(8) When processing flower nectar into honeybees extract, a considerable amount of water is added. How much flower nectar must be processed to yield 1 kg of honey, if nectar contains 70% water, and the honey obtained from this nectar contains 17% water?

(a) 2.77 kg (b) 1.54 kg (c) 4.11 kg (d) 3.5 kg (e) None

Hint: You need not apply any rule of alligation, but only equate the pure parts of nectar and that of honey because water will not yield honey during process

(9) A mixture contains brandy and water in the ratio 8 : x. When 33 litres of the mixture and 3 litres of water are mixed, the ratio of brandy and water becomes 2 : 1. The value of x is:

(a) 5 (b) 3 (c) 11 (d) 7 (e) 9

(10) Vessels A and B contains 4 litres and 6 litres of two different samples of acid. If the contents of two vessels are mixed, the result is 35% acid concentration. If one litre each from A and B are mixed, the result is 36% acid concentration. The amount of pure acid in A and B (in litres) are:

(a) 2.5, 3.6 (b) 3.5, 3.6 (c) 1.4, 2.16 (d) 3 , 2.2 (e) 1.64, 1.86

Hint: Use $\dfrac{\text{acid quantity}}{\text{mixture quantity}}$

Direction (11-12): Refer to the following table and answer the questions 11 and 12.

Relative Sweetness of Different Substances

Lactose	0.16	Maltose	0.32
Glucose	0.74	Sucrose	1.00
Saccharin	675.00		

(11) How many grams of sucrose must be added to one gram of saccharin to make a mixture that will be 100 times as sweet as glucose?

(a) 11.6 (b) 10 (c) 8.2 (d) 7.5 (e) 13.7

Hint: Let x = no. of grams of sucrose to be added. Use formula (4) for mean sweetness.

(12) What is the ratio of glucose to lactose in a mixture as sweet as maltose?

(a) 8 : 21 (b) 1 : 3 (c) 3 : 2 (d) 16 : 9 (e) 13 : 8

(13) A dishonest milkman fills up his bucket, which is only $\frac{4}{5}$th full of milk, with water. He again removes 5 litres of this mixture from the bucket and adds an equal quantity of water. If milk is now 60% of the mixture, the capacity of the bucket (in litres) is:

(a) 12 (b) 20 (c) 25 (d) 18 (e) 24

Hint: Assume x = capacity of bucket (= amount of mixture initially)

(14) There are two alloys of silver and copper. In the first alloy, there is 3 times as much copper as silver and in the second alloy there is one fourth as much copper as silver. How many times more must we take the first alloy in relation to the second alloy in order to obtain a new alloy in which there would be twice as much copper as silver?

(a) 11 (b) 1.5 (c) 5.6 (d) 3.5 (e) 7.2

(15) Two gallons of a mixture of spirit and water contains 12% of water. They are added to 3 gallons of another mixture, containing 7% of water and half a gallon of water is then added to the whole. The percentage of water in the resulting mixture is:

(a) $5\frac{1}{7}$ (b) 9 (c) $29\frac{1}{3}$ (d) $17\frac{3}{11}$ (e) $21\frac{3}{4}$

(16) An alloy contains copper and manganeese in the ratio of 8 : 5 and another alloy contains copper and tungsten in the ratio of 5 : 3. If equal weights of the two are melted together to form the third alloy, then the weight of manganeese per kg in the new alloy is:

(a) $\frac{79}{208}$ kg (b) $\frac{13}{5}$ kg (c) $\frac{5}{13}$ kg (d) $\frac{5}{26}$ kg (e) $\frac{8}{13}$ kg

(17) A cup of milk contains 3 parts of pure milk and 1 part of water. How much mixture must be withdrawn and water substituted in order that the resulting mixture may be half milk and half water?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$ (e) $\frac{1}{2}$

Answers

1. (b)	**2.** (d)	**3.** (e)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (b)
10. (e)	**11.** (c)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (a)	

10 CHAIN RULE

10.1 DEFINITION

When quantities of different kinds are connected to one another so that we know how much of one quantity is equivalent to a given quantity of a second, etc. we can determine how much of the last kind is equivalent to a given quantity of first kind by the *chain rule*.

10.2 DIRECT PROPORTION AND INDIRECT PROPORTION

If increase or decrease of a quantity Q_1 causes increase or decrease of another quantity Q_2 in the same extent, then, Q_1 is directly proportional to $Q_2 \Rightarrow Q_1 \propto Q_2$

I. Number of persons $\propto$ Amount of work done, i.e. more persons, more work
II. Number of days $\propto$ Amount of work, i.e. more days, more work
III. Working rate $\propto$ Amount of work, i.e. more working rate, more work
IV. Efficiency of man $\propto$ Amount of work, i.e. more efficiency of man, more work

Combining I, II, III, and IV, (Man × days × Workrate × Efficiency) $\propto$ Amount of work. If increase of a quantity Q_1 causes decrease of a quantity Q_2, in the same extent, then,

Q_1 is indirectly proportional to $Q_2 \Rightarrow Q_1 \propto \frac{1}{Q_2}$

V. Number of men $\propto \frac{1}{\text{No. of days}}$, i.e. more the men, less the no. of days required

10.2.1 Important Formula

(a) $$\frac{\text{Man}_1 \times \text{Days}_1 \times \text{Work rate}_1}{\text{Amount of work done}_1} = \frac{\text{Man}_2 \times \text{Days}_2 \times \text{Work rate}_2}{\text{Amount of work done}_2}$$

Remember, *"Man days" required per unit work is always same*. In fact, Man × Days specify the volume of job or work.

(b) If in place of men there are engines burning coal for certain number of hours, then, the above equation changes to

$$\frac{\text{No. of Engine}_1 \times \text{Hours}_1 \times \text{Consumption Rate}_1}{\text{Amount of coal burnt}_1} = \frac{\text{Engine}_2 \times \text{Hours}_2 \times \text{Consumption Rate}_2}{\text{Amount of coal burnt}_2}$$

because, here the job of engine is to burn the coal.

(c) If number of examiners examining a number of answer books in a number of days by working a number of hours or day, since the job of examiner is to check the answer books, then,

$$\frac{\text{No. of examiner}_1 \times \text{Days}_1 \times \text{Work rate}_1}{\text{No. of answer books checked}_1} = \frac{\text{Examiner}_2 \times \text{Days}_2 \times \text{Work rate}_2}{\text{No. of answer books checked}_2}$$

$\therefore$ From, (a), (b), and (c), it is concluded briefly, that,

$$\boxed{\frac{N_1 \times D_1 \times R_1 \times E_1}{W_1} = \frac{N_2 \times D_2 \times R_2 \times E_2}{W_2}}^{\dagger} \qquad \textbf{10.1}$$

Where, N_1, N_2 = number of workers; D_1, D_2 = time of work
R_1, R_2 = work rate of worker or machine; E_1, E_2^* = efficiency of worker or machine
W_1, W_2 = Amount of work (of same nature) done

* Efficiency of worker or machine is taken to be unity (=1) if other wise, not specified in the problem.

† This formula can be derived by combining I, II, III, IV and V from 10.2.

Solved Examples

E-1 If 120 men can do a job in 100 days, in how many days will 150 men do it?

S-1 See, 10.2.1, $W_1 = W_2$

Since same amount of job is done.

$\therefore \quad N_1 \times D_1 = N_2 \times D_2$

$\Rightarrow \quad 120 \times 100 = 150 \times N_2$

$\Rightarrow \quad N_2 =$ **80 days.**

E-2 If 18 men working 5 hours a day for 8 days can complete a job, how many men working 8 hours a day for 6 days will be needed?

S-2 Using, $\dfrac{N_1 \times D_1 \times R_1}{W} = \dfrac{N_2 \times D_2 \times R_2}{W}$, where $N_1 = 18$, $D_1 = 8$, $R_1 = 5$

$N_2 = ?$, $D_2 = 6$, $R_2 = 8$

$\Rightarrow \quad 18 \times 8 \times 5 = N_2 \times 6 \times 8 \qquad \Rightarrow \quad N_2 =$ **15 men.**

E-3 One thousand men in a fortress have provisions for 12 days. How long will the provisions last if 200 more men join them?

S-3 Here amount of work, i.e. amount of provisions is same (Since $N_1D_1 = N_2D_2$)

$\therefore \quad 1000 \times 12 = (1000 + 200) \times D_2 \qquad \Rightarrow \quad D_2 =$ **10 days.**

E-4 If 4 men reap 40 acres in 30 days, how many acres will 18 men reap in 12 days?

