(23) Two trains are travelling in opposite directions at uniform speeds of 50 km/hr and 60 km/hr respectively. They take 6 seconds to cross each other. If the two trains had travelled in the same direction, then a passenger sitting in the faster train would have overtaken the slower train in 27 seconds. The length of the faster train is more than the slower train by:

(a) 108.33 m (b) 75 m (c) 150 m (d) 183.33 m (e) 33.33 m

Directions (24-25) Answer the following questions on the basis of the following information:

(i) Trains *A* and *B* are travelling on the same route heading towards the same destination. Train *B* has already covered a distance of 220 km before train *A* started.

(ii) The two trains meet each other 11 hours after the start of train *A*.

(iii) Had the trains been travelling towards each other (from a distance of 220 km), they would have met after an hour.

(24) What is the speed of train *A* in kmph?

(a) 102 (b) 118 (c) 81

(d) Data inadequate (e) None

(25) What is the speed of train *B* in kmph?

(a) 150 (b) 136 (c) 100 (d) 120 (e) Data inadequate

Answers

1. (e)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (c)
10. (c)	**11.** (d)	**12.** (a)	**13.** (a)	**14.** (e)	**15.** (a)	**16.** (b)	**17.** (d)	**18.** (c)
19. (b)	**20.** (b)	**21.** (d)	**22.** (b)	**23.** (e)	**24.** (e)	**25.** (c)		

19 BOATS AND STREAMS

19.1 INTRODUCTION

The following terms are frequently used in the problems on boats and streams:

(a) **Still Water:** It implies that the speed of water in the river is zero.

(b) **Stream Water:** It implies that the water in the river is moving.

19.2 SPEED OF MAN (BOAT) AND STREAM

Direction →

Man's rate of rowing (or speed of boat) in **still** water = x km/h.

← UPSTREAM ROWING	DOWNSTREAM ROWING →

Speed of current (or stream) = y km/h.

Direction →

Let, the man's rate of rowing (or speed of boat) in **still** water = x km/h
and the speed of stream (or current) = y km/h

(a) **Downstream** (With the stream) **Rowing**:
It indicates that the stream **favours** the man's rowing (or boating).
i.e. direction of rowing and direction of flow (stream) is same.
$\therefore$ Man's rate of rowing downstream = **DOWNSTREAM RATE** = $x + y$

(b) **Upstream** (Against the stream) **Rowing**:
It indicates that the stream flows against the man's rowing (or boating)
i.e. direction of rowing and direction of stream (current) are opposite.
$\therefore$ Man's rate of rowing upstream = **UPSTREAM RATE** = $x - y$

19.3 IMPORTANT FORMULAE

Assumption:

Man's rate of rowing in still water = x
Speed of stream (current) = y

1. **Downstream** Rate $= x + y = \dfrac{\text{downstream distance}}{\text{time to cover it}}$ $\left[\because \text{ rate} = \dfrac{\text{distance}}{\text{time}}\right]$

$= \dfrac{d_{down}}{t_{down}}$

2. **Upstream** Rate $= x - y = \dfrac{\text{upstream distance}}{\text{time to cover it}}$ $\left[\because \text{ rate} = \dfrac{\text{distance}}{\text{time}}\right]$

$= \dfrac{d_{up}}{t_{up}}$

3. Man's rate in STILL water $= x = \dfrac{1}{2}$ [**DOWNSTREAM** rate + **UPSTREAM** rate]

$= \dfrac{1}{2}\left[\dfrac{d_{down}}{t_{down}} + \dfrac{d_{up}}{t_{up}}\right]$ since $x = \dfrac{1}{2}[(x + y) + (x - y)]$

4. Speed of stream $= y = \dfrac{1}{2}$ [**DOWNSTREAM** rate – **UPSTREAM** rate]

$= \dfrac{1}{2}\left[\dfrac{d_{down}}{t_{down}} - \dfrac{d_{up}}{t_{up}}\right]$ since $y = \dfrac{1}{2}[(x + y) - (x - y)]$

5. When downstream distance = upstream distance, then

$$\frac{\text{Man's rate in } \textbf{still water}}{\text{Speed of stream}} = \frac{t_{up} + t_{down}}{t_{up} - t_{down}}$$

6. When downstream distance = upstream distance, then
Average speed for total journey (UP + DOWN)

$$= \frac{\textbf{Upstream} \text{ rate} \times \textbf{Downstream} \text{ rate}}{\text{Man's rate in } \textbf{still water}}$$

7. When downstream distance = Upstream distance, then

$$\text{Total Journey time } (t_{up} + t_{down}) = \frac{\text{Man's rate in still water} \times \text{Total distance}^*}{\textbf{Upstream} \text{ rate} \times \textbf{Downstream} \text{ rate}}$$

* Total distance = downstream distance + upstream distance
= 2 × any one side distance [Since downstream distance = upstream distance]

Solved Examples

E-1 A man can swim downstream at 8 km/h and upstream at 2 km/h. Find man's rate in still water and the speed of current.

