More Differentiation

Chain Rule 2

if: $z = f(x, y)$, $x = g(s, t)$, $y = h(s, t)$

Question: what are $\frac{\delta z}{\delta s}$, $\frac{\delta z}{\delta t}$?
Answer:

$$\frac{\delta z}{\delta s} = \frac{\delta z}{\delta x} * \frac{\delta x}{\delta s} + \frac{\delta z}{\delta y} * \frac{\delta y}{\delta s}$$

▼ Click to show Example 3

Question 3

$z = e^x \sin(y)$, where $x = st^2$, $y = s^2t$. Find $\frac{\delta z}{\delta s}$

Solution

$$\frac{\delta z}{\delta s} = \frac{\delta z}{\delta x} * \frac{\delta x}{\delta s} + \frac{\delta z}{\delta y} * \frac{\delta y}{\delta s}$$

$$\frac{\delta z}{\delta s} = e^x \sin(y) * t^2 + e^x \cos(y) * 2st$$

This is not desirable because it depends on 4 variables, so plug st^2 and s^2t for x and y

$$\frac{\delta z}{\delta s} = e^{st^2} \sin(s^2t) * t^2 + e^{st^2} \cos(s^2t) * 2st$$

Implicit Differentiation

Implicit functions are not solve for one variable. This is useful because we can use functions that have not been solved or cannot be solved for y.

Example

Let y be given as a function depending on x implicitly by $F(x, y) = 0$. Find $\frac{dy}{dx}$.

If we denote $x = t$, $y = h(x) = h(t)$ Then, $g(t) = F(t, h(t)) = 0$ By Chain Rule 1,

$$\frac{dg}{dt} = \frac{\delta F}{\delta x} * \frac{dx}{dt} + \frac{\delta F}{\delta y} * \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{dy}{dx} = -\frac{\frac{\delta F}{\delta x}}{\frac{\delta F}{\delta y}} = -\frac{Fx}{Fy}$$

$$(Fy = 0)$$

▼ Click to show Example 8 (14.5)

Question

Given $x^3 + y^3 = 6xy$, Find $\frac{dy}{dx}$

Solution

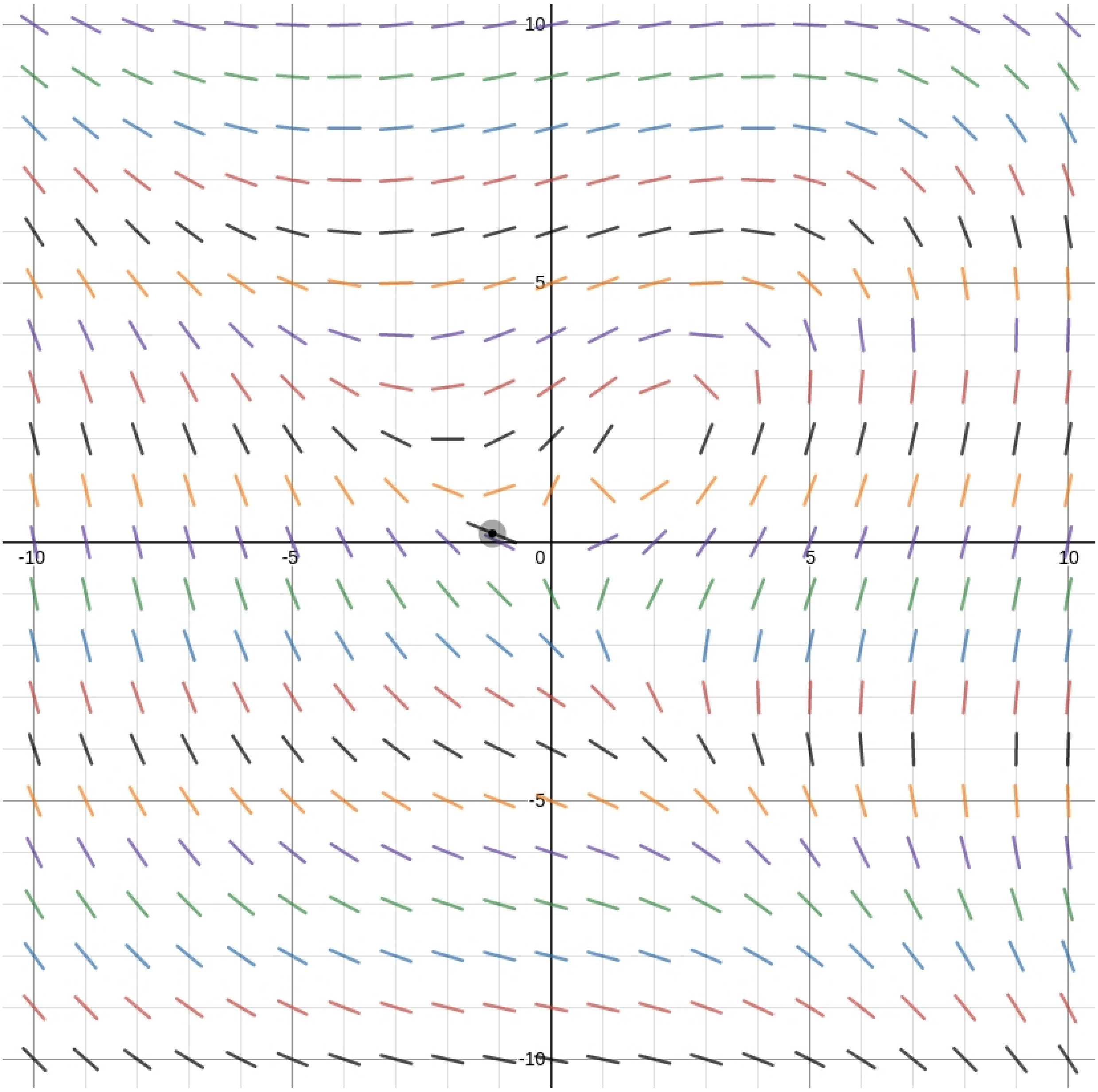
$$F(x, y) = 0 = x^3 + y^3 - 6xy$$

Therefore, from the above formula:

$$\frac{dy}{dx} - \frac{Fx}{Fy} = -\frac{3x^2 - 6y}{3y^2 - 6x} = -\frac{x^2 - 2y}{y^2 - 2x}$$

How it is used

Essentially, this is a representation of a slope field, related to differential equations.



Assume we want to find $\frac{dy}{dx}$ at the point$(0, 0)$

$$\frac{dy}{dx} = G(x, y) = -\frac{x^2 - 2y}{y^2 - 2x}$$

So, at this point, the derivative does not exist.

Find the value if $x = 1$, $1 + y^3 = 6y$.

There exists a formula to find y from this equation, so you can plug that in.

Directional Derivatives

Given a surface, we can define an arbitrary direction and find the derivative of the surface in that direction. That will represent the rate of change in the surface in that direction.

Definition

Given the function $z = f(x, y)$, and its partial derivatives $f_x(x_0, y_0, f_y(x_0, y_0)$. Let the directional vector $u = \langle a, b \rangle$ such that $\sqrt{a^2 + b^2} = 1$ (it has a length of 1)

$$D_u f(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + ha, y_0 + hb) - f(x_0, y_0)}{h}$$

This is showing the limit as a point displaced from (x_0, y_0) by h amount along the vector u, or $\langle a, b \rangle$,