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6-10 Quantitative Aptitude for Competitive Examinations

- (3) Out of four numbers, the average of the first three is 15 and that of the last three is 16. If the last number is 19, then first number is:
 (a) 15 (b) 16 (c) 18 (d) 19 (e) 22
- Tips:**

1	2	3	4
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 $\downarrow \qquad \qquad \downarrow$
 15 16
- (4) A cricket player makes 200 runs in the 15th over. In doing so, his average at the end of the 14th over increases by 10. What was his average at the end of 15th over? **(RRB Guwahati, '97)**
 (a) 60 (b) 50 (c) 40 (d) 45 (e) 35
- (5) The average of Suresh's marks in English and History is 55. His average marks in English and Science is 65. What is the difference between the marks that he obtained in History and Science? **(Baroda Bank PO, '97)**
 (a) 40 (b) 60 (c) 20 (d) data inadequate (e) None
- (6) A body weighs 121 grams and 125.44 grams on two different pans of a faulty balance. Its true weight will be:
 (a) 4.44 gms (b) 120 gms (c) 123.22 gms (d) 130 gms (e) 122 gms
- (7) The average of three consecutive odd numbers is 14 more than one-third of the first of these numbers. What is the last of these numbers?
 (a) 17 (b) 19 (c) 15
 (d) data inadequate (e) None of these
- (8) Ten years ago, the average age of a family of four members was 24 years. Three children having been born, the average age of the family is same today. What are the present ages of children, if two children are identical twins and differ by two years from the younger one?
 (a) 12, 12, 10 (b) 8, 8, 6 (c) 13, 13, 11 (d) 14, 14, 12 (e) None
- (9) The captain of a cricket team of 11 players is 25 years and the wicket keeper is 3 years older. If the age of these two are excluded, the average age of the remaining players is 1 year less than the average age of the whole team. The average age of the whole team is
 (a) 24 years (b) 21 years (c) 26 years (d) 22 years (e) 25 years
- (10) What fraction must be subtracted from the sum of $\frac{1}{4}$ and $\frac{1}{6}$ to have an average of $\frac{1}{12}$ of all the two fractions?
 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$ (e) None of these
- (11) In a coconut groove, $(x + 2)$ trees yield 60 nuts per year, x trees yield 120 nuts per year and $(x - 2)$ trees yield 180 nuts per year. If the average yield per year is per tree be 100. Then the value of x is
 (a) 8 (b) 4 (c) 12 (d) 10 (e) 14
- (12) If the average weight of boys of Ram's age and height is 105 kg, and if Ram weighs 110% of the average, then the weight of Ram is
 (a) 105 kg (b) data insufficient (c) 115.5 kg
 (d) 110 kg (e) 107.5 kg
- (13) After a certain number of matches, a bowler has had 200 runs knocked off him. In the next match, he takes 3 wickets for 52 runs and his average goes up by 1. The new average of the bowler is
 (a) 9 (b) $8\frac{1}{3}$ (c) $9\frac{1}{3}$ (d) 8 (e) either (a) or (c)

Hint: Bowler's average = $\frac{\text{runs knocked off}}{\text{no. of wickets taken}}$, Refer formula no. 4

- (14) A batsman has scored an average of 46 runs for a certain number of innings played in England. When he came back to India, he played another two test matches of two innings each and scored at an average of 55 runs. For the innings in England and in India taken together, he has improved his average by 2 runs over the matches played in England. The number of innings played in England was
 (a) 7 (b) 9 (c) 14 (d) 18 (e) 11

- (15) A batsman's scores in a particular innings is twice the average score in 4 previous innings. The percentage improvement in his average score now is:
 (a) 6 (b) 20 (c) 5 (d) 10 (e) 12

- (16) If the average of a, b, c be M and $ab + bc = -ca$, then the average of a^2, b^2, c^2 is:
 (a) M^2 (b) $3M^2$ (c) $9M^2$ (d) $27M^2$ (e) None of these

Hint: $a^2 + b^2 + c^2 + 2(ab + bc + ca) = (a + b + c)^2$

- (17) Among three numbers, the first number is thrice the second number and one-fourth of the third number. The average of all three numbers is 64. What is the average of first and third number?
 (a) 30 (b) 24 (c) 90 (d) 78 (e) 48

Tips: Since the first number is related to both the second and the third number in the given condition, so, it is better to assume the first number = x]

- (18) The average of ' n ' numbers is A . If the number Y is replaced by the number Y^0 , then the average becomes A^0 . Which of the following equations is true?

$$(a) A = A^0 \frac{(Y - Y^0)}{n} \quad (b) A = A^0 + n(Y - Y^0) \quad (c) A^0 = \frac{Y}{n} + \frac{Y^0}{A}$$

$$(d) A^0 + \frac{Y}{n} = A + \frac{Y^0}{n} \quad (e) \frac{A}{A^0} = n + \frac{Y}{Y^0}$$

Answers

1. (c) 2. (c) 3. (b) 4. (a) 5. (c) 6. (c) 7. (d) 8. (e) 9. (d)
 10. (c) 11. (b) 12. (c) 13. (e) 14. (c) 15. (b) 16. (b) 17. (c) 18. (d)

7.2 PROPERTIES OF RATIO

- (a) In a ratio, two quantities are compared. So, the quantities must be of the same kind, i.e. they must be expressed in the same units.
- (b) The ratio of two quantities determines how many times one quantity is contained by the other.
- (c) The order of the terms in a ratio $a : b$ is very important. Since $4 : 5$ is different from $5 : 4$.

7.3 DIVIDING A GIVEN NUMBER IN THE GIVEN RATIO

Let ' A ' be the given number. The given ratio is $a_1 : a_2$.

Here ' A ' is to be divided in the ratio $a_1 : a_2$.

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S-12 $\frac{\text{MEI of } A}{\text{MEI of } B} = \frac{\text{Profit of } A}{\text{Profit of } B}$

$$\Rightarrow \frac{\text{Capital of } A \times 8}{(\text{Capital of } A - 1000 \times 12)} \times = \frac{1}{1} \quad (\text{Since yearly profit share is same})$$

$$\therefore \text{Capital of } A = \text{Rs } 3,000.$$

E-13 A, B and C invest Rs 4,000, 5,000 and 6,000 respectively in a business and A gets 25% of profit for managing the business, the rest of the profit is divided by A, B and C in proportion to their investment. If in a year, A gets Rs 200 less than B and C together, what was the total profit for that year?

S-13 After giving 25% of the total profit amount to A for managing the business, the rest 75% of total profit is divided among A, B and C in proportion to their investments.

$$\text{In } 75\% \text{ of total profit, } A's \text{ share : } B's \text{ share : } C \text{ share} = 4,000 : 5,000 : 6,000$$

$$= 4 : 5 : 6$$

$$\therefore 75\% \text{ of total profit} = 4x + 5x + 6x$$

$$\therefore \text{Total profit} = \frac{15x}{75\%} = 20x$$

$$\therefore \text{Share of } A = 4x + 25\% \text{ of } 20x = 9x$$

$$\text{Share of } B = 5x$$

$$\text{Share of } C = 6x$$

$$\text{Given, } (5x + 6x) - 9x = 200 \Rightarrow x = 100$$

$$\therefore \text{Total profit} = 20x = 20 \times 100 = \text{Rs } 2,000.$$

E-14 Two partners invested Rs 1,250 and Rs 850 respectively in a business. Both the partners distribute 60% of the profit equally and distribute the rest 40% as the interest on their capitals. If one partner received Rs 30 more than the other, find the total profit.

S-14 Since 60 % of the profit is distributed equally so, one partner receives Rs 30 more than the other only due to distribution of rest 40 % of the basis of their invested capitals.

$$\therefore \frac{A's \text{ 40% profit}}{B's \text{ 40% profit}} = \frac{1250}{850} = \frac{25}{17}$$

$$\Rightarrow \frac{A's \text{ 40% profit} + B's \text{ 40% profit}}{A's \text{ 40% profit} - B's \text{ 40% profit}} = \frac{25+17}{25-17} \quad (\text{By Componendo and Dividendo})$$

$$\Rightarrow \frac{40\% \text{ of profit}}{30} = \frac{42}{8}$$

$$\Rightarrow \text{Profit} = \frac{42}{8} \times 30 \times \frac{100}{40} = \text{Rs } 393.75$$

\therefore Total profit is **Rs 393.75**.

