(19) A broker sold Rs 5,000 stock at $94\frac{5}{8}$. If the brokerage is $\frac{1}{8}\%$ find the amount of brokerage and sale proceeds of the stock holder.

(20) A man invested Rs 3,300 when he bought Rs 100 shares at Rs 110. If 15% dividend is declared, find his annual income.

Hint: Refer Table 2.

(21) A man invested Rs 12,000 in 11.5% debentures of face value Rs 100 and available market value is Rs 120. Find his annual income.

Hint: See E.17.

(22) Find the income per cent on 8% debenture of face value Rs 100 and available at Rs 80 each.

(23) A man sold 20% debentures worth Rs 7,500 at 5% premium. Find the cash released from this sale, if brokerage is 2%.

(24) A man buys 500 debentures of face value Rs 100 each at Rs 95 and sells the same when the price rises to Rs 98. If brokerage is 2% find his gain or loss.

Hint: Purchase cost = Market value$_1$, (1 + % brokerage)

Sell cost = Market value $_2$ (1 – % brokerage)

$\therefore$ Gain/Loss per share = Purchase Cost – Sell Cost.

Answers

1. (a) Rs 1, 900 (b) 1, 470 **2.** 400 **3.** $93\frac{5}{8}$ **4.** Rs 4,190 **5.** 1,580 **6.** 100 **7.** 120 **8.** Rs 450 **9.** $10\frac{1}{2}\%$ **10.** 63.75 **11.** $12\frac{1}{2}\%$ **12.** (ii) **13.** Rs 6,000 **14.** Rs 3,780, Rs 4,320 **15.** Rs 960 **16.** Rs 2,70,000 **17.** 3% **18.** 1,000 **19.** Rs 625, Rs 4,725 **20.** 450, **21.** 1,150 **22.** 10 **23.** Rs 7,717.50 **24.** 430

REAL PROBLEMS

(1) The cost of Rs S, $p\%$ stock at Rs M is

(a) $\frac{MS}{100}$ (b) $\frac{M}{S}$ (c) $\frac{p}{M} \times 100$ (d) $\frac{ps}{M}$ (e) $\frac{ps}{100}$

(2) The cost of Rs 1,500, $5\frac{1}{2}$ stock at 98, is

(a) Rs 1,550 (b) Rs 1,470 (c) Rs 1,420 (d) Rs 1,440 (e) Rs. 1,580

(3) The cash required to purchase Rs 1,600, $5\frac{1}{2}\%$ stock at $3\frac{1}{4}$ premium, $\left(\text{brokerage } \frac{1}{4}\%\right)$ is

(a) Rs 1,656 (b) Rs 1,650 (c) Rs 1,662 (d) Rs 1,590 (e) Rs 1,564

(4) The cash realised by selling Rs 2,500, 5% stock at $110\frac{1}{4}$ $\left(\text{brokerage } \frac{1}{4}\%\right)$ is **(RRB '84)**

(a) Rs 2,600 (b) Rs 2,650 (c) Rs 2,750 (d) Rs 2,350 (e) Rs 2,400

(5) The purchase price of a Rs 100 stock at 6 discount $\left(\text{brokerage } \frac{1}{2}\%\right)$ is

(a) Rs 100 (b) Rs 94.50 (c) Rs 106.5 (d) Rs 94 (e) Rs 106

(6) The income derived from a $6\frac{1}{2}\%$ stock at 94 is
(a) Rs 6 (b) Rs 6.50 (c) Rs 6.25 (d) Rs 5.80 (e) Rs 6.75

(7) A 3% stock yields 4%. The market value of the stock is **(SSC, '88)**
(a) 75 (b) 133 (c) 80 (d) 120 (e) 104

Hint: Market Value = $\frac{3}{4} \times 100 = 75$, Refer Eq. (3).

(8) A $4\frac{1}{2}\%$ stock should be purchased at what price to yield 5% interest?
(a) Rs 80 (b) Rs 111 (c) Rs 90 (d) Rs 75 (e) Rs 110

(9) If a man earns Rs 270 by investing Rs 7,800 in $4\frac{1}{2}\%$ stock, brokerage being $\frac{1}{8}\%$, the market value of stock is
(a) Rs 130 (b) Rs $130\frac{1}{8}$ (c) $129\frac{7}{8}$ (d) $129\frac{1}{8}$ (e) $131\frac{1}{8}$

(10) To produce an annual income of Rs 600 in a $4\frac{1}{2}\%$ stock at $94\frac{1}{2}$ the amount of stock needed is
(a) Rs 12,600 (b) Rs 13,000 (c) Rs 12,000 (d) Rs 10,000 (e) Rs 13,200

(11) Find the rate % on investment, of a 4% stock at $79\frac{7}{8}$, brokerage = $\frac{1}{8}\%$.
(a) 4 % (b) 5 % (c) $4\frac{1}{2}\%$ (d) $5\frac{1}{2}\%$ (e) $3\frac{1}{2}\%$

Hint: Rate % on investment = $\frac{\%\text{ rate of stock}}{\text{Market Value + brokerage}}$.

(12) Which is better investment, (i) $4\frac{1}{2}\%$ stock at 90 or (ii) 4% stock at 88? **(UDC, '81)**
(a) i (b) ii
(c) Data insufficient (d) None of these

(13) The rate % obtained from 4% stock at 88 and from $4\frac{1}{2}\%$ stock at M are equal. The value of M (purchase cost) is **(MBA, '85)**
(a) 92 (b) 99 (c) 96 (d) 102 (e) 111

Hint: $\frac{88}{4} = \frac{M}{4\frac{1}{2}}$ Refer Eq. (3).

(14) If the annual income from 5% stock at 112 is Rs 42 more than 4% stock at 98, then the investment is
(a) Rs 12,000 (b) Rs 11,000 (c) Rs 10,976 (d) Rs 12,032 (e) Rs 11,048

Hint: Investment = $\frac{\text{Income difference}}{\left(\frac{5}{112} - \frac{4}{98}\right)}$.

Answers

1. (a) **2.** (b) **3.** (a) **4.** (c) **5.** (b) **6.** (b) **7.** (a) **8.** (c) **9.** (c)
10. (a) **11.** (b) **12.** (a) **13.** (b) **14.** (c)

17 TIME AND DISTANCE

17.1 DEFINITION

The speed of a moving body is the distance travelled by it in unit time. Hence,

$$\text{Speed } V = \frac{\text{Total distance travelled } (d)}{\text{Total time taken } (t)}$$

or Total distance travelled = speed × total time

$$d = V \times t$$

17.2 AVERAGE SPEED

If a moving body travels $d_1, d_2, d_3, \ldots d_n$ metres, moving with different speeds $V_1, V_2, V_3, \ldots V_n$ metres/ seconds in time $t_1, t_2, t_3, \ldots t_n$ seconds respectively, then it is necessary to calculate the average speed of the body throughout the journey. If the average speed is denoted by V_a, then,

$$V_a = \frac{\text{Total Distance travelled}}{\text{Total Time taken}}$$

		Average speed for whole journey
If known values are $\begin{array}{c\|c\|c\|c} d_1 & d_2 & \ldots & d_n \\ \hline t_1 & t_2 & \ldots & t_n \end{array}$	then,	$V_a = \dfrac{d_1 + d_2 + \ldots + d_n}{t_1 + t_2 + \ldots + t_n}$
If known values are $\begin{array}{c\|c\|c\|c} V_1, t_1 & V_2, t_2 & \ldots & V_n, t_n \end{array}$	then,	$V_a = \dfrac{V_1 t_1 + V_2 t_2 + \ldots + V_n t_n}{t_1 + t_2 + \ldots + t_n}$ since $d = vt$
If known values, are $\begin{array}{c\|c\|c\|c} d_1 & d_2 & \ldots & d_n \\ \hline V_1 & V_2 & \ldots & V_n \end{array}$	then,	$V_a = \dfrac{d_1 + d_2 + \ldots + d_n}{\dfrac{d_1}{V_1} + \dfrac{d_2}{V_2} + \ldots + \dfrac{d_n}{V_n}}$ since $t = \dfrac{d}{V}$

Example: A car travels 600 km in 11 hours and another 800 km in 17 hours. Find the average speed of the during the entire journey.

Solution: Here, distance and time are known.

$$\therefore \quad \text{average speed} = \frac{d_1 + d_2 + \ldots + d_n}{t_1 + t_2 + \ldots t_n}$$

$$= \frac{600 + 800}{11 + 17}$$

$$= 50 \text{ km/hour}$$

$\therefore$ the average speed is 50 km per hour.

Example: A bus moves 300 km at a speed of 45 km per hour and then it increases its speed to 60 km per hour to travel another 500 km. Find the average speed of the bus.

Solution: Here distance and speed are known. Time is not known.

