Components of Acceleration

Physics application

A vector equation has properties of the parameterization, such as velocity.

Velocity: $\vec{v}(t) = \vec{r}'(t)$
Speed: $v(t) = |\vec{v}(t)|$
Acceleration: $\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t)$

Application:

The path of a projectile:

$$Acceleration:$$

$$\vec{a}(t) = \vec{g}$$

$$\vec{v}(t) = \int a(t) = \vec{g}t + v_0$$

$$\vec{r}(t) = \int v(t) = \frac{1}{2}\vec{g}t^2 + v_0 t + p_0$$

$$\vec{g} = -9.8\hat{j}$$

Components of Acceleration

Acceleration is on the inside of the curve (in the concave area)

The acceleration had two components: One along the velocity, and one perpendicular to it.

The one along the velocity is the tangential component, and the perpendicular one is the normal component. Tangential determines the speed, normal determines the "curviness"

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|r'(t)|} = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{\vec{v}(t)}{v(t)}$$

$$\vec{v}(t) = v(t) * T(t)$$

$$\vec{a}(t) = \vec{v}'(t) = v'(t) * T + v(t) * T'(t)$$

Note: $T(t)$ is orthagonal to $T'(t)$. This is because $|T(t)|$ is constant (see Example 12.4.2, p. 816)

Let N be the unti vector parallel to T'(t):

$$\vec{N}(t) = \frac{T'(t)}{|T'(t)|}$$

$$T'(t) = \vec{N} * |T'(t)|$$

Since

$$v(t) = |v(t)|$$

$$K = \frac{|T'|}{|r'|} = \frac{|T'|}{v}$$

$$|T'| = \kappa * v$$

$$T' = |T'| * \vec{N} = \kappa * v * \vec{N}$$

$$a(t) = v'T + \kappa * v^2 N$$