298			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95,0%	Upper 95,0%
Intercept	0.0002	0.0005	0.4803	0.6311	-0.0007	0.0012	-0.0007	0.0012
X Variable 1	1.0704	0.0362	29.5602	0.0000	0.9994	1.1414	0.9994	1.1414

Regression Analysis for Amazon Inc.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.7153
R Square	0.5117
Adjusted R Sq	0.5113
Standard Errc	0.0141
Observations	1258

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.2622	0.2622	1316.2530	0.0000
Residual	1256	0.2502	0.0002		
Total	1257	0.5124			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95,0%	Upper 95,0%
Intercept	0.0000	0.0004	0.0363	0.9710	-0.0008	0.0008	-0.0008	0.0008
X Variable 1	1.0783	0.0297	36.2802	0.0000	1.0200	1.1366	1.0200	1.1366

Regression Analysis for JP Morgan Chase & Co.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.7925
R Square	0.6281
Adjusted R Sq	0.6278
Standard Erro	0.0122
Observations	1258

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.3152	0.3152	2121.3577	0.0000
Residual	1256	0.1866	0.0001		
Total	1257	0.5018			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95,0%	Upper 95,0%
Intercept	0.0006	0.0003	1.8107	0.0704	-0.0001	0.0013	-0.0001	0.0013
X Variable 1	1.1822	0.0257	46.0582	0.0000	1.1318	1.2325	1.1318	1.2325

Regression Analysis for Apple Inc.

- The intercept term is insignificant across all stocks within the CAPM models, supporting the theory that firm-specific risks have been eliminated
- All the stock beta reject null hypothesis which therefore suggests that there is correlation within excess market return and excess stock return
- Our experiments with companies from three different industries — electronics, e-commerce, and banking — indicate that industry risk is also negligible in this model
- Our findings reveal that AAPL exhibits the highest sensitivity to market movements, with a multiplier of 1.182, compared to JPM and AMZN.

Finally, our 5 year CAPM model for three stocks is given as:

$$R(AAPL) - Rf = 1.182\,(Rm - Rf)$$

$$R(JPM) - Rf = 1.078\,(Rm - Rf)$$

$$R(AMZN) - Rf = 1.07\,(Rm - Rf)$$

While the decomposition of risks allows for deeper analysis and is necessary, restricting the analysis to a single systematic risk factor, as in the CAPM, **is insufficient**. Moreover, emperical evidence has challenged the validity of the single factor models. Therefore, to enhance predictive accuracy for stock prices, we need to adopt more robust models that incorporate multiple explanatory variables.

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II. Multifactor Models

In the 1970s and 1980s, researchers observed that value stocks consistently outperformed growth stocks, and small-cap stocks outperformed large-cap stocks, even after accounting for market exposure. This led to the exploration of multi-factor models, which incorporate a variety of elements linked to asset returns. These models are now widely employed by asset managers to make informed investment decisions and evaluate the associated risks.

We begin by defining the key variables in the multifactor model:

Let F_j where j = 1,2,… n be the values of various explanatory factors that are

statistically significant in influencing the asset price
B_{ij} *be the* ***factor loadings or factor beta*** *i.e. coeffecients for each* F_j *measuring how much the asset responds to that factor*
e_i *is defined as the* ***error term*** *or* ***idiosyncratic risk*** *for asset i, which represents returns unique to the asset that cannot be explained by the factor*

Thus, the multifactor model for asset i's returns is expressed as:

$$R_i = E(R_i) + \beta_{i1} * F_1 + \beta_{i2} * F_2 e_i + \ldots\ldots \beta_{in} * F_n + e_i$$

Q) Why are multifactor models widely adopted?

Multifactor models are widely employed because *factor betas provide a basis for developing effective hedging strategies*. Investors aiming to reduce specific risks can offset them by creating opposing exposures to the relevant factors, thereby neutralizing those risks.

These models serve three primary purposes:

1. **Risk Management:** Identifying the key sources of risk within a portfolio
2. **Portfolio Construction:** Building portfolios with dynamic hedging capabilities
3. **Performance Attribution:** Analyzing returns to distinguish a manager's skill from performance driven by factor exposures

We've explored how factor models dissect the sources of asset returns, but a key question remains: *how do macroeconomic factors influence the expected* *excess rate of return*? This is where Arbitrage Pricing Theory (APT) becomes crucial.

a) Arbitrage Pricing Theory

Arbitrage refers to the opportunity to earn risk-free profits by taking advantage of asset mispricing. In well-functioning capital markets, a common assumption is that arbitrage opportunities are ruled out.

Arbitrage Pricing Theory (APT), developed by economist Stephen Ross in the 1970s, offers an alternative to the Capital Asset Pricing Model (CAPM) for explaining asset or portfolio returns. APT is based on a multi-factor model that establishes a linear relationship between an asset's expected return and various macroeconomic factors, such as business-cycle risk, interest rate risk, inflation risk, or energy price fluctuations. In a single-factor market without additional risk factors, APT results in a mean return-beta equation that is identical to CAPM.

