1 NUMBER SYSTEM AND NUMBER SERIES

1.1 NUMBERS AND THEIR CLASSIFICATION

A number p may be,

(i) a **natural** number (N)
(ii) a **whole** number (W)
(iii) an **integer** (Z)
(iv) a **rational** number (Q)
(v) a **real** number (R)
(vi) an **irrational** number

For example,
set of **natural** numbers is {1, 2, 3, ...}
set of **whole** numbers is {0, 1, 2, 3, ...}
set of **integers** is { ... –3, –2, –1, 0, 1, 2, 3, ...}
set of **rational** numbers is $\frac{1}{2}, \frac{3}{5}, -\frac{8}{5}, 0, +3, -150, ...$
set of **irrational** numbers is $\pi, \sqrt{2}, \sqrt{5}, \sqrt{7}, ...$

Besides the above cited number, we often come across numbers like $\sqrt{-8}, \sqrt{-7}, \sqrt{-5}, \sqrt{-1}, 3+\sqrt{-7}$ and so on. These are undefined numbers, called **complex** numbers.

A positive integer, except 1, is called a **prime** number, if its factors are 1 and the number itself 2, 3, 5, 7, ... are prime numbers.

1.2 TEST FOR DIVISIBILITY OF NUMBERS

There are certain tests for divisibility of numbers by any of the numbers 2, 3, 4, 5, 6, 8, 9, 10 and 11 such that by simply examining the digits in the given number, one can easily determine whether or not a given number is divisible by any of these numbers. Such tests are detailed as follows:

i. Divisibility by 2

If the last digit is an even number or it has zero (0) at the end.

Example: 74, 148, 1210 are all divisible by 2.

ii. Divisibility by 3
If the sum of the digits of the given number is divisible by 3.

Example: The sum of the digits of number 3705 is 3 + 7 + 0 + 5 = 15
Since 15 (the sum of digits) is divisible by 3, the number 3705 is also divisible by 3.

iii. Divisibility by 4
If the number formed by the last two digits of the given number is divisible by 4, or if the last two digits are '00'.

Example: 216560 is a number whose last two digits are 60. Since 60 is divisible by 4, the given number 216560 is also divisible by 4.

iv. Divisibility by 5
If the last digit of the given number is 0 or 5.

Example: 865, 1705, 4270 are all divisible by 5.

v. Divisibility by 6
If the given number is divisible by both 2 and 3.

Example: Let us consider the number 629130. It has 0 as the last digit, so it is divisible by 2.
Sum of the digits = 6 + 2 + 9 + 1 + 3 + 0 = 21
This sum 21 is divisible by 3, so the number is divisible by 3.
Since, 629130 is divisible by both 2 and 3, the number is also divisible by 6.

vi. Divisibility by 8
If the number formed by the last three digits of the given number is divisible by 8 or if the last three digits are '000'.

Example: The number 81976 has 976 as the last three digits. Since 976 is divisible by 8, 81976 is also divisible by 8. The number 6145000 ends with '000' and so, it is divisible by 8.

vii. Divisibility by 9
If the sum of the digits of the given number is divisible by 9.

Example: 870111 is a number the sum of whose digits = 8 + 7 + 0 + 1 + 1 + 1 = 18.
Since 18 (sum of digits) is divisible by 9, the number 870111 is also divisible by 9.

viii. Divisibility by 10
If the last digit of the number is zero (0).

Example: 730 has 0 at the end, so it is divisible by 10.

ix. Divisibility by 11
If the difference of the sum of its digits in odd places (i.e. first, third, fifth . . .) and the sum of its digits in even places (i.e. second, fourth, sixth . . .) is either zero (0) or a multiple of 11.

Example: Let us consider the number 647053.
Sum of digits at odd places = 6 + 7 + 5 = 18
Sum of digits at even places = 4 + 0 + 3 = 7
Difference of the sums = 18 − 7 = 11
Since the difference (= 11) is a multiple of 11, 647053 is also divisible by 11
Let us consider another number 9610260.
Sum of digits at odd places = 9 + 1 + 2 + 0 = 12
Sum of digits at even places = 6 + 0 + 6 = 12

Difference of the sums = 12 − 12 = 0

Since the difference is 0, 9610260 is divisible by 11.

1.3 GENERAL PROPERTIES OF DIVISIBILITY

There are some general properties of divisibility that help in determining the divisibility of a natural number by other natural numbers (other than detailed in 1.2)

Property 1

If a number x is divisible by another number y, then any number divisible by x, will also be divisible by y and by all the factors of y.

Example: The number 84 is divisible by 6. Thus any number that is divisible by 84, will also be divisible by 6 and also by the factors of 6, i.e. by 2 and by 3.

Property 2

If a number x is divisible by two or more than two co-prime numbers then x is also divisible by the product of those numbers.

Example: The number 2520 is divisible by 5, 4, 13 that are prime to each other (i.e. co-prime), so, 2520 will also be divisible by 20 (= 5 × 4), 65 (= 5 × 13), 52 (= 4 × 13).

Property 3

If two numbers x and y are divisible by a number 'p', then their sum $x + y$ is also divisible by p.

Example: The numbers 225 and 375 are both divisible by 5. Thus their sum = 225 + 375 = 600 will also be divisible by 5.

Note: It is also true for more than two numbers.

Property 4

If two numbers x and y are divisible by a number 'p', then their difference $x \sim y$ is also divisible by p.

Example: The numbers 126 and 507 are both divisible by 3. Thus their difference = 507 − 126 = 381 will also be divisible by 3.

1.4 TEST OF A PRIME NUMBER

A prime number is only divisible by 1 and by the number itself. The first prime number is 2. Every prime number other than 2 is odd, but every odd number is not necessarily a prime number. Again any even number (other than 2) cannot be a prime number. To test whether any given number (if odd) is a prime number or not, following steps are to be considered:

Step 1 Find an integer (x) which is greater than the approximate square root of the given number.

Step 2 Test the divisibility of the given number by every prime number less than x.

Step 3
- If the given number is divisible by any of them in Step 2, then the given number is NOT a prime number.
- If the given number is not divisible by any of them in Step 2, then the given number IS a prime number.

Example: Consider a number 203. Test if it is a prime number or not.

Step 1 The approximate square root of 203 is 14 plus. Take x =

Step 2 Check the divisibility of 203 by the prime numbers less than 15 i.e. by 2, 3, 5, 7, 11, 13.

Step 3 203 is divisible by 7. Thus, it is not a prime number.

1.5 DIVISION AND REMAINDER

When a given number is not exactly divisible by any number, then there is a remainder number at the end of such division.

Suppose we divide 25 by 7 as.

7) 25 (3
   21

    4

then, divisor = 7, dividend = 25

quotient = 3, and $\boxed{\text{remainder} = 4}$

So, we can represent it as

divisor) dividend (quotient
remainder

Thus $\boxed{\text{dividend} = (\text{divisor} \times \text{quotient}) + \text{remainder}}$ **(i)**

So, if a number x is divided by k, leaving remainder 'r' and giving quotient 'q' then the number can be found by using (i)

$$x = kq + r$$

Hence, if the number x is exactly divisible by k, then remainder = $r = 0$

$\therefore$ $$x = kq$$

and so $\dfrac{x}{k} = q$, implying that x is divisible exactly by k and q is an integer.