E-15 In a partnership, A invests $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time, B invests $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time and C, the rest of the capital for whole time. Find A's share of the total profit of Rs 2,300.

$$\text{S-15 Capital of } C = 1 - \frac{1}{6} - \frac{1}{3} = \frac{1}{2}$$

Let the total time be 12 months

$\therefore A's \text{ profit} : B's \text{ profit} : C's \text{ profit} = \text{MEI of } A : \text{MEI of } B : \text{MEI of } C$

$$\begin{aligned} &= \frac{1}{6} \times \left(\frac{1}{6} \times 12 \right) : \frac{1}{3} \times \left(\frac{1}{3} \times 12 \right) : \frac{1}{2} \times 12 \\ &= \frac{1}{3} : \frac{4}{3} : 6 \\ &= 1 : 4 : 18 \end{aligned}$$

$$\therefore \text{Share of } A = \frac{1}{1+4+18} \times 2300 = \text{Rs } 100.$$

REGULAR PROBLEMS

Therefore,

- (1) The annual income of A is 10% less than that of B whose income is 20% more than that of C . If the monthly income of C is Rs 200, find the total annual income of A , B , and C together.
 (a) Rs 7,046 (b) Rs 7,772 (c) Rs 6,872 (d) Rs 7,872 (e) None of these
- (2) A sum of money is to be divided among A , B and C in the ratio 2 : 3 : 7. If the total share of A and B together is Rs 1,500 less than C , what is A 's share in it? (NIC, '80)
 (a) Rs 1,000 (b) Rs 1,500 (c) Rs 2,000
 (d) Data insufficient (e) None of these

Hint: $7x - (2x + 3x) = 1,500 \quad \therefore x = 750 \quad \therefore A = 2x = 1,500.$

- (3) Nirmal and Kapil started a business investing Rs 9,000 and Rs 12,000 respectively. After 6 months, Kapil withdrew half of his investment. If after a year, the total profit was Rs 4,600, what was Kapil's share in it?
 (a) Rs 2,000 (b) Rs 2,600 (c) Rs 1900 (d) Rs 2,300 (e) None of these
- (4) The ratio between the ages of Amar and Kabir at present is 2 : 3. After three years, the ratio of their ages will be 3 : 4. Find the present age of Amar in years.
 (a) 6 (b) 9 (c) 15
 (d) Data insufficient (e) None of these

Hint: $\frac{\text{Amar's age}}{\text{Kabir's age}} = \frac{2x - 3}{3x - 3} = \frac{3}{4}.$

- (5) Rs 750 is distributed among A , B and C such that A 's share : B 's share = 2 : 3 and B 's share : C 's share = 6 : 5. The share of A is,
 (a) Rs 150 (b) Rs 175 (c) Rs 200 (d) Rs 250 (e) None of these
- (6) The cost of a black and white TV and a colour TV are in 3 : 8 and total price of both is Rs 12,100. The difference in their prices is
 (a) Rs 6,600 (b) Rs 6,050 (c) Rs 5,500 (d) Rs 5,100 (e) Rs 5,000

Hint: $\frac{8x + 3x}{8x - 3x} = \frac{12,100}{?} \Rightarrow ? = \frac{5}{11} \times 12,100 = 5,500$

- (7) A profit of Rs 450 is divided between two partners, one of whom has contributed Rs 1,200 for 5 months and the other Rs 750 for 4 months. How much amount the second partner received?
 (a) Rs 300 (b) Rs 425 (c) Rs 150 (d) Rs 175 (e) None of these

- (8) A began business with Rs 1,250 and is joined afterwards by B with Rs 3,750. When did B join if the profits at the end of the year are divided equally.

- (a) After 6 months (b) After 8 months (c) After 4 months
 (d) After 7 months (e) None of these

Hint:
$$\frac{A's \text{ profit share}}{B's \text{ profit share}} = \frac{A's \text{ capital} \times \text{No. of months}}{B's \text{ capital} \times \text{No. of months}}$$

- (9) A and B rent a pasture for 10 months; A puts in 80 cows for 7 months. How many can B put in for the remaining 3 months, if he pays half as much again as A?

- (a) 120 (b) 180 (c) 200 (d) 240 (e) 280

- (10) A certain amount of money is divided among A, B and C so that for each, A has 1 rupee B has 65 paise and C has 40 paise. If C's share is Rs 8, find the sum of money.

- (a) Rs 14 (b) Rs 41 (c) Rs 37 (d) Rs 24 (e) None of these

Hint: If C's share is 40 paise, then sum of money = $1 + 0.65 + 0.40 = 2.05$.

- (11) Rs 120 is divided between X, Y and Z, so that X's share is Rs 20 more than Y and Rs 20 less than that of Z. The share of Y will be,

- (a) Rs 30 (b) Rs 60 (c) Rs 50 (d) Rs 20 (e) Rs 25

- (12) In a partnership between X and Y, X's capital is $\frac{2}{5}$ of total and is invested for $\frac{2}{3}$ year. If his share

of the profit is $\frac{4}{7}$ of the total, for how long is Y's capital in the business?

- (a) 1 year (b) $\frac{1}{8}$ year (c) $\frac{1}{3}$ year (d) $\frac{1}{4}$ year (e) $\frac{1}{2}$ year

- (13) X and Y put in Rs 3,000 and Rs 4,000 respectively into a business. X reinvests into the business his share of the first year's profit of Rs 2,100 whereas Y does not reinvest. In what ratio should they share the second year's profit?

- (a) 39 : 40 (b) 3 : 4 (c) 3 : 7 (d) 40 : 79 (e) 39 : 79

Hint:
$$\frac{X's \text{ profit share}}{Y's \text{ profit share}} = \frac{X's \text{ capital}}{Y's \text{ capital}}$$
 (Since, No. of months invested is same)

$$= \frac{3000 + \frac{3}{7} \times 2100}{4000} = \frac{39}{40}$$

- (14) In a partnership between A, B and C, A's capital is Rs 4,000. If in a profit of Rs 800, A's share is Rs 200 and C's share is Rs 100, then B's capital is

- (a) Rs 8,000 (b) Rs 2,000 (c) Rs 10,000 (d) Rs 5,000 (e) None of these

- (15) A workman earned Rs 180 in a certain number of days. If his daily wages had been Rs 2 less, he would take one more day to earn the same amount. Find how many days he worked at the higher rate?

- (a) 18 days (b) 10 days (c) 9 days (d) 6 days (e) 12 days

- (16) A, B and C start a business. A invests 3 times as much as B invests and B invests $\frac{2}{3}$ of what C invests.

If the total profit is Rs 1,320, find the share of A.

- (a) Rs 760 (b) Rs 620 (c) Rs 600 (d) Rs 660 (e) Rs 720

Hint: $3B : B : \frac{3}{2}B = A : B : C$

- (17) Rs 180 is to be divided among 66 men and women. The sums of men's share and women's share are in the ratio 5 : 4, but their individual shares are 3 : 2 respectively. The number of men are,

- (a) 26 (b) 30 (c) 40 (d) 5 (e) 10

Hint: Let the no. of men = $x \Rightarrow$ sum of shares of men = $3x$

No. of women = $66 - x \Rightarrow$ sum of shares of women = $2(66 - x)$

$$\therefore \frac{3x}{2(66 - x)} = \frac{5}{4} \text{ (given).}$$

- (18) Rs 36.60 is shared between two persons, and one of them gets five times as much as the other, then the smaller share is

- (a) Rs 6.09 (b) Rs 7.32 (c) Rs 3.66 (d) Rs 6.10 (e) Rs 7

- (19) The total amount with A and C is Rs 98 while with B and C is Rs 90. If the total amount with A, B and C is Rs 168, then what is the amount with C?

- (a) Rs 40 (b) Rs 50 (c) Rs 38 (d) Rs 78 (e) Rs 20

- (20) Three persons A, B and C distribute 2,600 chocolates in the ratio 11 : 18 : 23. Then B gets,

- (a) 1100 chocolates (b) 900 chocolates (c) 1800 chocolates
 (d) 800 chocolates (e) None of these

- (21) A group of boys contributed as many rupees as they were in number for their trip. The total collection was Rs 841. How many boys were in the group?

- (a) 21 (b) 31 (c) 29 (d) 19 (e) 41

- (22) A sum of money is divided among A, B and C so that to each rupees A gets, B gets 65 paise and C gets 40 paise. If C's share is Rs 40, then sum must be

- (a) Rs 105 (b) Rs 140 (c) Rs 205 (d) Rs 65 (e) Rs 165

- (23) A, B and C are partners in a business B's capital is $\frac{1}{6}$ of the total and A's is equal to that of B and

C together. How much does C receive out of a total profit of Rs 2,400?