S-4 $\dfrac{N_1 \times D_1}{W_1} = \dfrac{N_2 \times D_2}{W_2}$ where, $N_1 = 4$ $D_1 = 30$, $W_1 = 40$

$N_2 = 18$ $D_2 = 12$ $W_2 = ?$

$\Rightarrow \quad \dfrac{4 \times 30}{40} = \dfrac{18 \times 12}{W_2} \qquad \Rightarrow \quad W_2 =$ **72 acres.**

E-5 If 12 men can build a wall 100 metres long, 3 m high and 0.5 metre thick in 25 days, in how many days will 20 men build a wall 60 $m \times 4$ $m \times 0.25$ m.

S-5 Here the amount of job is the volume of the wall built.

Since $\dfrac{N_1 \; D_1}{W_1} = \dfrac{N_2 \; D_2}{W_2}$ [W_1, W_2 are the volume of wall]

$\Rightarrow \dfrac{12 \times 25}{100 \times 3 \times 0.5} = \dfrac{20 \times D_2}{60 \times 4 \times 0.25} \qquad \Rightarrow D_2 = \mathbf{6\ days.}$

E-6 Fifteen men take 21 days of 8 hours each to do a work. How many days of 6 hours each would 21 women take, if 3 women do as much work as 2 men?

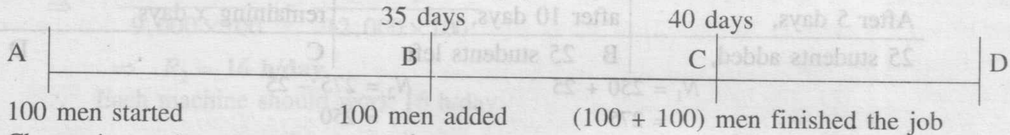
S-6 Here, nature of worker is different. In one case it is man and in other case it is woman. So, woman needs to be converted to man.

Since 3 women = 2 men. $\Rightarrow$ 21 women = 14 man.

$\therefore \quad 15 \times 21 \times 8 = 14 \times D_2 \times 6$ (Since $N_1 \times D_1 \times R_1 = N_2 \times D_2 \times R_2$)

$\Rightarrow D_2 = \mathbf{30\ days.}$

E-7 A contractor undertakes to do a piece of work in 40 days. He engages 100 men and after 35 days, he engaged an additional 100 men and completes the work. How many days behind the schedule would the work have been, if he had not engaged the additional men?



S-7 Change in number of men takes place at point *B*.

$\therefore$ Portion after point *B* is to be considered.

$100 \times D_1 = (100 + 100) \times 5 \Rightarrow D_1 = 10 \text{ days} = BD.$

$\therefore$ If 100 additional men were not engaged, 10 – 5 = **5 days** would the work have been behind the schedule. In diagram, *CD* represents the time behind the schedule.

E-8 A contract is to be completed in 56 days and 104 men were set to work, each working 8 hours a day. After 30 days, $\frac{2}{5}$ of the work is finished. How many additional men may be employed so that work may be completed on time, each man now working 9 hours per day? **(MBA, '87)**

S-8 $\dfrac{N_1 \times D_1 \times R_1}{W_1} = \dfrac{N_2 \times D_2 \times R_2}{W_2}$ *[Refer 10.1]*

$$\frac{104 \times 30 \times 8}{\frac{2}{5}} = \frac{(104 + x) \times \overbrace{(56 - 30)}^{\text{Remaining days}} \times 9}{\underbrace{\left(1 - \frac{2}{5}\right)}_{\text{Rest work to be done}}}$$

$$\Rightarrow \frac{104 \times 30 \times 8}{\frac{2}{5}} = \frac{(104 + x) \times 26 \times 9}{\frac{3}{5}}$$

$x = \mathbf{56\ men.}$

Hence 56 additional men should be employed.

E-9 Ten men begin to work together on a job, but after some days, 4 of them leave. As a result, the job which could have been completed in 40 days is completed in 50 days. How many days after the commencement of the work did the 4 men leave? **(Bank PO '81)**

S-9 Total job assigned = $10 \times 40 = 400$ men days.
Say, after x days of the commencement of work, 4 men leave.
$\therefore$ 4 men work for x days. $\therefore$ $M_1D_1 = M_2D_2 + M_3 \times D_3$.
$10 \times 40 = 4 \times x + 6 \times 50 \Rightarrow x = \mathbf{25}$. (Since 6 men work throughout 50 days)
They left 25 days after the commencement of work.

E-10 Four lorries carrying 4 tons each move 128 tons in 8 days. In how many days will 6 lorries carrying 3 tons each move 540 tons?

S-10 $\dfrac{N_1 \times D_1 \times R_1}{W_1} = \dfrac{N_2 \times D_2 \times R_2}{W_2}$ *[Refer 10.2.1]*

$\dfrac{4 \times 8 \times 4}{128} = \dfrac{6 \times D_2 \times 3}{540} \Rightarrow D_2 = \mathbf{30\ days.}$

E-11 A hostel has provisions for 250 students for 35 days. After 5 days, a fresh batch of 25 students were admitted to the hostel. Again after 10 days, a batch of 25 students left the hostel. How long will the remaining provisions survive?

S-11

	$D_1 = 10$	$D_2 = x$	
After 5 days,	after 10 days,	remaining x days	
25 students added,	B 25 students left	C	D
	$N_1 = 250 + 25 = 275$	$N_2 = 275 - 25 = 250$	

Remaining food provisions at B (after 5 days) = $ND = 250 \times (35 - 5)$ student days
$\therefore$ $ND = N_1D_1 + N_2D_2 \Rightarrow (250 \times 30) = 275 \times 10 + 250 \times x$
$\Rightarrow x = \mathbf{19}$. $\therefore$ The remaining provisions would last for 19 days.

E-12 A garrison of 3000 men has provisions for 25 days, when given at the rate of 900 g per head. At the end of 11 days, a reinforcement arrives and it was found that now the provision will last 10 days more, when given at the rate of 840 g per head. What is the strength of reinforcement?

(MBA, '82)

S-12 Let strength of reinforcement be x
Remaining food provisions after 11 days = $3000 \times (25 - 11) \times 900$
Total men after 11 days = $(3000 + x)$
$\therefore$ $3000 \times 14 \times 900 = (3000 + x) \times 10 \times 840$ *[Refer 10.1]*
$\Rightarrow x = \mathbf{1{,}500}$. $\therefore$ The reinforcement had 1,500 men.

E-13 Six diesel engines consume 900 litres of diesel, when each one is running for 5 h a day. How much diesel will be required by 9 engines, each running 8 h a day when 5 diesel engines of former type consume as much diesel as 8 diesel engines of the latter type.

S-13 Using the formula,

$$N_1 \times D_1 \times \frac{R_1}{W_1} = N_2 \times D_2 \times \frac{R_2}{W_2}$$

Since 5 diesel engines of I type = 8 diesel engines of II type

$\therefore$ $R_1 = \dfrac{1}{5}$ and $R_2 = \dfrac{1}{8}$

$W_1 = 900$ litres, $W_2 = ?$ *(Since Amount of work ≡ Diesel consumption)*

$$\Rightarrow \frac{6 \times 5 \times \frac{1}{5}}{900} = \frac{9 \times 8 \times \frac{1}{8}}{W_2} \Rightarrow W_2 = \mathbf{1{,}350 \text{ litres.}}$$ *[Refer 10.1]*

$\therefore$ The diesel required is 1,350 litres

E-14 Two coal loading machines each working 12 hours per day for 8 days handles 9,000 tonnes of coal with an efficiency of 90%. While 3 other coal loading machines at an efficiency of 80% set to handle 12,000 tonnes of coal in 6 days. Find how many hours per day each should work.

S-14 Here $\frac{N_1 \times D_1 \times R_1 \times E_1}{W_1} = \frac{N_2 \times D_2 \times R_2 \times E_2}{W_2}$ *[Refer 10.1]*

$N_1 = 2 \quad R_1 = 12$ h/day; $\quad N_2 = 3 \; R_2 = ?$

$E_1 = \frac{90}{100} \quad W_1 = 9{,}000;$

$E_2 = \frac{80}{100} \quad W_2 = 12{,}000$

$$\Rightarrow \frac{2 \times 8 \times 12 \times 90}{9{,}000 \times 100} = \frac{3 \times 6 \times R_2 \times 80}{12{,}000 \times 100}$$

$\Rightarrow R_2 = \mathbf{16 \text{ h/day.}}$

$\therefore$ Each machine should work 16 h/day.

E-15 'A' can do a piece of work in $2\frac{1}{2}$ days which 'B' can do in $3\frac{1}{3}$ days. If A's wages are Rs 50 per week and B's wages are Rs 42.50 per week,what 'A' would have charged for doing a piece of work for which B received Rs 340? **(ITI, '90)**

S-15 Total wage ≡ Total Amount of work; Wage rate ≡ Work Rate

$$\therefore \frac{N_1 \times D_1 \times R_1}{W_1} = \frac{N_2 \times D_2 \times R_2}{W_2}$$

$$\frac{1 \times \frac{5}{2} \times 50}{W_1} = \frac{1 \times \frac{10}{3} \times 42.50}{340} \Rightarrow W_1 = \mathbf{300.}$$

$\therefore$ 'A' should charge Rs 300 for the job.