1.5.1 Methods to Find a Number Completely Divisible by Another

Consider a given number x. When divided by d, it gives a quotient q and remainder 'r'.

It implies that the given number 'x' is not exactly divisible by 'd'

d) x (q
r

Now, to find a number exactly divisible by 'd', we can use either of the following two methods to reduce the remainder to zero. (If a number is exactly divisible, then remainder is zero).

Method 1

- By subtracting the remainder from the given number (dividend).

  $\therefore$ the required number that is exactly divisible by 'd' = $x - r$

  Hence 'remainder' is the **least number that can be subtracted** from any number to make it exactly divisible.

Method 2

- By adding the (divisor − remainder) to the given number.

  $\therefore$ the required number that is exactly divisible by $d = x +$ (divisor − remainder)

  Therefore, (divisor − remainder) is the **least number that can be added** to any given number to make it exactly divisible.

Example: Find the least number, that must be
(a) subtracted from
or (b) added to a given number 5029, to make it exactly divisible by 17.

Solution: On dividing 5029 by 17, we find that

```
17) 5029 (295
    34
    ---
    162
    153
    ---
     99
     85
    ---
     14
```

∴ remainder = 14.

(a) The least number to be subtracted to make it exactly divisible = remainder = 14. (By method 1)

(b) The least number to be added to make it exactly divisible = divisor − remainder = 17 − 14 = 3. (By method 2)

1.5.2 Greatest *n*-digit and Least *n*-digit Number Exactly Divisible by a Number

(a) To find out the **greatest *n*-digit** number exactly divisible by a divisor 'd', we use Method 1 (1.5.1)

∴ the required number = Greatest n-digit number − remainder.

(b) To find out the **least *n*-digit** number exactly divisible by a divisor 'd', we use Method 2 (1.5.1), because if we use method 1, then subtracting any number from the n-digit least number will reduce it to $(n - 1)$ digit number.

∴ the required number = Least n-digit number + (divisor − remainder)

Example: Find the (a) greatest 3-digit number divisible by 13.
(b) the least 3-digit number divisible by 13.

Solution: (a) 13) 999 (
11

∴ the required 3-digit greatest number
= 999 − 11 = 988

(b) 13) 100 (
9

∴ the required 3-digit least number
= 100 + (13 − 9)
= 104.

1.6 REMAINDER RULES

Rule 1

This rule is applied to find the **remainder for the smaller divison**, when the same number is divided by the two different divisors such that one divisor is a multiple of the other divisor and also the remainder for the greater divisor is known.

If the remainder for the greater divisor = r,
and the smaller divisor = d, then

Rule-I states, that **when $r > d$.** the required remainder for the smaller divisor will be the remainder found out by dividing the 'r' by 'd'. [Case I]
and when $r < d$, then the required remainder is 'r' it self. [Case II]

Example: If a number is divided by 527, the remainder is 42.
What will be the remainder if it is divided by 17?

Solution: Here the same number is divided by two divisors: 527 and 17.

Now, $\frac{527}{17} = 31$, so, 527 is a multiple of 17

Hence Rule 1 can be applied.
Remainder for the greater divisor (i.e., for 527) = 42
Smaller divisor = 17.

So, 17) 42 (
34
8 = required remainder for smaller divisor (i.e. 17)

Hence, if 527 is divided by 17, the remainder will be 8.

Rule 2

If two different numbers a and b, on being divided by the same divisor leave remainders r_1 and r_2 respectively, then their sum $(a + b)$ if divided by same divisor will leave remainder R, given by

$$R = (r_1 + r_2) - \text{divisor}$$

$\Rightarrow$ The required remainder R = sum of remainders − divisor
(When sum is divided)

Note: If R becomes negative in the above equation, then the required remainder will be the sum of the remainders.
$\therefore$ the required remainder = sum of remainders

Example: Two different numbers, when divided by the same divisor, leave remainders 15 and 39 respectively, and when their sum was divided by the same divisor, the remainder was 7. What is the divisor?

Solution: Using the Rule 2

$$7 = (15 + 39) - \text{divisor}$$

$$\Rightarrow \quad \text{divisor} = 47$$

Example: Two different numbers, when divided by 47, leave remainders 13 and 23 respectively. If their sum is divided by the same number 47, what will be the remainder?

Solution: Using Rule 2.

The required remainder = (13 + 23) − 47
= −11

Since the remainder is (−) ve, so, the actual remainder will be 23 + 13 = 36 (refer to NOTE under Rule 2)

Rule 3

When two numbers, after being divided by the same divisor leave the same remainder, then the difference of those two numbers must be exactly divisible by the same divisor.

Example: Two numbers 147 and 225, after being divided by a 2-digit number, leave the same remainder. Find the divisor.

Solution: By Rule 3, the difference of 225 and 147 must be perfectly divisible by the divisor.

The difference = 225 − 147 = 78

Now, 78 = 13 × 2 × 3.

Thus, 1-digit divisors = 2, 3 and 2 × 3

2-digit divisors = 13 × 2, 13 × 3, 13, 13 × 2 × 3

∴ the possible divisors are 26, 39, 13, 78.

Rule 4

If a given number is divided successively by the different factors of the divisor leaving remainders r_1, r_2 and r_3 respectively, then the true remainder (i.e. remainder when the number is divided by the divisor) can be obtained by using the following formula:

True remainder = (first remainder) + (second remainder × first divisor) + (third remainder × first divisor × second divisor)

Example: A number, being successively divided by 5, 7 and 11 leaves 3, 1, 2 as remainders respectively. Find the remainder if the same number is divided by 385.

Solution: Here, the divisor is 385, whose factors are 5, 7 and 11.

∴ by Rule 3,

True remainder (i.e. remainder when divided by 385) = 3 + (1 × 5) + (2 × 5 × 7)

= 3 + 5 + 70

= 78

Rule 5

When $(x + 1)^n$ is divided by x, the remainder is always 1, where x and n are natural numbers.

Example: What will be the remainder when $(17)^{21}$ is divided by 16?

Solution: $(17)^{21} = (16 + 1)^{21}$.

∴ when $(16 + 1)^{21}$ is divided by 16, the remainder = 1.

Rule 6

When $(x - 1)^n$ is divided by x, then

the remainder = 1, when n is an even natural number

but the remainder= $x - 1$, when n is an odd natural number.

Example: What will be the remainder when $(29)^{75}$ is divided by 30?

Solution: $(29)^{75} = (30 - 1)^{75}$, here index = 75 (which is odd) so, when $(30 - 1)^{75}$ is divided by 30, the remainder will be $x - 1 = 30 - 1 = 29$

1.7 NUMBER SERIES

In the number series, some numbers are arranged in a particular sequence. All the numbers form a series and change in a certain order. Sometimes, one or more numbers are wrongly put in the number series. One is required to observe the trend in which the numbers change in the series and find out which number/numbers misfit into the series. That number/numbers is the ODD NUMBER of the series.

