

What is a time complexity ?

" how long an algorithm takes to run based on the size of the input."

It does not measure time in seconds, because seconds can be change from , Computer to computer ,internet speed, processor speed , Instead we can measure time complexity in **number of operation**

How do we measure ?

we use **Big - O** notation like:

- $O(1)$ -> constant time
- $O(n)$ ->linear time
- $O(\log n)$ -->logarithmic times
- $O(n^2)$ --> Quadratic time
- $O(n^3)$ -->Cubic
- $O(2^n)$ --> exponential
- $O(n!)$ --> Factorial

Time complexity tell us how fast or slow an algorithm grows when the input size is increase

We do not count actual seconds

We count how many steps algorithm performs

Big O notation it can also shows worst case number of steps

12	8	20	9	15	
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--	--	--	--	--	--	--	--

$$A=[15,16,6,8,5]$$

$$n=4$$

For I in range($n-1$):

{

Flag=0;

For j in range($n-i-1$):

{

If $A[j]>A[j+1]$:

{

$A[j],a[j+1]=A[j+1],a[j]$

Flag=1

}

}

If(flag==0)

Break;

}

$$N=?$$

• Pass 1 -->(n-1) comparison

• Pass 2 -->(n-2) comparisons

• .

• .

• .

• Pass n-->1

• Total comparisons

• $(n-1)+(n-2)+\dots+1$

$n(n-1)/2$

$\dots n/2$

Time complexity of bubble sort

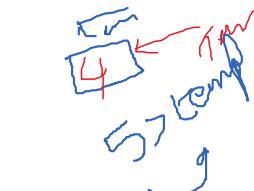
$O(n^2)$ --- without optimization

$O(n)$ --> with optimization

Insertion Sort (

$n=6$	$A[6]=5$	5	4	10	1	6	2
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Sorted sublist unsorted sublist-->



Time complexity :

Worst case:- list is given descending order wanted to make it as ascending order

$O(n^2)$

Best case is $O(n)$

2 3 5 6 8 9

5	4	10	1	6	2
5	4	10	1	6	2
4	5	10	1	6	2
4	5	10	1	6	2

4	5	?	10	6	2
---	---	---	----	---	---

4	?	5	10	6	2
---	---	---	----	---	---

1	4	5	10	6	2
---	---	---	----	---	---

1	4	5	6	10	2
---	---	---	---	----	---

1	4	5	6	?	10
---	---	---	---	---	----

2 ↘

1	4	?	5	6	10
---	---	---	---	---	----

1 ↗

1	?	4	5	6	10
---	---	---	---	---	----

1	2	4	5	6	10
---	---	---	---	---	----

```

for (i=1,i<n,i++)
{
    Temp= A[i]
    j=i-1;
    While(j>=0 && A[J]>temp)
    {
        A[j+1]=A[j],
        j--;
    }
    A[j+1]=temp
}
  
```

Algorithm:

Step 1: start

Step 2: divide sorted and unsorted list

Step 3: assume first element is sorted

Step 4:pick the next element (key)

Step 5: compare it with sorted element previous to the unsorted list

Step 6: shift elements greater than key one position to the right side

Step 7:insert the key in the correct empty position

Step 8: repeat until array is fully sorted

Step 9: stop

Searching Algorithms:
Linear search and binary search



Linear Search :



n=8

Searching element
Data=42

i=0 ;0<8;i++
A[0]==Data
15==42 f

1 If element found then return the location
2 if element is not found

```
Int flag=0;
For(i=0;i<n;i++)
{
    If(A[i]==Data)
    {
        Print the element found at ,i(index)
        Flag=1;
        Break;
    }
}
if (flag==0)
{
    print element not found
}
```

i=1 ;1<8; i++
A[1]==data
5==42 f

i=2;2<8; i++
A[2]==data
20==42 f

i=3;3<8; i++
a[3]==data
35==42 fa

i=4;4<8; i++
A[4]==42;
2==42 f

i=5;5<8; i++
A[5]==data
42==42
Element found at 5

i=6;6<8; i++
A[6]==data
67==42 f

i=7;7<8; i++
A[7]==data
17==42 f

i=8;8<8; i++
False ended the loop



Algorithm:

Step1 :start

Step2:read the Array

Step3: find the n value

Step4:read key, search data

Step5:flag=0,i=0

Step 6:I is value varies from 0 to n-1. for each value of I do step7

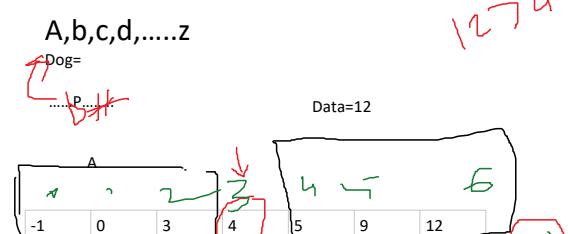
Step 7: check A[i] is equal to key. If yes set flag=1 and print element found ,otherwise goto step6;