REGULAR PROBLEMS

(1) If 12 men do a piece of work in 80 days, in how many days will 16 men do it?
(a) 45 (b) 50 (c) 55 (d) 60 (e) None of these

(2) If 5 men do a job in 6 days and 10 women can do it in 5 days, in how many days can 3 men and 5 women do the same? (Bengal Civil Service, '31)
(a) 3 days (b) 6 days (c) 5 days (d) 4 days (e) None of these

(3) If the rent for grazing 40 goats for 20 days is Rs 370, how many goats can graze for 30 days for Rs 111? (Calcutta Matriculation, '37)
(a) 5 (b) 8 (c) 11 (d) 14 (e) None of these

(4) If 10 men or 18 boys can do a job in 15 days, then 25 men and 15 boys together will do twice the work in how many days?
(a) $\frac{9}{2}$ days (b) 9 days (c) 18 days (d) 36 days (e) None of these

(5) In a camp, there was provision of food for 42 days. After 10 days, 300 more persons joined the camp, as result of which the food lasted only 24 days. The number of persons originally in the camp were

(a) 100 (b) 375 (c) 900 (d) 1,555 (e) 500

(6) If Rs 8,000 can maintain a family of 4 persons for 40 days, for how long will Rs 10,500 maintain a family of 6 persons?

(a) 30 days (b) 35 days (c) 25 days (d) 28 days (e) None of these

(7) A garrison of 2,000 men has provisions for 45 days. At the end of 15 days, a reinforcement arrives and it is found that now the food lasts for 20 days more. What is the strength of the reinforcement?

(a) 3,000 (b) 4,000 (c) 1,700 (d) 1,000 (e) 2,100

(8) A garrison has provisions for certain number of days. After 15 days, $\frac{1}{4}$ of the men left and if it is found that the provisions will now last just as before, how long was that? **(IA, '88)**

(a) 50 days (b) 60 days (c) 70 days (d) 120 days (e) 30 days

Hint: Required days $= 15 \div \frac{1}{4} = 60$ days.

(9) A man undertakes to do a piece of work in 150 days. He employs 200 men. He finds that only a quarter of work is done in 50 days. How many additional men must be employed so that the work may be finished on time?

(a) 100 men (b) 300 men (c) 150 men (d) 175 men (e) None of these

(10) A man and a boy together can do a certain amount of digging in 40 days. Their skills in digging are in the ratio of 8 : 5. How many days will the boy take, if engaged alone.

(a) 52 days (b) 104 days (c) 68 days (d) 80 days (e) 100 days

Hint: Since $\frac{(8+5)\,x \times 40}{1} = \frac{5\,x \times \text{days}}{1}$, $\therefore$ days $= 104$.

(11) If 36 men can dig a trench 200 metres long, 3 metres wide and 2 metres deep in 6 days working 10 hours a day, in how many days, working 8 h a day will 10 men dig a trench of 100 metres long, 4m wide and 3m deep.

(a) 20 days (b) 27 days (c) 15 days (d) 54 days (e) 18 days

(12) One man can set 180 tiles in one day. Five men were engaged for 2 days to set the floor with tiles completely. If area of each tile is $2' \times \frac{3'}{4}$, then area of floor is,

(a) 1500 ft^2 (b) Data insufficient (c) 2700 ft^2
(d) 2000 ft^2 (e) 675 ft^2

(13) If on burning 6 gas ovens for 6 h per day, an amount of Rs 45 is spent for 8 days, then by spending Rs 60, how many gas ovens can be burnt 3 h per day for 16 days?

(a) 8 (b) 16 (c) 12 (d) 6 (e) 18

(14) If 10 men weave 10 mats in 10 days, how many men will be required to weave 100 mats in 100 days? **(Bengal Civil Service, '31)**

(a) 100 (b) 1000 (c) 10 (d) 1 (e) None of these

(15) If 8 men or 17 boys can do a piece of work in 26 days, how many days will it take for 4 men and 24 boys to a piece of work 50 × 0.9 times as great? **(Calcutta Matriculation, '37)**

(a) 680 days (b) 612 days (c) 61 days (d) 68 days (e) 80 days

(16) If the cost of printing a book of 320 leaves with 21 lines on each page and on an average 11 words in each line is Rs 19, find the cost of printing a book with 297 leaves, 28 lines on each page and 10 words in each line.

(a) Rs $22\frac{3}{8}$ (b) Rs $20\frac{3}{8}$ (c) Rs $21\frac{3}{8}$ (d) Rs $21\frac{3}{4}$ (e) Rs $20\frac{3}{4}$

Hint: $\frac{\text{Cost of printing}}{\text{No. of words}}$ must be same in both cases. Here no. of words printed = volume of work.

By chain rule, $\frac{19}{320 \times 21 \times 11} = \frac{x}{297 \times 28 \times 10} \Rightarrow x = \text{Rs } 21\frac{3}{8}$.

(17) In a milk dairy, a gang of labourers promise to finish a piece of product in 10 hours. But five of them become absent, so the rest of gang finished the product in 12 hours. Find the original number of men in the gang.

(a) 30 men (b) 25 men (c) 40 men (d) 20 men (e) 60 men

(18) In order to carry out a contract completely in 40 days, 24 men were put to work. After 28 days 11 more men were engaged and the work was completed just in time. Had no extra men been engaged, how many days would the work have taken?

(a) $45\frac{1}{2}$ (b) $22\frac{3}{4}$ (c) 42 (d) 40 (e) 44

(19) A factory has 15 powerlooms. Due to failure of 3 powerlooms, during the previous week, only 168 sarees were made. If this week all powerlooms are running, the number of sarees made are:

(a) 70 (b) 168 (c) 196 (d) 210 (e) 252

(20) Jeetendra can wire 'x' radios in $\frac{3}{4}$ minutes. At this rate, how many radios can he wire in $\frac{3}{4}$ of an hour?

(a) $\frac{40}{x}$ (b) $\frac{4x}{3}$ (c) $\frac{9x}{16}$ (d) $60x$ (e) $45x$

(21) 'M' men agreed to purchase a gift for Rs D. If 3 men drop out, how much more will each have to contribute towards the purchase of the gift?

(a) $\frac{D-3}{M}$ (b) $\frac{D}{M^2-3}$ (c) $\frac{3D}{M-3}$ (d) $1+\frac{D}{M-3}$ (e) $\frac{3D}{M^2-3M}$

(22) A contractor undertakes to build a house in 8 months. He plans to complete the work in time with 210 men working 20 days in a month @ 6 hours a day. However, he manages to get only 140 men and is allowed 12 months to complete the work. If his men worked for 24 days a month, how many hours a day should they work to complete the work?

(a) 6 (b) 5 (c) 8 (d) 4 (e) $7\frac{1}{2}$

(23) If 38 men, working 6 hours a day, can do a piece of work in 12 days, find the number of days in which 57 men, working 8 hours a day, can do twice that piece of work, supposing that 2 men of the first set do as much work in one hour as 3 men of the second set do in $1\frac{1}{2}$ hours.

(a) 36 (b) 32 (c) 40 (d) 39 (e) 27

Answers

1. (d)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (a)
10. (b)	**11.** (b)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (a)	**18.** (a)
19. (d)	**20.** (d)	**21.** (e)	**22.** (b)	**23.** (e).				

11 TIME, WORK AND WAGES

In most of the problems on time and work, one of the following basic parameters is to be calculated:

(a) Time: Time needed by more than one person to complete a job or time for which a person(s) actually worked on the assigned job.

(b) Alone time: Time needed by single person to complete a job.

(c) Work: The amount of total work (assigned) or the part of total assigned work actually done.

11.1 BASIC CONCEPTS

Concept 1

Total amount of a complete job (or assigned job) = 1, always, unless otherwise specified.

Concept 2

If any person 'M' completes a job **alone** in t days, then **alone time** for 'M' = t

Concept 3

1 day's work by any person = $\left(\dfrac{1}{\text{alone time}}\right)^{\text{th}}$ part of total work

Example: Ram can polish the floor of a building in 16 days. Find the work done by Ram in one day.

Solution: Here, alone time for Ram = 16 days, so 1 day's work by Ram = $\dfrac{1}{16}$ th part of total work.

Concept 4

The reciprocal of 1 day's work gives the alone time i.e., alone time (or time to complete a job by a single person) = $\dfrac{1}{\text{1 day's work}}$

Example: Mukesh can do $\frac{2}{7}$th of an work in 1 day. In how many days can he complete the same work?