Arbitrage Pricing Theory (APT) operates under several key assumptions:
1. **Factor models can describe returns**: The returns of an asset can be explained by multiple risk factors, such as economic variables or market forces.
2. **Diversification eliminates specific risks**: By constructing a well-diversified portfolio, investors can reduce or eliminate firm-specific (idiosyncratic) risks, leaving only systematic risks.
3. **No arbitrage opportunities in well-functioning markets**: In efficient markets, any opportunity to earn risk-free profits through arbitrage is quickly exploited, ensuring asset prices reflect all available information

For a well-diversified portfolio, the Multi-factor Arbitrage Pricing Theory formula can be written as:

$$E(R_i) = R_f + \beta_1(F_1) + \beta_2(F_2) + ... + \beta_n(F_n)$$

where, $E(R_i)$ = Expected return
R_f = Risk-free return
B_i = The asset price sensitivity to a factor
F_i= Macroeconomic factor value

> *Unlike the capital asset pricing model, the arbitrage pricing theory does not specify the factors. However, Stephen Ross and Richard Roll in their paper suggest four economic variables — Change in Inflation, Change in levels of Industrial Production, Shifts in Risk Premiums & Change in the shape of the term structure interest rates (Roll & Ross, 1984)*

b) APT vs CAPM: Comparative Analysis

Both APT and CAPM distinguish between firm-specific and systematic risk. APT provides a benchmark for rates of return and is applicable in valuing securities such as stocks, bonds, real estate, and in capital budgeting decisions. It also emphasizes the difference between diversifiable and non-diversifiable risk.

APT is theoretically appealing because it does not require investors to have homogeneous expectations about the mean returns and variances of assets, relying instead on the reasonable assumption of no arbitrage.

In practice, APT offers a more flexible approach by establishing an expected return-beta relationship using a well-diversified index, such as the NASDAQ or S&P 500, as a benchmark — unlike CAPM, which assumes an idealized and unobservable market portfolio that includes all assets. However, this approach carries the risk of not capturing all relevant factors perfectly, which could make APT's expected return-beta relationship less precise than CAPM's.

Additionally, while CAPM provides an expected return-beta relationship for all assets, APT focuses on well-diversified portfolios and cannot ensure that this relationship holds for individual assets. Thus, APT guarantees the return-beta relationship for most portfolios but not necessarily for every single asset.

Despite APT's theoretical advantages, CAPM remains more popular in practice due to its simplicity as a single-factor model, whereas APT requires consideration of multiple factors to calculate expected returns.

c) Limitations

Arbitrage Pricing Theory (APT) offers a framework for understanding asset pricing through systematic risk factors, yet it faces several limitations that hinder its practical application.

1. *Lack of Consensus on Risk Factors*
   One significant limitation of APT is the absence of agreement among economists regarding the key systematic risk factors. This lack of consensus complicates the model's validity and applicability in real-world scenarios(Lekovic, 2019).
2. *Empirical Challenges*
   Empirical applications of APT often struggle with the estimation of risk premia, particularly when using individual securities. The presence of pricing errors necessitates a large number of diversified portfolios for consistent estimation, which can be impractical ("Arbitrage pricing

theory, the stochastic discount factor and estimation of risk premia from portfolios”, 2023).

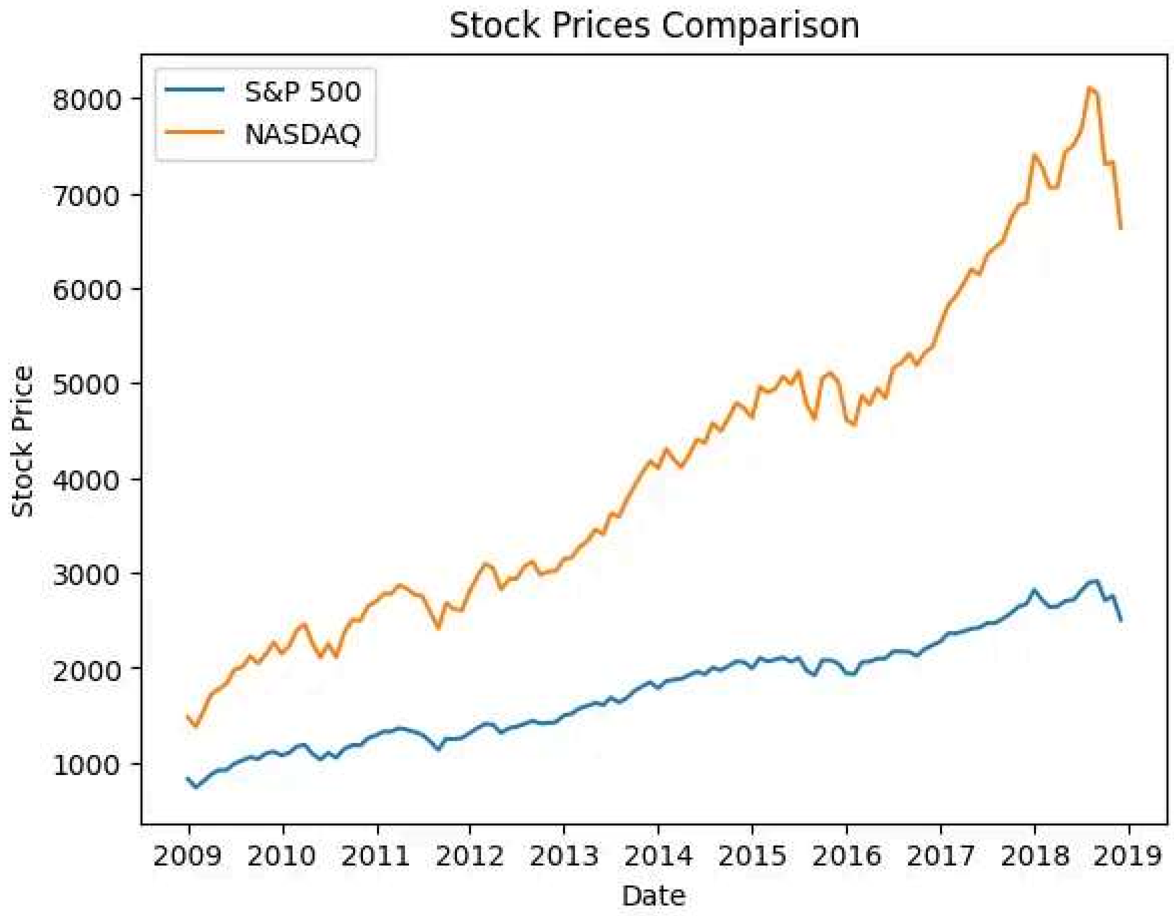
3. *Market Conditions and Assumptions*
APT assumes that markets are efficient and that arbitrageurs can exploit mispricings. However, factors such as illiquidity and idiosyncratic risk can impede arbitrage activities, leading to persistent mispricing (Godfrey & Brooks, 2015)