1.7.1 Important Number Series

Following are some of the important rules or order on which the number series can be made.

I. Pure Series

In this type of number series, the number itself obeys certain order so that the character of the series can be found out.

The number itself may be:

— perfect square
— perfect cube
— prime
— combination of above

II. Difference Series

Under this category, the **change** in **order** for the differences between each consecutive number of the serie is found out as shown in Table 1.1.

III. Ratio Series

Under this category, the change in order for the ratios between each consecutive number of the series is found out as shown in Table 1.2.

IV. Mixed Series

Here, the numbers obeying various orders of two or more different type of series are arranged alternately in a single number series.

V. Geometric Series

Under this category, each successive number is obtained by multiplying (or dividing) the previous number with a fixed number. (See Table 1.2).

Example: 5, 35, 245, 1715, 12005, . . .
43923, 3993, 363, 33, 3, . . .

VI. Two-tier Arithmetic Series

Under this category, the differences of successive numbers form an arithmetic series. (See Table 1.1)

Example: 4, 5, 9, 16, 26, 39, 55, 74.

+1 +4 +7 +10 +13 +16 +19 → Arithmetic Series

3 3 3 3 3 3

VII. Three-tier Arithmetic Series

The differences of successive numbers form a two-tier arithmetic series. The successive term difference in this, in turn form an arithmetic series.

Table 1.1 Change in Order for the Difference Series

Series Code	*Nature/Order of the Number Series*	*Examples for the Series*	
D1	Difference between consecutive numbers is same.	2, 7, 12, 17 +5, +5, +5	35, 27, 19, 11 –8, –8, –8
D2	Differences between consecutive numbers are in arithmetic progression (A.P.)	15, 26, 42, 63 +11, +16, +21 Here 11, 16, 21 are in A.P.	34, 25, 18, 13 –9, –7, –5 Here –9, –7, –5 are in A.P.
D3	Difference between consecutive numbers is a perfect square number.	7, 16, 41, 90 $+3^2$, $+5^2$, $+7^2$	70, 54, 45, 41 -4^2, -3^2, -2^2
D4	Differences between consecutive numbers are multiples of a number.	11, 23, 47, 95 +12, +24, +48 Here 12, 24 and 48 are multiples of 12.	204, 160, 127, 105 –44, –33, –22 Here –44, –33 and –22 are multiples of 11.
D5	Differences between consecutive numbers are prime numbers.	2, 3, 5, 8, 13 +1, +2, +3, +5 Here, 1, 2, 3 and 5 are prime numbers.	29, 22, 15, 4 –5, –7, –11 Here –5, –7 and –11 are all prime numbers.
D6	Difference between consecutive numbers is a perfect cube number.	7, 15, 42, 167 $+2^3$, $+3^3$, $+4^3$	121, 57, 30, 22 -4^3, -3^3, -2^3
D7	Difference between consecutive numbers are in geometric progression (G.P.).	1, 2, 5, 14, 41, 124 +1, +3, +9, +27, +81 Here 1, 3, 9, 27 and 81 are in G.P. 33, 17, 9, 5, 3, 2 –16, –8, –4, –2, –1 Here –16, –8, –4, –2 and –1 are in G.P.	

Table 1.2 Change in Order for the Ratio Series

Series Code	*Nature or Order of the Number Series*	*Examples for the Series*
R1	Ratio between each consecutive numbers is the same.	$32 \xrightarrow{\div 2} 16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4$ $3 \xrightarrow{\times 5} 15 \xrightarrow{\times 5} 75 \xrightarrow{\times 5} 375$
R2	Ratio between each consecutive numbers is in arithmetic progression (A.P.)	$2 \xrightarrow{\times 2} 4 \xrightarrow{\times 4} 16 \xrightarrow{\times 6} 96$ $420 \xrightarrow{\div 5} 84 \xrightarrow{\div 4} 21 \xrightarrow{\div 3} 7$
R3	Ratio between consecutive number is a perfect square number.	$3 \xrightarrow{\times 2^2} 12 \xrightarrow{\times 4^2} 192 \xrightarrow{\times 6^2} 6912$ $44100 \xrightarrow{\div 7^2} 900 \xrightarrow{\div 5^2} 36 \xrightarrow{\div 3^2} 4$
R4	Ratio between consecutive number is the multiple of a number.	$5 \xrightarrow{\times 2} 10 \xrightarrow{\times 4} 40 \xrightarrow{\times 8} 320$ Here 2, 4 and 8 are multiples of 2. $972 \xrightarrow{\div 9} 108 \xrightarrow{\div 6} 18 \xrightarrow{\div 3} 6$ Here 9, 6 and 3 are multiples of 3.
R5	Ratio between consecutive numbers is a prime number.	$5 \xrightarrow{\times 3} 15 \xrightarrow{\times 2} 30 \xrightarrow{\times 3} 90 \xrightarrow{\times 5} 450$ Here, 2, 3 and 5 are prime numbers. $770 \xrightarrow{\div 11} 70 \xrightarrow{\div 7} 10 \xrightarrow{\div 5} 2$ Here, 11, 7 and 5 are prime numbers.

(*Contd.*)

Table 1.2 (*Contd.*)

Series Code	*Nature or Order of the Number Series*	*Examples for the Series*
R6	Ratio between consecutive numbers is a perfect cube number.	$2 \xrightarrow{\times 1^3} 2 \xrightarrow{\times 3^3} 54 \xrightarrow{\times 5^3} 6750$ $13824 \xrightarrow{\div 4^3} 216 \xrightarrow{\div 3^3} 8 \xrightarrow{\div 2^3} 1$
R7	Ratios between consecutive numbers are in geometric progression (G.P.).	$1 \xrightarrow{\times 1} 1 \xrightarrow{\times 2} 2 \xrightarrow{\times 4} 8 \xrightarrow{\times 8} 64 \xrightarrow{\times 16} 1024$ Here 1, 2, 4, 8 and 16 are in G.P. $729 \xrightarrow{\times 27} 27 \xrightarrow{\times 9} 3 \xrightarrow{\times 3} 1$ Here 27, 9 and 3 are in G.P.

Example: 392, 245, 140, 70, 28, 7, 0 . . .

147 105 70 42 21 7

42 35 28 21 14 → Arithmetic Series

7 7 7 7

VII. Twin Series

Under this category, two series are alternatively placed in one.

Example: 1, 2, 3, –2, 5, –8, 7, –16, 9 . . . .

2 2 2 2

–4 –6 –8

–2 –2

(first, third, fifth, seventh terms are in A.P.)

(second, fourth, sixth, terms form two-tier Arithmetic Series).

1.8 THREE STEPS TO SOLVE A PROBLEM ON SERIES

Step 1 Determine whether the series is increasing, decreasing or alternating.