Solution: Time of completion by Mukesh alone = alone time = $\dfrac{1}{\text{1 day's work}}$

$$= \frac{1}{2/7}$$

$$= \frac{7}{2} \text{ days} = 3\frac{1}{2} \text{ days}$$

Therefore, Mukesh can complete the job alone in $3\frac{1}{2}$ days.

Concept 5

When more than one person are working on the same piece of work, then their combined 1 day's work = sum of 1 day's work by each person. i.e., If A, B and C are three persons working on a job, then $(A + B + C)$'s 1 day's work = A's 1 day work + B's 1 day work + C's 1 day work.

Example: A person 'P' can alone do a work in 10 days and 'Q' can do it in 15 days. What amount of work is done by P and Q together in one day?

Solution: $(P + Q)$'s 1 day work = P's 1 day work + Q's 1 day work. Now, using concept (3),

$$\text{1 day's work} = \frac{1}{\text{alone time}}$$

We can find

$$(P + Q)\text{'s 1 day work} = \left(\frac{1}{10} + \frac{1}{15}\right)\text{th part of total work.}$$

$$= \frac{3+2}{30} = \frac{1}{6}\text{th part of total work.}$$

Corollary

A's 1 day work = $(A + B + C)$'s 1 day work – B's 1 day work – C's 1 day work

Similarly, B's 1 day work = $(A + B + C)$'s 1 day work – A's 1 day work – C's 1 day work and so on.

Concept 6

It is the application of concept (4) for more than one person.

The reciprocal of combined 1 day's work gives the time for completion by the persons working together.

i.e., $$\text{time for completion} = \frac{1}{\text{combined 1 day's work}}$$

It implies that

if three persons, say, A, B and C are working together on a job, then

$$\text{time for completion by them} = \frac{1}{(A+B+C)\text{'s 1 day's work}}$$

Example: Three persons Ramesh, Suresh and Kana can do a job alone in 10 days, 12 days and 15 days respectively. In how many days they can finish the job working together?

Solution: Using concept (6),

$$\text{time for completion} = \frac{1}{\text{combined 1 day's work}}$$

Now, using concept (5),

combined 1 day's work = sum of 1 day's work by each person we can find,

(Ramesh + Suresh + Kana)'s 1 day work = Ramesh's 1 day work + Suresh's 1 day work + Kana's 1 day work

$$= \left(\frac{1}{10} + \frac{1}{12} + \frac{1}{15}\right)\text{th part of work}$$

$$= \frac{1}{4}\text{th part of work}$$

$$\therefore \text{time for completion by working together} = \frac{1}{1/4} = 4 \text{ days.}$$

Concept 7

Part of work done at any time 't' by one or more persons = $t \times$ (1 day's work)

Example: A person 'M' can do a job in 15 days. How much of the job is done by him in 7 days?

Solution: Part of workdone by M in 7 days $= 7 \times \left(\frac{1}{15}\right)$

$$= \frac{7}{15}\text{th part of work}$$

Example: Two friends A and B can do a work alone in 12 days and 8 days respectively. Find the amount of work done by them in 4 days.

Solution: Part of work done by $A + B$ in 4 days

$$= 4 \times (A + B)\text{'s 1 day work}$$

$$= 4\left(\frac{1}{12} + \frac{1}{8}\right)\text{th part of work}$$

$$= \frac{5}{6}\text{th}$$

Example: Two persons P and Q can do a piece of work alone in 10 days and 15 days respectively. If P works for 2 days and Q works for 5 days, then find the total amount of work done.

Solution: Part of work done by $P + Q$

= part of work done by P in 2 days + part of work done by Q in 5 days

$$= 2\left(\frac{1}{10}\right) + 5\left(\frac{1}{15}\right)$$

$$= \left(\frac{1}{5} + \frac{1}{3}\right)\text{th part of work}$$

$$= \frac{8}{15}\text{th}$$

Concept 8

If more than one persons are working for different time schedules to complete a piece of work, then

(i) Assume the time for completion = T
(ii) Number of days worked by each person is found with reference to T, if not mentioned in the problem.
(iii) Part of work done by each person is found out by using concept (7).
(iv) Sum of the parts of work done by each person = 1, since the job is complete [also, refer concept (1)]
(v) Solve to find out the unknowns.

Example: Dipa and Avik can do a piece of work in 20 days and 30 days respectively. They work together and Dipa leaves 5 days before the work is finished. Avik finishes the remaining work alone. In how many days is the total work finished?

Solution: Assume the time for completion = T
Since Dipa leaves 5 days before the work is finished, so, no. of days worked by Dipa = $T - 5$
and Avik works from the start to the finish of the work, so, no. of days worked by Avik = T
Now, the work is finished, so,

Dipa's work + Avik's work = 1

$$\Rightarrow \quad \frac{T-5}{20} + \frac{T}{30} = 1$$

$$\Rightarrow \quad T = 15$$

∴ Total work is finished in 15 days.

Concept 9

The ratio of the workdone by the two persons in the same time is the inverse ratio of their alone times.

e.g., If 'A' can do a work in 5 days and B can do in 9 days, then, in the same time, $\frac{A\text{'s work}}{B\text{'s work}} = \frac{9}{5}$ (inverse of alone times)

Concept 10

If a person 'P' is 'n' times as good a workman as Q, then alone time for $P = \frac{\text{alone time for } Q}{n}$

and after same time, $\frac{P\text{'s work}}{Q\text{'s work}} = \frac{n}{1}$ [using concept (9)]

e.g., If A is twice as good a workman as B, then A will take half of the time taken by B to do a certain piece of work. It also implies that during the same time, the ratio of the work done by A and B is 2 : 1.

Concept 11

If more than one person are engaged on payment basis for doing a work, then the total wages distributed to each person are:

(i) in proportion to the work done by each person, or
(ii) in proportion to the 1 day's work of each person, or
(iii) in inverse proportion to the alone time of each person.

Solved Examples

E-1 Tuktuki and Rasmani can do a job alone in 20 days and 30 days respectively. In how many days the job will be finished if they work together?

S-1 Here, we can use a direct formula, if $a = 20$, $b = 30$, then

$$\text{Combined required time} = \frac{ab}{a+b} \quad \text{[formula derived by using concept (6)]}$$

$$= \frac{20 \times 30}{20 + 30} = 12 \text{ days.}$$

E-2 Mohan and Sohan can do a job in 12 days. Sohan alone can finish it in 28 days. In how many days can Mohan alone finish the work?

S-2 Using concept (5),

(Mohan + Sohan)'s 1 day work = Mohan's 1 day work + Sohan's 1 day work

$$\Rightarrow \frac{1}{12} = \text{Mohan's 1 day work} + \frac{1}{28}$$

$$\Rightarrow \text{Mohan's 1 day work} = \frac{1}{12} - \frac{1}{28} = \left(\frac{1}{21}\right)\text{th of work}$$

$$\therefore \text{Mohan's alone time} = \frac{1}{1/21} \quad \text{[concept (4)]}$$

$$= 21 \text{ days}$$

Short-Cut if $T = 12$, $a = 28$, then

$$\text{Required time} = \frac{Ta}{a-T} = \frac{28 \times 12}{28 - 12} = 21 \text{ days.}$$

E-3 Mary and Maurice can do a piece of work in 10 days and 15 days respectively. They work together for 3 days and then Maurice leaves. Mary finishes the remaining work alone. In how many days is the total work finished?

S-3 Let the total work be finished in 'T' days.

Now, using the concept (8), and (7)

$$\frac{\text{no. of days Mary worked}}{\text{alone time}} + \frac{\text{no. of days Maurice worked}}{\text{alone time}} = 1$$

$$\Rightarrow \frac{T}{10} + \frac{3}{15} = 1$$

$$\Rightarrow \frac{T}{10} = 1 - \frac{1}{15}$$

$$\Rightarrow T = 10 \times \frac{4}{5} = 8$$

$\therefore$ Total work is finished in 8 days.

E-4 Singvi and Ravi can do a job alone in 10 days and 12 days respectively. Singvi starts the work and after 6 days Ravi also joins to finish the work together. For how many days Ravi actually worked on the job?

S-4 Let the work be finished in T days, then using the concept (8),

$$\frac{\text{no. of days Singvi worked}}{\text{alone time}} + \frac{\text{no. of days Ravi worked}}{\text{alone time}} = 1$$

$$\Rightarrow \quad \frac{T}{10} + \frac{T-6}{12} = 1$$

$$\Rightarrow \quad T = \frac{90}{11}$$

$\therefore$ Ravi worked for T – 6 i.e. $\frac{90}{11} - 6 = 2\frac{2}{11}$ days.

