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Amdhal's law proof

- define the meaning of a few terms first
  - S is speedup
  - T(1) is the time taken for the serial (1 process/thread) solution
  - T(N) is the time taken for the parallel solution
  - α is the proportion of code that is serial
  - $1 - \alpha$ is the proportion of code that is parallel

Amdhal's law proof

- Define $S = \frac{1}{\alpha}$
- Define $T(1) = \sigma + \pi$
- Define $T(N) = \sigma + \frac{\pi}{N}$
- Define $\alpha = \frac{\sigma}{\sigma+\pi}$
- Define $1 - \alpha = \frac{\pi}{\sigma+\pi}$

Amdhal's law proof

- First speedup is defined as the following
- $S(N) = \frac{T(1)}{T(N)}$
- Substitute in for T(1) and T(N) using definitions on previous slide
- $S(N) = \frac{\sigma+\pi}{\sigma+\frac{\pi}{N}}$

Amdhal's law proof

- Divide top and bottom by σ
- $S(N) = \frac{\frac{\sigma}{\sigma}+\frac{\pi}{\sigma}}{\frac{\sigma}{\sigma}+\frac{\pi}{N\sigma}}$
- When we clean it up we get the following
- $S(N) = \frac{1+\frac{\pi}{\sigma}}{1+\frac{\pi}{N\sigma}}$
- however we really don't like the σ and π in there but want α instead so lets define a formula to see if we can find a way to substitue them

Amdhal's law proof

- $\frac{1-\alpha}{\alpha} = \frac{\frac{\pi}{\sigma+\pi}}{\frac{\sigma}{\sigma+\pi}}$
- To divide a fraction flip it and multiply
- $\frac{1-\alpha}{\alpha} = \frac{\pi}{\sigma+\pi}\frac{\sigma+\pi}{\sigma}$
- the $\sigma + \pi$ cancel out and leave us with
- $\frac{1-\alpha}{\alpha} = \frac{\pi}{\sigma}$

Amdhal's law proof

- So now we can swap out the $\frac{\pi}{\sigma}$ for $\frac{1-\alpha}{\alpha}$
- $S(N) = \frac{1+\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{N\alpha}}$
- multiply top and bottom by α to remove the α on the bottom of the fractions
- $S(N) = \frac{\alpha+1-\alpha}{\alpha+\frac{1-\alpha}{N}}$

Amdhal's law proof

- The α and $-\alpha$ on the top line cancel out
- $S(N) = \frac{1}{\alpha + \frac{1-\alpha}{N}}$
- Nearly there but that last fraction on the bottom right is a touch annoying. However if we introduce limits we can deal with it
- $\lim_{N\to\infty} S(N) = \frac{1}{\alpha + \frac{1-\alpha}{N}} = \frac{1}{\alpha}$

Gustaffson's law proof

- Define $S = \frac{1}{\alpha}$
- Define $T(1) = \sigma + N\pi$
- Define $T(N) = \sigma + \pi$
- Define $\alpha = \frac{\sigma}{\sigma+\pi}$
- Define $1 - \alpha = \frac{\pi}{\sigma+\pi}$

Gustaffson's law proof

- First speedup is defined as the following
- $S(N) = \frac{T(1)}{T(N)}$
- Substitute in for T(1) and T(N) using definitions on previous slide
- $S(N) = \frac{\sigma + N\pi}{\sigma + \pi}$
- Split out the fraction into its two parts
- $S(N) = \frac{\sigma}{\sigma + \pi} + \frac{N\pi}{\sigma + \pi}$
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Gustaffson's law proof

- The first part of the fraction can be substituted out for α and the second part can mostly be substituted out for $1 - \alpha$
- $S(N) = \alpha + N(1 - \alpha)$
- Multiply out the brackets
- $S(N) = \alpha + N - N\alpha$

Gustaffson's law proof

- Reorder the terms
- $S(N) = N - N\alpha + \alpha$
- factor out $-\alpha$
- $S(N) = N - \alpha(N - 1)$