S-1 Man's rate in still water $= \dfrac{1}{2}$ [Up + Down] *[Refer formula (3)]*

$$= \frac{1}{2}(8 + 2) = 5 \text{ km/h}$$

Speed of current $= \frac{1}{2}$ [Downstream – Upstream rate] *[Refer formula (4)]*

$$= \frac{1}{2}(8 - 2) = \mathbf{3\ km/h}.$$

E-2 A man rows upstream 20 km and downstream 30 km taking 5 hours each. Find the speed of current.

S-2 Speed of current $y = \frac{1}{2}\left[\frac{d_{dn}}{t_{dn}} - \frac{d_{up}}{t_{up}}\right]$ *[Refer formula (4)]*

$$= \frac{1}{2}\left[\frac{30}{5} - \frac{20}{5}\right] = \mathbf{1\ km/h}.$$

E-3 A boat man can row 2 km against the stream in 20 min. and return in 15 min. Find the rate of rowing in still water and the speed of current.

S-3 Here distance is fixed, therefore, $d_{dn} = d_{up} = d$,
Using the formula (3), we get

x (Man's speed of rowing in Still water) $= \frac{d}{2}\left[\frac{1}{t_{dn}} + \frac{1}{t_{up}}\right]$ *[Refer formula (3)]*

$$= \frac{2}{2}\left[\frac{1}{20} + \frac{1}{15}\right] \text{ km/min} = \frac{7}{60} \text{ km/min} = 7 \text{ km/h}$$

y (Speed of current) $= \frac{d}{2}\left[\frac{1}{t_{dn}} - \frac{1}{t_{up}}\right]$ *[Refer formula (4)]*

$$= \frac{2}{2}\left[\frac{1}{20} - \frac{1}{15}\right] = \frac{1}{60} \text{ km/min} = \mathbf{1\ km/h}.$$

E-4 A man can row 4 km/h in still water and he finds that it takes him twice as long to row up as to row down the river. Find the rate of stream.

S-4 Using the formula (5),

$$\frac{\text{Man's rate in still water}}{\text{speed of stream}} = \frac{t_{dn} + t_{up}}{t_{up} - t_{dn}}$$

Since $t_{up} = 2 \times t_{dn}$

$$\therefore \quad \frac{4}{\text{speed of stream}} = \frac{t_{dn} + 2t_{dn}}{-t_{dn} + 2t_{dn}} = \frac{3}{1}$$

$$\therefore \quad \text{speed of stream} = \frac{4}{3} \text{ km/h} = \mathbf{1.3\ km/h}.$$

E-5 A man can row 6 km/h in the still water. If the river is running at 2 km/h, it takes him 3 hours to row to a place and back. How far is the place?

S-5 Using the formula, $t_{down} + t_{up}$ = total time,

$$\frac{d}{x+y} + \frac{d}{x-y} = \text{total time} \Rightarrow \frac{d}{8} + \frac{d}{4} = 3 \quad \therefore \quad d = \mathbf{8\ km}.$$

E-6 The speed of a boat in still water is 15 km/h and the rate of current is 13 km/h. Find the distance travelled downstream in 15 min.

S-6 Distance in downstream = downstream rate × time

$$= (15 + 13) \times \frac{15}{60} = \mathbf{7\ km}.$$

E-7 A man can row at a speed of 4.5 km/h in still water to a certain upstream point and back to the starting point in a river which flows at 1.5 km/h. Find his average speed for total journey.

S-7 x = 4.5 km/h (given) and y = 1.5 km/h

Using the formula (6),

$$\text{Average speed for total journey} = \frac{\text{Upstream rate} \times \text{Downstream rate}}{\text{Speed in still water}}$$

$$= \frac{(x+y)\times(x-y)}{x} = \frac{6\times 3}{4.5} = \mathbf{4\ km/h}.$$

E-8 A man rows 10 km upstream and back again to the starting point in 55 min. If the speed of stream is 2 km/h, find the speed of rowing in still water.

S-8 Let speed of rowing in still water = x

also, y = speed of stream = 2, total time $T = \frac{55}{60}$ h

Hence, using the formula (7), we get

$$\text{Total time} = \frac{\text{Speed in still water} \times \text{Total distance}}{\text{upstream rate} \times \text{downstream rate}}$$

$$\Rightarrow \quad \frac{55}{60} = \frac{x\times 2\times 10}{(x+2)(x-2)}$$

$$\Rightarrow \quad \frac{55}{60}(x^2 - 2^2) = 2 \times x \times 10 \Rightarrow \quad 11x^2 - 240x - 44 = 0$$

$\Rightarrow (x - 22)(11x + 2) = 0 \quad \therefore \quad x = \mathbf{22}$, Since (–)ve value of x is not admissible.

E-9 A motor boat can travel at 10 km/h in still water. It travelled 91 km downstream in a river and then returned, taking altogether 20 hours. Find the rate of flow of river.