- (a) Rs 1,200 (b) Rs 900 (c) Rs 1,600 (d) Rs 800 (e) Rs 400

Hint: $B = \frac{1}{6} \times \text{total}$ and $A = B + C$

$$\Rightarrow A = \frac{1}{6} \text{ total} + C$$

$$\therefore A + B + C = \left(\frac{1}{6} \text{ total} + C\right) + \left(\frac{1}{6} \times \text{total}\right) + C = \text{total}$$

$$\therefore C = \frac{1}{3} \times \text{total profit} = \frac{1}{3} \times 2,400 = \text{Rs } 800.$$

- (24) Out of a total amount of Rs 4,898, B receives 20% more than A and 25% more than C. What is B's share?

- (a) Rs 930 (b) Rs 1,860 (c) Rs 1,400 (d) Rs 1,540 (e) Rs 860

- (25) Rs 4,800 are distributed among *A*, *B* and *C* in the ratio of 6 : 5 : 4. The difference in the shares of *A* and *C* is
 (a) Rs 1,280 (b) Rs 400 (c) Rs 640 (d) Rs 1,200 (e) Rs 320
- (26) Ram and Shyam invest Rs 3,000 and Rs 4,000 respectively in a business. If Ram doubles his capital after 6 months, then in what proportion should Ram and Shyam divide that year's profit?
 (a) 3 : 4 (b) 4 : 3 (c) 16 : 9 (d) 9 : 8 (e) 9 : 16
- (27) *P* started a business by investing Rs 2,700. After sometime, *Q* joined him by investing Rs 2,025. At the end of one year, the profit was divided in the ratio 2 : 1. After how many months did *Q* join the business?
 (a) 3 (b) 4 (c) 6 (d) 9 (e) 8
- (28) *A*, *B* and *C* are partners. *A* receives $\frac{2}{3}$ of the profit, *B* and *C* dividing the remainder equally. *A*'s income increased by Rs 400 when the rate of profit rises from 5 to 7 per cent. The capital of *B* is
 (a) Rs 30,000 (b) Rs 5,000 (c) Rs 6,000 (d) Rs 15,000 (e) None
- (29) Divide Rs 671 among *A*, *B* and *C* such that if their shares be increased by Rs 3, Rs 7 and Rs 9 respectively, the result shall be in the ratio 1 : 2 : 3.
 (a) 118, 220, 333 (b) 110, 225, 336 (c) 115, 222, 334
 (d) 112, 223, 336 (e) 120, 220, 331

Answers

- | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (b) | 3. (d) | 4. (a) | 5. (c) | 6. (c) | 7. (c) | 8. (b) | 9. (e) |
| 10. (b) | 11. (d) | 12. (c) | 13. (a) | 14. (c) | 15. (c) | 16. (e) | 17. (b) | 18. (d) |
| 19. (e) | 20. (b) | 21. (c) | 22. (c) | 23. (d) | 24. (b) | 25. (c) | 26. (d) | 27. (b) |
| 28. (b) | 29. (d) | | | | | | | |

Let C_1 = cost price of 1st ingredient

C_2 = cost price of 2nd ingredient

Now, these two ingredients are mixed to produce a mixture such that

C_m = cost price of the mixture (mean price)

the ratio of the quantity of two ingredients needed to produce this mixture is given by

$$\frac{\text{quantity of 1}^{\text{st}} \text{ ingredient}}{\text{quantity of 2}^{\text{nd}} \text{ ingredient}} = C_2 - C_m$$

$$\frac{\text{quantity of 2}^{\text{nd}} \text{ ingredient}}{\text{quantity of 1}^{\text{st}} \text{ ingredient}} = C_m - C_1$$

Diagram Representation



9

MIXTURES

The proportion of 4 : 3 indicates that every 6 kg of cheaper salt is to be mixed with 4 kg of dearer salt (42 P) to produce the mean mixture (32 P).

(i) 20 kg of cheaper salt (given) is to be mixed with 20 kg of dearer salt (42 P).
 Hence required amount of dearer salt (42 P) is 20 kg.

9.1 Alligation Rule ~~What is a simple mixture? What is a compound mixture?~~

When three ingredients with cost prices C_1 , C_2 and C_m respectively are mixed to form a mixture of mean price (or cost price) C_m , then

9.1 DEFINITION

Mixtures are generally of two types. When two different **ingredients** are mixed together, it is known as **simple mixture**, e.g. a mixture of water and milk; water and pure spirit.

When two or more simple mixtures (made of same ingredients of same or different proportions) are mixed together to form another mixture, it is known as a **compound mixture**.

9.2 ALLIGATION RULE

Alligation literally means "linking". The **alligation rule** states that,

'When different quantities of same or different ingredients, of different cost (value) are mixed together to produce a mixture of a mean cost (value), the ratio of their quantities are inversely proportional to the differences in their cost from the mean cost (value)'.

$$\frac{\text{Quantity of smaller cost ingredient}}{\text{Quantity of larger cost ingredient}} = \frac{\text{Larger cost} - \text{Mean cost}}{\text{Mean cost} - \text{Smaller cost}}$$

Let C_1 = cost price of 1st ingredient

C_2 = cost price of 2nd ingredient

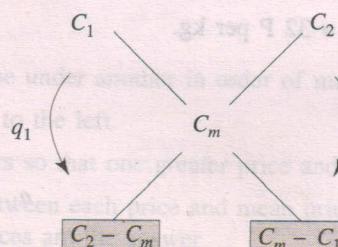
Now, these two ingredients are mixed to produce a mixture such that

C_m = cost price of the mixture (mean price)

then, the ratio of the quantity of two ingredients needed to produce this mixture is given by:

$$\frac{\text{quantity of 1st ingredient}}{\text{quantity of 2nd ingredient}} = \frac{C_2 - C_m}{C_m - C_1}$$

Diagram Representation



$$\frac{1}{2} = \frac{8}{16} =$$

$$23 \quad \frac{q_1}{q_2} = \frac{C_2 - C_m}{C_m - C_1}$$

(1)

e

9.2.1 To Find the Quantity of Ingredients in the Mixture of Given Amount

From formula (1), we have obtained the ratio of ingredients i.e. $\left(\frac{q_1}{q_2}\right)$.

Now, if the amount of the mixture = Q , then,

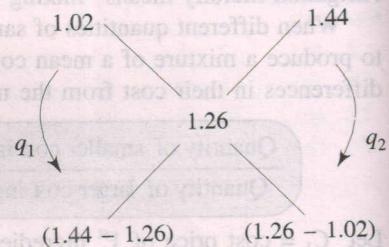
$$\begin{aligned} \text{quantity of 1st ingredient in the mixture} &= \frac{q_1}{q_1 + q_2} \times Q \\ \text{and} \quad \text{quantity of 2nd ingredient in the mixture} &= \frac{q_2}{q_1 + q_2} \times Q \end{aligned} \quad (2)$$

Formula 2 is known as **Ratio-Quantity Rule**.

Example: In what proportion must a grocer mix teas sold @ Rs 1.02/kg and Rs 1.44/kg so as to make a mixture worth Rs 1.26/kg ?

Solution: As per Alligation Rule,

$$\begin{aligned} \frac{\text{quantity of cheaper tea}}{\text{quantity of dearer tea}} &= \frac{q_1}{q_2} = \frac{(1.44 - 1.26)}{(1.26 - 1.02)} \\ &= \frac{.18}{.24} \\ &= \frac{3}{4} \end{aligned}$$



∴ proportion of two kinds of tea to be mixed (to get a mixture worth Rs 1.26/kg) is 3 : 4.

Example: How much salt (in kg) worth 42 P per kg must a man mix with 25 kg of salt worth 24 P per kg so that he may, on selling the mixture at 40 P per kg, gain 25% on the outlay?

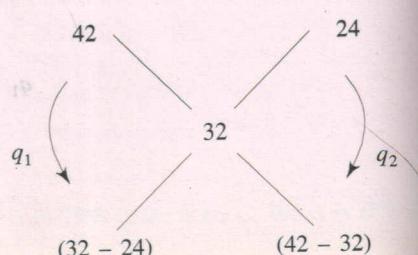
Solution: Here, selling price of the mixture has been given, but alligation rule is applied on the cost price of mixture as well as on cost price of ingredients.

Therefore, cost price of the mixture has to be found out first.

$$\text{Cost price of the mixture} = \frac{40}{125} \times 100 = 32 \text{ P per kg.}$$

By applying rule of Alligation now,

$$\begin{aligned} \frac{\text{quantity of dearer salt}}{\text{quantity of cheaper salt}} &= \frac{q_1}{q_2} \\ &= \frac{32 - 24}{42 - 32} \\ &= \frac{8}{10} = \frac{4}{5} \end{aligned}$$



The proportion of 4 : 5 indicates that every 5 kg of cheaper salt (24 P) is to be mixed with 4 kg of dearer salt (42 P) to produce the mean mixture (32 P).