While APT provides a theoretical foundation for asset pricing, its practical limitations highlight the need for alternative models or adjustments to enhance its robustness in diverse market conditions (French, 2017).

d) Implementation

[File Link] Implementing Arbitrage Pricing Theory (APT) involves several key steps:

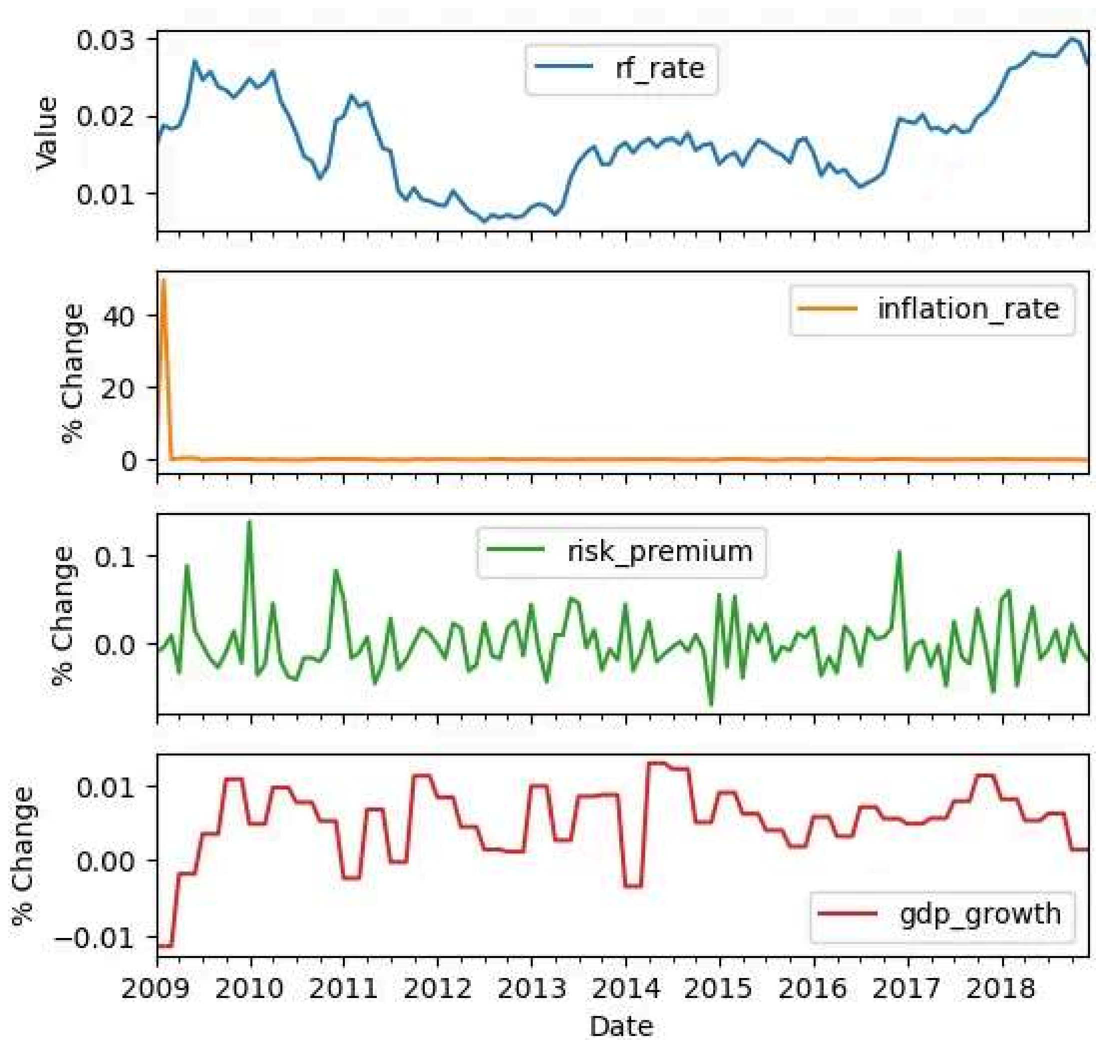
- **Select a Well-Diversified Portfolio**: Begin by choosing a well-diversified index such as the S&P 500 or NASDAQ. By using indices with a large number of constituents, diversifiable risks are minimized. Convert the stock prices into percentage changes (returns).
- **Select the Time Period**: For this example, consider the period from January 1, 2009, to January 1, 2019, with monthly intervals. Daily or weekly data are avoided to minimize noise.