S-9 Motor boat speed in still water (x) = 10 km/h

Let, the rate of flow of river = y, then,

Total journey time T = 20 h is given

Hence, using the formula (7), we get

$$\text{Total time} = \frac{\text{Speed in still water} \times \text{Total distance}}{\text{upstream rate} \times \text{downstream rate}}$$

$$\Rightarrow \quad 20 = \frac{10 \times 2 \times 91}{(10+y)(10-y)}$$

$$\Rightarrow \quad 91 \times 20 = 20\,(10^2 - y^2) \Rightarrow y^2 - 10^2 + 91 = 0$$

$$\Rightarrow \quad y^2 = 9 \text{ i.e., } y = 3 \text{ km/h} \quad \therefore \quad \text{Flow of river} = \mathbf{3\ km/h}.$$

REGULAR PROBLEMS

(1) A person can swim in still water at 4 km/h. If the speed of water is 2 km/h, how many hours will the man take to swim back against the current for 6 km. **(UTI, '90)**

(a) 3 (b) 4 (c) $4\frac{1}{2}$

(d) Insufficient data (e) None of these

(2) A boat goes 100 km downstream in 10 hours, and 75 km upstream in 15 hours. The speed of the stream is

(a) 7 km/h (b) 5 km/h (c) 3 km/h (d) $2\frac{1}{2}$ km/h (e) None of these

(3) A man can row his boat with the stream at 6 km/h and against the stream in 4 km/h. The man's rate is

(a) 1 km/h (b) 5 km/h (c) 8 km/h (d) 6 km/h (e) Data insufficient

(4) A man can swim in still water at 4.5 km/h, but takes twice as long to swim upstream than downstream. The speed of the stream is

(a) 3 (b) 7.5 (c) 2.25 (d) 1.5 (e) 6

(5) If a man takes 3 hours to row 3 km upstream or 15 km downstream, then the speed of the current is

(a) 4 (b) 9 (c) 2 (d) 6 (e) Data insufficient

(6) A man can row in still water at 7 km/h and the rate of stream is 3.5 km/h. A distance of 10.5 km in going upstream is covered in

(a) 1.5 h (b) 1 h (c) 3 h (d) 15 h (e) 10 h

(7) A boat goes downstream at U m/s and upstream at V m/s. Then the speed of the boat in still water is

(a) $\frac{1}{2}(U - V)$ (b) $U - V$ (c) $\frac{1}{2}(U + V)$

(d) $U + V$ (e) $2\,UV$

(8) A boat goes downstream at the rate of U m/s and upstream at V m/s. Then the speed of the stream is

(a) $\frac{1}{2}(U - V)$ (b) $U - V$ (c) $\frac{1}{2}(U + V)$

(d) $U + V$ (e) $2\,UV$

(9) A man rows d km upstream and back again downstream to the same point in T hours. The speed of rowing in still water is x km/h and the rate of stream is y km/h, then

(a) $(x^2 - y^2) = \frac{2xd}{T}$ (b) $(x + y) = \frac{dT}{x - y}$ (c) $xy = dT$

(d) $x^2 + y^2 = \frac{2xd}{T}$ (e) Data insufficient

(10) The speed of a boat in still water is 15 km/h and the rate of stream is 5 km/h. The distance travelled downstream in 24 min. is

(a) 4 km (b) 8 km (c) 6 km (d) 16 km (e) None of these

(11) If the speed of boat in still water, stream rate and time of travel are kept constant, and distance in downstream = d_{dn} and distance travelled in upstream = d_{up}, then

(a) $d_{dn} = d_{up}$ (b) $d_{dn} = \frac{d_{up}}{2}$ (c) $d_{dn} = 2d_{up}$

(d) $d_{dn} + d_{up} = 1$ (e) $d_{dn} \times d_{up}$ = constant

(12) A man rows 40 km upstream in 8 hours and a distance of 36 km downstream in 6 hours, then speed of stream is

(a) 0.5 km/h (b) 5.5 km/h (c) 6 km/h (d) 5 km/h

(13) A man rows 40 km upstream in 8 hours and a distance of 36 km downstream in 6 hours, then the speed of man in still water is

(a) 0.5 km/h (b) 5.5 km/h (c) 6 km/h (d) 5 km/h (e) None of these

(14) If a man's downstream rate is 10 km/h, and the rate of stream is 1.5 km/h, then the man's upstream rate is

(a) 13 km/h (b) 10 km/h (c) 3 km/h (d) 7 km/h (e) Data insufficient

(15) If a man rows at 8 km/h in still water and his upstream rate is 5 km/h, then the man's rate along the current (downstream) is

(a) 21 km/h (b) 8 km/h (c) 16 km/h (d) 11 km/h (e) 10 km/h

(16) The rowing speed of man in still water is 20 km/h. Going downstream, he moves at the rate of 25 km/h. The rate of stream is

(a) 45 km/h (b) 2.5 km/h (c) 12.5 km/h (d) 5 km/h (e) 10 km/h

Hint: Downstream Rate = (Speed in still water + Speed of stream).

(17) If a man goes upstream at 6 km/h and the rate of stream is 2 km/h, then the man's speed in still water is

(a) 4 km/h (b) 8 km/h (c) 2 km/h (d) 12 km/h (e) 3 km/h

Hint: Upstream rate = (Speed in still water – Speed of stream).