⇒ 25 kg of cheaper salt (given) is to be mixed with 20 kg of dearer salt (42 P).

Hence the required amount of dearer salt (42 P) is 20 kg.

9.2.2 Alligation Rule for a Mixture of Three Ingredients

When three ingredients with cost prices C_1 , C_2 and C_3 respectively are mixed to form a mixture of mean price (or cost price) C_m , then

$$q_1 = \text{proportion of 1st ingredient} = (C_2 - C_m) (C_3 - C_m)$$

Memory tips

(here, cost of q_1 absent)

$$q_2 = \text{proportion of 2nd ingredient} = (C_m - C_1) (C_3 - C_m)$$

(here, cost of q_2 absent)

$$q_3 = \text{proportion of 3rd ingredient} = (C_2 - C_m) (C_m - C_1)$$

(here, cost of q_3 absent)

So, we get

$$q_1 : q_2 : q_3 = (C_2 - C_m) : (C_m - C_1) : (C_3 - C_m) \quad (3)$$

Similarly, after finding out the proportion, for the different ingredients, the required quantities can be found out as:

$$q_1 = \text{quantity of 1st ingredient} = \frac{q_1}{(q_1 + q_2 + q_3)} \times \text{amount of mixture}$$

$$q_2 = \text{quantity of 2nd ingredient} = \frac{q_2}{(q_1 + q_2 + q_3)} \times \text{amount of mixture}$$

$$q_3 = \text{quantity of 3rd ingredient} = \frac{q_3}{(q_1 + q_2 + q_3)} \times \text{amount of mixture}$$

Example: In what proportion may three kinds of cashewnuts worth Rs 327, Rs 329 and Rs 332 per kg be mixed to produce a mixture worth Rs 330/kg?

Solution: Here, $C_1 = \text{Rs } 327$

$$C_2 = \text{Rs } 329 \quad C_m = \text{Rs } 330$$

$$C_3 = \text{Rs } 332$$

Using the formula (2), we have

$$\begin{aligned} q_1 : q_2 : q_3 &= (C_2 - C_m) : (C_m - C_1) : (C_3 - C_m) \\ &= 1 \times 2 : 3 \times 2 : 1 \times 3 \\ &= 2 : 6 : 3 \end{aligned}$$

This ratio of three kinds of cashewnuts can also be found out by the following short-cut method.

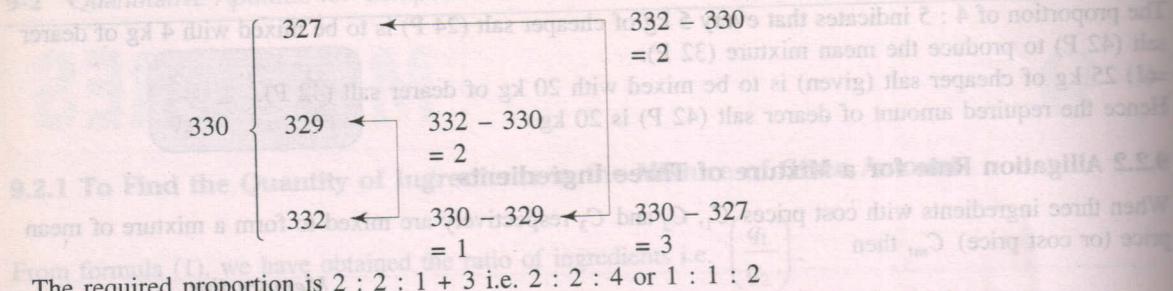
Short Cut Method

Step 1 Place all the prices one under another in order of magnitude, the least one at the top.

Step 2 Write the mean price to the left.

Step 3 Link the prices in pairs so that one greater price and one lesser price than mean go together.

Step 4 Find the difference between each price and mean price and place it opposite to the price with which it is linked. These differences are the answer.



Note: This proportion is different from the proportion found out by using formula (2).

In fact, we can use any amount (in kg) of the 1st or 2nd cashewnut as long as we use the necessary corresponding amount (in kg) of the 3rd variety of cashewnut and hence the number of proportions can be different.

9.3 MEAN VALUE (OR COST PRICE) OF THE MIXTURE

Case I

When two ingredients A and B are mixed

q_1 and $q_2 \rightarrow$ Quantities of A and B respectively

C_1 and $C_2 \rightarrow$ Cost price of A and B respectively

Mean value (cost price) is given by

$$C_m^* = \frac{C_1 \times q_1 + C_2 \times q_2}{q_1 + q_2} \quad (4)$$

* It is similar to finding the average of two ingredients with different cost price and quantities.

Note: (a) q_1 and q_2 may also be in ratio form (in proportion) in the above formula.

(b) Above formula can also be derived from the Alligation Rule

Case II

When more than two ingredients are mixed

$$C_m = \frac{(C_1 \times q_1 + C_2 \times q_2 + \dots + C_n \times q_n)}{(q_1 + q_2 + \dots + q_n)} \quad (5)$$

Example: A man mixes 6 kgs of tea worth Rs 6 per kg and 4 kgs of tea worth Rs 7 per kg and the mixture is sold at 10% profit. Find the selling price per kg of the mixture.

Solution: Using formula (5), we get

$$\text{Mean value (cost price) of the mixture} = \frac{6 \times 6 + 4 \times 7}{6 + 4}$$

$$= \text{Rs } 6.4 \text{ per kg.}$$

$$\begin{aligned} \text{Sale price per kg of the mixture} &= 1.1 \times 6.4 \\ &= \text{Rs } 7.04 \text{ per kg.} \end{aligned}$$

9.4 SIX GOLDEN RULES TO SOLVE PROBLEMS ON MIXTURE

Assumptions

- (a) *In Ratio form:* There is one given mixture of P kg (say) having two ingredients A and B in the ratio $a : b$
- OR
- (b) *In Percentage form:* There is one given mixture of P kg (say) containing two ingredients A and B . Ingredient A is $m\%$ of the mixture.

Rule 1

Quantity of each ingredient is to be found out by using the Ratio-Quantity Rule [Refer 9.2.1]

e.g. Quantity of $A = \frac{a}{a+b} \times P$, as per the assumption in (a)

$$\text{Quantity of } B = \frac{b}{a+b} \times P$$

Rule 2

This rule is applicable when the quantity of any ingredient is expressed as percentage of the mixture [as per the assumptions (b)]

e.g. quantity of $A = m\%$ of P , as per assumption (b) and quantity of $B = (100 - m)\%$ of P .

Rule 3

If x kg (say) of any ingredient is added to the mixture, then in the final, that particular ingredient increases by x kg and the amount of mixture also increases by x kg, but the quantity of other ingredient remains unchanged.

e.g. (a) **In Ratio form:** Let x kg of ingredient A be added to the mixture, then

$$\text{new ratio of } A \text{ and } B = \frac{\text{Quantity of } A}{\text{Quantity of } B} = \frac{\left(P \times \frac{a}{a+b} \right) + x}{P \times \frac{b}{a+b}} \quad [\text{Refer Rule-1}]$$

(b) **In Percentage form:** Let x kg of ingredient A be added to the mixture, then

$$\text{new \% of } A \text{ in the mixture} = \frac{\text{New quantity of } A}{\text{Amount of mixture}} \times 100\% \quad [\text{See Assumption (b)}]$$

$$\Rightarrow \% \text{ of } A \text{ in the mixture} = \frac{m\% \text{ of } P + x}{P + x} \times 100\% \quad [\text{See Assumption (b)}]$$

Similarly,

$$\text{new \% of } B \text{ in the mixture} = \frac{\text{Quantity of } B}{\text{Amount of the mixture}} \times 100\%$$

$$\text{new \% of } B \text{ in the mixture} = \frac{(100 - m)\% \text{ of } P}{P + x} \times 100\%$$

In Ratio Form

Example: The ratio of milk and water in a mixture of 35 litres is 4 : 1. How much water must be added to the mixture so that the ratio of milk and water be 3 : 2?

Solution: In this problem, we are concerned with ratio of milk and water,

i.e. $\frac{\text{quantity of milk}}{\text{quantity of water}}$

It is given that $\frac{\text{quantity of milk}}{\text{quantity of water}} = \frac{4}{1}$

Now, say, x litres of water is added to 35 litres of mixture to obtain the new ratio as 3 : 2

$$\Rightarrow \frac{\text{Quantity of milk}}{\text{Quantity of water}} = \frac{\frac{4}{1+4} \times 35}{\left(\frac{1}{1+4} \times 35\right) + x} = \frac{3}{2}$$

$$\Rightarrow \frac{28}{7+x} = \frac{3}{2}$$

$$\Rightarrow x = 11\frac{2}{3} \text{ litres}$$

Hence, $11\frac{2}{3}$ litres of water must be added to the mixture.