Stock Price Comparison of S&P 500 and NASDAQ from 2009–2019

- **Determine the Risk-Free Interest Rate**: For the given period, use the Market Yield on U.S. Treasury Securities (DGS5) as the risk-free rate. This data can be retrieved via the Federal Reserve Bank of St. Louis' FRED API.
- **Identify Systematic Factors**: For this example, systematic factors include changes in the inflation rate, changes in the risk premium, and GDP growth. While factors don't need to be perfectly independent, they should be sufficiently uncorrelated to ensure reliable beta estimates and clear interpretation of the relationship between factors and returns.
- **Fetch Factor Data**: Retrieve percentage changes for each systematic factor over the selected period. The Federal Reserve Bank of St. Louis' FRED API can be used to obtain this data. For factors with quarterly data, assume that the value remains constant throughout the quarter and align

it with the stock date index.
Notice that *inflation rate, risk premium, and GDP Growth* are percent changes and not direct values. At the same time, the risk-free interest rate is the direct value at the time



Graph of the risk-free rate, inflation rate, risk premium, and GDP Growth

- **Calculate Betas:** To determine each factor's beta (its sensitivity to the factor), perform an Ordinary Least Squares (OLS) regression. This involves regressing the portfolio returns against the data for the selected systematic factors.

By following these steps, APT can be effectively implemented to assess the relationship between asset returns and systematic risk factors.

e) Results & Implications

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 S&P500   R-squared:                       0.080
Model:                            OLS   Adj. R-squared:                  0.056
Method:                 Least Squares   F-statistic:                     3.331
Date:                Fri, 27 Sep 2024   Prob (F-statistic):             0.0220
Time:                        05:46:46   Log-Likelihood:                 222.10
No. Observations:                 119   AIC:                            -436.2
Df Residuals:                     115   BIC:                            -425.1
Df Model:                           3
Covariance Type:            nonrobust
==================================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
const             -0.0066      0.006     -1.148      0.253      -0.018       0.005
inflation_rate    -0.0024      0.001     -2.873      0.005      -0.004      -0.001
risk_premium      -0.0181      0.109     -0.167      0.868      -0.234       0.197
gdp_growth         0.1934      0.832      0.233      0.817      -1.454       1.841
==============================================================================
Omnibus:                        3.860   Durbin-Watson:                   1.920
Prob(Omnibus):                  0.145   Jarque-Bera (JB):                3.506
Skew:                          -0.269   Prob(JB):                        0.173
Kurtosis:                       3.645   Cond. No.                     1.08e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.08e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Regression Analysis of S&P 500 against Inflation, Risk Premium & GDP Growth

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 NASDAQ   R-squared:                       0.027
Model:                            OLS   Adj. R-squared:                  0.002
Method:                 Least Squares   F-statistic:                     1.080
Date:                Fri, 27 Sep 2024   Prob (F-statistic):              0.360
Time:                        05:46:46   Log-Likelihood:                 202.08
No. Observations:                 119   AIC:                            -396.2
Df Residuals:                     115   BIC:                            -385.0
Df Model:                           3
Covariance Type:            nonrobust
==================================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
const             -0.0038      0.007     -0.556      0.579      -0.017       0.010
inflation_rate    -0.0015      0.001     -1.565      0.120      -0.003       0.000
risk_premium      -0.0257      0.129     -0.200      0.842      -0.281       0.229
gdp_growth         0.2655      0.984      0.270      0.788      -1.684       2.215
==============================================================================
Omnibus:                        1.810   Durbin-Watson:                   1.971
Prob(Omnibus):                  0.405   Jarque-Bera (JB):                1.294
Skew:                          -0.181   Prob(JB):                        0.524
Kurtosis:                       3.360   Cond. No.                     1.08e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.08e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Regression Analysis of Nasdaq against Inflation, Risk Premium & GDP Growth

1. The findings reveal an insignificant relationship between GDP and risk premium with S&P500 and NASDAQ returns from the model.
2. A negative beta value for inflation indicates a negative correlation with the S&P500 returns. This result aligns with the real-world observation.
3. Furthermore, the **low R^2 value of 0.08 (8%)** in the APT model is not necessarily alarming, as it highlights that the model's effectiveness relies heavily on the chosen macroeconomic factors.

The final results of the APT model indicate that while it provides some insight into the relationship between macroeconomic factors and stock returns, the low R^2 value of 0.08 (8%) suggests that the selected factors may not fully capture the complexity of market dynamics. This implies the possibility that other unaccounted-for variables are playing a significant role in influencing asset returns. As a result, while the APT model offers a useful framework, it may require more refined factor selection to better explain the variability in returns.

Given the limitations observed in the APT model, it is useful to explore alternative approaches that have been developed to address similar concerns. One such model is the Fama-French Three-Factor Model (FF-3).

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III. Fama French Three Factor Model

The Fama-French Three-Factor Model (FF-3), developed by Eugene Fama and Kenneth French in 1993, is a renowned multi-factor model that extends the Capital Asset Pricing Model (CAPM) by incorporating two additional variables — size and value — alongside the market factor. This enhancement allows the model to more effectively capture the volatility of stock price returns.

> Fama & French argued that their three factor model which are not obvious represented deeper, harder-to-measure risks. For instance, companies with high book-to-market ratios (value stocks) often encounter financial difficulties, increasing their risk. Similarly, smaller firms are more susceptible to economic changes, heightening their vulnerability to market risks.