E-5 A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would each take separately to do the same work?

S-5 Since, $(A + B) + (B + C) + (C + A) = 2(A + B + C)$

So, using concept (5), we get,

$2(A + B + C)$'s 1 day work = $(A + B)$'s 1 day work + $(B + C)$'s 1 day work + $(C + A)$'s 1 day work

$$= \frac{1}{12} + \frac{1}{15} + \frac{1}{20}$$

$$= \frac{1}{5}$$

$\Rightarrow \quad (A + B + C)$'s 1 day work = $\frac{1}{5 \times 2} = \frac{1}{10}$ th part of work

Now, to find out alone time for A, we may write

$$A = (A + B + C) - (B + C)$$

Therefore, A's 1 day work = $(A + B + C)$'s 1 day work – $(B + C)$'s 1 day work

$$= \frac{1}{10} - \frac{1}{15}$$

$= \frac{1}{30}$ th part of work

So, Alone time for $A = \frac{1}{1/30}$ = **30 days.**

Similarly, B's 1 day work = $\frac{1}{10} - \frac{1}{20} = \frac{1}{20}$

Alone time for $B = \frac{1}{1/20}$ = 20 days.

and C's 1 day work = $\frac{1}{10} - \frac{1}{12} = \frac{1}{60}$

Alone time for $C = \frac{1 \cdot 1}{1/60} = 60$ days.

Short-Cut

$\because$ L.C.M. of 12, 15 and 20 = 60 = L, say

A and B can do in $T_1 = 12$ days $\rightarrow C$ absent

B and C can do in $T_2 = 15$ days $\rightarrow A$ absent

C and A can do in $T_3 = 20$ days $\rightarrow B$ absent

$$\text{Alone time for } A = \frac{2L}{\frac{L}{T_1} + \frac{L}{T_3} - \frac{L}{T_2}^{*}} = \frac{2 \times 60}{\frac{60}{12} + \frac{60}{20} - \frac{60}{15}} = \frac{120}{5+3-4} = 30 \text{ days}$$

$\left(* - \frac{L}{T_2} \right.$ because T_2 represents that A is absent and as alone time for A is to be found out.$\left. \right)$

Similarly, alone times for B and C can be found out.

E-6 P and Q can do a job in 15 days and 10 days respectively. They began the work together but P leaves after some days and Q finishes the remaining job in 5 days. After how many days did P leave?

S-6 In this problem, the total time for completion is neither known nor to be found out. Hence total time T is not be considered, but assume that P works for x days, then using the concept (8),

$$\frac{\text{no. of days } P \text{ worked}}{\text{alone time for } P} + \frac{\text{no. of days } Q \text{ worked}}{\text{alone time for } Q} = 1$$

$$\Rightarrow \quad \frac{x}{15} + \frac{x+5}{10} = 1$$

$$\Rightarrow \quad x = 3$$

$\therefore$ P leaves after 3 days.

E-7 Ramesh is thrice as good a workman as Bipan, and is therefore able to finish a piece of work in 40 days less than Bipan. Find the time in which they can do it working together.

S-7 Since Ramesh is thrice as good a workman as Bipan, then if Ramesh does a job in 1 day, Bipan will do the same in 3 days and the difference is $3 - 1 = 2$ days.

$\therefore$ For 40 days difference (20×2), Ramesh does in 20 days, Bipan in $20 \times 3 = 60$ days.

Now, (Ramesh + Bipan)'s 1 day work $= \frac{1}{20} + \frac{1}{60} = \frac{1}{15}$

$\therefore$ the required time is 15 days.

E-8 Two workers A and B working together completed a job in 5 days. If A worked twice as efficiently as he actually did and B worked $\frac{1}{3}$ as efficiently as he actually did, the work would have been completed in 3 days. Find the time taken by A to complete the job alone.

S-8 Let 'A' alone complete the job in 'a' days and B in 'b' days.

then, $\frac{1}{a} + \frac{1}{b} = \frac{1}{5}$ (1) (i.e. 1 day's combined work).

and in second case, A works twice efficiently, so A's 1 day work $= 2 \times \frac{1}{a}$

and B works $\frac{1}{3}$ as efficiently, so B's 1 day work $= \frac{1}{3} \times \frac{1}{b}$

$\therefore$ combined 1 day's work $= \frac{1}{3} = \frac{2}{a} + \frac{1}{3b}$

$$\Rightarrow \quad \frac{6}{a} + \frac{1}{b} = 1 \qquad (2)$$

Solving (1) and (2), we get, $a = 6\frac{1}{4}$

$\therefore$ A completes the job alone in $6\frac{1}{4}$ days.

E-9 Kaberi takes twice as much time as Kanti and thrice as much as Kalpana to finish a piece of work. They together finish the work in one day. Find the time taken by each of them to finish the work.

S-9 Here, the alone time of Kaberi is related to the alone times of other two persons, so assume the alone time of Kaberi $= x$,

then, alone time of Kanti $= \frac{x}{2}$ and of Kalpana $= \frac{x}{3}$

using concept (5),
Kaberi's 1 day work + Kanti's 1 day work + Kalpana's 1 day work = combined 1 day work

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{x/2} + \frac{1}{x/3} = \frac{1}{1}$$

$$\Rightarrow \quad x = 6$$

$\therefore$ Alone time for Kaberi = 6 days, for Kanti $= \frac{6}{2} = 3$ days, Kalpana $= \frac{6}{3} = 2$ days.

E-10 Anil and Sunil working separately can assemble a computer in 10 hours and 12 hours respectively. If they are working for 1 hour atternately, Anil beginning, in how hours will the computer be assembled?

S-10 After 10 hours of combined, but alternate working, we get the part of computer assembled = Anil's 5 hours work + Sunil's 5 hours work

$$= 5 \times \frac{1}{10} + 5 \times \frac{1}{12}$$

$$= \frac{11}{12} \text{th}$$

Remaining part $= 1 - \frac{11}{12} = \frac{1}{12}$ th

Now, at the start of 11th hour, Anil will work.

Time taken by Anil to do $\frac{1}{12}$ th work $= 10 \times \frac{1}{12}$ hour $= \frac{5}{6}$ hour

$\therefore$ Total time = 10 hours + $\frac{5}{6}$ hours = $10\frac{5}{6}$ hours.

$\therefore$ the computer will be assembled in $10\frac{5}{6}$ hours.

E-11 Two friends take a piece of work for Rs 960. One alone could do it in 12 days, the other in 16 days. With the assistance of an expert they finish it in 4 days. How much renumeration the expert should get?

S-11 First friend's 4 day's work = $\frac{4}{12} = \frac{1}{3}$ (Since, the work is finished in 4 days, when expert assists)

Second friend's 4 day's work = $\frac{4}{16} = \frac{1}{4}$

The expert's 4 day's work = $1 - \left(\frac{1}{3} + \frac{1}{4}\right) = \frac{5}{12}$

Now, total wages of Rs 960 is to be distributed among two friends and the expert in proportion to the amount of work done by each of them [Concept (11)]

So, 960 is to be divided in the proportion of $\frac{1}{3} : \frac{1}{4} : \frac{5}{12}$

or 4 : 3 : 5

$\therefore$ Share of expert = $\frac{5}{12} \times 960$ = Rs 400

Hence, the expert should get Rs 400.

REGULAR PROBLEMS

(1) Rajesh and Ajay can complete a job in 16 days. Rajesh alone can do it in 24 days. How long will Ajay alone take to finish the whole work? **(AGE, '82)**

(a) 20 days (b) 48 days (c) 30 days (d) 36 days (e) 28 days

(2) Kaberi can do a job in 10 days and Arati in 15 days. If both of them work together, how long will the work last?

(a) 8 days (b) 9 days (c) 4 days (d) 6 days (e) 11 days

(3) Ram, Shyam and Mohan can do a piece of work in 12, 15 and 20 days respectively. How long will they take to finish it together? **(LIC, '91)**

(a) 10 days (b) 12 days (c) 14 days (d) 8 days (e) 5 days

(4) Alok and Kaushik can do alone a holiday assignment in 25 days and 30 days respectively. They work together for 5 days and then Alok leaves due to his illness. Kaushik finishes the rest of the holiday assignment in x days. The value of 'x' is:

(a) 20 (b) 19 (c) 24 (d) 16 (e) 15

(5) X works twice as fast as Y. If Y can complete a job alone in 12 days, then in how many days can X and Y together finish the job?

(a) 18 (b) 4 (c) 6 (d) 8 (e) 2

(6) Sundar can copy 80 pages in 20 hours, Sundar and Purbasa can copy 135 pages in 27 hours. How many pages Purbasa can copy in 20 hours?

