

**What is a time complexity ?**

" how long an algorithm takes to run based on the size of the input."

It does not measure time in seconds , because seconds can be change from , Computer to computer ,internet speed, processor speed ,  
Instead we can measure time complexity in **number of operation**

**How do we measure ?**

we use **Big- O** notation like:

- $O(1)$  -> constant time
- $O(n)$  -> linear time
- $O(\log n)$  -> logarithmic times
- $O(n^2)$  -> Quadratic time
- $O(n^3)$  -> Cubic
- $O(2^n)$  -> exponential
- $O(n!)$  -> Factorial

Time complexity tell us how fast or slow an algorithm grows when the input size is increase

We do not count actual seconds

We count how many steps algorithm performs

Big O notation it can also shows worst case number of steps

12	8	20	9	15
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--	--	--	--	--	--	--	--	--	--	--

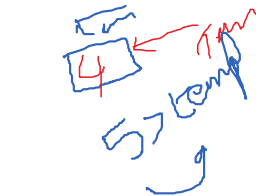
**Insertion Sort (**

**n=6**

A n=6

5	4	10	1	6	2
---	---	----	---	---	---

Sorted sublist    unsorted    sublist-->



5	4	10	1	6	2
---	---	----	---	---	---

4	5	10	1	6	2
---	---	----	---	---	---

4	5	10	1	6	2
---	---	----	---	---	---

4	5	?	10	6	2
---	---	---	----	---	---

4	?	5	10	6	2
---	---	---	----	---	---

1	4	5	10	6	2
---	---	---	----	---	---

1	4	5	6	10	2
---	---	---	---	----	---

1	4	5	6	?	10
---	---	---	---	---	----

1	4	5	?	6	10
---	---	---	---	---	----

1	4	?	5	6	10
---	---	---	---	---	----

1	?	4	5	6	10
---	---	---	---	---	----

1	2	4	5	6	10
---	---	---	---	---	----

Time complexity :

Worst case:- list is given descending order wanted to make it as ascending order

$O(n^2)$

Best case is  $O(n)$

2 3 5 6 8 9

**Algorithm:**

Step 1: start

Step 2: divide sorted and unsorted list

Step 3: assume first element is sorted

Step 4: pick the next element (key)

Step 5: compare it with sorted element previous to the unsorted list

Step 6: shift elements greater than key one position to the right side

Step 7: insert the key in the correct empty position

Step 8: repeat until array is fully sorted

Step 9: stop

A=[15,16,6,8,5]

n=4

For l in range(n-1):

{

    Flag=0;

    For j in range(n-i-1):

        {

            If A[j]>A[j+1]:

                {

                    A[j],a[j+1]=A[j+1],a[j]

                    Flag=1

                }

        }

    If(flag==0)

        Break;

}

N=?

- Pass 1 --> (n-1) comparison

- Pass 2 --> (n-2) comparisons

• .

• .

• .

• .

• .

- Pass n-->1

- Total comparisons

- $(n-1)+(n-2)+\dots+1$

$n(n-1)/2$

$\dots n^2/2$

Time complexity of bubble sort

$O(n^2)$  --> without optimization

$O(n)$  --> with optimization

for (i=1,i<n,i++)

{

    Temp= A[i]

    j=i-1;

    While(j>=0 && A[j]>temp)

    {

        A[j+1]=A[j],

        j--;

    }

    A[j+1]=temp

}

## Searching Algorithms:

Linear search and binary search

### Linear Search :



n=8

1 If element found then return the location  
2 if element is not found

Searching element  
Data=42

```

Int flag=0;
For(i=0;i<n;i++)
{
    If(A[i]==Data)
    {
        Print the element found at ,i(index)
        Flag=1;
        Break;
    }
}

if (flag==0)
{
    print element not found
}
    
```

### Time complexity:

Best case:  $O(1)$

Worst case is  $O(n)$

Average  $= 1+2+3+\dots+n/n$

$$\frac{n(n+1)}{2n} = O\left(\frac{n+1}{2}\right)$$

### Algorithm:

Step1 :start

Step2:read the Array

Step3: find the n value

Step4:read key, search data

Step5:flag=0,i=0

Step 6: i is value varies from 0 to n-1. for each value of i do step7

Step 7: check  $A[i]$  is equal to key. If yes set flag=1 and print element found ,otherwise goto step6;

Step 8: check flag=0 if yes data not found

Step9: stop

i=0 ;0<8;i++  
A[0]==Data  
15==42 f

i=1 ;1<8; i++  
A[1]==data  
5==42 f

i=2;2<8; i++  
A[2]==data  
20==42 f

i=3;3<8;i++  
a[3]==data  
35==42 fa

i=4;4<8;i++  
A[4]==42;  
2==42 f

i=5;5<8;i++  
A[5]==data  
42==42  
Element found at 5

i=6;6<8;i++  
A[6]==data  
67==42 f

i=7;7<8;i++  
A[7]==data  
17==42 f

i=8;8<8;i++  
False ended the loop

### Binary search:

- Binary search is a fast searching algorithm
- It follows divide and conquer method
- The data is in **sorted** format only (ASC ,DSC)
- Binary search looks for particular search data by comparing middle most item of the collection