$\therefore$ we use the relation,

$$\text{average speed} = \frac{d_1 + d_2 + \ldots + d_n}{\dfrac{d_1}{V_1} + \dfrac{d_2}{V_2} + \ldots + \dfrac{d_n}{V_n}}$$

As the time is not known, so the denominator of the previous problem, i.e. time, has been replaced by $\frac{\text{distance}}{\text{speed}}$. Hence, t_1, t_2, ... t_n have been replaced by $\frac{d_1}{V_1}, \frac{d_2}{V_2} \ldots \frac{d_n}{V_n}$

$$\Rightarrow \quad \text{average speed} = \frac{300 + 500}{\dfrac{300}{45} + \dfrac{500}{60}}$$

$$= \frac{800}{\dfrac{20}{3} + \dfrac{25}{3}}$$

$$= \frac{800 \times 3}{45}$$

$$= 53\frac{1}{3} \text{ km per hour.}$$

$\therefore$ the average speed is $53\frac{1}{3}$ km per hour.

17.3 DISTANCE COVERED IS SAME

Let a body, moving with two different speeds, say V_1 and V_2, cover the same distance d, in time t_1 and t_2 respectively then,

$$\text{distance } d = \text{speed} \times \text{time} = V_1t_1 = V_2t_2$$

(a) $d = V_1t_1 = V_2t_2 \quad \Rightarrow \quad d^2 = V_1V_2t_1t_2$

(b) Now, $V_1t_1 = V_2t_2$

$$\Rightarrow \quad \frac{V_1}{t_2} = \frac{V_2}{t_1}$$

$$\Rightarrow \quad \frac{V_1}{t_2} = \frac{V_2}{t_1} = \frac{V_1 + V_2}{t_2 + t_1} = \frac{V_1 - V_2}{t_2 - t_1} \quad \text{(By componendo and Dividendo)}$$

$$\Rightarrow \quad \frac{V_1}{t_2} = \frac{V_2}{t_1} = \frac{\text{sum of speed}}{\text{sum of time}} = \frac{\text{difference of speed}}{\text{difference of time}}$$

(c) Combining (a) and (b), we get another useful relation:

$$\frac{d}{\text{product of time}} = \frac{\text{product of speed}}{d} = \frac{\text{difference of speed}}{\text{difference of time}}$$

Memory Tip 'Speed' terms are in numerator and 'time' terms are in the denominator

(d) To find the distance when absolute values of speed and time are not known.
When speed is increased by x, the time reduces by (a), to cover the same distance (d) and when speed is decreased by y, the time increases by (b), to cover the same distance (d) then,

$$\text{distance} = d = \frac{(x+y)(a+b)}{\underbrace{(xb - ya)^2}_{\rightarrow \text{Follow the arrows}}} \times xyab$$

Example: An increase in the speed of a car by 10 km per hour saves 30 minutes in a journey of 100 km. Find the initial speed of the car.

Solution: Here, distance, difference in time and change (difference) in speed are known, but speed* is to be found out.

Let the initial speed of the car = V km/h

The new speed of the car = $(V + 10)$ km/hr.

Using the relation ('.3 – (c)), (as speed* is to be found out)

$$\frac{\text{product of speeds*}}{d} = \frac{\text{difference in speed}}{\text{difference in time}}$$

$$\Rightarrow \quad \frac{V(V+10)}{100} = \frac{10}{\frac{30}{60}} \qquad \left(\because 30 \text{ minutes} = \frac{30}{60} \text{ hour}\right)$$

$\Rightarrow \quad V(V + 10) = 2{,}000$

$\Rightarrow \quad V(V + 10) = 40 \times 50$

Without further solving, it appears that $V = 40$ km/h. Quadratic

$\therefore$ the initial speed of the car is 40 km per hour.

Example: Due to bad road, the speed of a tourist bus is reduced by 12 km/hr and it now takes $2\frac{1}{2}$ hours more to cover the same distance of 600 km. Find the time it now takes to cover the distance.

Solution: This problem is slightly different from the previous example, because here the time* is to be found out. So, the relation (19.3 – c) will be used, but, the parts to be equated will be different.

$$\frac{d}{\text{product of times}^*} = \frac{\text{difference in speed}}{\text{difference in time}}$$

Let the initial time = t hour

Then the new time = $\left(t + 2\frac{1}{2}\right)$ hour

$$\Rightarrow \quad \frac{600}{t\left(t+\frac{5}{2}\right)} = \frac{12}{\frac{5}{2}}$$

$$\Rightarrow \quad t\left(t+\frac{5}{2}\right) = 125$$

$$\Rightarrow \quad t = 10$$

$\therefore$ the required time = $10 + 2\frac{1}{2} = 12\frac{1}{2}$ hours

17.4 DISTANCE COVERED IS DIFFERENT

Let the moving body covers the distances d_1 and d_2 at speeds V_1 and V_2 in time t_1 and t_2 respectively. Using distance = speed × time, we can make the various arrangements as

① $d_1 = V_1 t_1$	② $d_2 = V_2 t_2$	→ **distance reference**	Specific use of these three references has been explained in the subsequent examples.
① $t_1 = \frac{d_1}{V_1}$	② $t_2 = \frac{d_2}{V_2}$	→ **time reference**	
① $V_1 = \frac{d_1}{t_1}$	② $V_2 = \frac{d_2}{t_2}$	→ **speed reference**	

Example: A car travels for 11 hours. Out of this, it travels 100 km at a certain speed and then it increases its speed by 15 km/hr to cover the remaining 280 km. Find the time it takes to travel the span of 280 km.

Solution: Given that

Speed difference	total time	d_1	d_2
$V_1 - V_2$	$t_1 + t_2$	100	280
15	11		

↓

Use $(V_1 - V_2)$, because, here V_1 and V_2 are *not* to be found out separately. (speed reference is used)

$$V_1 - V_2 = \frac{d_1}{t_1} - \frac{d_2}{t_2}$$

$$\Rightarrow \quad 15 = \frac{280}{x} - \frac{100}{11-x} \quad \text{(assume, } x = \text{time to travel 280 km)}$$

$$\Rightarrow \quad 3 = \frac{56}{x} - \frac{20}{11-x}$$

$$\Rightarrow \quad x = 7$$

$\therefore$ the car travels the span of 280 km in 7 hours

Example: A car travels a distance of 170 km in 2 hours partly at a speed of 100 km/hr and partly at 50 km hr. Find the distance travelled at speed of 100 km/hr.

Solution: Given that:

total distance	total time		
$(d_1 + d_2)$	$(t_1 + t_2)$	V_1	V_2
170	2	100	50

$\downarrow$

Use $(t_1 + t_2)$, because, here, t_1 and t_2 are *not* to be found out separately.

$$\therefore \quad t_1 + t_2 = \frac{d_1}{V_1} + \frac{d_2}{V_2} \quad \text{(Time reference is used)}$$

$$\Rightarrow \quad 2 = \frac{x}{100} + \frac{170 - x}{50} \quad \text{(assume } x = \text{distance covered at 100 km/hr)}$$

$$\Rightarrow \quad x = 60$$

$\therefore$ The car travelled 60 km at the speed of 100 km/hr.

140

Example: If a truck travels a distance of 240 km in 6 hours, partly at a speed of 60 km/hr and partly at 30 km/hr, then find the time for which it travels at 60 km/hr.

Solution: Given that

total distance	total time		
$d_1 + d_2$	$t_1 + t_2$	V_1	V_2
240	6	60	30

$\downarrow$

Use $(d_1 + d_2)$, because here d_1 and d_2 are not to be found out separately.

So, using distance reference, we find,

$$d_1 + d_2 = V_1 t_1 + V_2 t_2.$$

$$\Rightarrow \quad 240 = 60 \times x + 30 \times (6 - x), \quad \text{(assume } x = \text{time of travel at 60 km/hr)}$$

$$\Rightarrow \quad x = 2$$

$\therefore$ The truck travels 2 hours at 60 km/hr

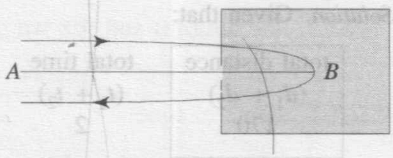
17.5 STOPPAGE TIME PER HOUR FOR A TRAIN

For the same distance of travel, if a train runs
at average speed V_1 km/hr, without stopping
and at average speed V_2 km/hr, with stoppage, then

$$\text{stoppage time per hour} = \frac{V_1 - V_2}{V_1} \text{ hour.}$$

$$= \frac{\text{difference in speed}}{\text{faster speed}} \text{ hour}$$

17.6 TIME TAKEN WITH TWO DIFFERENCE MODES OF TRANSPORT



A man uses one or two different kinds of transport for going from A to B and back to A again.
He has three options available for the above journey:
Option

1. He uses fast transport both ways (i.e. A to B and back to A)
2. He uses slow transport both ways
3. He uses mixed transport i.e. fast transport for one way and slow transport for the other way.