The three-factor model can be defined as follows:

$$R_{it} = E(R_i) + \beta_{iM}(R_{Mt}) + \beta_{iSMB}(SMB_t) + \beta_{iHML}(HML_t) + \epsilon_t$$

Here,

*R_{mt} is the **Market Risk Premium (Mkt-RF)** from CAPM, representing the excess return of the market over the risk-free rate*

*SMB_t is a factor measures the **size effect** — It is thedifference in returns between small-cap and large-cap stocks*

*HML_t is the factor that reflects the **value effect** — the tendency for value stocks (those with high book-to-market ratios) to outperform growth stocks (those with low book-to-market ratios)*

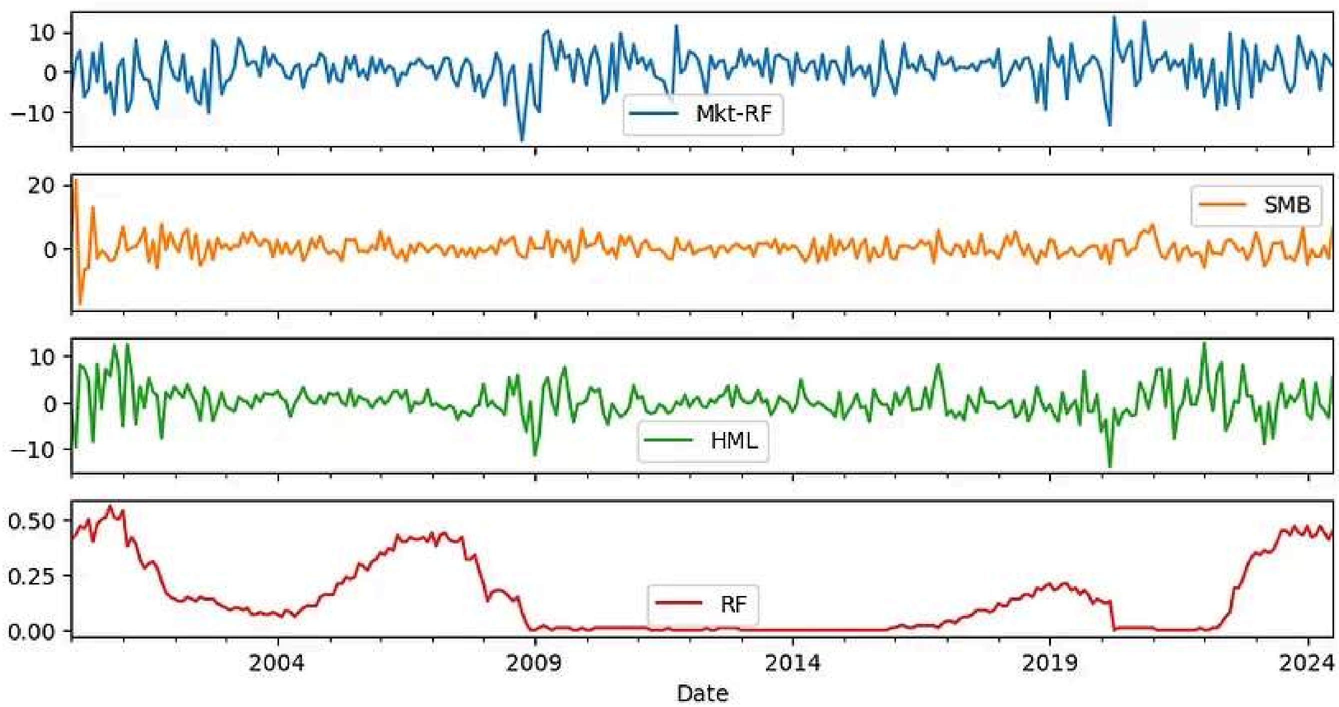
*B_{ij} represents the **price sensitivity** to the different j factors within the Fama French 3 factor model*

a) Implementation

[File Link] Implementing the Fama-French Three-Factor Model involves the following steps:

- **Select Stocks**: Choose three stocks, such as Apple Inc., Amazon Inc., and JPMorgan Chase & Co. By selecting stocks from different industries, we minimize the risk of selection bias. Convert the stock prices into percentage changes (returns).

- **Define the Time Period:** Consider the period from January 1, 2000, to July 1, 2024, aligning with the last reported dates in the Fama-French database, using monthly intervals to avoid noise from daily data.
- **Load Data:** Retrieve the Fama-French data using the Fama-French Reader dataset available in Python libraries.



Factors from Fama French Python Dataset, 2000–2024

- **Merge Datasets:** Integrate the Fama-French dataset with the selected stocks, calculating the excess returns for each stock using the risk-free rate (RF) provided in the Fama-French dataset.
- **Run Regression:** Perform an Ordinary Least Squares (OLS) regression on each stock's excess return data against the three factors from the Fama-French dataset: Market Risk Premium (Mkt-RF), Size Factor (SMB), and Value Factor (HML).

- **Perform Prediction:** Using the Fama-French Three-Factor model, we can generate predictions to assess how accurately it forecasts the daily returns of the stocks

b) Results & Implications

```
                            OLS Regression Results
==============================================================================
Dep. Variable:           Apple_Excess   R-squared:                       0.371
Model:                            OLS   Adj. R-squared:                  0.364
Method:                 Least Squares   F-statistic:                     56.94
Date:                Fri, 27 Sep 2024   Prob (F-statistic):           5.71e-29
Time:                        10:47:09   Log-Likelihood:                -1057.2
No. Observations:                 294   AIC:                             2122.
Df Residuals:                     290   BIC:                             2137.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.8130      0.523      3.464      0.001       0.783       2.843
Mkt-RF         1.3064      0.117     11.128      0.000       1.075       1.537
SMB            0.1293      0.169      0.764      0.446      -0.204       0.463
HML           -0.7664      0.149     -5.130      0.000      -1.060      -0.472
==============================================================================
Omnibus:                       33.526   Durbin-Watson:                   1.941
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              188.979
Skew:                          -0.108   Prob(JB):                     9.20e-42
Kurtosis:                       6.922   Cond. No.                         4.84
==============================================================================
```