Step 8: check flag=0 if yes data not found

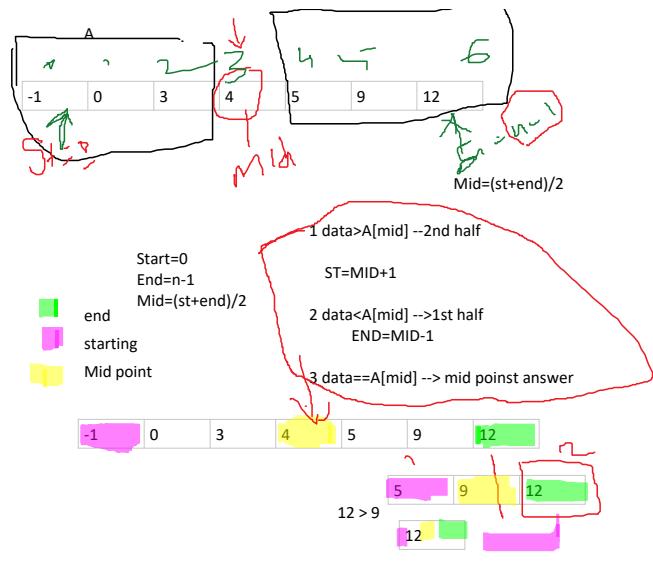
Step9: stop

Binary search:

- Binary search is a fast searching algorithm
- It follows divide and conquer method
- The data is in **sorted** format only (ASC ,DSC)
- Binary search looks for particular search data by comparing middle most item of the collection



- The data is in **sorted** format only (ASC ,DSC)
- Binary search looks for particular search data by comparing middle most item of the collection
- If match occurs , index of item is return
- If the middle item is greater than the search data , then item is searched in the sub-array to the left of the middle item .
- Otherwise ,item is searched for the sub-array to the right side of the middle item.
- This process is continues on the sub-array as well until the size of the subarray reduces to zero



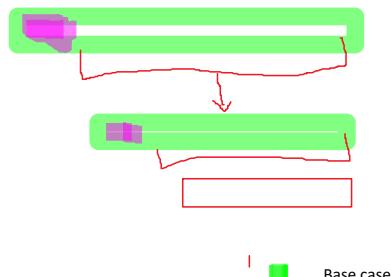
```

While (start<=end)
{
    Mid=(start + end)/2
    If(data > A[mid])
        Start=mid+1
    Else if( data < A[mid] )
        end=mid-1
    Else
        Return mid
}
Return -1

```

Recursion

Recursion is where a function can call itself to solve a smaller version of the same problem



Example: 4
N to 1

4	4	3	2	1	n=4
3	3	2	1	n=3	
2	2	1		n=2	
1	1			n=1	

We have 2 essential parts of the recursion

1. Base case: stops the recursion --> prevent infinite loop
2. Recursive case : function calling itself with smaller input

Factorial for recursion :

$$n!=n * (n-1) * (n-2) * \dots * 1$$

$$\begin{aligned}
f(n=4) &= 4 * f(n=3) \\
&3 * f(n=2) \\
&2 * f(n=1) \\
&1 * f(n=0)=1 \text{ ans } 1
\end{aligned}$$

```

def fact(n)
    if n==0;
        Return 1 //base case
    Return n * fact(n-1) //recursive case

```

$$\begin{aligned}
\text{Caluculate fact}(4) \text{ n=4} \\
&= 4 * \text{fact}(3) \\
&= 4 * (3 * \text{fact}(2)) \\
&= 4 * (3 * (2 * \text{fact}(1)))
\end{aligned}$$

```

= 4 * (3 * (2 * (1 * fact(0)))
= 4 * 3 * 2 * 1 * 1
= 24

```

```

Fact(4)
fact(3)
Fact(2)
fact(1)
Fact(0)

```

Fibonacci

0,1,1,2,3,5,8,....

```

0,1
1+0=1
1+ 1=2
2+1=3
3+2=5
5+3=8 .....n

```

Formula=

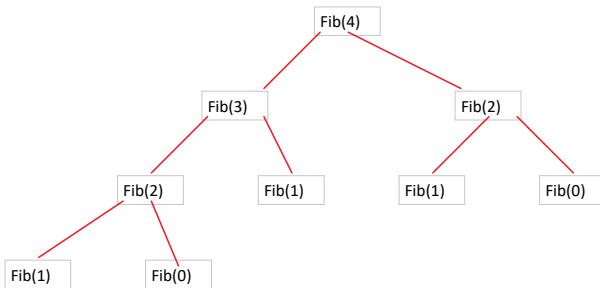
$$f(n) = f(n-1) + f(n-2)$$

```

def fib(n)
    if n <= 1;
        Return n //base case
    Return fib(n-1) + fib(n-2)

```

Calculate fib(4)



Fib(1)=1
Fib(0)=0

$$\begin{aligned} \text{Fib}(2) &= 1+0 = 1 \\ \text{Fib}(3) &= \text{fib}(2) + \text{fib}(1) = 1+1 = 2 \\ \text{Fib}(4) &= \text{fib}(3) + \text{fib}(2) = 2+1 = 3 \end{aligned}$$

Recursion is best for .

ds tree (binary tree traversal)
Backtracking (sudoku,maze)
Divide and conquer (sorting)
Mathematica functions

Time complexity

Space complexity

Recursive complexity (fib,factorial)

Time complexity:

How fast the code run when input grows

Example

For I in range (n) --> O(n)

For I in range(n * n) --> O(n^2)

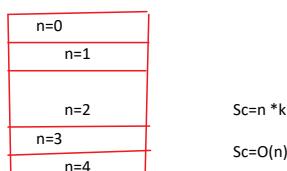
Binary search --> O (log n)

Def print_n(n):

For I in range(n):
Print(i) //O(n)

Space complexity :

Space complexity tells how much extra memory your program uses



x= 10 --> O(1) space

Arr= [1] * n // O(n) space

Recursive complexity (fib,factorial)

Time oplmexity --> O(n) ,space O(n) --fact

Fib-- O(2^n) , O(n)

Time complexity --> "how fast"

Space complexity --> "How much extra memory"

Recursive complexity --> "how many recursive calls ?"