(a) $\frac{55}{7}$ (b) 55 (c) 27 (d) 20 (e) 35

(7) Tapas works twice as much as Mihir. If both of them finish the work in 12 days, Tapas alone can do it in:

(a) 20 days (b) 24 days (c) 18 days (d) 36 days (e) 20 days

(8) Xavier can do a job in 40 days. He worked on it for 5 days and then Paes finished it in 21 days. In how many days Xavier and Paes can finish the work? **(Bank PO, '93)**

(a) 10 (b) 15 (c) 20 (d) $\frac{840}{61}$ (e) 12

Hint: To find the number of days required to finish a work, one day's work of each person is to be found out first. One day's job of Paes is to be found out with the given data as one day job of Xavier $= \frac{1}{40}$ (known)

(9) Mary has '*m*' minutes of home work in each of her '*s*' subjects. In one hour she completes what part of her home work?

(a) $\frac{m}{s}$ (b) $\frac{ms}{60}$ (c) $\frac{60}{ms}$ (d) $\frac{1}{ms}$ (e) $\frac{s}{m}$

(10) Gouri can knit a pair of socks in 3 days. Gita can knit the same in 6 days. If they are knitting together, in how many days will they knit two pairs of socks?

(a) 4 (b) 2 (c) $4\frac{1}{2}$ (d) 3 (e) 6

(11) Sun, Mon and Don can do a work in 6 days. Sun and Don can do it in 10 days. In how many days will Mon alone do the work?

(a) 20 (b) $3\frac{3}{4}$ (c) 15 (d) 4 (e) 16

Hint: Refer the text

(12) One man can paint a house in '*r*' days and another man can do it in '*t*' hours. If they can together do it in '*d*' days, then '*d*' is given by:

(a) $\frac{1}{r}+\frac{1}{t}$ (b) $\frac{rt}{r+t}$ (c) $\frac{24rt}{r+t}$ (d) $\frac{rt}{24(r+t)}$ (e) $\frac{rt}{t+24r}$

(13) Dinesh and Dipu can design an application software in 16 hours and 12 hours respectively. Dinesh joins Dipu 4 hours before completing the design, Dipu had started the design work alone. Find how many days Dipu has worked alone? **(IA, '93)**

(a) 5 (b) $\frac{5}{24}$ (c) 7 (d) $\frac{2}{7}$ (e) 9

(14) John can do a piece of work in 15 days. If he is joined by Jill who is 50% more efficient, in what time will John and Jill together finish the work?

(a) 10 days (b) 6 days (c) 18 days
(d) data insufficient (e) 8 days

(15) If M men can complete a job in H hours, then in how many hours 5 men will complete the job?

(a) $\frac{H}{5}$ (b) $\frac{M}{5H}$ (c) $\frac{MH}{5}$ (d) MH (e) $\frac{5H}{M}$

(16) If $\frac{4}{7}$ of a piece of work is completed in $\frac{7}{4}$ days, in how many days can rest of the work be completed?

(a) $\frac{7}{3}$ (b) $\frac{3}{7}$ (c) $\frac{21}{16}$ (d) 21 (e) $\frac{16}{21}$

(17) Mukesh, Anil and Sumit are three civil engineers. Mukesh can design a plan of a multi-storeyed apartment in 5 hours and Anil in 4 hours alone. Three together can do it in 2 hours. In what time can Sumit do it alone?

(a) 7 hours (b) 15 hours (c) 20 hours (d) 18 hours (e) 12 hours

Hint: $C = (A + B + C) - A - B$

(18) P can do a job in 3 days, Q can do 3 times that job in 8 days and R can do 5 times that job in 12 days. If all of them work together, they will finish one job in:

(a) $\frac{120}{47}$ days (b) $\frac{8}{9}$ days (c) 3 days (d) $2\frac{1}{2}$ days (e) data insufficient

(19) Miti and Aditi can do a piece of work in 45 days and 40 days respectively. They began to work together but Miti leaves after 'x' days and Aditi finished the rest of the work in $(x + 14)$ days. After how many days did Miti leave?

(a) 19 (b) 11 (c) 9 (d) 12 (e) 10

Answers

1. (b) **2.** (d) **3.** (e) **4.** (b) **5.** (b) **6.** (d) **7.** (c) **8.** (b) **9.** (c)
10. (a) **11.** (c) **12.** (e) **13.** (b) **14.** (b) **15.** (c) **16.** (c) **17.** (c) **18.** (b)
19. (c).

REAL PROBLEMS

(1) Mohan can mow his lawn in x hours. After 2 hours it begins to rain. The unmowed part of the lawn is

(a) $\frac{2}{x}$ (b) $\frac{2-x}{2}$ (c) $\frac{x}{2}$ (d) $\frac{x-2}{x}$ (e) $2x$

(2) Prakash has done $\frac{1}{3}$rd of a job in 8 days, Surya completes the rest of the job in 8 days. In how many days could Prakash and Surya together have completed the work?

(a) 16 (b) 8 (c) 24 (d) 4 (e) 20

(3) Working with a project, David and Sams together complete a piece of work in 6 days. If David works alone for 2 days and completes $\frac{1}{4}$th of the work, in how many days can Sams alone complete the rest of the work?

(a) 18 (b) 10 (c) 16 (d) 24 (e) 12

(4) The work done by $(x - 1)$ men in $(x + 1)$ days and the work done by $(x + 1)$ men in $(x + 2)$ days are in the ratio 5 : 6. Find x.

(a) 10 (b) 6 (c) 8 (d) 16 (e) 20

(5) Harish is thrice as good a workman as Manish and is therefore able to finish a piece of work in 60 days less than Manish. Find the time in which they can do it working together.

(a) 30 days (b) $22\frac{1}{2}$ days (c) 21 days (d) 36 days (e) 15 days

(6) Sreeji can stich a suit in 3 hours less time than Premji. Sreeji stiches it alone for 4 hours and then Premji takes over and completes it. If altogether 14 hours were required to stich the suit, how many hours would Premji take to stitch the same alone?

(a) 12 (b) 7 (c) 9 (d) 21 (e) 15

Tips: Assume Premji can do it alone in x hours.

(7) P alone would take 8 hours more to complete a job than both P and Q would together. If Q worked alone, he took $4\frac{1}{2}$ hours more to complete it than both P and Q worked together. What time would they take if both P and Q worked together?

(a) $\frac{72}{25}$ hrs. (b) $9\frac{1}{2}$ hrs. (c) 6 hrs. (d) $13\frac{1}{2}$ hrs. (e) 11 hrs.

(8) Kumar can alone audit the Company's accounts in 12 days, while Suresh alone takes 3 more days than Kumar. Kumar and Suresh undertook to do it for Rs 10,800 with the help of Vinod they finish it in 5 days. How much is paid to Vinod?

(a) Rs 4,500 (b) Rs 2,700 (c) Rs 3,600 (d) Rs 4,200 (e) Rs 3,000

Hint: Refer Text

(9) If a factory A turns out x cars an hour and factory B turns out Y cars every 2 hours, the number of cars that both factories turn out in 8 hours is:

(a) $8(x + Y)$ (b) $8x + \frac{Y}{2}$ (c) $16(x + Y)$ (d) $x + Y$ (e) $8\left(x + \frac{Y}{2}\right)$

(10) Ram, Shyam and Hari are employed to do a piece of work for Rs 529. Ram and Hari together are supposed to do $\frac{19}{23}$ of the work. What should Shyam be paid?

(a) Rs 82 (b) Rs 92 (c) Rs 437 (d) Rs 300 (e) Rs 425

(11) A man, a woman or a boy can do a job in 3, 4 or 12 days respectively. How many boys must assist 1 man and 1 woman to do the job in one and a half day?

(a) 3 (b) 2 (c) data insufficient
(d) 6 (e) 1

Hint: Find $1\frac{1}{2}$ days job by man, woman and x boys.

(12) X, Y, and Z can complete a job in 10 days. If Y does half what X and Z together do in 1 day, then in how may days can Y alone do the half work?

(a) 30 (b) 21 (c) 15 (d) 18 (e) 6

(13) A sum of Rs 300 was paid for a work that A can do in 32 days, B in 20 days, B and C in 12 days and D in 24 days. How much did C receive if all the four work together?

(a) Rs 96 (b) Rs 80 (c) Rs 108 (d) Rs 64 (e) Rs 72

(14) P and Q can weave a mat in 12 days, Q and R together do it in 15 days. If P is twice as good a workman as R, find in what time Q will do it alone?

(a) 30 days (b) 22 days (c) 18 days (d) 24 days (e) 20 days

Hint: $P = R/2$

(15) If 3 men and 2 boys together earn Rs 306 in 9 days while 7 men and 3 boys earn Rs 639 in the same time. In how many days will 8 men and 6 boys together earn Rs 376?