(Time taken by any one transport both ways – time taken by mixed transport) = time gained or lost*
[* in case of time loss, put a (–) ve sign before it.]

Example: A man takes 4 hours 30 minutes in walking to a certain place and riding back. He would have gained 1 hour 45 minutes by riding both ways. How long would he take to walk both ways?

Solution: Using the relation (17.6),
time for walking bothways – time by mixed (i.e. walking + riding) = time gained

$$\Rightarrow \quad \text{time for walking both ways } -4\frac{1}{2} = +1\frac{3}{4}$$

$$\Rightarrow \quad \text{time for walking both ways} = 4\frac{1}{2} + 1\frac{3}{4}$$

$$= 6\frac{1}{4} \text{ hours.}$$

$\therefore$ The man will take $6\frac{1}{4}$ hours to cover the same distance if he walks both ways.

17.7 TIME AND DISTANCE BETWEEN TWO MOVING BODIES

Let there be two persons, A and B.
Speed of $A = V_1$ km/hr
Speed of $B = V_2$ km/hr
If they walk in same direction,
A and B will be $(V_1 - V_2)$ km apart in 1 hour

$\Rightarrow$ A and B will be 1 km apart in $\boxed{\dfrac{1}{V_1 - V_2}}$ hour

$\Rightarrow$ A and B will be x km apart in $\boxed{\dfrac{x}{V_1 - V_2}}$ hour

Similarly, if they walk in opposite directions, then
A and B will be $(V_1 + V_2)$ km apart in 1 hour

$\Rightarrow$ A and B will be 1 km apart in $\boxed{\dfrac{1}{V_1 + V_2}}$ hour

$\Rightarrow$ A and B will be x km apart in $\boxed{\dfrac{x}{V_1 + V_2}}$ hour

Example: Two men, P and Q, start walking from an hotel at 2 km and $2\frac{1}{2}$ km an hour respectively. By how many km will they be apart at the end of $3\frac{1}{2}$ hours, if

(i) they walk in opposite directions
(ii) they walk in the same direction.

Solution:

(i) When they walk in opposite direction, then P and Q will be $\left(2 + 2\frac{1}{2}\right)$ km or $4\frac{1}{2}$ km apart in 1 hour.

$\therefore$ at the end of $3\frac{1}{2}$ hours, they will be $3\frac{1}{2} \times 4\frac{1}{2}$ km $= 15\frac{3}{4}$ km apart.

(ii) When they walk in the same direction, then P and Q will be $\left(2\frac{1}{2} - 2\right)$ km or $\frac{1}{2}$ km apart in 1 hour

$\therefore$ at the end of $3\frac{1}{2}$ hours, they will be $\frac{1}{2} \times 3\frac{1}{2}$ km $= 1\frac{3}{4}$ km, apart.

Solved Examples

E-1 Find the distance covered by a man walking for 12 min. at a speed of 3.5 km/h.

S-1 Distance = Speed × time $= 3.5 \times \frac{12}{60}$ $\left[12 \text{ minutes} = \frac{12}{60} \text{ hour}\right]$

$= \frac{7}{10}$ km = **700 metres.**

E-2 Find the time taken to cover a distance of 124 km by a train moving at 45 km/h.

S-2 Time $= \dfrac{\text{Distance}}{\text{speed}} = \dfrac{124}{45}$ hour

$= 2\frac{34}{45}$ hour $= 2 + \left(\frac{34}{45} \times 60\right)$ min.

= 2 hour 45 min.

E-3 Find the time taken to cover a distance of 360 km by a train moving at 20 metres/second.

S-3 $V = 20$ metre/second, $d = 360$ km $= 360 \times 1000$ metres

$\therefore$ Time taken

$= \dfrac{360 \times 1{,}000}{20}$ seconds.

$= \dfrac{360 \times 1{,}000}{20 \times 60 \times 60}$ hours = **5 hours**

E-4 A man is walking at a speed of 10 km per hour. After every kilometre, he takes rest for 5 min. How much time will be take to cover a distance of 5 km? **[RBI, '82]**

S-4 Rest time = Number of rest × Time for each rest

$= 4 \times 5 = 20$ min.

Total time to cover 5 km $= \left(\dfrac{5}{10} \times 60\right)$ min. + 20 min. = **50 minutes.**

E-5 Walking $\dfrac{5}{7}$ of his usual rate, a boy reaches his school 6 min. late. Find his usual time to reach school.

S-5 Since, the boy now walks at $\dfrac{5}{7}$ of usual speed, so he will take $\dfrac{7}{5}$ of his usual time

$\Rightarrow$ extra time $= \left(\dfrac{7}{5} - 1\right)$ usual time = 6 minutes (known)

$\Rightarrow$ $\dfrac{2}{5} \times$ usual time = 6

$\Rightarrow$ usual time = **15 minutes**

E-6 If I walk at 4 km/h, I miss the bus by 10 min. If I walk at 5 km/h, I reach 5 min. before the arrival of the bus. How far I walk to reach the bus stand ?

S-6 Here $V_1 = 4$ km/h $V_2 = 5$ km/h

$\therefore$ change in speed = 1

Here difference of time = 10 – (–5) (–ve sign indicates before the schedule time)

$= 15$ min. $= \dfrac{15}{60}\text{h} = \dfrac{1}{4}\text{h}$

distance $= d = ?$

Using the formula, $\dfrac{\text{product of speeds}}{d} = \dfrac{\text{difference in speeds}}{\text{difference in time}}$ *[Refer 17.3 (c)]*

$\Rightarrow \dfrac{4 \times 5}{d} = \dfrac{1}{\frac{1}{4}} \quad \Rightarrow \quad d = 5$

$\therefore$ The distance of bus stand is **5 km**.

E-7 A train does a journey without stopping in 8 hours. If it had travelled 5 km an hour faster, it would have done the journey in 6 hours 40 min. What is its slower speed? **[Bank PO, '86]**

S-7 Let its slower speed $= V$ km per hour, *[Refer 17.3]*

Here distance is same in both the cases.

Using the formula, $\Rightarrow$ $V_1 \times t_1 = V_2 \times t_2$ (= fixed distance)

$$\Rightarrow \quad V \times 8 = (V + 5) \times \frac{20}{3} \qquad \left(6 \text{ hour } 40 \text{ minutes} = \left(6 + \frac{40}{60}\right) \text{hour}\right)$$

$$\Rightarrow \quad 24V = (V + 5) \times 20 \quad \Rightarrow \quad V = 25 \text{ km/h}.$$

$\therefore$ Slower speed of train is **25 km/h.**

E-8 Without stoppages, a train travels certain distance with an average speed of 80 km/h, and with stoppages, it covers the same distance with an average speed of 60 km/h. How many min. per hour the train stops?

S-8 Here, distance to be covered is constant *[Refer 17.5]*
Hence, using the formula,

$$\text{Stoppage time/hour} = \frac{\text{Change in speed}}{\text{Faster speed}} \text{ hour}$$

$$= \frac{80 - 60}{80} \text{ hour} = \frac{1}{4} \text{ hour} = 15 \text{ minutes}$$

$\therefore$ The train stops **15 minutes/hour**

E-9 A man covers a certain distance on a toy train. If the train moved 4 km/h faster, it would take 30 min. less. If it moved 2km/h slower, it would have taken 20 min. more. Find the distance.

S-9 [Refer number 17.3 (d)] Using the formula,

$$d = \frac{(x + y)(a + b)\, abxy}{(bx - ay)^2}$$

where,

$$x = 4, \quad a = 30 \text{ minutes} = \frac{30}{60} \text{ hour} = \frac{1}{2}$$

$$y = 2, \quad b = 20 \text{ minutes } \frac{20}{60} \text{ hour} = \frac{1}{3}$$

$$d = \frac{(4+2) \times \left(\frac{1}{2} + \frac{1}{3}\right) \times 4 \times 2 \times \frac{1}{2} \times \frac{1}{3}}{\left(4 \times \frac{1}{3} - 2 \times \frac{1}{2}\right)^2}$$

$\Rightarrow$ $d = 60$. Hence the distance is **60 km.**

E-10 A thief is stopped by a policeman from a distance of 400 metres. When the policeman starts the chase, the thief also starts running. Assuming the speed of the thief as 10 km/h and that of police man as 15 km/h, how far the thief would have run, before he is overtaken?

S-10 Let the thief be overtaken after covering the distance = x metres
Both the policemen and the thief ran for same time.
Time of run for police = Time of run for thief

$$\Rightarrow \quad \frac{x+400}{15} = \frac{x}{10} \qquad \left(\text{Since time} = \frac{\text{distance}}{\text{speed}}\right)$$

$$\Rightarrow \quad x = 800$$

$\therefore$ The thief ran **800 metres** before he was over-taken.