Example: The ratio of milk and water-milk mixture is 2 : 3. How much water should be added to 60 litres of the mixture to make the ratio of milk and mixture as 1 : 3?

Solution: In this problem, we are concerned with the ratio of milk and mixture,

i.e. $\frac{\text{Quantity of milk}}{\text{Quantity of mixture}} = \frac{2}{3}$ (given)

Now, say, x litres of water is added to 60 litres of mixture to obtain the new ratio as 1 : 3. The quantity of mixture becomes $(60 + x)$

$$\Rightarrow \frac{\text{Quantity of milk}}{\text{Quantity of mixture}} = \frac{\frac{2}{3+2} \times 60}{60+x} = \frac{1}{3}$$

$$\Rightarrow x = 60 \text{ litres}$$

Therefore, 60 litres of water should be added.

In Percentage Form

Example: In 70 litres of a mixture of milk and water, the quantity of water is 10%. How much water should be added so that new mixture may contain 25%.

Solution: Quantity of water = 10% of mixture = 10% of 70 litres

Let the quantity of water to be added = x litres

$$\therefore \text{new percentage of water in the mixture} = \frac{(10\% \text{ of } 70) + x}{(70+x)} \times 100\%$$

$$= 25\%$$

New percentage of water = 25% (given)

$$\Rightarrow \frac{(10\% \text{ of } 70) + x}{70+x} \times 100 = 25$$

$$\Rightarrow \frac{7+x}{70+x} = \frac{1}{4}$$

$$\Rightarrow x = 14 \text{ litres}$$

Hence the quantity of water to be added = 14 litres.

Example: Ram adds 1 litre of water to 5 litres of a 20% solution of alcohol in water. What is the final strength of alcohol?

Solution: The problem asks to find the final strength of alcohol i.e. % of alcohol in solution. Here, one litre of water is only added.

Hence, we are concerned with the $\frac{\text{alcohol}}{\text{solution}}$

$$\therefore \text{Required final strength of alcohol} = \frac{\text{quantity of alcohol}}{\text{quantity of solution}} \times 100\%$$

$$= \frac{20\% \text{ of } 5}{5+1} \times 100\%$$

1 litre of water added increases
the solution by 1 litre

$$= \frac{1}{6} \times 100\%$$

$$= 16\frac{2}{3}\%$$

Rule 4

When two mixtures with same ingredients are mixed, then the quantity of each ingredient in each mixture is found separately and these are added to find the quantity in the final mixture.

e.g. One mixture of P kg contains A and B in the ratio $a : b$

Another mixture of Q kg contains A and B in the ratio $x : y$

Now, these two mixtures are mixed to form a new mixture.

In the new mixture (final mixture),

$$\frac{\text{Quantity of } A}{\text{Quantity of } B} = \frac{\text{Quantity of } A \text{ in first mixture} + \text{in second mixture}}{\text{Quantity of } B \text{ in first mixture} + \text{in second mixture}}$$

Case 1

Let the amount of mixture in each be denoted by M kg. Then, the application of Rule 4 gives us

$$\text{Amount of mixture} = M$$

In Ratio Form

$$= \left(\frac{a}{a+b} \right) P + \left(\frac{x}{x+y} \right) Q$$

Example: The ratio of milk to water in two mixtures is $4 : 1$. How much water should be added to the mixture so that the ratio becomes $2 : 1$? (Using Rule-1)

$$= \left(\frac{b}{a+b} \right) P + \left(\frac{y}{x+y} \right) Q$$

Solution: In this problem, we have to find the quantity of water to be added to the ratio of milk and water,

Similarly, in percentage form,

if ingredient A is $m\%$ in the P kg of first mixture
 and ingredient A is $n\%$ in the Q kg of second mixture,
 then, in the final mixture, containing these two mixtures,

$$\text{the new \% of } A = \frac{(\text{quantity of } A \text{ in first mixture} + \text{in second mixture})}{\text{amount of mixture}} \times 100\%$$

$$= \frac{(m\% \text{ of } P + n\% \text{ of } Q)}{(P+Q)} \times 100\% \quad (\text{Using Rule-2})$$

Example: In two alloys, the ratio of copper and zinc are $3 : 4$ and $5 : 8$. If 14 kg of first alloy and 26 kg of second alloy are mixed, then find the ratio of copper and zinc in the new alloy.

Solution:	Alloy 1	Alloy 2	New Alloy
Copper —	$\frac{3}{7} \times 14$	$\frac{5}{13} \times 26$	$\frac{3}{7} \times 14 + \frac{5}{13} \times 26$
Zinc —	$\frac{4}{7} \times 14$	$\frac{8}{13} \times 26$	$\frac{4}{7} \times 14 + \frac{8}{13} \times 26$

The required ratio of copper and zinc in the new alloy

$$= \frac{\text{Quantity of copper}}{\text{Quantity of Zinc}} = \frac{\frac{3}{7} \times 14 + \frac{5}{13} \times 26}{\frac{4}{7} \times 14 + \frac{8}{13} \times 26} = \frac{6+10}{8+16} = \frac{2}{3}$$

Rule 5*

When two mixtures are mixed, then the ratio of the quantities of each mixture in the final (new) mixture can be found by applying Alligation Rule to the parts of same ingredient in the two mixtures and also in the final mixture.

Example: Two vessels contain spirit and water mixed respectively in the ratios $3 : 1$ and $5 : 3$. Find the ratio in which these are to be mixed to get a new mixture in which the ratio of spirit to water is $2 : 1$.

Solution: Let us consider the parts of spirit in both the given mixtures and also in the new mixture.

$$\text{The parts of spirit in the first mixture} = \frac{3}{3+1} = \frac{3}{4}$$

$$\text{The parts of spirit in the second mixture} = \frac{5}{5+3} = \frac{5}{8}$$

$$\text{The parts of spirit in the new mixture} = \frac{2}{2+1} = \frac{2}{3}$$

*Note the difference in the application of Rule-4 and Rule-5. As the **Rule-4** is used to find the quantity of each ingredient in the final mixture, but the **Rule-5** is to find the quantity of each mixture in the final mixture.

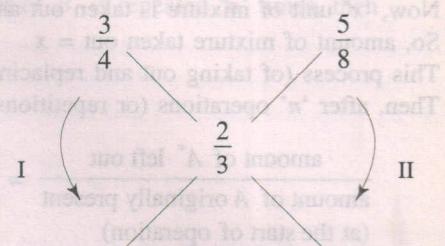
By applying Alligation Rule,

Quantity of Mixture I in the new mixture

Quantity of Mixture II in the new mixture

$$\begin{aligned} \text{Amount of } A \text{ remaining} &= \frac{2}{3} - \frac{5}{8} \\ &= \frac{3}{4} - \frac{2}{3} \\ &= \frac{1}{2} \end{aligned}$$

and,



$$\text{The required ratio of two mixtures is } 1 : 2$$

Rule 6

Two mixtures of the same ingredients are mixed.

If x kg (say) of any ingredient A is added to the above mixture, then the value of x can be found out by the following equation:

$$x + \text{quantity of } A \text{ in Mix I} + \text{quantity of } A \text{ in Mix II} = \text{quantity of } A \text{ in new mixture.}$$

Example: In two mixtures, spirit and water are related in the ratios of $3 : 5$ and $7 : 4$. 24 gallons of mixture I, 44 gallons of mixture II and 25 gallons of spirit are mixed together. What is the final ratio of spirit and water?

Solution: Here, 25 gallons of spirit is added to the mixture.

∴ quantity of spirit in both the mixtures and in the final mixture is to be considered.

Let the ratio of spirit and water in the final mixture be $m : n$.

Now, as per question,

quantity of spirit in Mix I + spirit in Mix II + pure spirit = quantity of spirit in final mixture

(By rule-6)

$$\Rightarrow \left(\frac{3}{3+5} \times 24 \right) + \left(\frac{7}{7+4} \times 44 \right) + 25 = \frac{m}{m+n} (24 + 44 + 25)$$

$$\Rightarrow \frac{9+28+25}{24+44+25} = \frac{m}{m+n}$$

$$\Rightarrow \frac{m}{n} = \frac{2}{3}$$

$$\Rightarrow \frac{m}{n} = \frac{2}{1}$$

Therefore, the final ratio of spirit and water is $2 : 1$

9.5 REMOVAL AND REPLACEMENT BY EQUAL AMOUNT

Case I

Removal of an amount from a mixture

Let the mixture contain A and B

Amount of mixture = M

Now, 'x' unit of mixture is taken out and replaced by an equal amount of any one ingredient, say *B*.