(18) A boat goes 12 km upstream in 48 min. The speed of stream is 2 km/h. The speed of boat in still water is

(a) 13 km/h (b) 2.25 km/h (c) 17 km/h (d) 15 km/h (e) Data insufficient

Answers

1. (a)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (a)
10. (b)	**11.** (c)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (c)

20 RACES

20.1 INTRODUCTION

(i) Any contest of speed in running, driving, riding or rowing is called a **Race**
(ii) The point from where a race begins is called the **Starting Point**
(iii) The point where the race finishes is called the **Finishing Point or Winning Post**
(iv) The person who first reaches the finishing point is called the **Winner**

20.2 TWO PARTICIPANTS

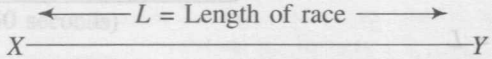
In a contest with 2 participants, one is the winner and the other is the loser.

(i) The winner can give/allow the loser a start of t seconds or x metres, i.e. start distance = x metres and start time = t seconds.

(ii) The winner can beat the loser by t seconds or x metres, i.e. beat distance = x metres and beat time = t seconds $\Rightarrow$ Loser can run x metres in t seconds.

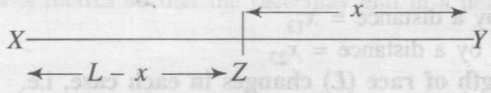
Let ***A* beats *B***

Winner's (A) distance = L



(a) ***A* beats *B* by x metre**

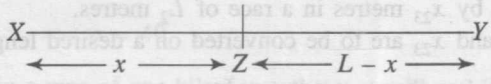
Loser's (B) distance = $L - x$



A and B start together at X
When A finishes at Y, B reaches Z.

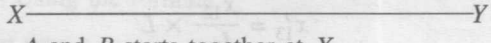
(b) ***A* gives *B* a start of x metre**

Loser's (B) distance = $L - x$



A starts at X, but B starts at Z i.e. x metres ahead of A at the same moment.

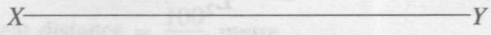
(c) ***A* beats *B* by t seconds**



A and B starts together at X
A finishes at Y, but t **seconds before** B finishes.

Winner's (A) time = Loser's (B) time $- t$

(d) ***A* gives *B* a start of t seconds**



A starts t seconds after B starts at X

From the above figures, following formulae have been derived for a race (contest) of two participants,

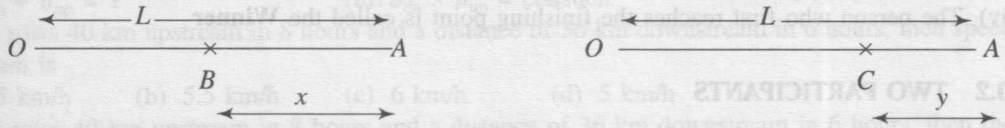
(a) Winner's distance = Length of race
(b) Loser's distance = Winner's distance – (beat distance + start distance)
(c) Winner's time = Loser's time – (beat time + start time)
(d) $\dfrac{\text{Winner's time}}{\text{Loser's distance}} = \dfrac{\text{Loser's time}}{\text{Winner's distance}} = \dfrac{\text{beat time} + \text{start time}}{\text{beat distance} + \text{start distance}}$
(e) If a race ends in a dead lock, i.e. both reach the winning post together then beat time = 0 and beat distance = 0

20.3 THREE PARTICIPANTS

Suppose, A, B, and C participate in a race. The length of the race is L metres.

Assuming, A as the winner, i.e. A gets the Ist position, in the race,

if A beats B by x metres and A beats C by y metres



The value of x and y will decide the IInd position in the race
If $x < y$, then B will beat C, i.e. B gets the IInd position
If $x > y$, then C will beat B, i.e C gets the IInd position
Now, the following relation is used for **three participants in a race of same length**

$$(L - x_{12})\, x_{23} = L\,(x_{13} - x_{12}) \qquad \textbf{(a)}$$

where,
Length of race = L
Ist beats IInd by a distance = x_{12}
Ist beats IIIrd by a distance = x_{13}
IInd beats IIIrd by a distance = x_{23}
But, if the length of race (L) changes in each case, i.e.
Ist beats IInd by x_{12} metres in a race of L_1 metres
Ist beats IIIrd by x_{13} metres in a race of L_3 metres
IInd beats IIIrd by x_{23} metres in a race of L_2 metres.
then, x_{12} , x_{13}, and x_{23} are to be converted on a desired length of race, say, L metres.

$$\therefore \quad x'_{12} = \frac{x_{12}}{L_1} \times L$$

$$x'_{13} = \frac{x_{12}}{L_3} \times L$$

$$x'_{23} = \frac{x_{23}}{L_2} \times L$$

Then using the formula $(L - x'_{12})\, x'_{23} = L\,(x'_{13} - x'_{12})$
the unknowns can be found out.

Solved Examples

E-1 In one kilometer race, A beats B by 36 metres or 9 seconds. Find A 's time over the course.

S-1 Here A is the winner and B is the loser.