Regression Analysis of Apple against Fama French Three Factors

```
                            OLS Regression Results
==============================================================================
Dep. Variable:            AMZN_Excess   R-squared:                       0.385
Model:                            OLS   Adj. R-squared:                  0.379
Method:                 Least Squares   F-statistic:                     60.52
Date:                Fri, 27 Sep 2024   Prob (F-statistic):           2.07e-30
Time:                        10:47:09   Log-Likelihood:                -1096.6
No. Observations:                 294   AIC:                             2201.
Df Residuals:                     290   BIC:                             2216.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.4305      0.598      2.390      0.017       0.253       2.608
Mkt-RF         1.4931      0.134     11.123      0.000       1.229       1.757
SMB           -0.1441      0.194     -0.744      0.457      -0.525       0.237
HML           -1.1919      0.171     -6.978      0.000      -1.528      -0.856
==============================================================================
Omnibus:                       62.871   Durbin-Watson:                   1.927
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              337.873
Skew:                           0.730   Prob(JB):                     4.28e-74
Kurtosis:                       8.045   Cond. No.                         4.84
==============================================================================
```

Regression Analysis of Amazon against Fama French Three Factors

```
                            OLS Regression Results
==============================================================================
Dep. Variable:             JPM_Excess   R-squared:                       0.578
Model:                            OLS   Adj. R-squared:                  0.574
Method:                 Least Squares   F-statistic:                     132.4
Date:                Fri, 27 Sep 2024   Prob (F-statistic):           4.82e-54
Time:                        10:47:09   Log-Likelihood:                -919.99
No. Observations:                 294   AIC:                             1848.
Df Residuals:                     290   BIC:                             1863.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0055      0.328      0.017      0.987      -0.640       0.651
Mkt-RF         1.2898      0.074     17.521      0.000       1.145       1.435
SMB            0.0603      0.106      0.568      0.571      -0.149       0.269
HML            0.7573      0.094      8.085      0.000       0.573       0.942
==============================================================================
Omnibus:                       40.171   Durbin-Watson:                   2.134
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              134.383
Skew:                           0.539   Prob(JB):                     6.59e-30
Kurtosis:                       6.132   Cond. No.                         4.84
==============================================================================
```

Regression Analysis of JPM Morgan Chase & Co. against Fama French Three Factors

Apple:

1. The results reveal that approximately *37.1% of the variability* in Apple's excess returns can be attributed to the three factors in the Fama-French Model, indicating a moderate fit. This suggests that while the model captures some of the factors driving Apple's returns, a substantial portion (62.9%) remains unexplained.
2. Notably, the model shows that Apple's returns have a *strong positive relationship* with Market Risk Premium exposure, reflected in a coefficient of 1.3064, while exhibiting a *significant negative relationship* with the Value Factor. However, there is an *insignificant exposure* to the Size Factor.
3. Lastly, the alpha for *apple's return is high* (1.813) & statistically significant representing superior performance than what is predicted by Fama

French model.

Amazon:

1. The analysis shows that around *38.5% of the variability* in Amazon's excess returns is explained by the three factors in the Fama-French Model, again indicating a moderate fit and suggesting that a considerable portion (61.5%) of the variability remains unexplained.

2. The model indicates that Amazon's returns are *positively and highly significant* in relation to Market Risk Premium exposure, with a coefficient of 1.4931, and are also *significantly negatively* related to the Value Factor. Similar to Apple, there is an *insignificant exposure* to the Size Factor.

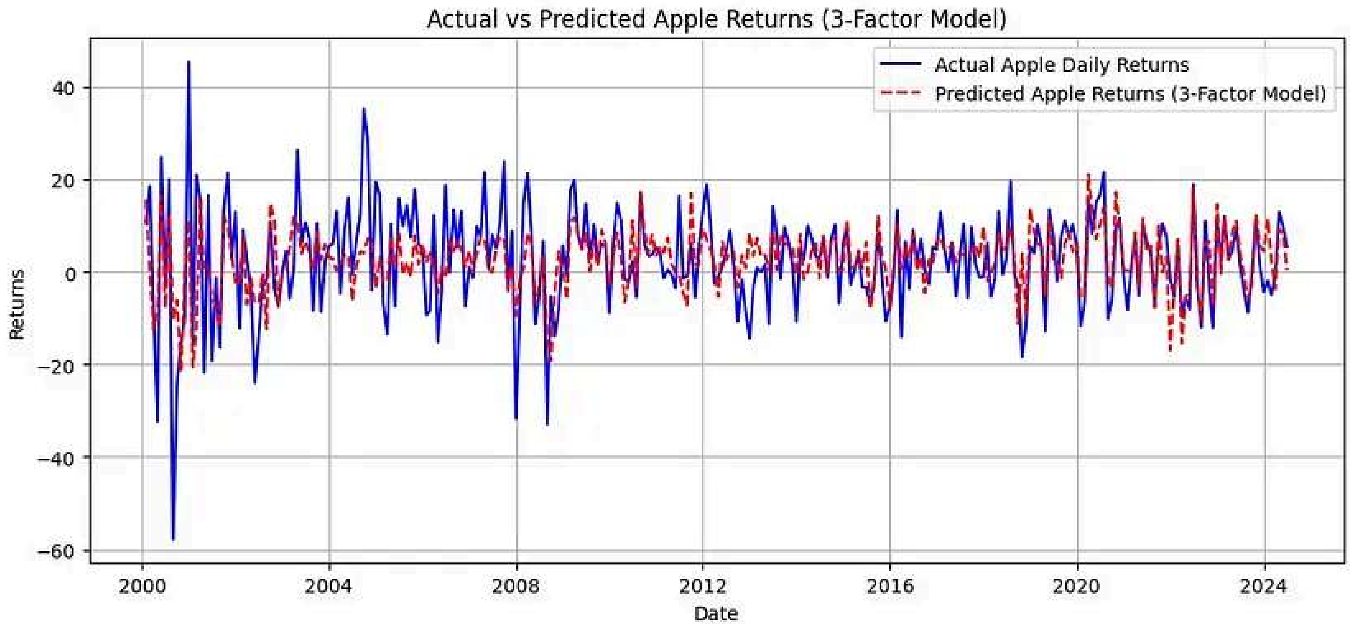
3. Lastly, the alpha for *amazon's return is high* (1.430) & statistically significant representing superior performance than what is predicted by Fama French model