Step 2 If the series is increasing or decreasing, then check:

(a) if change is slow or gradual, then it is a difference series.
(b) if the change is equally sharp, throughout, then it is a ratio series.
(c) if the rise is very sharp initially, but slows down later, then the series may be formed by adding squared, or cubed numbers.

If the series is alternating or irregular, there may be either a mix of two series or two different kinds of operations going on alternately.

Step 3 Complete the series accordingly.

1.9 TWO-LINE NUMBER SERIES

A two-line number series, as the name suggests, consists of number series in two lines. If one complete series is given in first line, with an incomplete series in second line, and it is given that the series in both the lines have the same definite rule, we need to work it out as follows:

Applying the very definite rule of the series in the first line, the series in second line can be completed. The pattern/type of series in the first line may be any of the types described in 1.7.

Example: 15 28 51 84 127 . . .
22 a b c d e . . .

In the first line, the differences of two successive terms of the series are 13, 23, 33, 43.

Hence following the pattern of first line series, the number series in second line are:

$a = 22 + 13 = 35$, $b = 35 + 23 = 58$, $c = 58 + 33 = 91$
$d = 91 + 43 = 134$, $e = 134 + 53 = 187$.

1.10 SUM RULES ON NATURAL NUMBERS

Rule 1

Sum of all the first n natural numbers $= \dfrac{n(n+1)}{2}$
(starting from 1)

Example: Sum of 1 to 74 $= \dfrac{74 \times 75}{2} = 2775$

Rule 2

Sum of first n odd numbers $= n^2$
(starting from 1)

Example: Sum of first 7 odd numbers $(1 + 3 + 5 + 7 + 9 + 11 + 13) = 72 = 49$

Rule 3

Sum of first n even numbers $= n\,(n + 1)$
(starting from 1)

Example: Sum of first 9 even numbers $(2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18) = 9\,(9 + 1) = 90$.

Rule 4

Sum of squares of first n natural numbers $= \dfrac{n(n+1)(2n+1)}{6}$
(starting from 1)

Example: Sum of squares of first 8 natural numbers

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 = \frac{8(8+1)(2 \times 8 + 1)}{6}$$

$$= \frac{8 \times 9 \times 17}{6} = 204$$

Rule 5

Sum of cubes of first n natural numbers $= \left[\dfrac{n(n+1)}{2}\right]^2$

Example: Sum of cubes of first 6 natural numbers $= 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = \left[\dfrac{6(6+1)}{2}\right]^2 = 441$

Note: For applying Rule 2 and Rule 3, it is required to find how many odd numbers or even numbers are there in the given series.

In the first 'n' natural numbers,

if n is even, then there are $\dfrac{n}{2}$ odd numbers and $\dfrac{n}{2}$ even numbers

if n is odd, then there are $\dfrac{n+1}{2}$ odd numbers and $\dfrac{n-1}{2}$ even numbers

Example: From 1 to 30, as 30 is even, there are 15 odd numbers and 15 even numbers.

From 1 to 29, as 29 is odd, there are $\dfrac{29+1}{2} = 15$ odd numbers and $\dfrac{29-1}{2} = 14$ even numbers

1.11 BASE AND INDEX

If a number 'b' is multiplied by itself 'n' times, then the product is called the nth power of b, i.e.

$b \times b \times b$ . . . . up to n times $= b^n$

Here, b is called the base and n is called the index.

1.11.1 Last Digit (Digit at Unit's Place) in $(xyz)^n$

Here the given number is $\underbrace{(xyz)}_{\text{base}}{}^{n} \longrightarrow$ index

z is the last digit of the base.

To find out the last digit in $(xyz)^n$, following steps are to be followed.

Divide the index (n) by 4, then

Case I

if remainder = 0

then check if z is **odd** (except 5), then last digit = 1.

and if z is **even**, then last digit = 6.

Case II

if **remainder = 1**, then required last digit = last digit of base (i.e. z)

if remainder = **2**, then required last digit = last digit of $(z)^2$

if remainder = **3**, then required last digit = last digit of $(z)^3$

Note: If z is 5, then last digit in the product = 5

Example: Find the last digit in $(295073)^{130}$ ($130 \rightarrow n$)

Solution: Dividing 130 by 4, the remainder = 2

∴ refering to Case II, the required last digit is the last digit of $(z)^2$, i.e. $(3)^2 = 9$, (because $z = 3$)

Example: Find the last digit in $(81678)^{199}$

Solution: Dividing 199 by 4, the remainder = 3

∴ the required last digit is the last digit of $(z)^3$, i.e. $(8)^3 = 512$ (as $z = 8$)

Hence the last digit is 2

1.11.2 Number of Zeroes at the End of a Product

On multiplying two or more given numbers, the zeroes are produced at the end of the resulting produc
to the following reasons:

(a) If there is any zero at the end of any of the factors (or numbers being multiplied)

Example: $7 \times 20 = 140$

This zero is produced at the end of the product also

(b) If 5 or a multiple of 5 is multiplied by any even number.

Example: $45 \times 12 = 540$

multiple of 5 (45), even (12) → 540

Combining the above two reasons, we may say that:

(i) Resolve all the given numbers into their factors.

(ii) Count the number of 2s and 5s.

i.e. $(5)^x \times (2)^y$, say.

(iii) No. of zeroes at the end of product = No. of 2s or no. of 5s, whichever is les

Example: Find the number of zeroes at the end of the product of :

$$15 \times 32 \times 25 \times 22 \times 40 \times 75 \times 98 \times 112 \times 125$$

Solution: We need not multiply these numbers to find the number of zeroes at the nd of the resultant product.

All we need to do is to find the numbers of 5s and 2s in the given numbers by resolving the numbers into their factors:

$$(5 \times 3) \times (2^5 \times 5^2) \times (2 \times 11) \times (2^3 \times 5) \times (5^2 \times 3) \times (2 \times 7 \times 7) \times (2^4 \times 7) \times 5^3$$

Only no. of 2s and 5s are relevant here, so, we have:

(i) No. of 5s in $(5^{1+2+1+2+3})$ i.e. $5^9 = 9$

(ii) No. of 2s in $(2^{5+1+3+1+4})$ i.e. $2^{14} = 14$.

Since $9 < 14$.

the no. of zeroes = 9 at the end of the product.

1.12 BINARY NUMBER SYSTEM

This system has a base 2 and uses only 0 and 1; whereas the conventional decimal system having a base 10, uses 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

For any number system it is common that

— the position of each digit within a number affects the total value of the number. In fact, the position of each digit carries a specific weight according to the base of the system.

— each digit has a distinct value and it cannot equal or exceed the base of the system.

In the binary number system, the base = 2 and the number of digits used in the binary system is 2 (0 and 1). So the digits can have value, 0 or 1.

The positional weights of a binary number have been indicated in Table 1.3.