So, amount of mixture taken out = *x*

This process (of taking out and replacing) is repeated '*n*' times

Then, after '*n*' operations (or repetitions),

$$\frac{\text{amount of } A^* \text{ left out}}{\text{amount of } A \text{ originally present}} = \left[1 - \frac{x}{M} \right]^n$$

(at the start of operation)

Amount of *B* left out = *M* – amount of *A* left out.

* because replacement is done by equal amount of *B*, so, the formula finds 'amount of *A* left out' first.

Example: A seven litre vessel contain a mixture of milk and water, milk being 49% of total volume. A few litres of the mixture is taken out and replaced by equal amount of water. This process is repeated twice and now the milk is 9% only. How many litres of mixture is taken out each time?

Solution: Let the amount of mixture taken out each time = *x* litres.

Number of operations = 2 [Here, mixture is replaced by water].

Using the formula,

$$\begin{aligned} \frac{\text{Amount of milk left}}{\text{Amount of milk originally}} &= \left(1 - \frac{x}{M} \right)^2 \\ \Rightarrow \frac{9\%V}{49\%V} &= \left(1 - \frac{x}{7} \right)^2 \\ \Rightarrow \frac{9}{49} &= \left(1 - \frac{x}{7} \right)^2 \\ \Rightarrow 1 - \frac{x}{7} &= \frac{3}{7} \\ \Rightarrow x &= 4 \end{aligned}$$

Therefore, 4 litres of mixture are taken out each time.

Case II

Removal of an amount from an ingredient

Unlike case I, here, initially there is one ingredient only and a certain amount of it is taken out at the first instance and replaced by an equal amount of another ingredient. Then again, same amount of mixture is taken out in the second operation and replaced by the same ingredient.

Let the vessel contain only ingredient *A*.

Amount of *A* initially present = *a*, say

In first operation, amount of *A* taken out = *x* and replaced by another ingredient, say *B*

In second operation, amount of mixture taken out = *x* and replaced by *B*



(after first operation, amount of *A* removed is replaced by equal amount of second ingredient *B*, so before second operation, *A* gets mixed with the second ingredient, *B*. Hence, in second operation, we cannot take only *A*, but we take out a mixture of *A* and *B*).

If this operation of taking out and replacing with equal amount of B^* is repeated ' n ' times, then

$$\frac{\text{Amount of } A^* \text{ remaining}}{\text{Amount of } A \text{ initially present}} = \left[1 - \frac{x}{a}\right]^n$$

$$\therefore \text{Amount of } A \text{ remaining} = a \left[1 - \frac{x}{a}\right]^n$$

and,

$$\frac{\text{amount of } A \text{ remaining}}{\text{amount of } B \text{ remaining}} = \frac{\left[1 - \frac{x}{a}\right]^n}{1 - \left[1 - \frac{x}{a}\right]^n}$$

*Because replacement is done by equal amount of B , so, the formula finds 'amount of A remaining' first

Example: Nine litres are drawn from a cask full of wine and it is then filled with water. Nine litres of the mixture are withdrawn and the cask is again filled with water. The ratio of quantity of wine now remaining in the cask to that of the water in it is $16 : 9$. How much does the cask hold?

Solution: Let the capacity of cask = a litres (which is equal to amount of wine initially present).

Using the formula,

$$\frac{\text{Amount of } A \text{ remaining}}{\text{Amount of } B \text{ remaining}} = \frac{\left(1 - \frac{x}{a}\right)^n}{1 - \left(1 - \frac{x}{a}\right)^n}$$

The process is repeated twice, i.e. $n = 2$
 Amount withdrawn each time = $x = 9$

$$\Rightarrow \frac{16}{9} = \frac{\left(1 - \frac{9}{a}\right)^2}{1 - \left(1 - \frac{9}{a}\right)^2}$$

$$\Rightarrow \frac{16}{9} - \frac{16}{9} \left(1 - \frac{9}{a}\right)^2 = \left(1 - \frac{9}{a}\right)^2$$

$$\Rightarrow \frac{16}{9} = \frac{25}{9} \left(1 - \frac{9}{a}\right)^2$$

$$\Rightarrow \left(1 - \frac{9}{a}\right)^2 = \frac{16}{25}$$

$$\Rightarrow a = 45$$

Therefore, the capacity of cask = 45 litres.

Solved Examples

E-1 In what proportion should one variety of oil at Rs 9.50 per kg be mixed with another at Rs 10 per kg to get a mixture worth Rs 9.60 per kg?

S-1 By alligation rule [9.2] and using the formula, we get

$$\frac{q_1}{q_2} = \frac{c_2 - C_m}{C_m - c_1} = \frac{10 - 9.6}{9.6 - 9.5} = \frac{4}{1}$$

∴ Two varieties of oil are mixed in the ratio 4 : 1.

E-2 In what ratio must 25% alcohol be mixed with 50% alcohol to get a mixture of 40% alcohol strength?

S-2 Here two same ingredients (i.e. alcohol), but of different strength are mixed. Hence, by the alligation rule,

$$\frac{q_{25\%}}{q_{50\%}} = \frac{50 - 40}{40 - 25} = \frac{10}{15} = \frac{2}{3}$$

∴ 25% alcohol is mixed with 50% alcohol in 2 : 3.

E-3 A merchant lent out Rs 1,000 in two parts, one at 8% and the other at 10% interest. The yearly average comes out to be 9.2%. Find the amount lent in two parts.

S-3 $\frac{\text{Quantity lent at } 8\%}{\text{Quantity lent at } 10\%} = \frac{10 - 9.2}{9.2 - 8} = \frac{0.8}{1.2}$, by alligation rule

$$= \frac{2}{3}$$

∴ Dividing Rs 1,000 in $\frac{2}{3}$ as,

$$\text{Quantity of money lent at } 8\% = \frac{2}{2+3} \times 1000 = \text{Rs } 400.$$

$$\text{Quantity of money lent at } 10\% = \frac{3}{2+3} \times 1000 = \text{Rs } 600.$$

E-4 Pure milk costs Rs 3.60 per litre. A milkman adds water to 25 litres of pure milk and sells the mixture at Rs 3 per litre. How many litres of water does he add? (UDC, '82)

S-4 $\frac{q_{\text{pure milk}}}{q_{\text{water}}} = \frac{3.00 - 0}{3.6 - 3.0} = \frac{3}{0.6} = \frac{5}{1}$ (Since cost of water = 0)

For every 5 litres of milk, he adds 1 litre of water,

For every 25 litres of milk, he adds 5 litres of water.

E-5 How many kg of salt at 42 paise per kg must a man mix with 25 kg of salt at 24 paise per kg so that he may, on selling the mixture at 40 paise per kg gain 25% on the outlay? (IITCE, '89)

S-5 $\frac{q_1}{q_2} = \frac{c_2 - C_m}{C_m - c_1}$, by Alligation Rule,

where c_1 , c_2 , and C_m are always cost price of the ingredients and mixture.

$$\therefore \text{Cost price of mixture} = C_m = 40 \times \frac{100}{(100+25)} \quad (\text{Since } 25\% \text{ gain}) \\ = 32 \text{ paise}$$

$$\therefore \frac{q_{42}}{q_{24}} = \frac{32-24}{42-32} = \frac{8}{10} = \frac{4}{5}$$

$$\therefore q_{42} = \frac{4}{5} \times 25 = 20 \text{ kg.} \quad (\text{Since } q_{24} = 25 \text{ kg given})$$

E-6 A mixture of 40 litres of milk and water contains 10% water. How much water must be added to make 20% of water in the new mixture? (FT, '92)

$$\text{S-6} \quad \frac{q_{\text{milk}}}{q_{\text{water}}} = \frac{90\% \text{ of } 40 \text{ litres}}{10\% \text{ of } 40 \text{ litres}}$$

Let x litres of water be added, then,

$$\frac{q_{\text{milk}}}{q_{\text{water}}} = \frac{(0.9 \times 40)}{(0.1 \times 40) + x} = \frac{80}{20} \quad (\text{Since } 20\% \text{ water in new mixture})$$

$$\Rightarrow x = 5, \text{ hence, 5 litres of water is added.}$$

E-7 Five litres of water is added to a certain quantity of pure milk costing Rs 3 per litre. If by selling the mixture at the same price as before, a profit of 20% is made, what is the amount of pure milk in the mixture? (SSC, '80)

S-7 Sale price of mixture = Cost price of pure milk

$$\Rightarrow \frac{120}{100} \times \text{CP of mixture} = 3.0 \quad (\text{Since a profit of } 20\% \text{ is made})$$

$$\Rightarrow \text{CP of mixture} = \frac{3 \times 100}{120} = \text{Rs } 2.5 \text{ per litre}$$

