NUS, FE5108 (Portfolio Theory and Investments): practitioner segment class project

Fall, 2024

- Teams of 4 or 5 students. Write-up due in full over email two weeks after the last class, by Sunday Nov 10. PDF or Word, where supporting model/code can be sent as separate attachments or as an appendix to the main write-up. On the cover page, succinctly state the contributions of each team member (coding, data analysis, write-up, etc.).
- Concise write-up, in your own words paraphrasing descriptions you have been asked to find. No more than 7 pages of text, not counting exhibits (tables, charts).
- In the header or footnote for any exhibit you include, please state clearly (but again concisely) what is being shown, including data source, time period, investable universe, etc. Similarly, make sure to label any chart axes.
- Bonus credit awarded when the exhibits are easy to interpret and support your stated conclusions, so please give some thought about how to best display your numerical results for each question. In your career, you will find that an effective exhibit is much more powerful and compelling than long descriptions, a habit worth forming now.

The objective of this assignment is to illustrate the notions around factor investing, portfolio construction, and portfolio analytics that we have discussed in class, as well as provide hands-on experience with the types of projects you may encounter in your future careers.

A new part of the project will be made available after each class, tracking the course outline to the extent possible.

For your convenience, the data sets you are asked to download have already been provided on the class website, but I recommend you take a look at the original source to familiarize yourself with it.

Part 1: factor investing

The first part of this project aims to provide insight into different factor construction methodologies used in academic finance research and by practitioners of factor investing. It will highlight how design choices have meaningful impact, yet the return profiles of the underlying factors remain recognizable.

In 1992, University of Chicago professors Eugene Fama and Kenneth French wrote a highly influential paper positing that three factors, rather than just a single market-beta factor (CAPM) helped explain the cross-section of expected stock returns. These included the market factor ("Mkt – RF", the equity market's return in excess of the risk-free rate), a size factor ("SMB", "small minus big", the return of small-cap stocks over large-capitalization stocks), and a value factor ("HML", "high minus low", the return of stocks with a high book-to-price ratio relative to those with a low book-to-price ratio). All newly proposed factors in the subsequent academic literature were evaluated against these to gauge if the historical premium went beyond what this model already predicted. This included all factor premia that had been identified before the Fama-French 3-factor model but not gained academic acceptance due to the strong belief in CAPM. Only in the past decade, after the global financial crisis, have alternative factor models started to gain traction, and professors Fama and French updated their model

with two more factors (high profitability and conservative investment). However, the original 3-factor model is still widely referenced and remains important to understand.

1A) Factor premiums and factor decay

Download the history of monthly "Fama-French 3-factor model" returns from the "U.S. Research Returns Data" tab on:

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html

Read the description of how these factor returns are calculated. We shall later return to how the "value factor" (high to low book to market, abbreviated as HML[1]) return is calculated.

- Calculate the t-stat of the average return to the market, value, and small-size factors both for the time period from 1927 through 1992, as well as for January 1993 through July of 2023 (post-publication of the original paper). Since these are return spreads, not actual portfolio returns, think of what type of mean is appropriate (geometric? arithmetic?). For purposes of this question, you can ignore any autocorrelation in factor returns in the calculation of the t-stat (that is, no need for Newey West adjustment).
- Comment on the difference in statistical significance of the estimated factor premia you are seeing. Do they all remain high-conviction investment ideas as they may have seemed at the time of the publication of this research?

From the same website, also download the univariate monthly returns of (US) portfolios sorted by price momentum and beta. These will be provided on both a value-weighted and an equal-weighted factor portfolio construction. Calculate the time-series of monthly spreads between the top decile of lowest-beta and highest momentum portfolio versus the worst decile by those metrics on a value-weighted basis for use later in this assignment.

1B) Factor index construction in practice

While Fama-French factor spreads have dominated academic discourse, practitioners have created their own versions of "factor indexes," paper portfolios that tilt to certain factors that can be tracked by investors. A big index provider whose indexes are the basis for many exchange-traded factor funds is MSCI, which makes index returns available on: https://www.msci.com/end-of-day-data-search

From the MSCI website, download historical (monthly, total, gross) returns for:

- MSCI USA index (standard, meaning for large-cap and mid-cap stocks)
- MSCI USA value index (standard, this is a legacy "style" index)
- MSCI USA minimum volatility index (standard, this is a "factor-high exposure" suite index; select USD version, meaning optimized for USD returns)
- MSCI USA equal-weighted index (standard, this is a "factor-high exposure" suite index)
- MSCI USA momentum index (standard, this is a "factor-high exposure" suite index)

[1] While Fama-French refer to this value factor as 'book to market', practitioners more commonly use 'book to price' to refer to the same ratio.

