Mathematics Worksheet

Algebra: Solving Equations

This section focuses on solving linear and quadratic equations. Complete the following exercises and show all steps.

1. Solve the linear equation: $3x + 5 = 17$.
   - Subtract 5 from both sides: $3x = 12$.
   - Divide both sides by 3: $x = 4$.
2. Solve the quadratic equation: $x^2 - 5x + 6 = 0$.
   - Factorize the quadratic: $(x - 2)(x - 3) = 0$.
   - Set each factor to zero: $x - 2 = 0$ or $x - 3 = 0$.
   - Solutions: $x = 2$ or $x = 3$.
3. Solve for x: $\frac{2x+1}{3} = 5$.
   - Multiply both sides by 3: $2x + 1 = 15$.
   - Subtract 1: $2x = 14$.
   - Divide by 2: $x = 7$.

Geometry: Area and Perimeter

This section explores basic geometric calculations. Compute the following quantities.

1. Find the area and perimeter of a rectangle with length 8 cm and width 5 cm.
   - Area: length $\times$ width $= 8 \times 5 = 40\,\text{cm}^2$.
   - Perimeter: $2(\text{length} + \text{width}) = 2(8 + 5) = 26\,\text{cm}$.
2. Calculate the area of a circle with radius 4 cm. Use $\pi \approx 3.14$.
   - Area: $\pi r^2 = 3.14 \times 4^2 = 3.14 \times 16 = 50.24\,\text{cm}^2$.
3. Determine the volume of a cylinder with radius 3 cm and height 10 cm.
   - Volume: $\pi r^2 h = 3.14 \times 3^2 \times 10 = 3.14 \times 9 \times 10 = 282.6\,\text{cm}^3$.

Calculus: Differentiation

This section introduces basic differentiation. Find the derivatives of the following functions.

1. Differentiate $f(x) = 3x^2 + 2x + 1$.
   - Apply the power rule: $f'(x) = \frac{d}{dx}(3x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(1)$.
   - Result: $f'(x) = 6x + 2$.
2. Find the derivative of $g(x) = \sin(x) + e^x$.
   - Use standard derivatives: $\frac{d}{dx}(\sin(x)) = \cos(x)$, $\frac{d}{dx}(e^x) = e^x$.
   - Result: $g'(x) = \cos(x) + e^x$.
3. Compute the derivative of $h(x) = \frac{1}{x^2}$.
   - Rewrite as $h(x) = x^{-2}$.
   - Apply the power rule: $h'(x) = -2x^{-3} = -\frac{2}{x^3}$.