(a) 7 days (b) 5 days (c) 3 days (d) 6 days (e) 4 days

Answers

1. (d)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (e)	**7.** (c)	**8.** (b)	**9.** (e)
10. (b)	**11.** (e)	**12.** (c)	**13.** (d)	**14.** (e)	**15.** (e)			

12 PIPES AND CISTERNS

The problems often asked on pipes and cisterns have the similarity with those of time and work. Hence, the approach to solve the problems became quite easy if one has a good command on the problems of time and work. In pipes and cisterns, the following basic parameters are often asked to be calculated:

(a) Time—It means time for **filling** or **emptying** pipes that have been actually kept open

(b) Work— It means the part of the cistern to be filled or emptied or the part of cistern actually filled or emptied. In actual practice, while solving the problems on pipes and cisterns, the amount or part of cistern assigned for filling or emptying (is always) is a unit, unless otherwise specified.

If a **filling** pipe can fill a cistern **alone** in f_1 min., then **Alone Fill Time** for that filling pipe = f_1

If an **emptying** pipe can empty a cistern **alone** in e_1 min., then

Alone Emptying Time for that emptying pipe = $-e_1$

(–)ve is used for all cases of emptying work.

When a cistern is filled completely, amount of work done (filling) = 1

When a cistern full of water is **emptied** completely. Amount of **Fill Work** = Amount of **Empty Work**

$\therefore$ Amount of **Fill Work – Empty Work** = 0

12.1 BASIC CONCEPTS

The concepts used to solve problems on time and work can also be applied to the problems on pipes and cistern; because here the work refers to the work of filling or emptying the cistern and 'persons' have been replaced by the fill pipes/empty pipes/leaks that are doing the work.

(A) For one Pipe

Concept A-1

$$\text{Fill or empty work done} = \frac{\text{time of work}}{\text{alone time of the pipe}}$$

Concept A-2

$$\text{Fill or empty work done in 1 unit}^{*}\text{ time} = \frac{1}{\text{alone time}}$$

Concept A-3

Alone time of the pipe = $\dfrac{1}{\text{work done in 1 unit time}}$

*1 unit time means 1 hour, or 1 minute or 1 second depending upon the unit of the alone time.

Example: A pipe can fill a cistern in 4 hours. What part of the cistern is filled up in 40 minutes?

Solution: Using concept (A-1),

$$\text{fill work done} = \frac{\text{time of work}}{\text{alone time}}$$

$$= \frac{40}{4 \times 60} \qquad [4 \text{ hrs.} = 4 \times 60 \text{ minutes}]$$

$$= \frac{1}{6}$$

Therefore, $\frac{1}{6}$ th of the cistern is filled.

Note: fill work done = part of the cistern filled
empty work done = part of the full cistern emptied

(B) For Two or More Pipes

Concept B-1

For only fill pipes,
total fill work done by fill pipes (or inlets) in 1 unit time
= sum of work done by each inlet in 1 unit time

$$= \frac{1}{\substack{\text{alone time}\\ \text{for first fill}\\ \text{pipe}}} + \frac{1}{\substack{\text{alone time}\\ \text{for second}\\ \text{fill pipe}}} + \ldots \qquad \text{[Concept (A-2)]}$$

Concept B-2

For only empty pipes/leaks,
total empty work done by empty pipes (outlets) in 1 unit time
= sum of work done by each outlet in 1 unit time

$$= \frac{1}{\substack{\text{alone time}\\ \text{for first}\\ \text{empty pipe}}} + \frac{1}{\substack{\text{alone time}\\ \text{for second}\\ \text{empty pipe}}} + \ldots \qquad \text{[Concept (A-2)]}$$

Concept B-3

For fill pipes and empty pipes/leaks together.
Net work done by all the pipes in 1 unit time
= (sum of work done by inlets in 1 unit time) – (sum of work done by outlets in 1 unit time)

Note: Net work done by all the pipes = part of cistern emptied or filled in one minute
This net work done is (–)ve if the cistern gets emptied, otherwise it is (+)ve.
If the net work done is (+)ve, then the cistern is being filled and vice versa.
If the net work done is (–)ve, then the cistern is being emptied and vice versa.

Concept B-4

Time to fill/empty a cistern = $\dfrac{1}{\text{net work done in 1 minute}}$ [using Concept (A-3)]

Example: A pipe can fill a cistern in 12 minutes and another pipe in 15 minutes, but a third pipe can empty it in 6 minutes. The three pipes are kept open together. Find when the cistern is emptied/filled.

Solution: Net work done by all the pipes in 1 minute

= (sum of work done by inlets in one minute) – (sum of work done by outlets in one minute)

here, fill pipes are inlets and empty pipes are outlets

Work done by inlets in one minute = $\dfrac{1}{12}+\dfrac{1}{15}$

Work done by outlets in one minute = $\dfrac{1}{6}$

Net work done by all the pipes in 1 minute = $\left(\dfrac{1}{12}+\dfrac{1}{15}\right)-\left(\dfrac{1}{6}\right)$

$= -\dfrac{1}{60}$

(–)ve sign indicates the cistern is getting emptied.

i.e. $\dfrac{1}{60}$ part is emptied in 1 minute.

$\Rightarrow$ time to empty the complete cistern = $\dfrac{1}{\text{net work done in 1 minute}}$

$= \dfrac{1}{\frac{1}{60}}$

= 60 minutes.

(C) Flow Rate and Work Done of Same Pipe

If flow rate per minute of a fill pipe or waste pipe and work done per 1 minute for the same pipe are known, then,

Capacity of cistern = $\dfrac{\text{flow rate in 1 minute of the pipe}}{\text{work done by the pipe in 1 minute*}}$

[net work done can be found out by using concept (B-3)]
*work done/minute and flow rate/minute are to be calculated for the same pipe, may be fill or waste pipe.

Example: Two pipes A and B can separately fill a cistern in $7\frac{1}{2}$ and 5 minutes respectively and a waste pipe C can carry off 14 litres per minute. If all the pipes are opened when the cistern is full, it is emptied in 1 hour. How many litres does it hold?

Solution: After opening all three pipes, the full cistern gets emptied in 1 hour = 60 minutes.

$\Rightarrow$ Net work done by all three pipes in 1 minute = $-\frac{1}{60}$ (–)ve, for cisterm in emptied.

(or part or cistern emptied/filled in 1 minute)

As per concept (B-3),

$$(\text{work done by inlets in 1 minute}) - (\text{work done by outlets in 1 minute}) = -\frac{1}{60}$$

Here, the outlet is the waste pipe C.

$$\Rightarrow \left(\frac{1}{7\frac{1}{2}}+\frac{1}{5}\right) - (\text{work done by outlets in 1 minute}) = -\frac{1}{60}$$

$$\Rightarrow \text{work done by waste pipe in 1 minute} = \frac{2}{15}+\frac{1}{5}+\frac{1}{60}$$

$$= \frac{7}{20}$$

Using concept (C),

$$\text{Capacity of the cistern} = \frac{\text{flow rate in 1 min by waste pipe}}{\text{work done by waste pipe in 1 min}}$$

$$= \frac{14}{\frac{7}{20}} = \mathbf{40\ litres.}$$

Solved Examples

E-1 A fill pipe can fill $\frac{3}{4}$ of cistern in 12 minutes. In how many minutes can it fill $\frac{1}{2}$ of cistern?

S-1 Alone time = $\frac{\text{time of open}}{\text{work done}}$ (Concept A-1)

For the same pipe, alone time will be fixed, so,

$$\frac{\text{time of open}_1}{\text{work done}_1} = \frac{\text{time of open}_2}{\text{work done}_2}$$

$$\Rightarrow \frac{12}{\frac{3}{4}} = \frac{x}{\frac{1}{2}}$$

$$\Rightarrow x = \mathbf{8\ minutes.}$$

E-2 A tap can fill a cistern in 20 minutes and another can fill in 30 minutes. If both are opened simultaneously, find the time when the cistern will be full.

S-2 Work done per minute = $\dfrac{1}{\text{alone time}}$

For two fill pipes, *[Refer concept (B-2)]*

$$\text{total work done in 1 minute} = \frac{1}{20} + \frac{1}{30}$$

$$= \frac{1}{12}$$

$$\text{Time to fill} = \frac{1}{\text{work done in 1 minute}}$$

$$= \mathbf{12\ minutes.}$$

Note: In fact, if total work done in 1 minute = $\dfrac{x}{y}$, then total time to complete it = $\dfrac{y}{x}$.

Short Cut

If two pipes can fill a cistern in 'a' and in 'b' minutes respectively, then both pipes opened together can fill it in $\dfrac{ab}{a+b}$ minutes.