Table 1.3 Positional Weights of a Binary Number

Positions from Right to Left ←	*8th* 7	*7th* 6	*6th* 5	*5th* 4	*4th* 3	*3rd* 2	*2nd* 1	*1st* 0 ← *value*
Positional weight of a	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Binary number	128	64	32	16	8	4	2	1

Using Table 1.3, the decimal equivalents of any binary number can be found out.

$$(10101)_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 16 + 0 + 4 + 0 + 1 = (21)_{10}$$

Alternatively,

1	0	1	0	1
16	$\cancel{8}$	4	$\cancel{2}$	1

Hence, $(10101)_2 = 16 + 4 + 1 = (21)_{10}$

1.12.1 Conversion of a Decimal Number to a Binary Number

To convert a decimal number to a binary number, the following steps are to be considered.

Step 1 Divide the decimal number by 2.

Step 2 Go on dividing the quotients (obtained at each stage) by 2 till the quotient is 0.

Step 3 Write down the remainders on the right side after each of the above divisions.

Step 4 Arrange the remainders (as obtained in Step 3) in the reverse order to get the equivalent bir number.

For example, conversion of 25 to a binary number is given as:

		Remainder
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0 ⇒ last quotient	Arrange the remainders

The equivalent binary number = $(11001)_2$

1.13 CALCULATION IN THE BINARY SYSTEM

Mathematical calculations (i.e. addition, subtraction and multiplication) in the binary system follow t own rules and are similar to those in the decimal system.

1.13.1 Binary Addition

It is easy to add two binary numbers. The rules for binary addition are as follows:

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 10$ (Put 0 in the same column and carry 1 to the **next left column**)

What is carry 1?

In the decimal system, when two numbers are added and the sum of two digits exceeds the highest (i.e. 9), then 1 is carried to the next higher digit position (left column).

Similarly, in binary system, when the sum of two digits (bits) exceeds the highest digit (i.e. 1), the is carried to the next higher digit (bit) position (next left column).

In binary addition, the numbers are written one below the other with their rightmost digits aligned. case of the numbers having fractional parts, the binary points are aligned. Adding is started from right left, as done in the decimal system.

For example, 1011 + 111

carry	→	1	1	1	1	
(Augend) upper row	→		1	0	1	1
(Addend) lower row	→	+	0	1	1	1
		1 ←	0 ←	0 ←	1 ←	0

Carry 1 Carry 1 Carry 1 Carry 1

Explanation of Addition (column-wise)

Column 1 (rightmost column) 1 + 1 = 1 + $\boxed{0}$

carry result

Column 2 (from rightmost) 1 (carry) + 1 (upper row) = 1 0 carry

0 + 1 (lower row) = $\boxed{1}$ result

Column 3 (from rightmost) 1 (carry) + 0 (upper row) = 1

1 + 1 (lower row) = 1 $\boxed{0}$ result

← carry

Column 4 (from rightmost) 1 (carry) + 1 (upper row) = 1 0

0 + 0 (lower row) = $\boxed{0}$ result carry

and put in the left-most column

1.13.2 Binary Subtraction

It is easy to subtract a binary number from another binary number. The rules for binary subtraction are follows:

0 − 0 = 0
1 − 1 = 0
1 − 0 = 1

But to find 0 − 1, we write 1 in the result and also we borrow 1 from the next left column.

What is Borrow 1?

In decimal system, when one number is subtracted from another and it happens that a greater digit is to subtracted from a smaller one, then 10 is borrowed from the next digit position (next left column).

Similarly, in the binary system if it is required to subtract 1 from 0, then 1 is borrowed from the n digit position (next left column) and the result is 1.

For example, to find $(1000)_2 - (11)_2$, we proceed as follows.

Upper Row	1	0	0	0
Lower Row	0	0	1	1
Borrow	1	1	1	
		1	0	1

Explanation of Subtraction (Column-wise)

Column 1 (rightmost) — 0 – 1 yields a result $\boxed{1}$ and a borrow 1 which is placed below column 2.

Column 2 (from rightmost) — 0 – 1 yields a **temporary result 1** and a borrow 1 (placed below column 3). From this temporary result 1, subtract borrow 1 which has already been placed below column 2. So, 1 – 1 yields a result $\boxed{0}$.

Column 3 (from rightmost) — 0 – 0 yields a **temporary result 0**. From this temporary result, **subtract borrow 1** (which has already been placed below column 3). So, 0 – 1 yields a result $\boxed{1}$ and a borrow 1 (placed below column 4).

Column 4 (from rightmost) — 1 – 0 yields a temporary result 1. From this temporary result 1, subtract borrow 1 (which has already been placed below column 4). So, 1 – 1 yields a result $\boxed{0}$.

1.13.3 Binary Multiplication

Binary multiplication is as simple as multiplication in decimal system. The four rules that are followed multiplication of two binary numbers are summarised below.

0×0	= 0
0×1	= 0
1×0	= 0
1×1	= 1

For example, $(11011)_2 \times (101)_2$, we write as

			1	1	0	1	1
					1	0	1
			1	1	0	1	1
		0	0	0	0	0	
	1	1	0	1	1		
1	0	0	0	0	1	1	1

carry (1) to the next higher digit position (next left column) and add, because

$1 + 1 = 1\ 0$

carry (1), result (0)

1.13.4 Binary Division

In binary division, the method that is applied is similar to that in decimal system. The two rules which are followed here are,

$$\frac{0}{1} = 0 \quad \text{and} \quad \frac{1}{1} = 1$$

Here also, the value of $\frac{1}{0}$ is undefined.

For example, to divide 100111 by 111

```
111 ) 1 0 0 1 1 1 ( 1 0 1
      0 1 1 1
     ----------
          1 0 1 1
            1 1 1
         ---------
            1 0 0
```

∴ Quotient = 101
Remainder = 100

Note: In the problems on binary system it has been found in the examinations that
0 is written as *
1 is written as •

Hence it is to be remembered that all the rules of binary system are applicable, but only type of coding may vary. Write * for 0 and • for 1.

Solved Examples

E-1 On dividing 15625 by 41, what is the quotient and the remainder?

S-1 Divisor = 41, dividend = 15625, quotient = 381 and remainder = **4.**

E-2 On dividing 397246 by a certain number, the quotient is 865 and the remainder is 211. Find the divisor.

S-2 Divisor $= \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$$= \frac{397246 - 211}{865} = 459 \qquad \therefore \text{ The divisor is } \mathbf{459.}$$

E-3 What is the number which on dividing $(x + ak)$ gives 'a' as the quotient and x as the remainder?

S-3 Divisor $= \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$$= \frac{x + ak - x}{a} = k \qquad \therefore \text{ the divisor is } \boldsymbol{k}.$$

E-4 Find the least number, that must be subtracted from 87375, to get a number exactly divisible by 698.

S-4 On dividing 87375 by 698, the remainder is 125

By Method 1 (Refer 1.5.1)

The least number to be subtracted from the dividend is the remainder

∴ the least number to be subtracted = **125.**

E-5 What least number must be added to 49123 to get a number exactly divisible by 263

S-5 On dividing 49123 by 263, the remainder is 205.

By Method 2 (Refer 1.5.1)

the least number to be added to the dividend = divisor – remainder = 263 – 205 = 58

∴ the least number to be added = **58.**

E-6 Find the greatest number of 3 digits, which is exactly divisible by 35.