E-11 A train travels 450 km in 7 hours and another 740 km in 10 hours. Find the average speed of train.

S-11 Using the formula, for two moving bodies, *[Refer 17.2]*

$$\text{Average speed} \quad V_a = \frac{d_1 + d_2}{(t_1 + t_2)}$$

$$= \frac{450 + 740}{(7 + 10)} = \frac{1{,}190}{17} \text{ km/h}$$

$$= \mathbf{70 \text{ km/h}}.$$

E-12 If a car moves from A to B at a speed of 60 km/h and comes back from B to A at a speed of 40km/h, then find its average speed during the journey.

S-12 Here both distances are same.
So, using the formula,

$$V_a = \frac{2V_1V_2}{V_1 + V_2} = \frac{2 \times 60 \times 40}{100} = \mathbf{48 \text{ km/h}}.$$

E-13 A car during its journey travels 30 min. at a speed of 40 km/h, another 45 min. at a speed of 60 km/h and 2 hours at a speed of 70 km/h. Find its average.

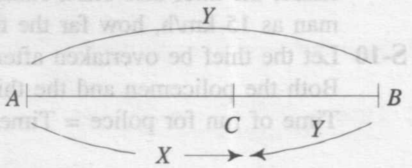
S-13 Using the formula, $V_a = \dfrac{V_1t_1 + V_2t_2 + V_3t_3}{t_1 + t_2 + t_3}$ *[Refer 17.2]*

we get,
$$V_a = \frac{\left(\frac{30}{60} \times 40\right) + \left(\frac{45}{60} \times 60\right) + (2 \times 70)}{\frac{30}{60} + \frac{45}{60} + 2}$$

$$\Rightarrow \quad V_a = \frac{20 + 45 + 140}{\frac{30 + 45 + 120}{60}} = \frac{205}{195} \times 60 \text{ km/h} = \mathbf{63 \text{ km/h}}.$$

E-14 Two friends X and Y walk from A to B at a distance of 39 km, at 3 km an hour and $3\frac{1}{2}$ km an hour respectively. Y reaches B, returns immediately and meet X at C. Find the distance from A to C.

S-14 When Y meets X at C, Y has walked the distance $AB + BC$ and X has walked the distance AC.
So, both X and Y have walked together a distance
$= 2 \times AB = 2 \times 39 = 78$ km

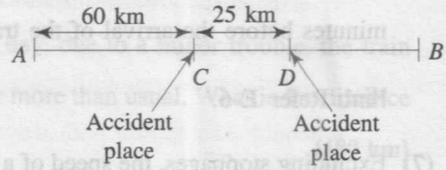


The ratio of the speeds of X and Y is $3 : 3\frac{1}{2}$ i.e. $\frac{6}{7}$

Hence, the distance travelled by $X = AC = \dfrac{6}{6+7} \times 78 = \mathbf{36 \text{ km}}.$

E-15 A train after travelling 60 km meets with an accident and then proceeds at $\frac{3}{4}$ of its former rate and arrives at the terminus 40 minutes late. Had the accident happened 25 km further on, it would have arrived 10 minutes sooner. Find the speed of the train and the distance.

S-15 By travelling at $\frac{3}{4}$ of its original rate, the train would take $\frac{4}{3}$ of its original (usual) time



$\Rightarrow$ the train takes $\frac{1}{3}$ of original (usual) time more.

When the accident takes place at C, then after the accident, distance covered = CB

When the accident takes place at D, then after the accident, distance covered = DB

As per question, $CB - DB = 25$ km $= CD$

and $\frac{1}{3}$ of usual time to cover $CD = 10$ minutes (because it reaches 10 minutes sooner for not covering CD in second case)

$\Rightarrow$ Usual time to cover $CD = 10 \times 3 = 30$ minutes

$\Rightarrow$ In 30 minutes, train covers $CD = 25$ km

$\therefore$ speed of train per hour $= \frac{25}{30} \times 60 = 50$ km

Now, $\frac{1}{3}$ of usual time to cover $CB = 40$ min

$\Rightarrow$ usual time to cover $CB = 120$ min $= 2$ hours.

$\therefore$ distance CB = speed × time $= 50 \times 2 = 100$ km.

Total distance, $AB = AC + CB = 60 + 100 = 160$ km.

$\therefore$ speed of train = 50 km/hr.

distance = 160 km.

REGULAR PROBLEMS

(1) Find the distance covered by a car moving at 20 metres per second for 3 hours. **{216 km}**

Hint: Remember 1 metre per second $= \frac{18}{5}$ km/h.

$\therefore \quad d = Vt$

$= 20 \times \frac{18}{5} \times 3$ km, = 216 km.

(2) Find the time taken to cover a distance of 0.9 km by a bullock cart moving at 0.25 metre/s. **{1 h}**

(3) A train travels at 90 km/h. How many metres will it travel in 15 minutes? **{22,500 m}**

(4) If a man walks to his office at $\frac{3}{4}$ of his usual rate, he reaches office $\frac{1}{3}$ of an hour later than usual. What is his usual time to reach office? **{1 h}**

Hint: Refer E-5.

(5) If a man walks to his office at $\frac{5}{4}$ of his usual rate, he reaches office 30 min. early than usual. What is his usual time to reach office. $\left[2\frac{1}{2}\right]$

Hint: See **E-5.**

(6) If I walk at 3 km/h, I miss a train by 2 minutes. If however, I walk at 4 km/h, I reach the station 2 minutes before the arrival of the train. How far do I walk to reach the station? $\left[\frac{4}{5}\text{ km}\right]$

Hint: Refer **E-6.**

(7) Excluding stoppages, the speed of a train is 45 km/h, and including stoppages, it is 36 km/h. For how many minutes does the train stop per hour? **{12 min.}**

Hint: Refer **E-9.**

(8) A man covers a certain distance on auto rikshaw. Had it moved 3 km/h faster, he would have taken 40 minutes less. If it had moved 2 km/h slower, he would have taken 40 minutes more. Find the distance.
{40 km}

Hint: Refer 17.3 (e).

(9) A thief goes away with a Maruti car at a speed of 40 km/h. The theft has been discovered after half an hour and the owner sets off in another car at 50 km/h. When will the owner overtake the thief from the start. $\left\{2\frac{1}{2}\textbf{ h after the theft}\right\}$

Hint: Distance to be covered by the thief and by the car owner is same.
Let after time 't' owner catches the thief, $V_1t_1 = V_2t_2$

$$\therefore \quad t \times 40 = \left(t - \frac{1}{2}\right) \times 50 \; t = 2\frac{1}{2} \text{ hours.}$$

(10) A man travels on a car from x to y at a speed of 77 km/h and returns back at 33 km/h from y to x. Find the average speed of the journey. **{46.2 km/h}**

Hint: Refer E-13.

(11) A man travels on a scooter from A to B at a speed of 30 km/h and returns back from B to A at 20 km/h. The total journey was performed by him in 10 hours. Find the distance from A to B.

Hint: Let the distance be d km **{120 km}**

$$\therefore \frac{d}{30} + \frac{d}{20} = t_1 + t_2 = 10 \text{ (given)} \Rightarrow d = \frac{10 \times 30 \times 20}{(30 + 20)} = 120 \text{ km.}$$

(12) A long distance runner runs 9 laps of a 400 metres track everyday. His timings (in min.) for four consecutive days are 88, 96, 89, and 87 respectively. On an average, how many metres/minute does the runner cover? **(MBA, '85) {40 m}**

Hint: Average speed (metre/minutes) $= \frac{\text{Total distance}}{\text{Total time}} = \frac{9 \times 400 \times 4}{(88 + 96 + 89 + 87)} = 40$ metre/minutes

(13) A man performs $\frac{2}{25}$ of his total journey by bus, $\frac{21}{50}$ by car and the remaining 2 km on foot. Find the total journey. **(Bank PO, '90) {2.5 km}**

Hint: Let total journey = x km.

$$\therefore \quad \frac{2}{25}x + \frac{21}{50}x + 2 = x \quad \Rightarrow \quad x = 2.5.$$

(14) Normally it takes 3 hours for a train to run from A to B. One day, due to a minor trouble, the train had to reduce the speed by 12 km/h and so it took $\frac{3}{4}$ of an hour more than usual. What is the distance from A to B. **{180 km}**

Hint: Let d be the distance from A to B.

$$\frac{d}{t_1} - \frac{d}{t_2} = V_1 - V_2$$ *[Refer 17.4]*

$$\Rightarrow \quad \frac{d}{3} - \frac{d}{3\frac{3}{4}} = 12 \quad \Rightarrow \quad d = 180 \text{ km}.$$

REAL PROBLEMS

(1) A car takes 3 hours to cover a distance of 180 km. If the distance is to be covered in $2\frac{1}{2}$ hours, what should be the speed of the car?
(a) 90 (b) 60 (c) 36 (d) 72 (e) None of these

Hint: Takes 3 hours is useless data.