Note: the website will provide monthly index levels which you will have to transform to monthly returns by taking their month-end to month-end percent change. Download the return data since January 1995.

Look up the methodology of the momentum index and answer the following questions in the context of what we discussed in class about factor portfolio construction methods:
https://www.msci.com/eqb/methodology/meth_docs/MSCI_Momentum_Indices_Methodology.pdf

Please paraphrase in your own words and do not copy exhibits or text literally from the factsheet

- What is the investable universe of the index?
- On what basis is price momentum measured?
- How is price momentum standardized across stocks to create the momentum index? How are outliers dealt with?
- How are "high momentum" stocks selected for inclusion in the factor index?
- How are the selected stocks weighted? Comment on whether this just looks at price momentum, or whether the weighting methodology also includes market cap? If so, give two reasons why this would be beneficial and one reason why this may be detrimental.
- Comment on the target rebalance frequency of the index.
- Comment on when the index may see trigger-based ad-hoc reconstitutions and why this may benefit investors, as per the rationale provided in the factsheet?
- Comment on any cap (i.e., upper limit) that may or may not be applied to individual security weights?

1C) Benefits of factor diversification

Create a multi-factor index return for each month by taking the average return over the value, minimum-volatility, equal-weighted, and momentum indexes. For each of the 6 indexes you now have (the overall MSCI USA market index, the four smart-beta indexes, and the multi-factor index) as well as for a risk-free investment (using the Fama-French monthly "RF" returns for 1-month US treasury bills) calculate the annualized average return from January 1995 through July 2023. Also calculate the Sharpe Ratio for each[2]. Create exhibits that compare the average return level and risk-adjusted return (Sharpe Ratio) for each of these and comment on which factor indexes did best, did worst, and the relative performance of the multi-factor index.

1D) Sensitivity of factor returns to factor portfolio construction

Note that the above comparison is based on factor index returns, not yet factor spreads as provided by Fama-French. For each month (January 1995-July 2023) calculate the spread of the 4 individual smart-beta strategies over the broad USA equity market index. Calculate the correlation of each of those four factor spreads to each of the four Fama-French monthly return spreads (HML, SMB, high versus low momentum, low versus high beta, all on a capitalization-weighted basis). Present the results in a (4 by 4) table. Comment on the similarity between the Fama-French factor return spreads and those you calculated based on the MSCI smart-beta indexes.

[2] The Sharpe Ratio is defined as the annualized return to the strategy minus the annualized return of the risk-free investment, divided by the annualized volatility of strategy returns.

Part 2: Hands-on analysis of security-level data

The second part of this project aims to provide experience working with security-level data. Guillaume Coqueret and Tony Guida have generously made their excellent book "Machine Learning for Factor Investing" referenced later in class available on the website:

http://www.mlfactor.com/

They have kindly provided actual stock-level historical data with this, an extract of which is available on the class website. Note that the stocks have been 'anonymized' and all but the return data are provided in standardized form. The reason is that this type of market data is generally expensive and sourced from vendors like Refinitiv, Factset, Bloomberg, and S&P. It therefore cannot be simply made available online in untransformed form even for educational purposes. The data dictionary for the full data set is available at: http://www.mlfactor.com/data-description.html

Note that the provided returns field is in decimal points (0.01 is 1%) and '1 month forward,' meaning the value for 12/31/1998 is the return for January of 1999. This is a standard convenience for back-testing of systematic investment strategies: you can create investment portfolios using data known as of 12/31/1998 (the investable universe, market valuations, factor values, etc.) and calculate the subsequent return thereof without suffering from look-ahead bias, provided the data are truly point in time.

2A) Data cleaning and validation

As discussed in class, it's good practice to take a look at any new data set to understand its scope, identify any potential issues including necessary data cleaning steps, and do sanity checks that give confidence the data is what its vendor claims it to be. This is before even analyzing the data for being useful in your investment process and testing whatever investment hypothesis you have in mind.

- Plot the number of stocks included in the data for each month of the provided history. Does the investable universe appear to be consistently defined every month? What months are particularly questionable?
- Visualize the distribution of the provided returns field. Are there outliers on the upside or the downside? Clean the return data in each period in the provided history by winsorizing at the 99% level.
- Plot the coverage (% not missing) of the return data field each period. Are there stocks with missing return data in any period? If there are, how would you treat these stocks in your analysis and why?
- Visualize the distribution of the provided standardized price to book field. Are there any values that stand out? Given your insights previously given on how Fama-French define their HML value factor, what do you believe this indicates about the data and how would you treat these stocks below, when construction your own value portfolios?
- The data include only standardized (percentiled: 100 is highest) fields for price-to-book ratio (P/B), book value of equity, and market capitalization, not their actual values. Given this limitation, show how you might do a sanity check that the P/B value is indeed the ratio between market cap and book value?