Using the formula, 20.2

$$\frac{\text{Winner's time}}{\text{Loser's distance}} = \frac{\text{beat time} + \text{start time}}{\text{beat distance} + \text{start distance}}$$

$$\Rightarrow \quad \frac{A\text{'s time}}{1{,}000 - 36} = \frac{9+0}{36+0}$$

$$\Rightarrow \quad A\text{'s time} = \frac{9}{36} \times 964 \text{ s} = 241 \text{ s}.$$

A 's time over the course is **241 s**.

E-2 A runs $1\frac{1}{3}$ as fast as B. If A gives B a start of 30 metres, how far must be the winning post, so that the race ends in a dead heat?

S-2 Assuming L = distance of the winning post such that the race ends in a dead heat, i.e. both the participants A and B reach the winning post at the same time.

$\therefore$ time taken by A = time taken by B

$$\Rightarrow \quad \frac{L}{\frac{4}{3}V} = \frac{L-30}{V} \qquad \left(\text{Since } t = \frac{d}{V}\right)$$

$$\Rightarrow \quad L = 120$$

$\therefore$ length of race (distance) of winning post is **120 m**.

E-3 P runs a kilometer in 4 min. and Q in 4 min. 10 seconds. How many metres start can P give Q in a kilometer race so that the race may end in a dead heat?

S-3 P runs a kilometer in 4 min. (= 240 seconds)

Q runs a kilometer in 4 min. 10 s (= 250 seconds)

$\therefore$ P can beat Q by 10 seconds

But if P gives Q a start of 10 seconds or x metres so that the race may end in a dead heat, i.e, beat time = 0, beat distance = 0.

Then, using the formula 20.2

$$\frac{\text{Loser's time}}{\text{Winner's distance}} = \frac{\text{beat time} + \text{start time}}{\text{beat distance} + \text{start distance}}$$

$$\Rightarrow \quad \frac{250}{1{,}000} = \frac{0+10}{0+x} \quad \Rightarrow \quad x = 40$$

Hence if P gives Q a start of **40 metres** in a race of one kilometer, the race will end in a dead heat.

E-4 P can run a kilometer in 4 min. 50 seconds and Q in 5 min. By what distance can P beat Q?

S-4 Here, P is the winner and Q is the loser. Using the formula,

$$\frac{\text{Loser's time}}{\text{Winner's distance}} = \frac{\text{beat time} + \text{start time}}{\text{beat distance} + \text{start distance}}$$

$$\Rightarrow \quad \frac{300}{1000} = \frac{10+0}{\text{beat distance} + 0} \quad \Rightarrow \quad \text{beat distance} = \frac{100}{3} \text{ metre}$$

$\therefore$ P beats Q by $\mathbf{33\frac{1}{3}}$ metres in a kilometer race.

E-5 In a race of 100 metres, A beats B by 4 metres and A beats C by 2 metres. By how many metres would C beat B in a 100 metres race?

S-5 Here, the number of participants = 3
Length of race = L = 100 metres
A becomes the winner (I^{st}) and
C gets IInd position (Since 2 metres < 4 metres)
B gets IIIrd position
Using the formula, *[Refer 20.3]*
$(L - x_{12})\ x_{23} = L\ (x_{13} - x_{12})$ where
Ist (A) beats IInd (C) by x_{12} = 2 metres
Ist (A) beats IIIrd (B) by x_{13} = 4 metres
IInd (C) beats IIIrd (B) by x_{23} = ?
Length of race L = 100 metres
$\Rightarrow \quad (100 - 2) \times x_{23} = 100\ (4 - 2) \quad \Rightarrow \quad x_{23} = 2.04$ metres
Hence C would beat B by **2.04 metres** in a 100 metres race.

E-6 x, y, and z are the three contestants in a kilometer race. If x can give y a start of 50 metres and x can also give z a start of 69 metres, how many metres start y can give z?

S-6 Here x becomes Ist, y becomes IInd and z becomes IIIrd in the race (Since 50 < 69)
Using the formula *[Refer 20.3]*
$(L - x_{12})\ x_{23} = L\ (x_{13} - x_{12})$ where,
Ist (X) gives IInd (Y) a start of x_{12} = 50 metres
Ist (X) gives IIIrd (Z) a start of x_{13} = 69 metres
IInd (Y) gives IIIrd (Z) a start of x_{23}
Length of race L = 1,000 metres

$\Rightarrow \quad (1{,}000 - 50)\ x_{23} = 1{,}000\ (69 - 50)$

$\Rightarrow \quad x_{23} = 20$ metres
Hence Y gives Z a start of **20 metres**

E-7 In a 100 metres race, A runs at a speed of 2 metres/s. If A gives B a start of 4 metres and still beats him by 10 seconds, find the speed of B.

S-7 Here A is the winner and B is the loser.
Using the formula [20.2]
Loser's time – Winner's time = beat time + start time
$\Rightarrow$ B's time – A's time = 10 + 0

$$\Rightarrow \quad \frac{B\text{'s distance}}{B\text{'s speed}} - \frac{A\text{'s distance}}{A\text{'s speed}} = 10$$

$$\Rightarrow \quad \frac{100 - 4}{B\text{'s speed}} - \frac{100}{2} = 10 \quad \Rightarrow \quad B\text{'s speed} = 1.6 \text{ metre/s}$$

Hence the speed of B is **1.6 metre/s**.