S-6 The greatest 3 digit number = 999.

On dividing 999 by 35, remainder = 19 Now by applying *Method* 2, we obtain

the required number = (dividend) – (remainder) = 999 – 19 = **980.**

E-7 Find the least number of 3 digits, which is exactly divisible by 14.

S-7 The least number of 3 digits = 100

On dividing 100 by 14, remainder = 2

To determine exactly divisible least number, we follow the method 2.

∴ The required number = Dividend + (Divisor – Remainder) = 100 + (14 – 2) = **112.**

E-8 A number when divided by 602 leaves a remainder 36. What remainder would be obtained dividing the same number by 14?

S-8 Here divisor is a multiple of the other divisor, we find that one

i.e. $\frac{602}{14}$ = an integer. Rule 1 of Remainder Rules is to be used. (Refer 1.6) and so.

```
14) 36 (2
    28
    --
     8
```

∴ the required remainder = 8

E-9 A number, when divided by 357, leaves a remainder 5. What remainder would be obtained dividing the same number by 17?

S-9 Here, $\frac{357}{17}$ = an integer. i.e. one divisor is multiple of the other divisor.

∴ But the remainder by the greater divisor = 5, which is even less than the smaller divisor (=

so using Case II of Rule-1, we find as

the required remainder by smaller divisor = the remainder by greater divisor = 5.

E-10 In each of the following number series, one number is wrong. Find out the odd number.

(i) 2, 8, 20, 44, 92, 184, 380

(ii) 60, 48, 38, 28, 24, 20, 18

(iii) 380, 188, 92, 48, 20, 8, 2

(iv) 3, 4.5, 9, 22.5, 67.5, 270, 945

(v) 7, 9, 17, 42, 91, 172, 293

(vi) 5, 15, 30, 135, 405, 1215, 3645

(vii) 2, 9, 28, 65, 126, 216, 344

(viii) 1, 2, 6, 21, 86, 445, 2676

(ix) 3, 5, 12, 38, 154, 914, 4634

(x) 696, 340, 168, 80, 36, 14, 3

(xi) 445, 221, 109, 46, 25, 11, 4
(xii) 1, 2, 4, 12, 36, 71, 144, 432
(xiii) 7, 10, 12, 14, 17, 19, 22, 24
(xiv) 1, 3, 10, 21, 64, 129, 356, 777
(xv) 200, 165, 148, 118, 104, 77, 68
(xvi) 2, 6, 24, 96, 285, 568, 567
(xvii) 6072, 1008, 200, 48, 14, 5, 3
(xviii) 3, 10, 36, 180, 1080, 7560
(xix) 318, 368, 345, 395, 372, 422, 400, 449
(xx) 54, 9, 15, 6, 24, 4, 16
(xxi) 444, 153, 156, 52, 60, 20, 28
(xxii) 2, 6, 12, 27, 58, 121, 248
(xxiii) 3, 9, 18, 54, 110, 324, 648
(xxiv) 5, 6, 15, 41, 89, 170, 291
(xxv) 8544, 1420, 280, 44, 18, 5
(xxvi) 1, 1, 4, 36, 586, 14400
(xxvii) 812, 398, 190, 90, 40, 16
(xxviii) 7, 8, 12, 24, 37, 62, 98
(xxix) 2, 2, 4, 12, 66, 420, 4620
(xxx) 7, 8, 10, 18, 17, 22, 28
(xxxi) 1, 3, 4, 8, 16, 36, 64
(xxxii) 12, 20, 19, 26, 24, 31, 30
(xxxiii) 4, 8, 11, 22, 18, 33, 25, 50
(xxxiv) –1, 2, 7, 14, 28, 34, 47
(xxxv) 100, 50, $33\frac{1}{3}$, 22, 20
(xxxvi) 1, 7, 16, 2, 8, 15, 3, 9, 18
(xxxvii) $\frac{1}{8}, \frac{1}{4}, \frac{3}{4}$, 4, 15, 90
(xxxviii) 6, 15, 19, 30, 32, 43, 45
(xxxix) 2348, 3437, 4346, 5436, 6344, 7433
(xxxx) 87, 86, 82, 75, 57, 32, –4
(xxxxi) 0, 3, 9, 15, 24, 35, 48
(xxxxii) $-\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{2}, 2$
(xxxxiii) 2, 4, 6, 3, 9, 13, 4, 16, 20
(xxxxiv) 15, 1, 14, 2, 12, 4, 9, 11, 5
(xxxxv) 21, 28, 29, 36, 38, 46, 48, 55
(xxxxvi) 9, 7, 64, 6, 3, 18, 1, 8, 9
(xxxxvii) 27, 54, 58, 116, 232, 240, 244
(xxxxviii) –3, 9, 41, 113, 262, 577

S-10 (i)

2   8   20   44   92 $\boxed{184}$ 380

+6   +12   +24   +48   +96   +192

Wrong number = **184**, correct number = **188** (Refer D7, Table 1.1)

(ii)

60   48   38 $\boxed{28}$ 24   20   18

–12,   –10,   –8   –6   –4   –2

Wrong number = **28**, correct number = **30** (Refer D2, Table 1.1)

(iii)

380   188   92 $\boxed{48}$ 20   8   2

–192   –96   –48   –24   –12   –6

Wrong number = **48**, correct number = **44** (Refer D7, Table 1.1)

(iv)

3   4.5   9   22.5   67.5 $\boxed{270}$ 945

× 1.5   × 2   × 2.5   × 3   × 3.5   × 4

Wrong number = **270**, correct number = **236.25** (Refer R2, Table 1.2)

(v)

7   $\boxed{9}$ 17   42   91   172   293

$+ 1^2$ $+ 3^2$ $+ 5^2$ $+ 7^2$ $+ 9^2$ $+ 11^2$

(1)   (9)   (25)   (49)   (81)   (121)

Wrong number = **9**, correct number = **8** (Refer D3, Table 1.1)

(vi)

5   15 $\boxed{30}$ 135   405   1215   3645

× 3   × 3   × 3   × 3   × 3   × 3

Wrong number = **30**, correct number = **45** (Refer R1, Table 2.1)

(vii)

2 9 28 65 126 $\boxed{216}$ 344

$1^3 + 1$ $2^3 + 1$ $3^3 + 1$ $4^3 + 1$ $5^3 + 1$ $6^3 + 1$ $7^3 + 1$

Wrong number = **216**, correct number = **217** (It is a pure number serie

(viii)

1 2 6 21 $\boxed{86}$ 445 2676

$\times 1 + 1$ $\times 2 + 2$ $\times 3 + 3$ $\times 4 + 4$ $\times 5 + 5$ $\times 6 + 6$

Wrong number = **86**, correct number = **88** (It is a mixed series).

