



DATA STRUCTURES AND ALGORITHMS

Sep22 : Day 4

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stable and unstable:

2 5 7 9 4 6 5 1

1 2 4 5 5 6 7 9

1 2 4 5 5 6 7 9

stable sorting

not changing the sequence of arrangement

unstable sorting

changing the sequence of arrangement

-Main meory + auxillary memory

stable and unstable:

2 5 7 9 4 6 5 1

1 2 4 5 5 6 7 9

1 2 4 5 5 6 7 9

stable sorting

not changing the sequence of arrangement

unstable sorting

changing the sequence of arrangement

Efficiency of Algorithms:

- Best case
- Worst case:
- Averge case:

Complexity

$O(n \log n)$

↓
 $o(n^2)$

Size : n

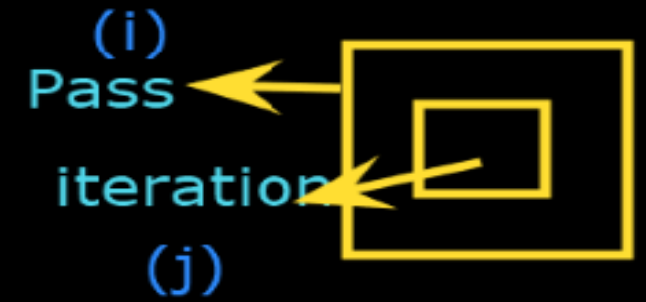
comparision

-Bubble sorting
-comparision

5	3	8	4	6
3	5	8	4	6
3	5	8	4	6
3	5	4	8	6
3	5	4	6	8

max

3	5	4	6	8
3	5	4	6	8
3	4	5	6	8
3	4	5	6	8
3	4	5	6	8



```

class Sorting{
    static void bsort(int a1[])
    {
        int n =a1.length;
        for(int i=0;i<n-1;i++)//pass
        {
            for(int j=0;j<n-i-1;j++)//internal iterations
            {
                if(a1[j] > a1[j+1])
                {
                    //exchange
                    int temp = a1[j];
                    a1[j]=a1[j+1];
                    a1[j+1]=temp;
                    flag = 1;
                }
            }
        }
    }
    static void display(int a1[])
    {
        int n =a1.length;
        for(int i=0;i<n;i++)
        {
            System.out.print(a1[i]+" ");
        }
    }

    public static void main(String args[]){
        //int a1[] = new int[10];
        int a1[]={2,13,24,45,9,30}:
    }
}

```

boolean flag=0;

save

Output

```

int n = a1.length;
for(int i=0; i<n-1; i++) // pass
{
    for(int j=0; j<n-i-1; j++) // internal iterations
    {
        if(a1[j] > a1[j+1])
        {
            //exchange
            int temp = a1[j];
            a1[j] = a1[j+1];
            a1[j+1] = temp;
        }

        display(a1);
        System.out.println();
    }
    //display(a1);
}
}

```

1 2 3 4 5 6

Best case: $O(n)$

n: Elements
n-1: comparisons

6 5 4 3 2 1

Worst case: $O(n^2)$

Average case: $O(n^2)$

```

static void display(int a1[])
{
    int n = a1.length;
    for(int i=0; i<n; i++)
    {
        System.out.print(a1[i] + " ");
    }
}

```

Algorithm 1: Bubble sort

Data: Input array $A[]$

Result: Sorted $A[]$

int i, j, k ;

$N = \text{length}(A)$;

for $j = 1$ **to** N **do**

for $i = 0$ **to** $N-1$ **do**

if $A[i] > A[i+1]$ **then**

$temp = A[i]$;

$A[i] = A[i+1]$;

$A[i+1] = temp$;

end

end

end

Selection sort:

min=

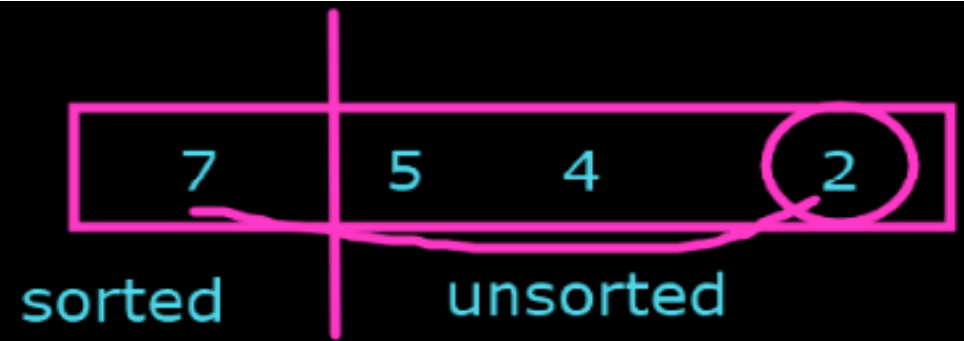
7 5 4 2

2 5 4 7

2 4 5 7

2 4 5 7

2 4 5 7



min=2 4+

2 4 5 7

Algorithm:

SelectionSort(A)

```
{  
    for( i = 0; i < n ; i++)  
    {  
        least=A[i];  
        p=i;  
        for ( j = i + 1; j < n ; j++)  
        {  
            if (A[j] < A[i])  
                least= A[j]; p=j;  
        }  
    }  
    swap(A[i],A[p]);  
}
```

```
void ssort(int a1[])
{
    int n=a1.length;
    for(int i=0;i<n-1;i++)
    {
        int min = i;
        for(int j=i+1;j<n;j++)
        {
            if(a1[j] < a1[min])
                min =j;
        }
        //swapping
        int temp = a1[min];
        a1[min]=a1[i];
        a[i]=temp;
    }
}
```