E-8 *A* can run 330 metres in 41 seconds and *B* in 44 seconds. By how many seconds will *B* win if he has 30 metres start.

S-8 *B* runs 330 metres in 44 seconds

$\therefore$ *B* runs (330 – 30) metres in $\frac{44}{330} \times 300$s

i.e., 40 s

But *A* runs 330 metres in 41 seconds

So, *B* wins by (41 – 40) seconds, i.e. **1 s**.

E-9 *A* can give *B* 40 metres start and *A* can give *C* 50 metres start in a 200 metres race, while *B* can give *C* two seconds over the course. How long does each take to run 200 metres?

S-9 Here, *A* comes Ist, *B* comes IInd and *C* comes IIIrd in the contest.

Using the formula, *[Refer 20.3]*

$(L - x_{12})\, x_{23} = L(x_{13} - x_{12}) \Rightarrow (200 - 40) \times x_{23} = 200\ (50 - 40)$

$\Rightarrow \quad x_{23} = \frac{25}{2}$ metres

Hence, *B* can beat *C* by $\frac{25}{2}$ metres or 2 seconds,

i.e. *C* can run $\frac{25}{2}$ metres in 2 seconds

$\therefore$ *C* can run 200 metres in $\frac{2}{25} \times 200 = 32$ seconds

$\therefore$ *B* takes (32 – 2), i.e. 30 seconds.

and *A* takes $\frac{30}{200} \times (200 - 40)$, i.e. 21 seconds.

Hence *A* , *B* and *C* take **21 seconds**, **30** and **32 seconds** respectively to run 200 metres.

E-10 *A*, *B*, and *C* are three participants in a kilometre race. If *A* can give *B* a start of 40 metres and *B* can give *C* a start of 25 metres, how many metres *A* can give *C* a start?

S-10 Here *A* is the winner (I)

Since *B* can give *C* a start, therefore *B* becomes II and *C* becomes III in the race.

Using the formula,

$$(L - x_{12})x_{23} = L(x_{13} - x_{12})$$

we get $(1{,}000 - 40) \times 25 = 1{,}000 \times (x_{13} - 40)$

$\Rightarrow \quad 960 \times 25 = 1{,}000 \times (x_{13} - 40) \Rightarrow x_{13} = 64$ metres

Hence, *A* can give *C* a start of **64 metres**.

E-11 In a race of 600 metres, *A* can beat *B* by 60 metres and in a race of 500 metres, *B* can beat *C* by 25 metres. By how many metres will *A* beat *C* in a 400 metres race?

S-11 Here, length of race is different in each race. So, respective beat distance (given) is to be converted to the desired length of race *L* (400 metres).

A is the winner (Ist)

Since *B* can beat *C*, therefore *B* becomes IInd and *C* becomes IIIrd in the race

$\therefore \quad x'_{12} = \frac{x_{12}}{L_1} \times L = \frac{60}{600} \times 400 = 40$ metres

$$x'_{23} = \frac{x_{23}}{L_2} \times L = \frac{25}{500} \times 400 = 20 \text{ metres}$$

$x'_{13} = ?$

Using the formula, *[Refer 20.3]*

$$(L - x'_{12})\, x'_{23} = L\,(x'_{13} - x'_{12})$$

$$\Rightarrow \quad (400 - 40) \times 20 = 400\,(x'_{13} - 40)$$

$$\Rightarrow \quad x'_{13} = \frac{360 \times 20}{400} + 40 \Rightarrow \quad x'_{13} = 58 \text{ metre.}$$

Hence A will beat C by **58 metres** in a 400 metres race .

E-12 In a 400 metres race, A gives B a start of 5 seconds and beats him by 15 metres. In another race of 400 metres, A beats B by $7\frac{1}{7}$ seconds. Find their speeds.

S-12 In a 400 metres race,

B takes $7\frac{1}{7}$ seconds more time than A

In another 400 metres race,
B takes 5 seconds more time and runs 15 metres less distance than A.

$\therefore$ B can run in $\left(7\frac{1}{7} - 5\right)$ seconds a distance of 15 metres.

B can run in 1 seconds a distance of $\dfrac{15}{2\frac{1}{7}}$

$\therefore$ speed of B is 7 metres/s, i.e. 7 metres

B takes $7\frac{1}{7}$ seconds more time than A to run 400 metres.

also time = $\dfrac{\text{distance}}{\text{speed}}$

Lets the speed of A be V_A metres/s

then $\dfrac{400}{\text{speed of } B} - \dfrac{400}{\text{speed of } A} = 7\frac{1}{7}$

$$\Rightarrow \quad \frac{400}{7} - \frac{400}{V_A} = \frac{50}{7} \Rightarrow \quad V_A = 8 \text{ metres/s}$$

Hence, speed of A is **8 metres/s**.
Speed of B is **7 metres/s**

REGULAR PROBLEMS

(1) In a kilometre race, A beats B by 5 seconds or 40 metres. How long does B take to run the kilometre?