(ix)

3 5 12 38 154 $\boxed{914}$ 4634

$\times 1 + 2$ $\times 2 + 2$ $\times 3 + 2$ $\times 4 + 2$ $\times 5 + 2$ $\times 6 + 2$

Wrong number = **914**, correct number = **772.**

(x)

696 $\boxed{340}$ 168 80 36 14 3

– 352 – 176 – 88 – 44 – 22 – 11

Wrong number = **340**, correct number = **344** (Refer D7, Table 1.1)

(xi)

445 221 109 $\boxed{46}$ 25 11 4

– 224 – 112 – 56 – 28 – 14 – 7

Wrong number = **46**, correct number = **53** (Refer D7, Table 1.2)

(xii)

1 2 4 12 36 $\boxed{71}$ 144 432

$\times 2$ $\times 2$ $\times 3$ $\times 3$ $\times 2$ $\times 2$ $\times 3$

Here, one series is multiple of 2 and other is multiple of 3 and both are repeated in a batch of two numbers.

Wrong number = **71**, correct number = **72**. It is a mixed series (Refer R4, Table 1.2)

(xiii)

7 10 12 14 17 $\boxed{19}$ 22 24

+ 3 + 2 + 2 + 3 + 3 + 2 + 2

Wrong number = **19**, correct number = **20**. It is a mixed series.

(xiv)

1 3 10 21 64 129 $\boxed{356}$ 777

$\times 2 + 1$ $\quad \times 3 + 1$ $\quad \times 2 + 1$ $\quad \times 3 + 1$ $\quad \times 2 + 1$ $\quad \times 3 + 1$ $\quad \times 2 + 1$

Wrong number = **356**, correct number = **388**. It is a mixed series.

(xv)

200 165 148 $\boxed{118}$ 104 77 68

$14^2 + 4$ $\quad 13^2 - 4$ $\quad 12^2 + 4$ $\quad 11^2 - 4$ $\quad 10^2 + 4$ $\quad 9^2 - 4$ $\quad 8^2 +$

Wrong number = **118**, correct number = **117**. It is a pure series (Refer 1.3.1 (I)).

(xvi)

2 6 $\boxed{24}$ 96 285 568 567

$\times 6 - 6$ $\quad \times 5 - 5$ $\quad \times 4 - 4$ $\quad \times 3 - 3$ $\quad \times 2 - 2$ $\quad \times 1 - 1$

Wrong number = **24**, correct number = **25**.

(xvii)

6072 $\boxed{1008}$ 200 48 14 5 3

$- 12 \div 6$ $\quad - 10 \div 5$ $\quad - 8 \div 4$ $\quad - 6 \div 3$ $\quad - 4 \div 2$ $\quad - 2 \div 1$

Wrong number = **1008**, correct number = **1010**.

(xviii)

3 $\boxed{10}$ 36 180 1080 7560

$\times 3$ $\quad \times 4$ $\quad \times 5$ $\quad \times 6$ $\quad \times 7$

Wrong number = **10**, correct number = **9** (Refer R2, Table 1.1).

(xix)

318 368 345 395 372 422 $\boxed{400}$ 44

+ 27   + 27   + 27

+ 27   + 27   + 27

Wrong number = **400**, correct number = **399**.

(xx)

54   9 $\boxed{15}$ 6   24   4   16

÷ 6   ÷ 6   ÷ 6

Wrong number = **15**, correct number = **36**.

(xxi)

444   $\boxed{153}$ 156   52   60   20   28

÷ 3   ÷ 3   ÷ 3

Wrong number = **153**, correct number = **148**.

(xxii)

2   $\boxed{6}$ 12   27   58   121   24

× 2 + 1   × 2 + 2   × 2 + 3   × 2 + 4   × 2 + 5   × 2 + 6

Wrong number = **6**, correct number = **5**.

(xxiii)

3   9   18   54 $\boxed{110}$ 324   64

× 3   × 2   × 3   × 2   × 3   × 2

Wrong number = **110**, correct number = **108**.

(xxiv)

5   6   15 $\boxed{41}$ 89   170   291

$+ 1^2$ $+ 3^2$ $+ 5^2$ $+ 7^2$ $+ 9^2$ $+ 11^2$

Wrong number = **41**, correct number = **40** (Refer D3, Table 1.1).

(xxv)

8544   1420   280 $\boxed{44}$ 18   5

÷ 6 − 4   ÷ 5 − 4   ÷ 4 − 4   ÷ 3 − 4   ÷ 2 − 4

Wrong number = **44**, correct number = **66**.

(xxvi)

1   1   4   36 $\boxed{586}$ 14400

$\times 1^2$ $\times 2^2$ $\times 3^2$ $\times 4^2$ $\times 5^2$

Wrong number = **586**, correct number = **576** (Refer R3, Table 1.2).

(xxvii)

812 398 $\boxed{190}$ 90 40 16

÷ 2 – 8 ÷ 2 – 7 ÷ 2 – 6 ÷ 2 – 5 ÷ 2 – 4

Wrong number = **190**, correct number = **192**.

(xxviii)

7 8 12 $\boxed{24}$ 37 2 98

$+ 1$ $+ 2^2$ $+ 3^2$ $+ 4^2$ $+ 5^2$ $+ 6^2$

Wrong number = **24**, correct number = **21** (Refer D3, Table 1.1).

(xxix)

2 2 4 12 $\boxed{66}$ 420 4620

× 1 × 2 × 3 × 5 × 7 × 11

Wrong number = **66**, correct number = **60** (Refer R5, Table 1.2).

(xxx)

7 8 10 $\boxed{18}$ 17 22 28

+ 1 + 2 + 3 + 4 + 5 + 6

Wrong number = **18**, correct number = **13** (Refer D2, Table 1.1).

(xxxi)

1 3 4 8 16 $\boxed{36}$ 64

1 + 3 1 + 3 + 4 1 + 3 + 4 + 8 1 + 3 + 4 + 8 + 16 1 + 3 + 4 + 8 + 16 + 32

Wrong number = **36**, correct number = **32**.

(xxxii)

12 20 19 26 $\boxed{24}$ 31 30

+ 8 – 1 + 7 – 1 + 6 – 1

Wrong number = **24**, correct number = **25**. It is a mixed series.

(xxxiii)

4 8 11 22 18 $\boxed{33}$ 25 50

+ 7 + 7 + 7

+ 14 + 14 + 14

Wrong number = **33**, correct number = **36**.

(xxxiv)

–1 2 7 14 $\boxed{28}$ 34 47

+ 3 + 5 + 7 + 9 + 11 + 13

Wrong number = **28**, correct number = **23**.

(xxxv)

100 50 $33\frac{1}{3}$ $\boxed{22}$ 20

$100 \div 1$ $100 \div 2$ $100 \div 3$ $100 \div 4$ $100 \div 5$

Wrong number = **22**, correct number = **25**.

(xxxvi)

1 7 16 2 8 $\boxed{15}$ 3 9 18

x y z x y z x y z

There are three series x, y and z. mixed

x series contains 1, 2, 3

y series contains 7, 8, 9

z series contains 16, 15, 18. (Wrong number = **15**, correct number = **17**)

(xxxvii)

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{4}$ $\boxed{4}$ 15 90

× 2 × 3 × 4 × 5 × 6

Wrong number = **4**, correct number = **3** (Refer R2, Table 1.2).