JPMorgan Chase & Co.:

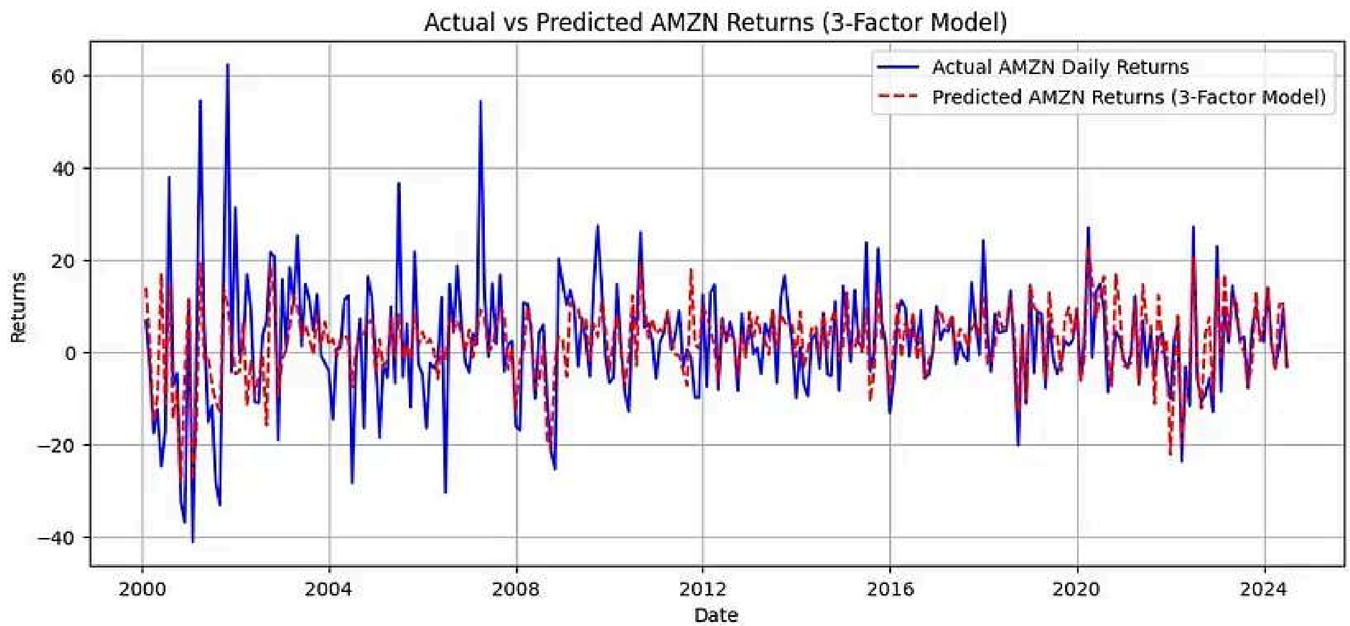
1. The findings indicate that approximately 57.8% of the variability in JPMorgan Chase & Co.'s excess returns is explained by the three factors in the Fama-French Model, reflecting a higher fit compared to the other two stocks and suggesting that the model captures more than half of the factors influencing JPM's returns.

2. The model shows a strong *positive and highly significant* relationship between JPM's returns and Market Risk Premium exposure, with a coefficient of 1.2898, along with a *significant negative relationship* with the Value Factor. However, there is again an *insignificant exposure* to the Size Factor.

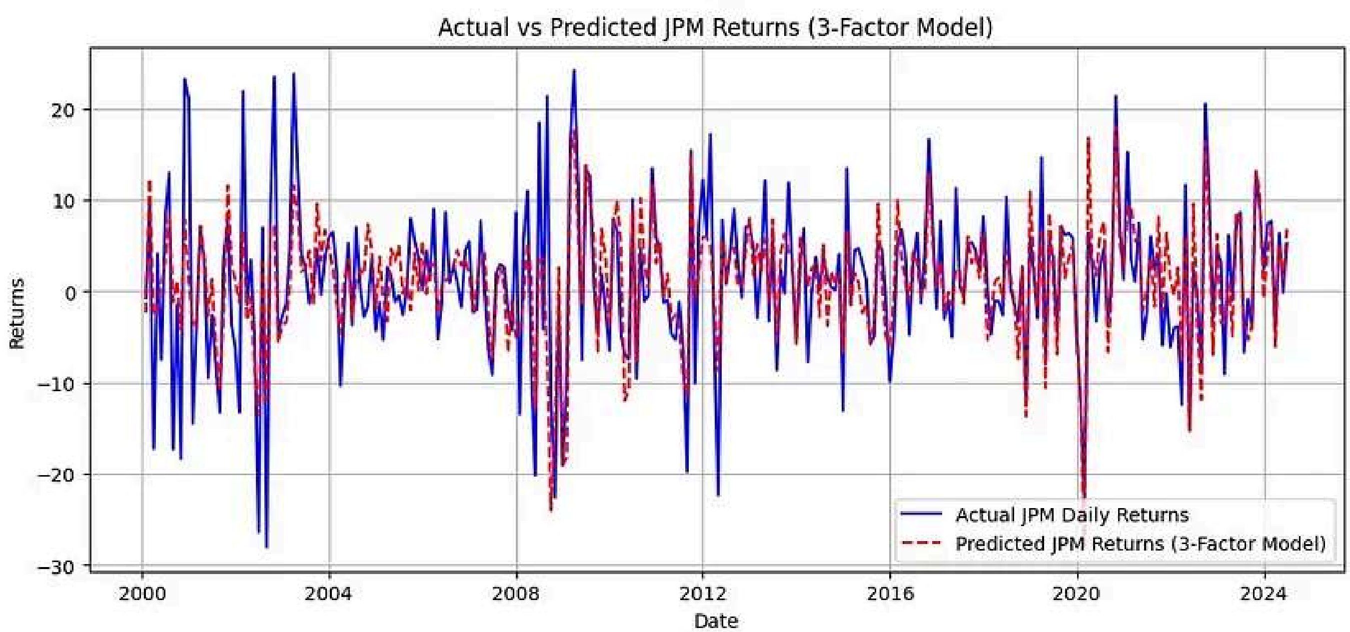
3. Lastly, the alpha for *JP Morgan Chase & Co. return is* statistically insignificant representing that the stock's performance is adequately explained by the three Fama-French factors