Hint: B is the loser

$$\therefore \quad \frac{\text{Loser's time}}{\text{Winner's distance}} = \frac{\text{Beat time + Start time}}{\text{Beat distance + Start distance}}.$$

(2) A runs $2\frac{1}{3}$ times as fast as B. If A gives B a start of 60 metres, how far must be the winning post, so that the race ends in a dead heat?

Hint: See E-2.

(3) P can run a kilometre in 3 minutes 10 seconds and B in 3 minutes 20 seconds. By what distance can A beat B?

Hint: $\therefore \quad \dfrac{\text{Loser's time}}{\text{Winner's distance}} = \dfrac{\text{Beat time} + \text{Start time}}{\text{Beat distance} + \text{Start distance}}$.

(4) A can run one kilometer in half a minute less time than B . In a kilometre race, B gets a start of 100 metres and loses by 100 metres. Find the time A and B take to run a kilometre.

Hint: $\dfrac{\text{Winner's time}}{\text{Loser's distance}} = \dfrac{\text{Loser's time}}{\text{Winner's distance}}$

A is winner and B is the loser

$$\frac{\left(t - \frac{1}{2}\right)}{1{,}000 - (1{,}000 + 100)} = \frac{t}{1{,}000}$$

$$\therefore \qquad t = \frac{5}{2} \text{ minutes}$$

(5) A and B can run 200 metres in 22 and 25 seconds respectively. How far is B from the finishing line when A reaches it?

(6) A and B take part in a 100 metres race. A runs at 5 km/h. A gives B a start of 8 metres and still beats him by 8 seconds. Find the speed of B.

Hint: See E-7.

(7) In a flat race, A beats B by 15 metres, and C by 29 metres. When B and C run over the course together, B wins by 15 minutes Find the length of the course.

Hint: Using $(L - x_{12})\, x_{23} = L\,(x_{13} - x_{12})$

Where $x_{12} = A$ beats $B = 15$ metres

$x_{23} = B$ beats $C = 15$ metres

$x_{13} = A$ beats $C = 29$ metres

L = Length of course = ? $\quad \therefore \quad (L - 15)15 = L(29 - 15)$

$\Rightarrow \qquad L = 225$ metres.

(8) A can give B a start of 20 metres and C a start of 39 metres in a walking race of 400 metres. How much can B give C start?

Hint: $(L - x_{12})\, x_{23} = L(x_{13} - x_{12})$

$(400 - 20)\, x_{23} = 400\,(39 - 20)$

$\therefore \qquad x_2 = 20$ metres.

(9) A can run 100 metres in 11 seconds, and B 100 metres in 12 seconds. What start can be given to B to make the race a dead heat?

Hint: See E-3.

(10) A can run a kilometre in half a minute less time than B. In a kilometre race, B gets a start of 100 metres and loses by 25 metres. Find the time A and B take to run a kilometre.

(11) A and B ran a race which lasted a minute and a half. A gave B a start of 9 metres and beat him by 1 metre. A ran 40 metres while B ran 39 metres. Find the length of the course.

Hint: Let L = length of course

$$\therefore \quad \frac{L}{40} = \frac{L-(9+1)}{39} \quad \therefore \quad L = 400 \text{ metres.}$$

(12) In a 500 metres race, B gives A a start of 160 metres. The ratio of the speeds of A and B is 2 : 3. Who wins and by how much?

Hint: Let after time t seconds, B reaches 500 metres, then, A reaches x_A metres

$$\therefore \quad \frac{x_A - 160}{500} = \frac{V_A \times t}{V_B \times t}$$

(Since B gives A a start of 160 m)

$$\Rightarrow \quad \frac{x_A - 160}{500} = \frac{2}{3}$$

$$x_A = 493\frac{1}{3} \text{ metres}$$

$$\therefore \quad B \text{ beats } A \text{ by } \left(500 - 493\frac{1}{3} \text{ metres}\right) = 6\frac{2}{3} \text{ metres.}$$

(13) Two men A and B ran 100 metres in 11.25 and 12.5 seconds respectively. If they start together, find how far was B from the finish time, when A completed his 100 metres.

Hint: See E-4.

(14) P can run 440 metres in 51 seconds and Q in 55 seconds. By how many seconds will B win if he has 40 metres start?

Hint: See E-8.

(15) A takes 4 minutes 50 seconds while B takes 5 minutes to complete the race. A beats B by $33\frac{1}{3}$ metres. Find the length of the course.

Hint:
$$\frac{\text{Loser's time}}{\text{Winner's distance}} = \frac{\text{Beat time}}{\text{Beat distance}}$$

Since Winner's distance = Length of course

$$\therefore \quad \frac{5 \times 60}{L} = \frac{10}{33\frac{1}{3}} \quad \therefore \quad L = 1{,}000 \text{ metres.}$$

(16) P and Q run a kilometre and P wins by 1 minute. P and R run a kilometre, and P wins by 375 metres. Q and R run a kilometre and Q wins by 30 seconds. Find the time taken by P, Q and R to run a kilometre.