(2) A train runs at 45 km/h. How far does it go in 6 seconds?
(a) 72 metres (b) 60 metres (c) 75 metres (d) 70 metres (e) 150 metres

Hint: 1 km/h = $\frac{5}{18}$ m/s.

(3) Joseph walked 1 km/h slower than usual and he could return home in $\frac{9}{8}$ of his usual time. His normal walking rate is
(a) 8 km/h (b) 9 km/h (c) 10 km/h (d) 11 km/h (e) None of these

Hint: $\left(1 - \frac{8}{9}\right)$ usual speed = 1 km/h.

(4) A car travels a certain distance at 60 km/h and comes back at 50 km/h. Find the average speed for total journey.
(a) 50 km/h (b) 45 km/h (c) 48 km/h (d) 55 km/h (e) None of these

(5) A motor cyclist travels for 10 hours, the first half at 21 km/h and the other half at 24 km/h. Find the distance travelled.
(a) 225 km (b) 224 km (c) 200 km (d) 324 km (e) 350 km

(6) A train travels at 90 km/h. How many metres does it travel in one second?
(a) 20 metres (b) 30 metres (c) 25 metres (d) 40 metres (e) 28 metres

Hint: 1 km/h $\frac{5}{18}$ metre/s.

(7) A monkey ascends a greased pole 12 metres high. He ascends 2 metres in first minute and slips down 1 metre in the alternate minute. In which minute, he reaches the top?
(a) 10th (b) 21st (c) 12th (d) 13th (e) 14th

(8) A man walks a certain distance and rides back in $6\frac{1}{4}$ h. He can walk both ways in $7\frac{3}{4}$ h. How long it would take to ride both ways?
(a) 5 hours (b) $4\frac{1}{2}$ hours (c) $4\frac{3}{4}$ hours (d) 6 hours (e) $5\frac{1}{2}$ hours

(9) If a man walks at 4 km/h, he misses the bus by 10 min. If he walks at 5 km/h, he reaches 5 min. before the arrival of the bus. How far is the bus stand?
(a) 10 km (b) 12 km (c) 15 km (d) 5 km (e) Data insufficient

Hint: Let d = Distance of bus stand, $\frac{d}{V_1} - \frac{d}{V_2} = t_1 - t_2$

$\therefore \quad d = \frac{10-(-5)}{(5-4)\times 60} \times 5 \times 4 = 5$ km. [(–)ve sign indicates before time]

(10) A car travels from A to B at V_1 km/h and travels back from B to A at V_2 km/h. The average speed of the car is

(a) $\frac{V_1 + V_2}{2V_1 V_2}$ (b) $\frac{\frac{1}{V_1} + \frac{1}{V_2}}{2}$ (c) $\frac{V_1 + V_2}{2}$ (d) $\frac{2V_1 V_2}{V_1 + V_2}$ (e) $\frac{V_1 + V_2}{\sqrt{V_1 V_2}}$

(11) A man walks a km in b hours. The time taken to walk 200 metres is
(a) $\frac{ab}{200}$ hours (b) $\frac{200b}{a}$ hours (c) $\frac{b}{5a}$ hours
(d) $\frac{a}{5b}$ hours (e) $\frac{a}{200b}$ hours

(12) A man travels at the rate of 'x' cm per minute, how many km does he travel in 'y' hours? **(SSC, '87)**
(a) $\frac{2xy}{5,000}$ km (b) $\frac{y}{1,000x}$ km (c) $\frac{xy}{60,000}$ km
(d) $\frac{3xy}{5,000}$ km (e) $\frac{xy}{5,280}$ km

(13) A train does a journey without stoppage in 8 hours, if it had travelled 5 km/h faster, it would have done the journey in 6 hours 40 min. Find its original speed.
(a) 25 km/h (b) 40 km/h (c) 45 km/h (d) 36.5 km/h (e) 50 km/h

Hint: Using $\frac{\text{Difference of two speeds}}{\text{Difference of two timings}} = \frac{\text{slower speed}}{\text{faster time}} = \frac{\text{faster speed}}{\text{slower time}}$.

(14) A car driver completes a 180 km trip in 4 hours. If he averages 50 km/h during the first 3 hours of trip, what was his speed in the final hour.
(a) 44 km/h (b) 45 km/h (c) 35 km/h (d) 30 km/h (e) 26 km/h

(15) *A* and *B* start running towards each other at 6 km/h and 9 km/h respectively. What was the distance between them when they started if they met after 16 minutes?
(a) 800 m (b) $2\frac{1}{2}$ km (c) 900 m (d) 4 km (e) 14 km

(16) A man wants to cover a distance of 50 km by bicycle. He goes at the speed of 12.5 km/h but takes rest for 20 min. after covering each 12.5 km. How much time will he take to cover the whole distance?
(a) 4 mg (b) $4\frac{1}{3}$ hrs (c) 5 hrs (d) $4\frac{2}{3}$ hrs (e) $4\frac{1}{2}$ hrs

(17) On a journey across Mumbai, tourist bus averages 10 km/hr for 20% of the time, 30 km/hr for 60% of it and 20 km/hr for the remainder. The average speed for the whole journey was:
(a) 10 km/hr (b) 30 km/hr (c) 5 km/hr (d) 24 km/hr (e) 25 km/hr

(18) A bullock-cart, covers a distance of 80 km in 10 hours. If half of the distance is covered in $\frac{3}{5}$ time, then what will be speed of bullock-cart to cover the rest of the distance in the remaining time?
(a) 8 km/hr (b) 6.4 km/hr (c) 10 km/hr (d) 20 km/hr (e) 12 km/hr

(19) The distance between two cities is 800 km. A motor car starts from the first city at the speed of 30 km/hr. At the same time, another car starts from the second city towards the first at the speed of 50 km/hr. The distance of the point from the first city where both the cars meet is : **(RBI, '95)**
(a) 200 km (b) 300 km (c) 400 km (d) 500 km (e) None

(20) The first third of a 75 km trip took twice as long as the rest of the trip. If the first third took '*h*' hours, then the average speed for the whole trip was:
(a) $\frac{50}{h}$ (b) $\frac{75}{2h}$ (c) $\frac{25}{2h}$ (d) $\frac{225}{2h}$ (e) data insufficient

(21) Two cars, C_1 and C_2, travel to a place at a speed of 30 and 45 km/hr respectively. If C_2 takes $2\frac{1}{2}$ hours less time than C_1 for the journey, then the distance of the place is:
(a) 300 km (b) 400 km (c) 600 km (d) 225 km (e) 350 km

Hint: Refer the text to find the distance if time difference, and both speeds are known.

(22) The average speed of a train is 20% less on the return journey than on the onward journey. The train halts for half an hour at the destination station before starting on the return journey. If the total time taken for the to and fro journey is 23 hours, covering a distance of 1,000 km, the speed of the train on the return journey is:
(a) 60 km/hr (b) 50 km/hr (c) 40 km/hr (d) 46 km/hr (e) 55 km/hr

Hint: Distance covered on each side journey = $\frac{1,000}{2}$ = 500 km.

(23) Xavier travels from Kisanpet to Mohanpet by train at 32 km/hr. But before reaching Mohanpet, he returns from Anandpet to Kisanpet by car at 72 km/hr. If the time for the whole journey is 13 hours, find the distance of Anandpet from Kisanpet.
(a) 312 km (b) 374 km (c) 288 km (d) 392 km (e) 224 km.

(24) To travel 600 km, train X takes 8 hours more than train Y. If however, the speed of train X is doubled, it takes 2 hours less than train Y. The speed of train Y (in km/hr) is :

(a) 50 (b) 55 (c) 45 (d) 60 (e) 30

Tips: Do not waste your time by forming two linear equations with the speeds of trains. Rather use the logic that for train X, if speed is doubled, the time (difference) reduces by 8 – (–2) = 10 hours in 600 km i.e. at new speed train X takes 10 hours to travel 600 km. Therefore, Y takes 10 + 2 = 12 hrs. to travel 600 km.

(25) Nair can walk a certain distance in 52 days when he rests 10 hours a day. How long will he take for twice the distance, if he walks twice as fast and rests twice as long each day?

(a) 104 days (b) 26 days (c) 78 days (d) 182 days (e) 136 days.

Hint: Chain rule may also be used to find the number of days required. In rest hours, no distance is covered, so it is not to be considered for calculation

(26) A train travelling at the speed of 90 km/hr follows a goods train after 6 hours from a station and overtakes it in 4 hours. What is the speed of the train?

(a) 30 km/hr (b) 60 km/hr. (c) 36 km/hr. (d) 42 km/hr (e) data insufficient

(27) Two trains move from station A and station B towards each other at the speed of 50 km/hr and 60 km/hr. At the meeting point, the driver of the second train felt that the train has covered 120 km more. What is the distance between A and B?