2B) Rules-based portfolio construction: broad market

Limit the data history to December 1998 through November 2018, so that any performance analysis below covers 1999-2018 in full. In this and the following questions, we focus on long-only (no negative weight) and fully invested (no leverage or cash holding: sum of weights equals 1) portfolios.

- In each period, approximate the capitalization-weighted market index by weighting stocks in proportion to their standardized market cap and calculate the monthly return history.
- Compare the resultant monthly returns to those of the MSCI USA market index used in Part 1. Show how you might gain confidence that the stock-level data you have been provided cover a broad universe of US stocks and the return data are accurate.

2C) Factor portfolio construction

We will now compare different factor portfolio methodologies for their relative merits. In each period, create the following long-only portfolios investing in the Book to Price (!!) value factor and calculate their month return series. For each portfolio, most weight is allocated to those stocks that are the cheapest on book to price.

1. Equally weight the top 20% of stocks by their book-to-price ratio
2. Weight stocks in proportion to the provided standardized (percentiled) exposure to the book-to-price ratio.
3. Weight stocks in proportion to the provided standardized (percentiled) exposure to the book-to-price ratio TIMES their provided standardized market cap.

Please answer the following questions:

- Report the average annualized return to each of these three value indexes here and compare to that of your estimated return on the cap-weighted broad market. Which is highest and why do you believe this is the case?
- Also report their annualized standard deviation and compare to the cap-weighted broad market. Be mindful of what type of 'average' is appropriate here! Which is highest and why do you believe this is the case?
- Looking at the portfolio weights for the last period in your analysis window, note which of the three has the least stocks held in the portfolio and which has the highest single-stock weight. Comment, based on the discussion in class, on whether this is as expected.

2D) For each month, calculate the excess return (return differential) of each of your three value strategies versus your estimated return on the cap-weighted broad universe.

- Calculate the annualized standard deviation of each of the three excess return time-series: this is their annualized tracking error to the cap-weighted broad market. Which is lowest, and why do you believe this is the case?
- Calculate the return correlation of each of your value strategies excess returns over the broad cap-weighted market to each other and to the Fama-French monthly value premia (HML return) you considered in Part 1. Comment on the magnitude of the correlations you are seeing: are they all conceivably investing in the same style factor? Comment on which of your three value strategies is most similar to the HML factor. Given your previous answers on how the HML factor is constructed, is this as expected?

Part 3: portfolio construction

The third part of this project aims to provide hands-on experience with portfolio optimization and the impact of design choices. From the Fama-French website, download the history of monthly industry returns (use 49 industries, total returns [meaning including dividend payouts which are assumed reinvested], value-weighted [meaning, each constituent weighted by its market capitalization]) back to January 1995. Note how -99.99% return reflects missing data (in this case, no stock available at the time in that industry yet): double-check that this does not affect the analysis period! Also note that Fama-French keep updating their data so make sure to cut out any periods outside the stated periods in the assignments.

3A) portfolio optimization

Using the first 10 years (January 1995 – December 2004) as a "seed period" to estimate the covariance matrix, set up an optimization to calculate the minimum-variance portfolio as of the close of 12/31/2004[3]. The portfolio needs to be fully invested but may include short positions (meaning, negative weights). Do the same for the minimum-variance portfolio but now subject to long-only constraints (that is, no negative weights allowed). Lastly, calculate the "inverse-volatility" portfolio which weights each industry in proportion to the inverse of its historical volatility (1/sigma, with sigma the return volatility). The latter is a simple heuristic to give less risky assets a higher weight in the portfolio.

Show the mathematical formulation of the long-only minimum-variance problem and explain how you solved it (using any tool of your choice; include your code as an appendix to the write-up, not counting it for the stated page limit). Show a table with the optimal industry weights for each of the above 3 portfolio construction methods, as well as the total sum of industry weights as a sanity check. For each strategy, highlight the top 5 weights in green and the bottom 5 weights in red to allow easy comparison.

3B) in-sample effectiveness

For each of the 3 low-volatility portfolios created earlier, show the in-sample annualized return volatility for the period January 1995 through December 2004. Comment on which one resulted in the lowest realized volatility and on the portfolio with the highest realized volatility. In particular, is the order "as expected" and why might this have been the case, in the context of our review of optimization in class?