E-3 Two fill pipes A and B can fill a cistern in 12 and 16 minutes respectively. Both fill pipes are opened together, but 4 minutes before the cistern is full, one pipe A is closed. How much time will the cistern take to fill?

S-3 Let the cistern be filled in T minutes,

$\therefore$ Pipe B is opened for T minutes and pipe A is opened for $T-4$ minutes. Using the concept (C) of time and work,

A's amount of work + B's amount of work = 1 (Since work is completed)

$$\Rightarrow \frac{A\text{'s open time}}{\text{Alone time for } A} + \frac{B\text{'s open time}}{\text{Alone time for } B} = 1$$

$$\Rightarrow \frac{T-4}{12} + \frac{T}{16} = 1$$

$$\Rightarrow T = \frac{64}{7}$$

$\therefore$ The cistern is filled in $9\frac{1}{7}$ minutes.

E-4 Two pipes A and B can fill a cistern in 10 and 15 minutes respectively, but an empty pipe C can empty it in 5 minutes. The pipes A and B are kept open for 4 minutes and then the empty pipe C is also opened. In what time is the cistern emptied?

S-4 Let the cistern be emptied in T minutes after opening the empty pipe C.

$\therefore$ Pipe C is opened for 'T' minutes and Pipes A and B are opened for $(T + 4)$ minutes.

Now, since all the pipes are not opened for the same time, so, concept (C) of 'Time and Work' is to be used.

Here, fill pipes and empty pipe are connected to same cistern, and cistern gets emptied, so, it implies that
total fill work done = total empty work

$$\Rightarrow \frac{A\text{'s open time}}{\text{alone time for } A} + \frac{B\text{'s open time}}{\text{alone time for } B} = \frac{C\text{'s open time}}{\text{alone time for } C}$$

$$\Rightarrow \frac{T+4}{10} + \frac{T+4}{15} = \frac{T}{5}$$

$$\Rightarrow T = 20$$

The cistern emptied **20 minutes** after opening the pipe C.

E-5 Two pipes P and Q can fill a cistern in 3 and 6 minutes respectively, while an empty pipe R can empty the cistern in 4 minutes. All the three pipes are opened together and after 2 minutes pipe R is closed. Find when the tank will be full.

S-5 Here, all three pipes are not opened for same time, so, concept (C) of 'Time and Work' is to be used.
Let the tank be filled up in 'T' minutes. Since the tank is getting filled up, so
total work done = 1

$\Rightarrow$ P's work + Q's work + R's work = 1

$$\Rightarrow \frac{P\text{'s open time}}{\text{alone time of } P} + \frac{Q\text{'s open time}}{\text{alone time of } Q} - \frac{R\text{'s open time}}{\text{alone time of } R} = 1 \quad \text{[(–)ve for empty work]}$$

$$\Rightarrow \frac{T}{3} + \frac{T}{6} - \frac{2}{4} = 1$$

$$\Rightarrow T = 3$$

$\therefore$ The cistern is filled in **3 minutes** from the start.

E-6 There is a leak in the bottom of a cistern. Before the leak, it could be filled in $4\frac{1}{2}$ hours. It now takes $\frac{1}{2}$ hour longer. If the cistern is full, how long would the leakage empty the full cistern?

S-6 The leak is the outlet
Using concept (B-3),
(work done by inlet in 1 hour) – (Work done by outlet in 1 hour) = Net work done in 1 hour

$$\Rightarrow \frac{1}{4\frac{1}{2}} - \text{(work done by leak in 1 hour)} = + \frac{1}{\left(4\frac{1}{2} + \frac{1}{2}\right)} \quad \text{[(+)ve, for cistern gets filled up]}$$

$$\Rightarrow \text{work done by leak in 1 hour} = \frac{1}{4\frac{1}{2}} - \frac{1}{5}$$

$$\Rightarrow \text{work done by leak in 1 hour} = \frac{1}{45}$$

$\therefore$ The leak can empty the full cistern in $\frac{45}{1}$ i.e. **45 hours.**

E-7 Two pipes can fill a cistern in 14 and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom, 32 minutes extra are taken for the cistern to be filled up. If the cistern is full, in what time would the leak empty it?

S-7 Normal fill time, without leak = $\dfrac{1}{\frac{1}{14}+\frac{1}{16}} = \dfrac{14\times 16}{14+16} = \dfrac{112}{15}$ hour.

Extra time, due to leak = 32 minutes = $\dfrac{32}{60}$ hour = $\dfrac{8}{15}$ hour.

The leak is the outlet. Fill pipes are inlet
Using concept (B-3),
(work done by inlet in 1 hour) – (work done by outlet in 1 hour) = Net work done in 1 hour

$$\Rightarrow \left(\frac{1}{14}+\frac{1}{16}\right) - \text{(work done by leak in 1 hour)} = \frac{1}{\underbrace{\left(\frac{112}{15}+\frac{8}{15}\right)}_{\text{total fill time, with leak}}} = \frac{1}{8}$$

$$\Rightarrow \text{work done by leak in 1 hour} = \frac{1}{14}+\frac{1}{16}-\frac{1}{8} = \frac{1}{112}$$

$\therefore$ The leak can empty the full cistern in **112 hours.**

E-8 Two pipes A and B can fill a tank in 20 and 30 hours respectively. Both the pipes are opened to fill the tank but when the tank is one-third full, a leak develops in the tank through which one-fourth water supplied by both pipes goes out. Find the total time taken to fill the tank.

S-8 Time taken to fill $\frac{1}{3}$rd of tank = $\dfrac{1}{3}\left(\dfrac{20\times 30}{20+30}\right)$ = 4 hrs.

Now, a leak develops, and $\frac{1}{4}$ th of water supplied by A and B is drained out through leak. So, $\left(1-\frac{1}{4}\right)$ $= \frac{3}{4}$ th of water supplied by A and B is actually filling the tank.

$$\therefore \text{Time taken to fill the remaining } \frac{2}{3}\text{rd of tank} = \frac{\frac{2}{3}\left(\frac{20\times 30}{20+30}\right)}{\underbrace{\frac{3}{4}}_{\text{(since } \frac{3}{4} \text{ th of water supplied by full pipes is actually used to fill)}}}$$

$$= \frac{32}{3} \text{ hour}$$

Total time taken to fill the tank $= 4 + \frac{32}{3} = 14\frac{2}{3}$ hour.

E-9 A ship 55 kms. from the shore springs a leak which admits 2 tonnes of water in 6 min.; 80 tonnes would suffer to sink her, but the pumps can throw out 12 tonnes an hour. Find the average rate of sailing that she may just reach the shore as she begins to sink. **(MBA '92)**

S-9 Rate of admission of water $= \frac{2}{6}$ tonnes/min. $= \frac{1}{3}$ tonnes/min.

Rate of pumping out of water $= \frac{12}{60}$ tonnes/min. $= \frac{1}{5}$ tonnes/min.

Rate of accumulation $= \left(\frac{\cancel{2}^{1}}{\cancel{6}_{3}} - \frac{\cancel{12}^{1}}{\cancel{60}_{5}}\right)$ tonnes/min.

Time to accumulate 80 tonnes of water $= \frac{\text{Amount of water}}{\text{Accumulation rate}}$

$$= \frac{80}{\left(\frac{1}{3} - \frac{1}{5}\right)} = 600 \text{ min.} = 10 \text{ hours}$$

$\therefore$ Average sailing rate so as avoid sinking $= \frac{\text{Distance}}{\text{Time}} = \frac{55}{10}$ km/h = **5.5 km/h**.

E-10 A cistern can be filled by two pipes filling separately in 12 and 16 minutes respectively. Both pipes are opened together for a certain time but being clogged, only $\frac{7}{8}$ of full quantity water flows through the former and only $\frac{5}{6}$ through the latter pipe. The obstructions, however, being suddenly removed, the cistern is filled in 3 minutes from that moment. How long was it before the full flow began? **(AAO '82)**

S-10 Let both pipes remain clogged for x minutes and hence full flow began after x minutes only.

$\therefore$ Part of cistern filled in x minutes + Part of cistern filled in 3 minutes = Cistern filled

$$\Rightarrow \quad \left[\frac{7}{8} \times \left(\frac{x}{12}\right) + \frac{5}{6} \times \left(\frac{x}{16}\right)\right] + \left[\frac{3}{12} + \frac{3}{16}\right] = 1$$

$$\Rightarrow \quad \frac{12x}{96} + \frac{7}{16} \Rightarrow x = \mathbf{4.5 \text{ minutes}}.$$

REGULAR PROBLEMS

(1) A cistern is filled in 9 hours and it takes 10 hours when there is a leak in its bottom. If the cistern is full, in what time shall the leak empty it?

(a) 90 h (b) 94 h (c) 92 h (d) 91 h (e) None of these