(a) 1,320 km (b) 1,200 km (c) 1,100 km (d) 960 km (e) None of these

(28) Shan starts 3 minutes after Bonny from the same point, for a place at a distance of $4\frac{1}{2}$ miles. Bonny on reaching her destination immediately returns and after walking a mile meets Shan. If Shan's speed be a mile in 18 minutes, what is Bonny's speed in miles per minute?

(a) $\frac{1}{6}$ (b) $\frac{1}{24}$ (c) $\frac{2}{15}$ (d) $\frac{1}{4}$ (e) $\frac{1}{12}$

(29) A Shatabdi Express train, owing to a defect in the engine, goes at $\frac{5}{8}$ of its proper speed and arrives at 6.49 pm instead of 5.55 pm. At what hour did it start?

(a) 4.15 pm (b) data insufficient (c) 4.35 pm
(d) 4.25 pm (e) 5.19 pm

(30) A man travels 300 kilometers in 13 hours, partly by rail and partly by steamer. If he had gone all the way by rail he would have ended his journey 8 hours sooner, and saved $\frac{4}{5}$ of the time he was on steamer. How far did he go by rail?

(a) 150 km (b) 130 km (c) 180 km (d) 205 km (e) 120 km.

Tips: Do not try to form linear equations. It is to better to use the basic concept of unitary method and reasoning for the given problem.
The time taken all the way by rail = 13 – 8 = 5 hrs.

$\therefore$ speed of rail $= \frac{300}{5} = 60$ km/hr

Since, $\frac{4}{5}$ of time on steamer = 8 hours

$\Rightarrow$ time on steamer $= 8 \div \frac{4}{5} = 8 \times \frac{5}{4} = 10$ hrs.

$\therefore$ time on rail $= 13 - 10 = 3$ hours

$\therefore$ distance by rail $= 3 \times 60 = 180$ km

(31) The distance from P to Q is 25 km; 6 km of which are uphills; 12 km downhill and the rest 7 km in level ground. The time in which a man would go from P to Q and back again, supposing his speed uphill is 2 km, downhill 3 km and on the level ground $3\frac{1}{2}$ km per hour, is:

(a) 9 hours (b) 10 hours (c) 15 hours (d) 18 hours (e) 19 hours

(32) A hare pursued by a greyhound, is 50 of her own leaps ahead of him. While the hare takes 4 leaps, the greyhound takes 3 leaps. In one leap, the hare goes $1\frac{3}{4}$ metres and the greyhound $2\frac{3}{4}$ metres. In how many leaps will the greyhound overtake the hare?

(a) 210 leaps (b) 180 leaps (c) 200 leaps (d) 224 leaps (e) 315 leaps.

Answers

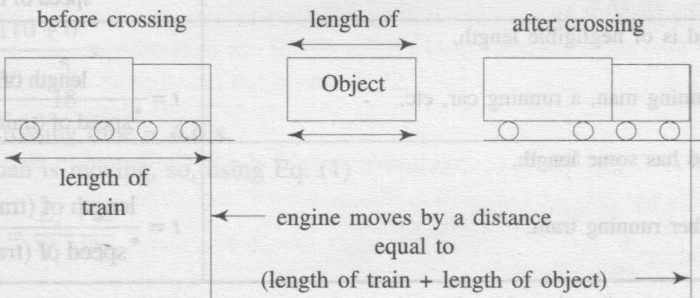
1. (d)	**2.** (c)	**3.** (b)	**4.** (e)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (d)
10. (d)	**11.** (c)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (c)	**17.** (d)	**18.** (c)
19. (b)	**20.** (a)	**21.** (d)	**22.** (c)	**23.** (c)	**24.** (a)	**25.** (d)	**26.** (c)	**27.** (a)
28. (e)	**29.** (d)	**30.** (c)	**31.** (e)	**32.** (a)				

18 TRAINS

18.1 CONCEPT

A train is said to have crossed an object (stationary or moving) only when the last coach (end) of the train crosses the said object completely. It implies that the total length of the train has crossed the total length of the object.

Hence, the distance covered by the train = length of train + length of object



18.2 BASIC FORMULA

Applying the well-known relation, time = $\dfrac{\text{Distance}}{\text{Speed}}$,

we can find the basic formula for the time required for a train to cross different type of objects.

$$\text{Time to cross an object moving in the direction of train} = \frac{\text{Length of train} + \text{Length of object}}{\text{Speed of train} - \text{Speed of object}}$$

Assume length of train = L_t, speed of train = V_t, t = time to cross

length of object = l, speed of object = V, then

basic formula can be represented as

$$t = \frac{L_t + L}{V_t - V} \tag{1}$$

Note:

1. If the object is of negligible length, then, put length of object, $L = 0$
2. If the object is stationary, then, put speed of object, $V = 0$
3. If the object is moving in opposite direction, then, put (–)ve sign before V, so the denominator of the formula becomes $V_t - (-V)$, i.e. $(V_t + V)$

18.3 DIFFERENT TYPES OF OBJECTS

On the basis of various types of objects that a train has to cross, we find the following different cases:

Case	*Type of Objects*	*Time to cross*⁺
1.	Object is stationary and is of negligible length, eg, train crosses lamp post, pole, standing man, etc.	$t = \dfrac{\text{length of train}}{\text{speed of train}}$
2.	Object is stationary and is of some length, eg, train crosses a bridge, a tunnel, platform, or another train at rest.	$t = \dfrac{\text{length of (train + object)}}{\text{speed of train}}$
3.	Object is moving* and is of negligible length, eg, train crosses a running man, a running car, etc.	$t = \dfrac{\text{length of train}}{^*\text{speed of (train – object)}}$
4.	Object is moving* and has some length, eg, train crosses another running train.	$t = \dfrac{\text{length of (train + object)}}{^*\text{speed of (train – object)}}$

*if the object is moving in opposite direction, then denominator becomes speed of (train + object) (Refer Note-3 in 18.2)

⁺all formula for time to cross have been derived by putting the values of speed of object and length of object (as per the important note in 18.2) in the basic formula itself.

18.4 TWO TRAINS CROSSING EACH OTHER IN BOTH DIRECTIONS

Two trains are crossing each other

Length of one train = L_1

Length of second train = L_2

They are crossing each other in **opposite direction** in t_1 seconds

They are crossing each other in **same direction** in t_2 seconds then,

$$\text{Speed of } \textbf{Faster Train} = \frac{L_1 + L_2}{2}\left[\frac{1}{t_1} + \frac{1}{t_2}\right]$$

$$\text{Speed of } \textbf{Slower Train} = \frac{L_1 + L_2}{2}\left[\frac{1}{t_1} - \frac{1}{t_2}\right]$$

Solved Examples

E-1 A train 110 metres long travels at 60 km/h. How long does it take to cross,

(a) a telegraph post
(b) a man running at 6 km/h in the same direction
(c) a man running at 6 km/h in the opposite direction
(d) a platform 240 metres long
(e) another train 170 metres long standing on another parallel track
(f) another train 170 metres long, running at 54 km/h in same direction
(g) another train 170 metres long, running at 80 km/h in opposite direction.

S-1 Since 1 km/h = $\frac{5}{18}$ m/s

$\therefore$ Speed of train = 60 km/h = $60 \times \frac{5}{18}$ m/s

(a) The telegraph post is a stationary object, so, using Eq. (1)

$$t = \frac{L_t + L}{V_t}, \text{ Since } V = 0, \text{ for stationary object}$$

$$t = \frac{110 + 0}{60 \times \frac{5}{18}} \text{ s}$$

$\therefore$ Crossing time = **6.6 s.**

(b) The man is moving, so, using Eq. (1)

$$t = \frac{L_t + L}{V_t - V}$$

$$\Rightarrow t = \frac{110 + 0}{(60 - 6) \times \frac{5}{18}} \text{ s} \Rightarrow t = 7.33 \text{ s}$$

$\therefore$ Crossing time = **7.33 s.**

(c) The man is moving, so using Eq. (1),

$$t = \frac{L_t + L}{V_t - V}$$

But here man is moving in opposite direction so, put $-V$ in place of V

$$\Rightarrow \quad t = \frac{110 + 0}{[60 - (-6)] \times \frac{5}{18}} \text{ s}$$

$$\Rightarrow \quad t = \frac{110}{66 \times \frac{5}{18}} \text{ s} \Rightarrow t = 6 \text{ s}$$

$\therefore$ Crossing time = **6 s.**

(d) The platform is stationary, so using Eq. (1),

$$t = \frac{L_t + L}{V_t}, \text{ since } V = 0$$

Here, length of platform $L = 240$ metres

$$\Rightarrow \quad t = \frac{110 + 240}{60 \times \frac{5}{18}} \text{ s} \Rightarrow t = 21 \text{ s}$$