(xxxviii)

6 15 19 30 32 $\boxed{43}$ 45

+ 13 + 13 + 13

+ 15 + 15

Wrong number = **43**, correct number = **45**. It is a mixed series.

(xxxix)

2348 3437 4346 $\boxed{5436}$ 6344 7433

+ 1089 + 909 + 1089 + 909 + 1089

Here, 1089 and 909 are added alternately to continue the series.

Wrong number = **5436**, correct number = **5435**

(xxxx)

87 86 82 $\boxed{75}$ 57 32 -4

-1^2 -2^2 -3^2 -4^2 -5^2 -6^2

Wrong number = **75**, correct number = **73** (Refer D3, Table 1.1).

(xxxxi)

0 3 $\overline{\underline{9}}$ 15 24 35 48

+ 3 + 5 + 7 + 9 + 11 + 13

Wrong number = **9**, correct number = **8**.

(xxxxii)

$-\frac{1}{2}$ 0 $\frac{1}{2}$ $\overline{\underline{\frac{1}{4}}}$ $\frac{3}{2}$ 2

$+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{1}{2}$

Wrong number = $\mathbf{\frac{1}{4}}$, correct number = **1**.

(xxxxiii)

2 4 6 3 9 $\overline{\underline{13}}$ 4 16 20

$2 + 4$ $3 + 9$ $4 + 16$

Wrong number = **13**, correct number = **12**. It is a mixed series.

(xxxxiv)

+ 1 + 2 + 3

15 1 14 2 12 4 9 $\boxed{11}$ 5

-1 -2 -3 -4

There are two series arranged alternately.

Series 1 contains 1, 2, 4, 11

Seies 2 contains 15, 14, 12, 9, 5

Wrong number = **11**, correct number = **7** (Refer D2, Table 1.1).

(xxxxv)

21 28 29 36 38 $\boxed{46}$ 48 55

+ 7 + 1 + 7 + 2 + 7 + 3 + 7

Wrong number = **46**, correct number = **45**.

(xxxxvi)

9 7 64 6 3 $\boxed{18}$ 1 8 9

$9 \times 7 + 1$ $6 \times 3 + 1$ $1 \times 8 + 1$

Wrong number = **18**, correct number = **19**.

(xxxxvii)

27 54 58 116 $\boxed{232}$ 240 244

× 2 + 4 × 2 + 4 × 2 + 4

Wrong number = **232**, correct number = **120**.

(xxxxviii)

– 3 9 41 113 $\boxed{262}$ 577

+ 12 + 32 + 72 + 152 + 312

+ 20 + 40 + 80 + 160

Here, 20, 40, 80 and 160 are in G.P.

Wrong number = **262**, correct number = **265**.

E-11 7 13 78 83 415

3 *a* *b* *c* *d* *e*

What should replace *c*?

S-11 The pattern of the first line series is obtained as:

7, 7 + 6, 13 × 6, 78 + 5, 83 × 5

Therefore, the second line series can be completed on the basis of the same pattern, as:

3, $a = 3 + 6$, $b = 9 \times 6$, $c = 54 + 5$, . . .

∴ The value of *c* is 59.

E-12 3 6 24 72 144 576

1 *a* *b* *c* *d* *e*

What should replace *d*?

S-12 The pattern of the first line series is obtained as:

3, 3 × 2, 6 × 4, 24 × 3, 72 × 2, 144 × 4

Therefore, the second line series can be completed on the basis of the same pattern as:

1, $a = 1 \times 2$, $b = 2 \times 4$, $c = 8 \times 3$, $d = 24 \times 2$

∴ The value of *d* is 48

E-13 4 6 15 49 201 1011

15 *a* *b* *c* *d* *e*

What should replace *e*?

S-13 The pattern of the first line series is obtained as:

4, 4 × 1 + 2, 6 × 2 + 3, 15 × 3 + 4, 49 × 4 + 5, 201 × 5 + 6

Therefore, the second line series can be completed on the basis of the same pattern as:
15, $a = 15 \times 1 + 2$, $b = 17 \times 2 + 3$, $c = 37 \times 3 + 4$, $d = 115 \times 4 + 5$, $e = 465 \times 5 + 6$
$\therefore$ The value of e is 2331

E-14 –1 0 10 65 345 1750
–2 a b c d e
What should come in place of d?

S-14 The pattern of the first line series is obtained as:
–1, $(-1 + 1) \times 5$, $(0 + 2) \times 5$, $(10 + 3) \times 5$, $(65 + 4) \times 5$, $(345 + 5) \times 5$
Therefore, the second line series can be completed on the basis of the same pattern as:
–2, $a = (-2 + 1) \times 5$, $b = (-5 + 2) \times 5$, $c = (-15 + 3) \times 5$, $d = (-60 + 4) \times 5$
$\therefore$ the value of d is –270

E-15 –1 0 –8 3 –52 –135
21 a b c d e
What should come in place of c?

S-15 The pattern of the first line series is obtained as:
–1, $(-1 \times 1) + 1^3$, $(0 \times 2) - 2^3$, $(-8 \times 3) + 3^3$, $(3 \times 4) - 4^3$, $-52 \times 5 - 5^3$.
Therefore, the second line series can be completed on the basis of the same pattern as :
21, $a = 21 \times 1 + 1^3$, $b = 22 \times 2 - 2^3$, $c = 36 \times 3 + 3^3$
$\therefore$ the value of c is 135.

E-16 3000 191 2216 847 1688 959
3435 a b c d e
What should come in place of b?

S-16 The pattern of the first line series is obtained as:
3000, $3000 - 53^2$, $191 + 45^2$, $2216 - 37^2$, $847 + 29^2$, $1688 - 21^2$
Therefore, the second line series can be completed on the basis of the same pattern as:
3435, $a = 3435 - 53^2$, $b = 626 + 45^2$
$\therefore$ the value of b is 2651.

Directions (17–21): In each of the following questions, a number series is established if the positions of two out of the five marked numbers are interchanged. The position of the first unmarked number remains the same and it is the beginning of the series. The earlier of the two marked numbers whose positions are interchanged is the answer.

For example, if an interchange of number marked 1 and the number marked 4 is required to establish the series, your answer is 1. If it is not necessary to interchange the position of the numbers to establish the series, give 5 as your answer. Remember that when the series is established, the numbers change from left to right (i.e. from the unmarked number to the last marked number) in a specific order.

E-17 1200 40 1000 50 750 75
1) 2) 3) 4) 5)

S-17 The pattern of the series is:
1200, $1200 \div 30$, 40×25, $1000 \div 20$, 50×15, $750 \div 10$
Here, it is not necessary to interchange the position of any number to establish the series.
Hence, 5) is the answer.

E-18 2 5 26545 177 4424 44
1) 2) 3) 4) 5)

S-18 The pattern of the series is:
2, $2 \times 2 + 1$, $\underline{5 \times 3^2 - 1}$, $44 \times 4 + 1$, $177 \times 5^2 - 1$, $\underline{4424 \times 6 + 1}$
5) ——to interchange—— 2)