# Summary, sections 12.1-13.4

This is mostly a list of useful properties and formulas. FOr the concepts behind them, I have provided links to the pages that explain each topic

## Vectors

Vectors are 3-Dimensional, and represent a point or direction in space:

$$\vec{v} = \langle x, y, z \rangle$$

### Operations

$$-\vec{v} = \langle -x, -y, -z \rangle$$

$$\vec{v} + \vec{u} = \langle x_v + x_u, y_v + y_u, z_v + z_u \rangle$$

$$\vec{v} \cdot \vec{u} = x_v * x_u + y_v * y_u + z_v * z_u$$

$$\vec{v} \times \vec{u} = \langle y_v z_u - z_v y_u, z_v x_u - x_v z_u, x_v y_u - y_v x_u \rangle$$

### Properties

$$\vec{a} \cdot \vec{b} = \mid \vec{a} \mid * \mid \vec{b} \mid * \cos(\theta)$$

$$\mid \vec{a} \times \vec{b} \mid = \mid \vec{a} \mid * \mid \vec{b} \mid * \sin(\theta)$$

Dot product is associative, cross product is not.

### Uses

Finding perpendicular/parallel vectors

$$iff \;\; \vec{a} \cdot \vec{b} = 0$$

$$\vec{a} \;/\!/\; \vec{b} \;\; iff \;\; \vec{a} \times \vec{b} = 0$$

Projections

Scalar: (the length of the projection)

$$comp_a \vec{b} = \mid \vec{b} \mid \cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{\mid \vec{a} \mid}$$

Vector: (the vector representation of the projection)

$$proj_a \vec{b} = comp_a \vec{b} * \frac{\vec{a}}{\mid \vec{a} \mid} = (\frac{\vec{a} \cdot \vec{b}}{\mid \vec{a} \mid}) \frac{\vec{a}}{\mid \vec{a} \mid} = \frac{\vec{a} \cdot \vec{b}}{\mid \vec{a} \mid^2} \vec{a}$$

### Unit Vectors

$$\hat{\imath} = \langle 1, 0, 0 \rangle$$

$$\hat{\jmath} = \langle 0, 1, 0 \rangle$$

$$\hat{k} = \langle 0, 0, 1 \rangle$$

$$\vec{a} = \langle a_1, a_2, a_3 \rangle = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

## Lines

Lines are infinite, one-dimensional objects that can be described by an equation

All lines are curves (but not all curves are lines)

Essentially, they are defined by an initial point (like a y-intercept) and a direction/vector (like a slope) that maps $t$ (time) to each point on the line

### Vector Equation

$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$

$$\vec{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$$

### Parametric Equation

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

### Symmetric Equations

$$t = \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

### Line Segments

$$r(t) = \vec{r}_0 + t(\vec{r}_1 - \vec{r}_0) = (1 - t)\vec{r}_0 + t\vec{r}_1$$

$$0 \le t \le 1$$

## Planes

Planes are infinite, two-dimensional objects that can be defined by an equation

All planes are surfaces (but not all surfaces are lines)

Planes are defined by a vector that is perpendicular to the plane and a point on that plane

### Scalar Equation

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

### Linear form

$$ax + by + cz = d$$

$$where$$

$$d = ax_0 + by_0 + cz_0$$

## Curves

Curves are equations that describe the movement of a line in space. They are usually parametric, with the form:

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

or

$$\vec{r}(t) = \begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}$$

## Calculus with vectors and equations in 3D

TL;DR: To take the limit, derivative, or integral of a vector function, do the same to each component function.

### Limits and Continuity

$$\lim_{t \to a} \vec{r}(t) = \langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \rangle$$

The function is continuous if the individual pieces are continuous:

$$\lim_{t \to a} f(t) = f(a)$$

$$\lim_{t \to a} g(t) = g(a)$$

$$\lim_{t \to a} h(t) = h(a)$$

### Derivatives

$$\vec{r}^{'}(t) = \langle f^{'}(t), g^{'}(t), h^{'}(t) \rangle$$

Rules for dot and cross products

$$\frac{d}{dt}[\vec{u}(t) \cdot \vec{v}(t)] = \vec{u}^{'}(t) \cdot \vec{v}^{'}(t) + \vec{u}(t) \cdot \vec{v}(t)$$

$$\frac{d}{dt}[\vec{u}(t) \times \vec{v}(t)] = \vec{u}^{'}(t) \times \vec{v}^{'}(t) + \vec{u}(t) \times \vec{v}(t)$$