$\therefore$ Crossing time is **21 s.**

(e) Another train is stationary, so using Eq. (1),

$$t = \frac{L_t + L}{V_t}, \text{ since } V = 0$$

Here length of another train $L = 170$ metres

$$\Rightarrow \quad t = \frac{110 + 170}{60 \times \frac{5}{18}} \Rightarrow t = 16.8 \text{ s}$$

$\therefore$ Crossing time is **16.8 s.**

(f) Another train is moving, so using Eq. (1),

$$t = \frac{L_t + L}{V_t - V}$$

Another train is moving in the same direction at a speed $V = 54$ km/h, and its length = 170 metres

$$\Rightarrow \quad t = \frac{110 + 170}{(60 - 54) \times \frac{5}{18}} \text{ s} \Rightarrow t = 168 \text{ s}$$

$\therefore$ Crossing time is 2 minutes **48 s.**

(g) Here, another train is moving in opposite direction, so, putting $-V$ in place of V, in Eq. (1)

$$t = \frac{L_t + L}{V_t - (-V)}$$

$$\Rightarrow \quad t = \frac{110 + 170}{(60 + 80) \times \frac{5}{18}} \text{ s} \Rightarrow t = 7.2 \text{ s}$$

$\therefore$ Crossing time is **7.2 s.**

E-2 A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, find the length of the platform. **(AGE, '82)**

S-2 Let 'l' be the length of the platform in metres.

Since time is indicated in seconds, so, speed of train $V_t = 54$ km/hr $= 54 \times \frac{5}{18} = 15$ metre/sec

Using the basic formula (18.2)

$$t = \frac{L_t + L}{V_t - V},$$

Since the platform is stationary, put $V = 0$, in the basic formula then we have

$$36 = \frac{L_t + L}{15} \quad \text{(i)}$$

for standing man, put $L = 0$ and $V = 0$, in the basic formula then we get

$$20 = \frac{L_t}{15}$$

$$\Rightarrow \quad L_t = 300 \quad \text{(ii)}$$

From (i) and (ii)

$$36 = \frac{300 + L}{15}$$

$$\Rightarrow \quad L = 240$$

$\therefore$ length of the platform is **240 metres.**

E-3 The train crosses a man standing on a platform 150 metre long in 10 seconds and crosses the platform completely in 22 seconds. Find the length of train and speed of train.

S-3 Let L_t and V_t be the length and the speed of the train

Using the basic formula (18.2)

$$t = \frac{L_t + L}{V_t - V}$$

For standing man, put $L = 0$ and $V = 0$, then, we get

$$10 = \frac{L_t}{V_t} \quad \text{(i)}$$

For the stationary platform, put $V = 0$, and $L = 150$ (given), then we get

$$22 = \frac{L_t + 150}{V_t}$$

$$\Rightarrow \quad 22 = \frac{L_t}{V_t} + \frac{150}{V_t} \quad \text{(ii)}$$

From (i) and (ii),

$$22 = 10 + \frac{150}{V_t} \qquad \left[\text{since } \frac{L_t}{V_t} = 10 \text{ in (i)}\right]$$

$$\Rightarrow \quad V_t = 12.5 \text{ m/sec.}$$

$\therefore$ speed of train = 12.5 metre/sec.

From (i) $\quad 10 = \frac{L_t}{12.5} \qquad \because V_t = 12.5$

$$\Rightarrow \quad L_t = 125$$

$\therefore$ length of train = **125 metres.**

E-4 A train running at 25 km/hr takes 18 seconds to pass a platform. Next it takes 10 seconds to pass a man walking at the rate of 7 km/hr in the same direction. Find the length of the platform and length of train.

S-4 Let L_t and L be the length of train and the length of the platform respectively.

Speed of train $= V_t = 25$ km/hr $= 25 \times \frac{5}{18} = \frac{125}{18}$ m/sec.

Speed of man $= V = 7$ km/hr $= 7 \times \frac{5}{18} = \frac{35}{18}$ m/sec.

Using the basic formula (18.2)

$$t = \frac{L_t + L}{V_t - V}$$

For the stationary platform, put $V = 0$

$$18 = \frac{L_t + L}{\frac{125}{18}}$$

$$\Rightarrow \quad L_t + L = 125 \qquad \text{(i)}$$

For the moving person in the same direction, put $L = 0$, $V = +\frac{35}{18}$

$$10 = \frac{L_t}{\frac{125}{18} - \frac{35}{18}}$$

$$\Rightarrow \quad L_t = \frac{90}{18} \times 10 = 50 \qquad \text{(ii)}$$

$\therefore$ length of train = 50 metres

Put $L_t = 50$ in (i), $L = 75$.

$\therefore$ length of platform = **75 metres**.

E-5 A toy train crosses 210 and 122 metre long tunnels in 25 and 17 seconds respectively. Find the length of train and speed of train. **(MBA, '88)**

S-5 Let L_t and V_t be the length of train and speed of train respectively.

Using the basic formula (18.2)

$$t = \frac{L_t + L}{V_t - V}$$

For tunnel, put $V = 0$

$$\therefore \quad 25 = \frac{L_t + 210}{V_t}$$

$$\Rightarrow \quad L_t + 210 = 25\, V_t \qquad \text{(i)}$$

and

$$17 = \frac{L_t + 122}{V_t}$$

$\Rightarrow \quad L_t + 122 = 17\ V_t$ **(ii)**

Subtracting (ii) from (i)

$210 - 122 = 8\ V_t$

$\Rightarrow \quad V_t = 11$ m/sec.

Putting $V_t = 11$ in equation (i),

$L_t + 210 = 25 \times 11$

$\Rightarrow \quad L_t = 275 - 210 = 65$ m.

$\therefore$ speed of the train = 11 m/sec.

length of the train = **65 m.**

E-6 A train with 90 km/hr crosses a bridge in 36 seconds. Another train 100 metres shorter crosses the same bridge at 45 km/hr. Find the time taken by the second train to cross the bridge.

S-6 Let t_2 be the time taken by the second train to cross the bridge.

Using the basic formula (18.2),

$$t = \frac{L_{t_1} + L}{V_t - V}$$

For bridge, put $V = 0$

Now, for the first train,

$$36 = \frac{L_{t_1} + L}{90 \times \frac{5}{18}} \qquad \left[\because 1 \text{ km/hr} = \frac{5}{18} \text{ m/sec}\right]$$

$\Rightarrow \quad L_{t_1} + L = 900$ **(i)**

For the second train,

$$t_2 = \frac{L_{t_2} + L}{45 \times \frac{5}{18}}$$

$\Rightarrow \quad L_{t_2} + L = \frac{25}{2} t_2$ **(ii)**

Subtracting (ii) from (i),

$$L_{t_1} - L_{t_2} = 900 - \frac{25}{2} t_2$$

$\Rightarrow \quad 100 = 900 - \frac{25}{2} t_2 \qquad [\because$ difference in length of two trains = 100 m (given)]

$\Rightarrow \quad t_2 = \frac{800 \times 2}{25} = 64$

$\therefore$ Required time is **64 seconds.**

E-7 A train 75 metres long overtook a man who was walking at the rate of 6 km/hr and crossed him in 18 seconds. Again, the train overtook a second person in 15 seconds. At what rate was the second person travelling?

S-7 Let V_t be the speed of the train and V_2 be the speed of the second person
Using the basic formula (18.2)

$$t = \frac{L_t + L}{V_t - V}$$

For the first man, put $L = 0$, $V = 6$ km/hr $= 6 \times \frac{5}{18} = \frac{5}{3}$ m/sec, $t = 18$ sec.

$$\Rightarrow \quad 18 = \frac{75}{V_t - \frac{5}{3}}$$

$$\Rightarrow \quad V_t - \frac{5}{3} = \frac{75}{18}$$

$$\Rightarrow \quad V_t = \frac{75}{18} + \frac{5}{3} = \frac{105}{18} \text{ m/sec.}$$

For the second person, put $L = 0$, $V_t = \frac{105}{18}$ m/sec.

$$\Rightarrow \quad 15 = \frac{75}{\frac{105}{18} - V_2}$$

$$\Rightarrow \quad \frac{105}{18} - V_2 = 5$$

$$\Rightarrow \quad V_2 = \frac{15}{18} \text{ m/sec} = \frac{15}{18} \times \frac{18}{5} = 3 \text{ km/h.}$$

$\therefore$ the speed of the second person is 3 km/hr.

E-8 Two trains of lengths 190 m and 210 m respectively, are running in opposite directions on parallel tracks. If their speeds are 40 km/hr and 32 km/hr respectively, in what time will they cross each other?