\therefore Using formula (a), after assuming x = amount of pure milk

$$\text{mean CP of mixture} = 2.5 = \frac{x \times 3 + 5 \times 0}{x + 5} \quad (\text{Since CP of water} = 0)$$

$$\Rightarrow x = 25$$

\therefore amount of pure milk = **25 litres**

NB: When water is mixed with any other ingredient to form a mixture such that

Sale price of mixture = Cost price of ingredient,

Then,

$$\frac{\text{Quantity of water}}{\text{Quantity of ingredient}} = \% \text{ profit made due to mixing of water}$$

E-8 To 5 litres, of 20% sulphuric acid, 5 litres of 100% pure sulphuric acid is added. What is the strength of the acid in the mixture now?

$$\text{S-8} \quad \% \text{ strength of acid in mixture} = \frac{5 \times 20 + 5 \times 100}{5 + 5} \quad (\text{here, quantity is in \%}) [Refer 9.3] \\ = 60\%.$$

E-9 Two liquids are mixed in the proportion of 3 : 2 and the mixture is sold at Rs 11 per litre at a 10% profit. If the 1st liquid costs Rs 2 more per litre than the second, what does it cost per litre? (HMAN, '88)

S-9 Since the mixture is sold at 10% profit,

$$\therefore \text{Cost price of the mixture } (C_m) = \frac{11}{1+0.1} = \text{Rs } 10$$

Cost price of 1st liquid c_1 = Rs x per litre

Cost price of 2nd liquid c_2 = Rs $(x - 2)$ per litre

By alligation rule [9.2], since proportion of two liquids are known.

$$\begin{aligned} \frac{\text{Quantity of liquid}_1 (q_1)}{\text{Quantity of liquid}_2 (q_2)} &= \frac{C_m - c_2}{c_1 - C_m} \\ \Rightarrow \frac{3}{2} &= \frac{10 - (x - 2)}{x - 10} \end{aligned}$$

$$\Rightarrow x = \text{Rs } 10.80 \text{ per litre.}$$

\therefore Cost price of 1st liquid is Rs 10.80 per litre

Cost price of 2nd liquid is Rs 8.80 per litre

E-10 Two equal glasses filled with mixtures of alcohol and water in the proportions of 2 : 1 and 1 : 1 respectively were emptied into a third glass. What is the proportion of alcohol and water in the third glass? (Bank PO, '90)

S-10 Using Rule 4 we get

$$\begin{aligned} q_1 &= \frac{M \left(\frac{a}{a+b} \right) + N \left(\frac{x}{x+y} \right)}{M \left(\frac{b}{a+b} \right) + N \left(\frac{y}{x+y} \right)}; \\ q_2 &= \end{aligned}$$

$$\text{Given that, } \frac{a}{b} = \frac{2}{1} = \frac{\text{alcohol (in 1st mixture)}}{\text{water (in 1st mixture)}}$$

$$\frac{x}{y} = \frac{1}{1} = \frac{\text{alcohol (in 2nd mixture)}}{\text{water (in 2nd mixture)}}$$

$M = N$ [Since glasses (filled with two mixture) are equal]

$$\therefore \frac{\text{Quantity of alcohol in 3rd glass}}{\text{Quantity of water in 3rd glass}} = \frac{q_1}{q_2} = \frac{\frac{2}{(2+1)} + \frac{1}{(1+1)}}{\frac{1}{(2+1)} + \frac{1}{(1+1)}} = \frac{7}{5}$$

\therefore In third glass, alcohol and water are in the proportion of 7 : 5.

E-11 A butler stole wine from a butt of sherry which contained 32% of spirit and then replaced what he stole, by wine containing only 18% spirit. The butt was then of 24% strength only. How much of the butt had he stolen? (FT, '92)

S-11 By alligation rule, [9.2]

$$\frac{\text{Quantity of } 32\% \text{ spirit}}{\text{Quantity of } 18\% \text{ spirit}} = \frac{24 - 18}{32 - 24} = \frac{3}{4}$$

(Since the strength of resultant butt is 24%)

∴ Quantity of 18% spirit is $\frac{4}{3+4}$ of the butt, after replacement

Hence quantity of 32% spirit stolen is $\frac{4}{7}$ of the butt.

E-12 Fresh fruit contains 72% water and dry fruit contains 20% water. How much dry fruit from 100 kg of fresh fruit can be obtained? (MBA, '91)

S-12 We are concerned with solid part of the fruit (pure portion). Assume x kg of dry fruit is obtained.

∴ Solid part in fresh fruit = Solid part in dry fruit

$$\Rightarrow 0.28 \times 100 = 0.8 \times x$$

$$\Rightarrow x = 35 \text{ kg.}$$

∴ 35 kg of dry fruit can be obtained from 100 kg fresh fruit.

E-13 Sea water contains 5% salt by weight. How many kg of fresh water must be added to 60 kg of sea water for the content of salt in solution to be made 3%. (MBA, '95)

S-13 Let x kg of fresh water is added to sea water (By rule-3),

$$\frac{q_{\text{salt}}}{(q_{\text{salt}} + q_{\text{water}})} = \frac{5\% \text{ of } 60}{60 + x} = \frac{3}{100}$$

(given, 3% salt in solution)

$$\Rightarrow \frac{3}{60 + x} = \frac{3}{100} \Rightarrow x = 40 \text{ kg.}$$

∴ 40 kg of fresh water must be added to sea water.

E-14 In two alloys, the ratio of copper to zinc are 5 : 2 and 3 : 4. How many kg of the first alloy and of the second alloy should be melted together to obtain 28 kg of a new alloy with equal contents of copper and zinc.

S-14 Here, two alloys having same ingredients (copper and zinc) are mixed to obtain a new alloy. Individual amount of each alloy is to be found out. So, the Rule-5 is to be used.

Considering the quantity of copper in all the three alloys, we get

$$\text{Copper in the 1st alloy} = \frac{a}{a+b} = \frac{5}{5+2} = \frac{5}{7}$$

$$\text{Copper in 2nd alloy} = \frac{x}{x+y} = \frac{3}{3+4} = \frac{3}{7}$$

$$\text{Copper in (1st + 2nd) new alloy} = \frac{q_A}{q_A + q_B} = \frac{1}{1+1} = \frac{1}{2}$$

By alligation rule,

$$\therefore \frac{q_{\text{I alloy}}}{q_{\text{II alloy}}} = \frac{\frac{1}{2} - \frac{3}{7}}{\frac{5}{7} - \frac{1}{2}} = \frac{1}{3}$$

$$\text{So, amount of 1st alloy} = \frac{q_1}{q_1 + q_2} \times 28 = \frac{1}{4} \times 28 = 7 \text{ kg}$$

$$\text{Amount of 2nd alloy} = 28 - 7 = 21 \text{ kg}$$

E-15 A sum of Rs 39 was divided among 45 boys and girls. Each girl gets 50 paise, whereas a boy gets one rupee. Find the number of boys and girls. (CG, '90)

$$\text{S-15} \quad \text{Average amount of money received by each} = \text{Rs } \frac{39}{45} = \text{Rs } \frac{13}{15}$$

$$\text{Amount received by each girl} = 50 \text{ paise} = \text{Rs } \frac{1}{2}$$

$$\text{Amount received by each boy} = \text{Re. 1}$$

By alligation rule,

$$\frac{\text{Number of boys}}{\text{Number of girls}} = \frac{q_{\text{boy}}}{q_{\text{girl}}} = \frac{\frac{13}{15} - \frac{1}{2}}{1 - \frac{13}{15}} = \frac{11}{4}$$

S-10 Using Rule 4 we get

$$\therefore \text{Number of boys} = \frac{11}{11+4} \times 45 = 33$$

$$\text{Number of girls} = 45 - 33 = 12.$$

E-16 An 8 litre cylinder contain a mixture of oxygen and nitrogen, the volume of oxygen being 16% of total volume. A few litres of the mixture is released and an equal amount of nitrogen is added. Then the same amount of the mixture as before is released and replaced by nitrogen for the second time. As a result, the oxygen content becomes 9% of total volume. How many litres of mixture is released each time?

S-16 The cylinder originally contains a mixture of oxygen and nitrogen. An equal amount of released mixture is replaced by an equal amount of nitrogen. So, using the formula, [Refer 9.5]

$$\frac{\text{Amount of } A \text{ (oxygen) left}}{\text{Amount of } A \text{ (oxygen) originally present}} = \left(1 - \frac{x}{M}\right)^n$$

where, total volume of mixture = volume of cylinder = $M = 8$

released amount of mixture = x litres

number of operations done (n) = 2

$$\therefore \frac{0.09 \times 8}{0.16 \times 8} = \left(1 - \frac{x}{8}\right)^2$$

$$\Rightarrow x = 2.$$

\therefore 2 litres of mixture is released each time.