Actual vs Predicted Apple Returns (3-Factor Model)
Actual Apple Daily Returns
Predicted Apple Returns (3-Factor Model)
Returns
Date
40
20
0
−20
−40
−60
2000
2004
2008
2012
2016
2020
2024



Actual vs Predicted AMZN Returns (3-Factor Model)
Actual AMZN Daily Returns
Predicted AMZN Returns (3-Factor Model)
Returns
Date
60
40
20
0
−20
−40
2000
2004
2008
2012
2016
2020
2024



Actual vs Predicted JPM Returns (3-Factor Model)
Actual JPM Daily Returns
Predicted JPM Returns (3-Factor Model)
Returns
Date
20
10
0
−10
−20
−30
2000
2004
2008
2012
2016
2020
2024

c) Key Findings in the Industry on New Factor Modelling

Factor modelling is widely utilised across the finance industry, with key professionals and researchers engaging in backtesting and analysing stock return components to create well-designed and dynamic portfolios.

MSCI Barra stands out as a leading provider of data on factor indices, categorizing global equity factors into eight groups comprising 16 distinct factors that significantly influence portfolio performance. Portfolio managers worldwide develop strategies based on one or more of these factors to effectively price in the associated risks and returns of securities.

MSCI Global Equity Factor Structure

Ongoing research in the field of finance has sparked extensive discussions regarding the optimal number of factors needed to accurately predict stock prices. Notable models, including the four-factor momentum model (Carhart, 1997), five-factor model (Fama and French, 2015), Q-Factor model (Hou, Xue, and Zhang, 2015), six-factor model (Barillas et al., 2019), and the Pastor-Stambaugh model (2003), emphasize the significance of developing unique factor-based strategies. The introduction of new factors such as Profitability, Investment, and Liquidity has contributed to what is known as the “Factor Zoo” in academic literature, highlighting the ongoing pursuit of more robust models while minimizing overfitting.

d) Shortcomings & Limitations of Factor Models

While factor models offer valuable insights for predicting and analyzing stock or portfolio returns, they come with several limitations:

1. **Oversimplicity**: These models tend to oversimplify the complex interdependencies among various factors affecting stock prices. Geopolitical and macroeconomic variables are inherently complex and do not exhibit a linear relationship with stock prices, as assumed by these models. Additionally, the static nature of factor models restricts their adaptability, as the factors driving returns can evolve over time due to economic shifts, regulatory changes, or fluctuations in market sentiment.
2. **Subjectivity**: The daily return data of a stock or portfolio is influenced by numerous factors. However, determining the exact number of factors to consider in a model can be challenging and lacks universal consensus. Moreover, interpretations of these factors can vary significantly among researchers, leading to potential bias and inconsistency in findings.
3. **Technical Issues**: Many factor models require extensive historical data to establish statistically significant evidence of a manager’s skill — or the lack thereof. A manager may outperform their regression-based benchmark over a decade yet still fail to demonstrate statistical significance. This limitation underscores the challenges of relying on historical data, as finance researchers often argue that it is difficult to definitively ascertain a manager’s skill, even when analyzing numerous years of performance data.

4. **Data Snooping and Survivorship Bias:** Past performance does not guarantee future results. Researchers searching for patterns often engage in data snooping, leading to the discovery of correlations that may be purely coincidental. Additionally, constructing factor models may involve survivorship bias, where only currently successful companies are included in the dataset, potentially skewing factor effects and resulting in inaccurate predictions

5. **Multicollinearity:** In real-world scenarios, factors are frequently interconnected and interdependent. For instance, the size and value factors in the Fama-French model may be correlated, introducing multicollinearity that can diminish the precision of the model's estimates.

Despite these challenges, factor models remain invaluable for understanding asset return distributions. They break down returns into various components attributable to specific factors, offering insights into the elements that influence stock prices within a controlled framework. Furthermore, factor models shed light on portfolio managers' strategies and thinking processes when managing assets. Ultimately, they serve as a useful tool for evaluating a manager's skill by comparing alphas. If a manager achieves impressive results marked by a high alpha, yet this success can be traced back to significant exposure to a particular factor, it suggests that the manager may not be uniquely skilled but rather benefiting from advantageous factor exposure.

Portfolio Management Econometrics Capm Three Factor Model Finance