3C) out-of-sample effectiveness

For each of the 3 low-volatility portfolios created earlier, show the out-of-sample annualized return volatility for the period January 2005 through July 2023. Comment on which one resulted in the lowest realized volatility and on the portfolio with the highest realized volatility. In particular, is the order "as expected" and why might this have been the case, in the context of our review of optimization in class?

3D) the value of re-optimization

[3] Empirical research has shown that the performance benefits of low-volatility investing can be accomplished equally well through investing in defensive industries as through individual stock selection (see De Boer, Campagna, and Norman, "Country and Sector Drive Low-Volatility Investing in Global Equity Markets," Journal of Index Investing, Spring 2014)

Now create an updated long-only minimum-variance portfolio as of Dec 31, 2015, using again 10 years of historical returns for the covariance matrix estimation (Jan 2006 – Dec 2015). Compare the realized volatility of this portfolio for Jan 2016 through July 2023 to that of the long-only minimum-variance portfolio you created as-of Dec 31, 2004. Comment on the difference and why this may or may not be as expected?

3E) performance expectations

Focusing again on the long-only minimum-variance industry portfolio you created as-of Dec 31, 2004, plot the cumulative value of $1 invested in this strategy at its inception date (12/31/2004) through July 31, 2023 using monthly returns (so the final data point for July 2023 is the value of that initial investment at that date). Plot the value on a log-2 scale of the y-axis. In the same chart, add the cumulative value of $1 invested in the MSCI US index. Comment on the performance of the minimum variance strategy relative to the overall equity market (MSCI US index) during both the Global Financial Crisis market downturn (Sept 2008 through Febr 2009) as well as during the COVID market meltdown (February and March 2020). In particular, did the defensive strategy perform as-expected during these periods?

Digging deeper, identify the 10 industries that held up best (that is, had the highest cumulative return) during each of these two market crises (separately) and comment on whether these are traditional defensive industries or whether some at first sight are unexpected (hint: what types of stocks actually benefitted from the COVID crisis and are those historically low-volatility stocks?)

Part 4) product innovation

Quantitative investment analysts will often get involved in new product development, helping 'engineer' the desired investment outcome and evaluate if the final version delivers on it. While ETFs (Exchange Traded Funds) have long been a convenient way to invest in diversified portfolios of individual securities, the past few years have seen the launch of single-stock leveraged ETFs. In investing, 'leverage' typically refers to investors borrowing money to be able to invest more money in some investment opportunity they find attractive. If the investment works out, this will multiply their profits. If the investment loses money, the losses will be multiplied since the loans will still have to be paid back in full. Leverage is therefore a dangerously double-edged sword and brokers to individual investors are cautious in making it available to them. Single-stock leveraged ETFs aim to offer their investors some multiple of the daily return of the underlying stock. The ETF company will take on the necessary leverage to do so, so the investor in the ETF avoids the inconvenience in having to do so themselves.

In this part of the assignment we focus on GraniteShares 2x Long NVDA Daily ETF (US ticker NVDL), which on any given US trading day aims to deliver twice the return of NVDIA (ticker NVDA). (Prior to January 22 of 2024, it was only 1.5 times the return.)

https://graniteshares.com/institutional/us/en-us/etfs/nvdl/

A file with daily total returns for both NVDL and NVDA from January through August of 2024 is provided, as is the ETF's factsheet.

4A) For both NVDA and NVDL, calculate and report the Standard Deviation, 95% Value at Risk, and 95% Conditional Value at Risk. (no need to annualize) Comment on whether these risk metrics corroborate that leverage increases the risk of an investment.

4B) Run a regression (with an intercept) of the daily returns of NVDL on the daily returns of NDVA. Report the estimated coefficients and their confidence intervals. How does this analysis show or fail to show if NVDL delivers on its stated objective in the factsheet of *"The Fund seeks daily investment results, before fees and expenses, of 2 times (200%) the daily percentage change of the common stock of NVIDIA Corp. "*

4C) The estimated intercept of the regression analysis is a crude estimate of by how much the ETF return lags the NVDA-related return component on a daily basis (that is, performance drag). Annualizing it by multiplying by 252 trading days in a year, does this roughly align with the stated net operating expense ratio (after a temporary discount offered by the ETF provider) of 1.15% per annum? Keeping in mind that the ETF borrows money and trades NVDA stock on a daily basis (see below), name two other types of expenses that may be reflected in the estimated annual performance drag.