Answers

1. 125 s **2.** 105 m **3.** 50 m **4.** 2 min, $2\frac{1}{2}$ min **5.** 24 m

6. 4.14 km/hr **7.** 225 m **8.** 20 m **9.** $8\frac{1}{3}$ m **10.** $3\frac{1}{2}$ min, 4 min

11. 400 m **12.** $6\frac{2}{3}$ m **13.** 10 m **14.** 1 sec **15.** 1,000 m

16. 150 s, 210 s, 240 s.

REAL PROBLEMS

(1) In a 200 metres race, A beats B by 20 metres, while in a 100 metres race, B beats C by 5 metres. A beats C in a kilometre race by

(a) 105 metres (b) 25 metres (c) 150 metres
(d) 145 metres (e) None of these

(2) X runs $1\frac{1}{2}$ times as fast as Y. If X gives Y a start of 300 metres,. how far must X run before he catches up with Y.

(a) 1 km (b) 400 metres (c) 450 metres
(d) 900 metres (e) 600 metres

(3) X runs $1\frac{1}{4}$ times as fast as Y. If X gives Y a start of 200 metres, how far Y has run before he is caught by X?

(a) 1 km (b) 500 metres (c) 400 metres
(d) 800 metres (e) 50 metres

(4) In a flat race, A beats B by 15 metres and C by 29 metres. When B and C run over the course together, B wins by 15 metres. The length of the course is

(a) 150 metres (b) 225 metres (c) 275 metres
(d) 300 metres (e) 250 metres

Hint: $\therefore (L - x_{12})\, x_{23} = L\,(x_{13} - x_{12})$.

(5) In a flat race of L metres, A beats B by x_1 metres, beats C by x_3 metres, B beats C by x_2 metres, then

(a) $(L - x_3)\, x_2 = L \times (x_3 - x_1)$ (b) $(L - x_3)\, x_2 = L \times (x_1 - x_3)$
(c) $(L - x_1)\, x_2 = L \times (x_3 - x_1)$ (d) $(L - x_1)\, x_2 = L \times (x_1 - x_3)$

(6) In a flat race of L metres, A beats B by x_1 metres, beats C by x_3 metres but C beats B by x_2 metres, then

(a) $(L - x_3)\, x_2 = L \times (x_3 - x_1)$ (b) $(L - x_3)\, x_2 = L \times (x_1 - x_3)$
(c) $(L - x_1)\, x_2 = L \times (x_3 - x_1)$ (d) $(L - x_1)\, x_2 = L \times (x_1 - x_3)$

(7) In a flat race of L metres A beats B by x_1 metres and beats C by x_3 metres, then B will beat C, if

(a) $x_1 = x_3$ (b) $x_1 > x_3$ (c) $x_3 > x_1$ (d) $x_1\, x_3 = 1$ (e) $x_1 + x_3 = 0$

(8) In a flat race of L metres, A beats B by x_1 metres, and beats C by x_3 metres; then C will beat B if

(a) $x_1 = x_3$ (b) $x_1 > x_3$ (c) $x_3 > x_1$ (d) $x_1\, x_3 = 1$ (e) $x_1 + x_3 = 0$

(9) J is $1\frac{3}{8}$ times as fast as K. If J gives K a start of 150 metres, how far must be the winning post so that the race ends in a dead heat?

(a) 100 metres (b) 440 metres (c) 200 metres
(d) 550 metres (e) 1 km

Hint: Race ends in dead heat, i.e. time taken by J and K are same.

$$\therefore \quad \text{use } t = \frac{J\text{'s distance}}{J\text{'s speed}} = \frac{K\text{'s distance}}{K\text{'s speed}}$$

$$\therefore \quad \frac{x}{\frac{11}{8}k} = \frac{x-150}{k} \qquad \therefore \quad x = \text{Length of race} = 550 \text{ metres.}$$

(10) In a 500 metres race, A beats B by 50 metres or 8 seconds. Find A's time over the course.
(a) 2 minutes (b) 1 minutes 12 seconds
(c) 2 minutes 35 seconds (d) 1 minutes

(11) In a 300 metre race, A beats B by 5 seconds or 15 metres. A's time over the course is
(a) 95 s (b) 100 s (c) 85 s (d) 90 s (e) 105 s

(12) X can run 20 metres while Y runs 25 metres. In a kilometre race, Y beats X by
(a) 150 metres (b) 200 metres (c) 250 metres
(d) 175 metres (e) 300 metres

(13) In a 500 metres race, the ratio of speeds of two contestants A and B is 3 : 4. A has a start of 140 metres, then **(MBA '80)**
(a) B wins by 20 metres (b) A wins by 20 metres
(c) B wins by 40 metres (d) B wins by 25 metres
(e) A wins by 40 metres

(14) If A can run a kilometre in $3\frac{1}{6}$ minutes and B in $3\frac{1}{3}$ minutes, by what distance can A beat B?
(a) 100 metres (b) 75 metres (c) 60 metres
(d) 50 metres (e) 125 metres

Answers

1. (d)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (d)
10. (b)	**11.** (a)	**12.** (b)	**13.** (b)	**14.** (d)				