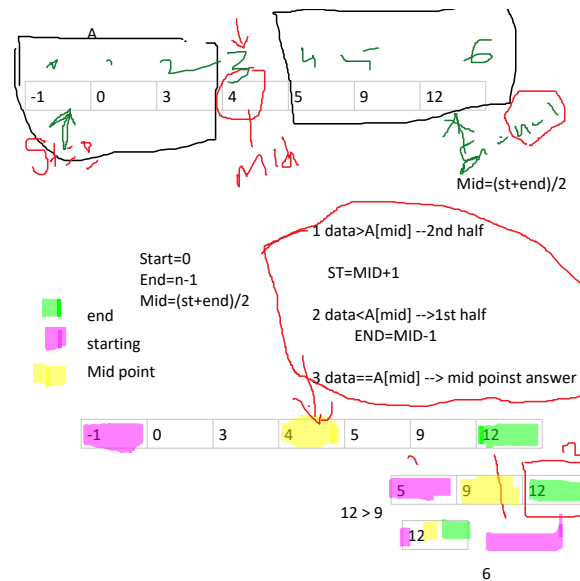
A,b,c,d,....z

Dog=  
P

Data=12



- The data is in **sorted** format only (ASC ,DSC)
- Binary search looks for particular search data by comparing middle most item of the collection
- If match occurs , index of item is return
- If the middle item is greater than the search data , then item is searched in the sub-array to the left of the middle item .
- Otherwise ,item is searched for the sub-array to the right side of the middle item.
- His process is continues on the sub-array as well until the size of the subarray reduces to zero



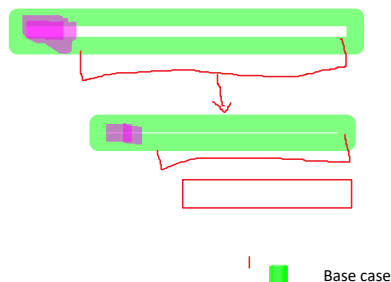
```

While (start <= end)
{
    Mid=(start + end)/2
    If(data > A[mid])
        Start=mid+1
    Else if( data < A[mid] )
        end=mid-1
    Else
        Return mid
}
Return -1

```

### Recursion

Recursion is where a function can call itself to solve a smaller version of the same problem



Example: 4  
N to 1

4	4	3	2	1	n=4
3	3	2	1		n=3
2	2	1			n=2
1	1				n=1

We have 2 essential parts of the recursion

1. Base case: stops the recursion --> prevent infinite loop
2. Recursive case : function calling itself with smaller input

**Factorial for recursion :**

$$n! = n * (n-1) * (n-2) * \dots * 1$$

$$\begin{aligned}
 f(n=4) &= 4 * f(n=3) \\
 &= 4 * (3 * f(n=2)) \\
 &= 4 * (3 * (2 * f(n=1))) \\
 &= 4 * (3 * (2 * (1 * f(n=0)))) \\
 &= 4 * (3 * (2 * 1)) \\
 &= 4 * 6 \\
 &= 24
 \end{aligned}$$

```

def fact(n)
    If n==0;
        Return 1 //base case
    Return n * fact(n-1) //recursive case

```

Caluculate fact(4) n=4

$$\begin{aligned}
 &= 4 * \text{fact}(3) \\
 &= 4 * (3 * \text{fact}(2)) \\
 &= 4 * (3 * (2 * \text{fact}(1)))
 \end{aligned}$$

```

= 4 * (3 * (2 * (1 * fact(0)))
= 4 * 3 * 2 * 1 * 1
=24

```

```

Fact(4)
  fact(3)
    Fact(2)
      fact(1)
        Fact(0)

```

## Fibonacci

0,1,1,2,3,5,8,....

```

0,1
1+0=1
1+ 1=2
2+1=3
3+2=5
5+3=8 .....n

```

## Formula=

**f(n) =f(n-1)+f(n-2)**

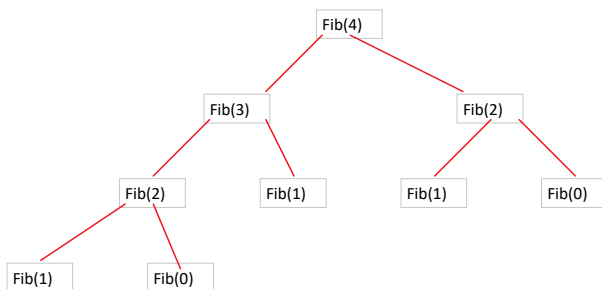
```

def fib(n)
  If n <= 1;
    Return n //base case

  Return fib(n-1) + fib(n-2)

```

Calculate fib(4)



Fib(1)=1  
Fib(0)=0

Fib(2) =1+0 =1  
Fib(3) = fib(2) + fib(1) =1+1=2  
Fib(4) =fib(3)+fib(2)=2+1 =3

Recursion is best for .

ds tree (binary tree traversal)  
Backtracking (sudoku,maze)  
Divide and conquer (sorting)  
Mathematica functions

## Time complexity

## Space complexity

## Recursive complexity (fib,factorial)

### Time complexity:

How fast the code run when input grows

Example

For l in range (n) --> O(n)

For l in range(n \* n) --> O(n^2)

Binary search --> O (log n)

Def print\_n(n):

For l in range(n):

Print(i) //O(n)

### Space complexity :

Space complexity tells how much extra memory your program uses

n=0
n=1
n=2
n=3
n=4

Sc=n \*k

Sc=O(n)

$x = 10 \rightarrow O(1)$  space

$Arr = [1] * n$  //  $O(n)$  space

### **Recursive complexity (fib, factorial)**

Time complexity  $\rightarrow O(n)$ , space  $O(n)$  --fact

Fib --  $O(2^n)$ ,  $O(n)$

**Time complexity  $\rightarrow$  "how fast"**

**Space complexity  $\rightarrow$  "How much extra memory"**

**Recursive complexity  $\rightarrow$  "how many recursive calls ?"**