E-17 A bottle is full of dettol. One-third of it is taken out and then an equal amount of water is poured into the bottle to fill it. This operation is done four times. Find the final ratio of dettol and water in the bottle.

S-17 The bottle originally contains dettol only

Let the bottle contain 1 litre of dettol originally,

So, using the formula [Refer 9.5 (Case II)]

$$\frac{\text{Amount of } A \text{ (dettol) left}}{\text{Amount of } B \text{ (water) left}} = \frac{\left(1 - \frac{x}{a}\right)^n}{1 - \left(1 - \frac{x}{a}\right)^n}$$

$$\Rightarrow \frac{\text{Dettol}}{\text{Water}} = \frac{\left(1 - \frac{1}{3}\right)^4}{1 - \left(1 - \frac{1}{3}\right)^4}$$

$$= \frac{\left(\frac{2}{3}\right)^4}{1 - \left(\frac{2}{3}\right)^4}$$

Finally, the bottle contains dettol and water in the ratio **16 : 65**.

REGULAR PROBLEMS

(1) How much chicory at Rs 4 a kg should be added to 15 kg of tea at Rs 10 a kg so that the mixture be worth Rs 6.50 per kg?

- (a) 15 kg (b) 18 kg (c) 12 kg (d) 21 kg (e) 10 kg

Hint: Do not waste time by applying alligation rule to find the ratio of quantity of chicory and quantity of tea. It is better to use formula (4) of mean cost price directly. Assume quantity of chicory = x kg

(2) Six litres of a 20% solution of alcohol in water are mixed with four litres of 60% solution of alcohol in water. The alcoholic strength of the mixture is:

- (a) 40% (b) 36% (c) 26% (d) 30% (e) 32%

Hint: Use formula (4) directly to find mean strength of the mixture.

- (3) One litre of water was mixed to six litres of sugar solution containing 4% of sugar. What is the percentage of sugar in the solution? (UTI, '90)

(a) 3 (b) 4 (c) 6 (d) insufficient data (e) none of these

Hint: We need to find out % of sugar in the new solution and the quantity of sugar in the original sugar solution has also been given in % form (as 4%). So, do not convert the % into fraction, rather

$$\text{use directly as: new \% of sugar in } (3 + 1) \text{ litre solution} = \frac{4\% \times 6}{(3+1)} = 6\%$$

↗ quantity of water added

- (4) An ore contains 5% of aluminium. To get 52 kg of aluminium, the quantity of the ore required is:

(a) 520 kg (b) 780 kg (c) 1,040 kg (d) 1,560 kg (e) 260 kg

- (5) Three lumps of gold, weighing respectively 6, 5, 4 gms and of 15, 14, $12\frac{1}{2}$ carats fineness are mixed together. The fineness of the resulting compound is:

(a) 14 carats (b) 20 carats (c) 16 carats (d) 13 carats (e) 11 carats

- (6) The quantity of water (in ml) added to reduce the strength of 9 ml of an after-shave lotion, from 50% alcohol to 30% alcohol, is:

(a) 3 (b) 4 (c) 5 (d) 6 (e) none of these

Hint: See question (3)

- (7) A mixture contains milk and water in the ratio 5 : 1. On adding 5 litres of water, the ratio of milk to water becomes 5 : 2. The quantity of milk in the mixture is: (RBI, '91)

(a) 5 litres (b) 15 litres (c) 25 litres (d) 20 litres (e) 10 litres.

- (8) Sameer bought 10 kg of tomatoes at Rs 45 per kg and 8 kg at Rs 50 per kg. He mixed both the grades of tomatoes and sold it at a total profit of Rs 32. The selling price per kg of the mixed tomatoes was (LIC, '91)

(a) Rs 79 (b) Rs 48 (c) Rs 49 (d) Rs 46 (e) Rs 54

Hint: Total cost + profit = Total sale price

- (9) Gold is 19 times as heavy as water and copper is 9 times heavy. In what ratio must these metals be mixed so that the mixture may be 15 times as heavy as water?

(a) 2 : 3 (b) 3 : 2 (c) 1 : 3 (d) 2 : 1 (e) 4 : 3

Hint: The heaviness of gold, copper mixture has been compared with the same compound (here, it is water), so, apply alligation rule

- (10) Two equal-sized glasses are respectively $\frac{1}{4}$ and $\frac{1}{3}$ full of orange juice. Then they are filled with water and the contents are mixed in a tumbler. The ratio of the orange juice to water in the tumbler is (a) 3 : 11 (b) 7 : 17 (c) 9 : 23 (d) 11 : 23 (e) 7 : 24

Hint: Here glasses are of equal size, so only by adding the parts of contents in two glasses, we can take it as total content

- (11) A mixture is composed of 8 parts of whisky and 3 parts of water. After adding 28 litres of water, if the mixture contains whisky one half as much as water, then how many litres of whisky does it contain? (NDA, '82)

(a) 96 (b) 48 (c) 192 (d) 72 (e) 108

- (12) A bottle is full of dettol. One-third of it is taken out and then an equal amount of water is poured into the bottle to fill it. This operation is done four times. Find the final ratio of dettol and water in the bottle.

(a) 49 : 65 (b) 65 : 16 (c) 9 : 16 (d) 16 : 65 (e) 9 : 25

- (13) A dishonest hairdresser uses a mixture having 5 parts after-shave lotion and 3 parts water. After taking out some portion of the mixture, he adds equal amount of water to the remaining portion of mixture such that the amount of after-shave lotion and water become equal. Find the part of mixture taken out.

(a) $\frac{4}{5}$ (b) $\frac{1}{3}$ (c) $\frac{3}{5}$ (d) $\frac{1}{5}$ (e) $\frac{1}{2}$

Answers

1. (d) 2. (b) 3. (c) 4. (c) 5. (a) 6. (d) 7. (c) 8. (c) 9. (b)
 10. (b) 11. (a) 12. (d) 13. (d)

REAL PROBLEMS

- (1) The ratio of milk and water in 55 litres of adulterated milk is 7 : 4. How much water must be added to make the mixture's ratio 7 : 6?

(a) 5 kg (b) 10 kg (c) 15 kg (d) 25 kg (e) 30 kg

- (2) A mixture of an herbal liquid and a base oil contains 45% herbal liquid by weight. Twenty five grams of base oil is added to such 200 grams of mixture. What % of herbal liquid by weight is there in the new mixture?

(a) 25 (b) 60 (c) 80 (d) 40 (e) 35

Hint: Here % of herbal liquid in mixture is to be found out. So, use, $\frac{\text{herbal liquid quantity}}{\text{mixture quantity}}$

- (3) How many litres of pure glycerine must be added to 10 litres of a mixture, which is 15% glycerine, to make a mixture that will be 25% glycerine?

(a) $\frac{5}{4}$ (b) $\frac{5}{2}$ (c) 2 (d) $\frac{3}{4}$ (e) $\frac{4}{3}$

Hint: Pure glycerine means 100% glycerine

- (4) Two varieties of white edible oils are mixed in the proportion of 3 : 2 and the mixture is sold at Rs 11 per kg at a 10% profit. If the first variety of oil costs Rs 2 more per kg than the second, what does it cost per litre?

(a) Rs 11 (b) Rs 10.50 (c) Rs 11.50 (d) Rs 10.80 (e) Rs 10

Hint: Use formula (4) only. No other rule is to be used so as to save your time

- (5) A man buys milk at a certain rate per kg and, after mixing it with water, sells it again at the same rate. Find how many grams of water there are in every kg of milk if the man makes a profit of 20%?

(a) 20 gm (b) 200 gm (c) 250 gm (d) 150 gm (e) None of these

- (6) How much water must be added to 12 litres of beer at Rs 10 a litre to reduce the price to Rs 8 a litre?

(a) 4 litres (b) 15 litres (c) 3 litres (d) 5 litres (e) None

Hint: Cost price of water = 0, always to be remembered in solving the problem of mixture containing water as one of the ingredients

- (7) A milk vendor receives two adulterated supplies that contains 10% and 3% of water. How much of each kind must be taken so as to form 84 litres of a mixture containing 95% milk?

(a) 34 l, 50 l (b) 24 l, 60 l (c) 54 l, 30 l (d) 38 l, 46 l (e) 30 l, 54 l

Hint: Do not try to find the ratio of their quantity, rather use direct formula (4)