FE5209 Homework Assignment 1

Notes:

1. All tests are based on the **1% significance level** ($\alpha = 1\%$).
2. For the report (soft copy), do not hand in whole R outputs, use copy-and-paste to **summarise the outputs**.
3. Save the corresponding R commands and outputs (soft copy).
4. Submit (both 2 and 3) by **23:59, 8 Oct 2024** online through Canvas "HW1 Submissions".
5. **Only ONE** assignment solution will be submitted through Canvas by each group of **THREE students**.
6. In addition to the specified time series models in some of the problems, you can try your own models to gain further experience.

1 Assignment: Answer the following questions without R.

1.1 Probability and Statistics

1. Suppose that $X_1, \cdots, X_n$ are i.i.d. exponential(θ), i.e. $f(x) = \frac{1}{\theta} e^{-x/\theta}$ for $x > 0$.
(a) Find the MLE $\hat{\theta}$ of θ.
(b) Is $\hat{\theta}$ unbiased?

2. Suppose we observe $X_1, X_2, \ldots, X_n$ i.i.d. from a distribution F. The empirical distribution $F_n(x)$ is defined to be

$$F_n(x) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq x)$$

Show that

$$E(F_n(x)) = F(x), \ \mathrm{Var}(F_n(x)) = \frac{1}{n} F(x)(1 - F(x)).$$

and

$$F_n(x) \to F(x),$$

in probability as $n \to \infty$.

1.2 Linear Regression

2. Suppose that $\epsilon_1, \cdots, \epsilon_n$ are i.i.d. $N(0, \sigma_\epsilon^2)$, find the MLE of α and β in the simple regression model.
Hint: In the simple regression model $y_1 - \alpha - \beta x_1, \cdots, y_n - \alpha - \beta x_n$ are independent $N(0, \sigma_\epsilon^2)$.

3. (Brooks's book) Suppose that a researcher wants to test whether the returns on a company stock (y) show unit sensitivity to two factors (factor x_2 and factor x_3) among three considered. The regression is carried out on 144 monthly observations. The regression is

$$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \epsilon \tag{1.1}$$

(a) What are the restricted and unrestricted regressions?
(b) If the two RSS are 436.1 and 397.2, respectively, perform the test.

1.3 Time Series Models

1. Let Y_t be a stationary AR(2) process,

$$(Y_t - \mu) = \phi_1(Y_{t-1} - \mu) + \phi_2(Y_{t-2} - \mu) + \epsilon_t$$

(a) Show that the ACF of Y_t satisfies the equation

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2}$$

for all values of $k > 0$.
(b) Use part (a) to show that (ϕ_1, ϕ_2) solves the following system of equations: (This is a special case of the Yule - Walker equations.)

$$\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

(c) Suppose that $\rho_1 = 0.4$ and $\rho_2 = 0.2$. Find ϕ_1, ϕ_2, and ρ_3.

2 R Assignment: Answer the following questions with R (or other softwares).

1. Consider the monthly returns for Abbott Laboratories (ABT), CRSP value-weighted index (VW), CRSP equal-weighted index (EW), and S&P composite index from January 1972 to December 2012. The returns include dividend distributions. Data file is **m-abt3dx.txt** (date, bat, vwretd, ewretd, sprtrn).

(a) Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of each simple return series. (Hint: use the R command *basicStats* of fBasics)

(b) Transform the simple returns to log returns. Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of each log return series.

(a) Test the null hypothesis that the mean of the log returns of ABT stock is zero. (Hint: use the R command *t.test*)

(b) Obtain the histogram (with nclass=40) and sample density plot of the daily log returns of ABT stock.

2. Consider the monthly stock returns of value-weighted index (VW) from January 1972 to December 2012 in Problem 1. Perform the tests and draw conclusions using the 1% significance level.

(a) Test $H_0 : \mu = 0$ versus $H_a : \mu \neq 0$, where μ denotes the mean return.

(b) Test $H_0 : m_3 = 0$ versus $H_a : m_3 < 0$, where m_3 denotes the skewness.

(c) Test $H_0 : K = 3$ versus $H_a : K > 3$, where K denotes the kurtosis.

(d) Test $H_0 : \mu = 0$ versus $H_a : \mu > 0$, where μ denotes the mean return.

3. Consider the growth rates of the U.S. real gross domestic product (GDP) from 1947.I to 2012.IV. The original data, from Federal Reserve Bank of St Louis, are in the file **q-gdpmc1.txt** (year, month, day, gnp), and the GDP are in millions of 2005 chained dollars. The growth rate is the first differenced series of the log(GDP).

(a) Plot the GDP growth rates. The reduction in volatility, starting in the 1980s, is referred to as the *great moderation* in the economic literature.

(b) Denote by ρ_i the lag-i autocorrelation coefficient of the GDP growth rates. Test the null hypothesis $H_0 : \rho_1 = \rho_2 = \cdots = \rho_{12} = 0$ versus the alternative $H_a : \rho_i \neq 0$ for some $i = 1, \cdots, 12$. Draw your conclusion.

(c) Let μ be the mean of U.S. GDP growth rates. Test $H_0 : \mu = 0$ versus $H_a : \mu \neq 0$. Draw your conclusion.

(d) Find the order of an AR model for the growth rate series. (Hint: use the command *ar* with method "mle" for the order)

4. Again, consider the U.S. quarterly GDP growth rates of Problem 3.

(a) Build an AR model for the growth rate series. Perform model checking to validate the fitted model. Write down the model.

(b) Does the model confirm the existence of business cycles? Why? (Hint: use the command *polyroot* to find roots of a polynomial.)

(c) Obtain 1-step to 8-step ahead point and 99% interval forecasts for the U.S. quarterly GDP growth rate at the forecast origin October 1, 2012 (the last data point).

5. Consider, again, the quarterly U.S. real GDP growth rates from 1947 to 2012 in Problem 3.

(a) Fit a simple AR(1) model to the series. Write down the model.

(b) Is the model adequate? Why?

(c) Compare the AR(1) model with the AR model built in Problem 4. Which model is preferred? Why?