AMDHAUS LAW

$$S = \frac{1}{q} \qquad T(1) = \sigma + \pi \qquad T(N) = \sigma + \frac{\pi}{N}$$

$$q = \frac{\sigma}{\sigma + \pi} \qquad 1 - q = \frac{\pi}{\sigma + \pi}$$

(SUB IN FOR T(1) AND T(N))

$$S = \frac{T(1)}{T(N)} = \frac{\sigma + \pi}{\sigma + \frac{\pi}{N}}$$

(DIVIDE TOP AND BOTTOM BY σ)

$$S = \frac{1 + \frac{\pi}{\sigma}}{1 + \frac{\pi}{N\sigma}}$$

(SUB $\frac{\pi}{\sigma}$ FOR $\frac{1-q}{q}$)

$$S = \frac{1 + \frac{1-q}{q}}{1 + \frac{1-q}{Nq}}$$

(MULTIPLY TOP AND BOTTOM BY q)

$$S = \frac{\cancel{q} + 1 - \cancel{q}}{q + \frac{1-q}{N}}$$

(SIMPLIFY)

$$S = \frac{1}{q + \frac{1-q}{N}}$$

(USE LIMITS)

$$\lim_{N \to \infty} S = \frac{1}{q + \frac{1-q}{N}} = \frac{1}{q}$$

$\therefore$ PROVED AMDHAUS LAW

(DEFINE $\frac{1-q}{q}$)

$$\frac{1-q}{q} = \frac{\frac{\pi}{\sigma + \pi}}{\frac{\sigma}{\sigma + \pi}}$$

(FLIP BOTTOM FRACTION AND MULTIPLY)

$$\frac{1-q}{q} = \frac{\pi}{\cancel{\sigma + \pi}} \cdot \frac{\cancel{\sigma + \pi}}{\sigma}$$

(CANCEL OUT)

$$\frac{1-q}{q} = \frac{\pi}{\sigma}$$

GUSTAFFSON'S LAW

$$S(N) = \frac{T(1)}{T(N)}$$

$\cancel{T(1)} \cancel{= \sigma + N\pi}$

$$T(1) = \sigma + \cancel{\ } N\pi \qquad T(N) = \sigma + \pi$$

$$S = N - a(N-1)$$

$$a = \frac{\sigma}{\sigma + \pi} \qquad 1-a = \frac{\pi}{\sigma + \pi}$$

(SUB IN FOR T(1) AND T(N))

$$S\cancel{(N)} = \frac{T(1)}{T(N)} = \frac{\sigma + N\pi}{\sigma + \pi}$$

(SPLIT FRACTION IN TWO)

$$S = \cancel{\sigma} \frac{\sigma}{\sigma + \pi} + \frac{N\pi}{\sigma + \pi}$$

(SUB IN a AND $1-a$)

$$S = a + N(1-a)$$

(MULTIPLY IN N)

$$S = a + N - Na$$

(REARRANGE)

$$S = N - Na + a$$

(FACTORISE $-a$)

$$S = N - a(N-1)$$

$\therefore$ PROVED GUSTAFFSONS LAW