S-8 Using the basic formula (18.2).

$$t = \frac{L_t + L}{V_t - V}$$

Since $L_t = 190$ m, $L = 210$ m

$V_t = 40$, $V = -32$

$$\Rightarrow \quad V_t - V = 40 - (-32) = 72 \text{ km/hr}$$

$$= 72 \times \frac{5}{18} = 20 \text{ m/sec}$$

Hence $$t = \frac{190 + 210}{20}$$

$$\Rightarrow \quad t = 20 \text{ secs}$$

$\therefore$ The two trains cross each other in 20 seconds.

E-9 When two trains were running in the same direction at 90 km/hr and 60 km/hr respectively, the faster train crossed a man in the slower train in 27 seconds. Find the length of the faster train.

S-9 It is given that the faster train crosses a man in the slower train in same direction. It imples that a train crosses a man moving in the same direction.

Speed of slower train = speed of the man (because the man is in the slower train)

Now, using the basic formula, we get

$$t = \frac{L_t - L}{V_t - V}\text{, where } L_t = \text{length of faster train}$$

for the man, put $L = 0$

$$27 = \frac{L_t}{(90-60) \times \frac{5}{18}}$$

$$\Rightarrow \quad L_t = 30 \times \frac{5}{18} \times 27 = 225$$

Hence, the length of the faster train is **225 metres**.

E-10 Two trains, 130 m and 110 m long, are going in the same direction. The faster train takes one minute to pass the other completely. If they are moving in opposite direction, they pass other completely in 3 seconds. Find the speed of trains.

E-10 Using the basic formula

$$t = \frac{L_t + L}{V_t - V}$$

When two trains are moving in same direction,

$$60 = \frac{130 + 110}{V_t - V}$$

$$\Rightarrow \quad V_t - V = 4 \qquad \text{(i)}$$

When two trains are moving in opposite direction,

$$3 = \frac{130 + 110}{V_t + V}$$

$$\Rightarrow \quad V_t + V = 80 \qquad \text{(ii)}$$

From (i) and (ii), we get

$V_t = 42$ metre/sec. $\quad\therefore$ speed of faster train = **42 metre/sec.**

$V = 38$ metre/sec. $\quad\therefore$ speed of slower train = **38 metre/sec.**

REGULAR PROBLEMS

(1) A 120 metres long train speeds past a pole in 10 seconds. What is the speed in km/h?

(a) 60 km/h (b) 75 km/h (c) 50 km/h
(d) Data insufficient (e) None of these

Hint: 1 metre/s = $\frac{18}{5}$ km/h

(2) If a train running at 72 km/h crosses a coconut tree standing by the side of the track in 7 seconds, the length of train is
(a) 104 metres (b) 140 metres (c) 504 metres
(d) 540 metres (e) None of these

(3) A man running at 18 km/h crosses a bridge in 2 min. The length of the bridge is
(a) 1.2 km (b) 0.6 km (c) 1 km (d) 3.6 km (e) None of these

(4) A train 110 metres long passes a telegraph pole in 3 seconds. How long will it take to cross a railway platform 165 metres long?
(a) 4 s (b) 10 s (c) 5 s (d) $7\frac{1}{2}$ s (e) 12 s

(5) A train 100 metres long running at 36 km/h can cross a bridge 150 metres long in,
(a) 20 s (b) 25 s (c) 15 s (d) 22 s (e) 30 s

(6) In how many seconds will a train 105 metres long, running at 51 km/h, cross a man walking at 3 km/h in opposite direction?
(a) 12 s (b) 2.75 s (c) 5 s (d) 7 s (e) 10 s

(7) A train 120 metres long, travelling at 45 km/h, overtakes another train travelling in the same direction at 36 km/h and passes it completely in 80 seconds. The length of the second train is
(a) 100 metres (b) 75 metres (c) 80 metres
(d) 120 metres (e) 110 metres

(8) A train of unknown length crosses a platform L_1 metres in t_1 seconds and also crosses a telegraph post in t_0 seconds. Then speed of train V_t is
(a) $L_1(t_1 - t_0)$ (b) $\dfrac{L_1}{t_1 - t_0}$ (c) $\dfrac{L_1\, t_1}{t_0^2}$
(d) $\dfrac{L_1\, t_1}{t_1 + t_0}$ (e) Data insufficient

(9) A train crosses two platforms of L_1 metres and L_2 metres in t_1 seconds and t_2 seconds respectively. If the length of train is L_t ,then
(a) $\dfrac{L_t}{L_1 + L_2} = t_1 + t_2$ (b) $L_t\,(L_1 - L_2) = t_1 \times t_2$
(c) $\dfrac{L_t + L_1}{t_1} = \dfrac{L_t + L_2}{t_2}$ (d) $L_t\, L_1\, L_2 = t_1 - t_2$ (e) None of these

(10) A train consists of 12 boggies, each boggy 15 metres long. The train crosses a telegraph post in 18 seconds. Due to some problem, two boggies were detached. The train now crosses a telegraph post in
(a) 18 s (b) 12 s (c) 15 s (d) 20 s (e) None of these

(11) A train of unknown length crosses two platforms of L_1 metres and L_2 metres in t_1 seconds and t_2 seconds respectively. Then the speed of the train (V_t) is
(a) $\dfrac{L_1 L_2}{V_t} = t_1 + t_2$ (b) $\dfrac{1}{L_1} + \dfrac{1}{L_2} = \left[\dfrac{1}{t_1} + \dfrac{1}{t_2}\right] V_t$
(c) $L_1 t_1 + L_2 t_2 = V_t$ (d) $V_t = \dfrac{L_1 - L_2}{t_1 - t_2}$ (e) Data insufficient

(12) A train of length L_t crosses a man moving in opposite direction at V_m metre/s in t_m seconds. If the speed of train is V_t m/s, then

(a) $t_m = \dfrac{L_t}{V_t + V_m}$ (b) $t_m = \dfrac{L_t}{V_t - V_m}$ (c) $V_t V_m = \dfrac{L_t}{t_m}$

(d) $V_t = \dfrac{L_t}{t_m} + V_m V_t$ (e) Data insufficient

(13) Two trains running at V_1 m/s and V_2 m/s crosses the same tunnel in t_1 seconds and t_2 seconds respectively. If the difference of lengths of two trains is x metres then

(a) $x = V_1 t_1 - V_2 t_2$ (b) $\dfrac{1}{x} = \dfrac{1}{V_1 t_1} + \dfrac{1}{V_2 t_2}$ (c) $x = V_1 t_1 + V_2 t_2$

(d) $\dfrac{x}{V_1 t_1} = V_2 t_2 + 1$ (e) None of these

(14) A train running at a speed of 84 km/h crosses a man running at a speed of 6 km/h in the opposite direction in 4 seconds. The length of the train in metres is

(a) 180 (b) 75 (c) 200 (d) 150 (e) None of these

(15) Two trains 180 and 220 metres long are running in opposite directions at 40 and 50 km/h respectively. They cross each other in

(a) 16 s (b) 17 s (c) 18 s (d) 22 s (e) 20 s

(16) The driver of a maruti car driving at the speed of 68 km/h locates a bus 40 metres ahead of him. After 10 seconds, the bus is 60 metres behind. The speed of the bus is

(a) 30 km/h (b) 32 km/h (c) 25 km/h (d) 38 km/h (e) 6.8 km/h

(17) Two trains of equal length are running on parallel lines in the same direction at the rate of 65 and 44 km/h. The faster train passes the slower train in 48 seconds. The length of each train is

(a) 120 metres (b) 150 metres (c) 100 metres
(d) 140 metres (e) 720 metres

(18) A train 150 metres long passes a tree in 12 seconds. It will pass a tunnel 250 metres long in

(a) 20 s (b) 25 s (c) 32 s (d) 26 s (e) 36 s

(19) Rajdhani Express train travelling at a uniform speed clears a platform 200 metres long in 10 seconds and passes a telegraph post in 6 seconds. The speed of train is

(a) 150 km/h (b) 180 km/h (c) 200 km/h (d) 175 km/h (e) 220 km/h

(20) A train 100 metres long meets a man going in opposite direction at 5 km/h and passes him in $7\frac{1}{5}$ seconds. The speed of the train is

(a) 40 km/h (b) 45 km/h (c) 36 km/h (d) 52 km/h (e) 42 km/h

(21) A passenger in train '*P*' travelling at 1 km/min uses his stop watch and finds that another train '*Q*' travelling in the opposite direction, completely passed his windows in 3 seconds. If the length of the train '*Q*' is 87.50 m. Find its speed (km/hr).

(a) 36 (b) 54 (c) 48 (d) 45 (e) 50

(22) Local trains leave from a station at an interval of 15 minutes at a speed of 16 km/hr. A man moving from opposite side meets the trains at an interval of 12 minutes. The speed of the man is

(a) 5 km/hr (b) 4 km/hr (c) 5.5 km/hr (d) 3 km/hr (e) None