### Integrals

$$\int_a^b \vec{r}(t)dt = \langle \int_a^b f(t)dt, \int_a^b g(t)dt, \int_a^b h(t)dt \rangle$$

If $\vec{R}(t)$ is an anti-derivative of $\vec{r}(t)$ (i.e. $\vec{R}^{'}(t) = \vec{r}(t)$) then

$$\int_a^b \vec{r}(t)dt = \vec{R}(t) \mid_a^b = \vec{R}(b) - \vec{R}(a)$$

## Arc Length

$$length = \int_a^b |\vec{r}'(u)|\, du = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}\, du$$

$$s(t) = \int_0^t |\vec{r}'(u)|\, du$$

Natural Equation

$$\vec{r}_n(s) = \vec{r}(t(s))$$

Where $t(s)$ = the inverse of $\int_0^t |\vec{r}'(u)|\, du$

## Curvature

$\kappa$ (curvature) is defined as:

$$\kappa = |\vec{r}_0''(s)|$$

Or (easier to compute):

$$\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|r'(t)|^3}$$

The concept of first and second drivative can be applied to the physics equation for the movement of a particle:

$$\vec{a}(t) = \vec{g}$$

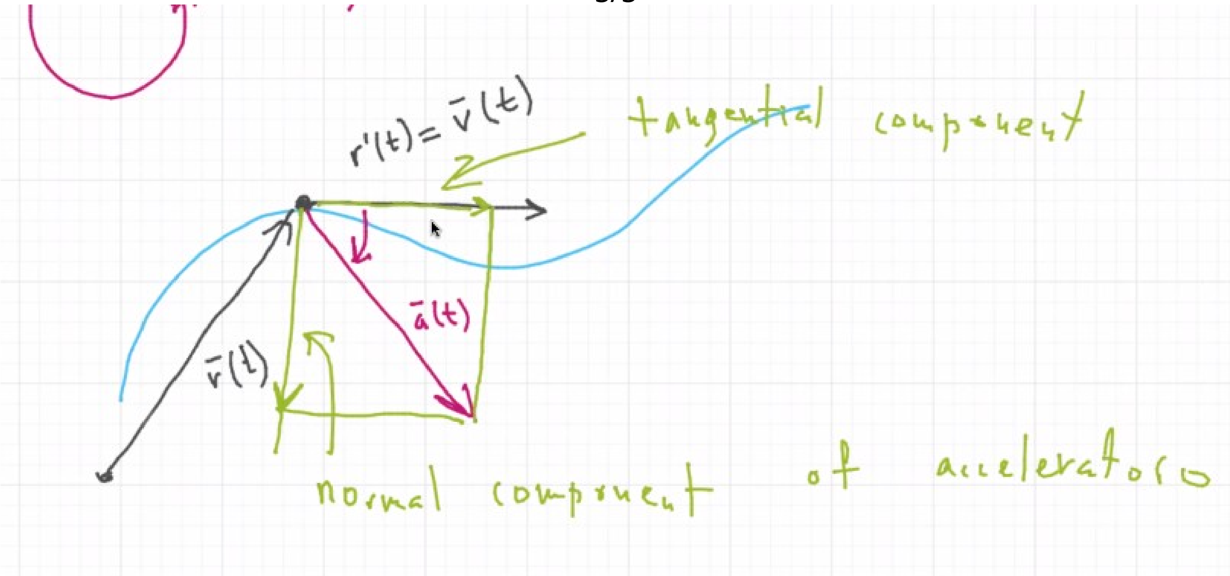
$$\vec{v}(t) = \int a(t) = \vec{g}t + v_0$$

$$\vec{r}(t) = \int v(t) = \frac{1}{2}\vec{g}t^2 + v_0 t + p_0$$

$$\vec{g} = -9.8\hat{j}$$

## Components of Acceleration

There are two components to the acceleration, one that determines change in speed, and another which determines change in direction.




### Tangential - speed

It has the same direction as velocity, and is the component of acceleration with respect to the velocity (vector projection of acceleration on velocity)

### Normal - direction

It is perpendicular to velocity, and is the component of acceleration with respect to vector perpendicular to velocity

### Application

It seems to help define acceleration in another way:

$$v = \mid \vec{v}(t) \mid$$

$$\vec{N}(t) = \frac{T^{'}(t)}{\mid T^{'}(t) \mid}$$

$$\vec{a}(t) = \vec{v}^{'}(t) = v^{'}(t) * T + v(t) * T^{'}(t)$$

$$a(t) = v^{'}T + \kappa * v^{2}N$$