4D) For each calendar month in the analysis period (February, …, August) as well as for the entire analysis window (February through August), calculate and compare the total return for both NVDL and NVDA. Comment on how this supports or fails to support the warning on the fund factsheet that 'the Fund should not be expected to provide 2 times the cumulative return of NVDA for periods greater than one day.'

4E) Assume that NVDA is up 10% on one day, and down 9% the next trading day. Over these two trading days, what is the two-day return on NVDA, and what is the two return on NVDL (ignoring any of the daily drag)? Comment on how this supports or fails to support the warning on the fund factsheet that *'The Fund will lose money if the underlying stock's performance is flat over time, and (…) it is even possible the Fund will lose money (…) while the underlying stock's performance increases over a period longer than a single day.'*

4F) If you're a trader who is bullish on NVDA on a day it will present its new chips to the public, would you prefer to buy the stock or the ETF? Same question if you're a long-term investor who believes NVDA will do well over the coming year but may see some price volatility.

The above results may seem puzzling but have to do with the leverage of the ETF being dynamic. After a day on which NVDA rallies, the ETF will have to borrow money and buy more shares of NVDA to maintain the two-times exposure for each new investor that buys the ETF. Conversely, when the NVDA share price drops, the ETF will have to sell part of its NVDA shares to bring its exposure back in line. In a volatile market, this 'buy high, sell low' mechanic needed to maintain the '2x long' exposure on every new trading day will be very costly for the ETF.

Part 5) portfolio analytics

In the last part of this project we focus on how asset owners can use factor attribution to understand the performance drivers of an investment strategy and whether the portfolio manager delivers "alpha" (risk-adjusted excess return) over generic factor strategies that are available for very low fees.

5A) Factor composite definitions

Looking up the methodology for the MSCI style indexes (link below), comment on how MSCI defines "value" for purposes of selecting stocks from the overall core universe. Which value indicator(s) are included and which growth metrics are used to define "growth" stocks (the opposite of value). Also comment on whether the same metrics are used for all stocks or whether there are exceptions, and on what basis? (for this question, a high-level description of the indicators suffices, no need for implementation details) Contrast this to how Fama-French (FF) define their "value" (HML) metric, which you reviewed in Part 1 of this project.

https://www.msci.com/eqb/methodology/meth_docs/MSCI_GIMIVGMethod_Feb2021.pdf

5B) Identifying strategy factor exposures based on return patterns

Run a pooled panel regression of the monthly total gross returns of MSCI's value index (Jan 1995 - July 2023) on the same-month returns of the risk-free asset (RF, sourced from FF), the FF market factor, FF value return (HML), and the FF return to the small size factor (SMB). Include an intercept in this regression. Report the regression coefficients and their t-stat. For purposes of this exercise, there is no need to adjust the t-stats for the impact of any auto-correlation in the residuals (i.e., no need for Newey-West adjustment).

Comment on whether the MSCI US Standard (Large+Mid Cap) Value equity index is "as-advertised" based on this analysis, meaning has the stated exposures.

5C) Enhancing factor performance

The intercept in this regression equates the monthly average return of the MSCI Value Index over its "replication" using generic Fama-French factors. Comment on whether the more elaborate definition of "value" by MSCI seems to have improved performance over its original academic version used by Fama-French.

5D) Fund replication

The estimated coefficients in this regression (except the intercept) can be interpreted as the weights of a "replicating portfolio" that invests in the Fama-French generic factor portfolios rather than in the MSCI Value index. Calculate the time-series of monthly returns to this "replicating" portfolio and its return correlation to the original MSCI Value Index: was the replication arguably effective? Then calculate and compare the annualized return, annualized volatility, and Sharpe Ratios of both the MSCI Value Index and this replicating portfolio. Which of these was historically the best investment option?

Keeping in mind that these are returns gross of any type of cost, provide at least one (up to 3) examples of what else you might have to evaluate before choosing between these two.

5E) Relative performance analysis

Consider the time-series of monthly returns of the multi-factor index you calculated in Part I of this assignment using the average return over the value, minimum-volatility, equal-weighted, and

momentum indexes. For the multi-factor index and each of its 4 components, calculate the monthly "active" return (return difference) to the MSCI USA standard index, which we'll use as their benchmark.

Calculate the annualized excess return of the multi-factor index compared to its benchmark over our analysis period. Also calculate the annualized tracking error of your multi-factor index to its benchmark. Use this to then calculate the annualized information ratio and comment on its magnitude given the context provided in class: is the excess return of the multi-factor index over its benchmark high relative to its tracking error? (recall that in Part I of this assignment, you compared their total returns, rather than benchmark-relative)