**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans :** B. 0.2676

1-pnorm(50,45,8)= 0.265985529048701

Therefore the probability that the service manager cannot meet is commitment

will be 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans :** False

Around 70% of the data falls within one standard deviation of the mean

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans :** True

P(x≤30)=p(z≤(30-38)/6)=p(z≤-1.33)=0.0918 (using z table)

Expected count = 0.918\*400=36.72

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans :**  2 *X*1  one will be the greatest scale vision then *X*1  + *X*2 if X1 + X2 are normally distributed then the sum of the random sample will be exactly same

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans :** Given µ = 100

σ²=20

P(random variable i.e., X) = 0.99

From Z-table for 99% significance level, i.e., at 0.995, X = 2.58

Interval Estimate = Point Estimate ± Margin of Error

= Mean ± Z(1-α) σ/√n

= 100 ± 2.58(20)

= [48.4, 151.6] =D

qnorm(0.99,100,20)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans :** > qnorm(0.025,45\*5,3)

[1] 219.1201

> qnorm(0.975,45\*5,3)

[1] 230.8799

> qnorm(0.025,45\*7,3)

[1] 309.1201

> qnorm(0.975,45\*7,3)

[1] 320.8799

So the Rupee range with 95% probability for the annual profit of the company is given by,

=Profit1 + Profit2

= [219.12, 230.87] + [309.12, 320.87]

= [528.24, 551.74]

So Profit of the company in Rupees(in Million) is between range **[528.24, 551.74]**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans :** Specify the 5th percentile of profit (in Rupees) for the company

> qnorm(0.05,45\*7,3)

[1] 310.0654

> qnorm(0.05,45\*5,3)

[1] 220.0654

5th percentile of profit = 310.0654+ 220.0654 = **530.1308 Million Rs**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans :** Division 2 with distribution N(7, 42)