Insertion sort:

4	3	2	10	12	1	5	6
4	3	2	10	12	1	5	6
3	4	2	10	12	1	5	6
2	3	4	10	12	1	5	6
2	3	4	10	12	1	5	6
1	3	4	10	12	1	5	6

temp

1

INSERTION-SORT(A)

for $j \leftarrow 2$ to n

do $\text{key} \leftarrow A[j]$

▷ Insert $A[j]$ into the sorted sequence $A[1 \dots j-1]$

$i \leftarrow j - 1$

while $i > 0$ and $A[i] > \text{key}$

do $A[i + 1] \leftarrow A[i]$

$i \leftarrow i - 1$

$A[i + 1] \leftarrow \text{key}$

t_j : # of times the while statement is executed at iteration j



```
static void isort(int a1[])
```

```
{
```

```
    int n=a1.length;
```

```
    for(int i=1;i<n;++i)
```

```
    {
```

```
        int key = a1[i];
```

```
        int j=i-1;
```

```
        while(j>=0 && a1[j] > key)
```

```
        {
```

```
            a1[j+1]=a1[j];
```

```
            j=j-1;
```

```
        }
```

```
        a1[j+1]=key;
```

```
    }
```

```
}
```

```
static void isort(int a1[])
{
    int n=a1.length;
    for(int i=1;i<n;++i)
    {
        int key = a1[i];
        int j=i-1;

        while(j>=0 && a1[j] > key)
        {
            a1[j+1]=a1[j];
            j=j-1;
        }
        a1[j+1]=key;
        display(a1);
        System.out.println();
    }
}
```

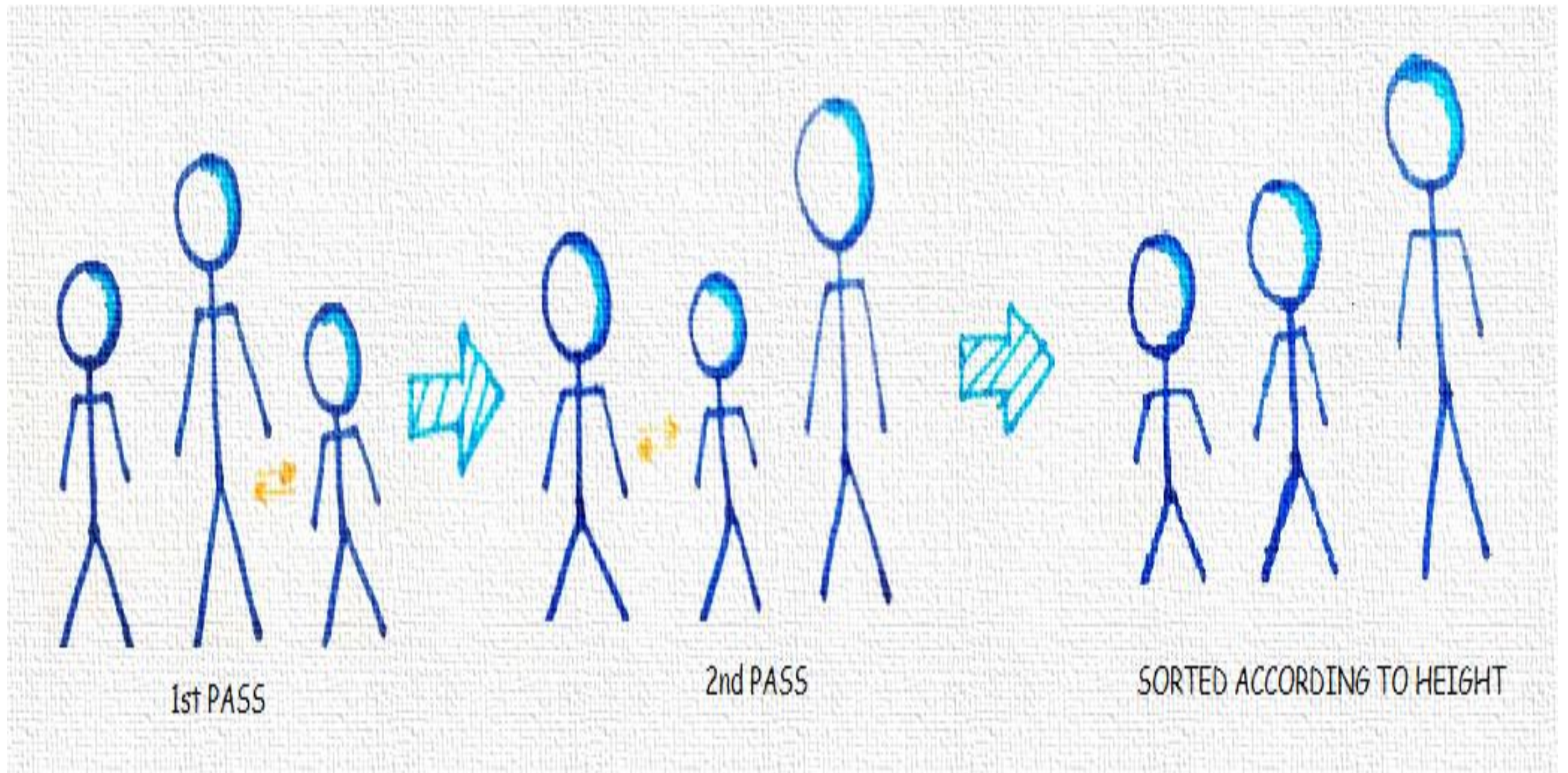
Average case

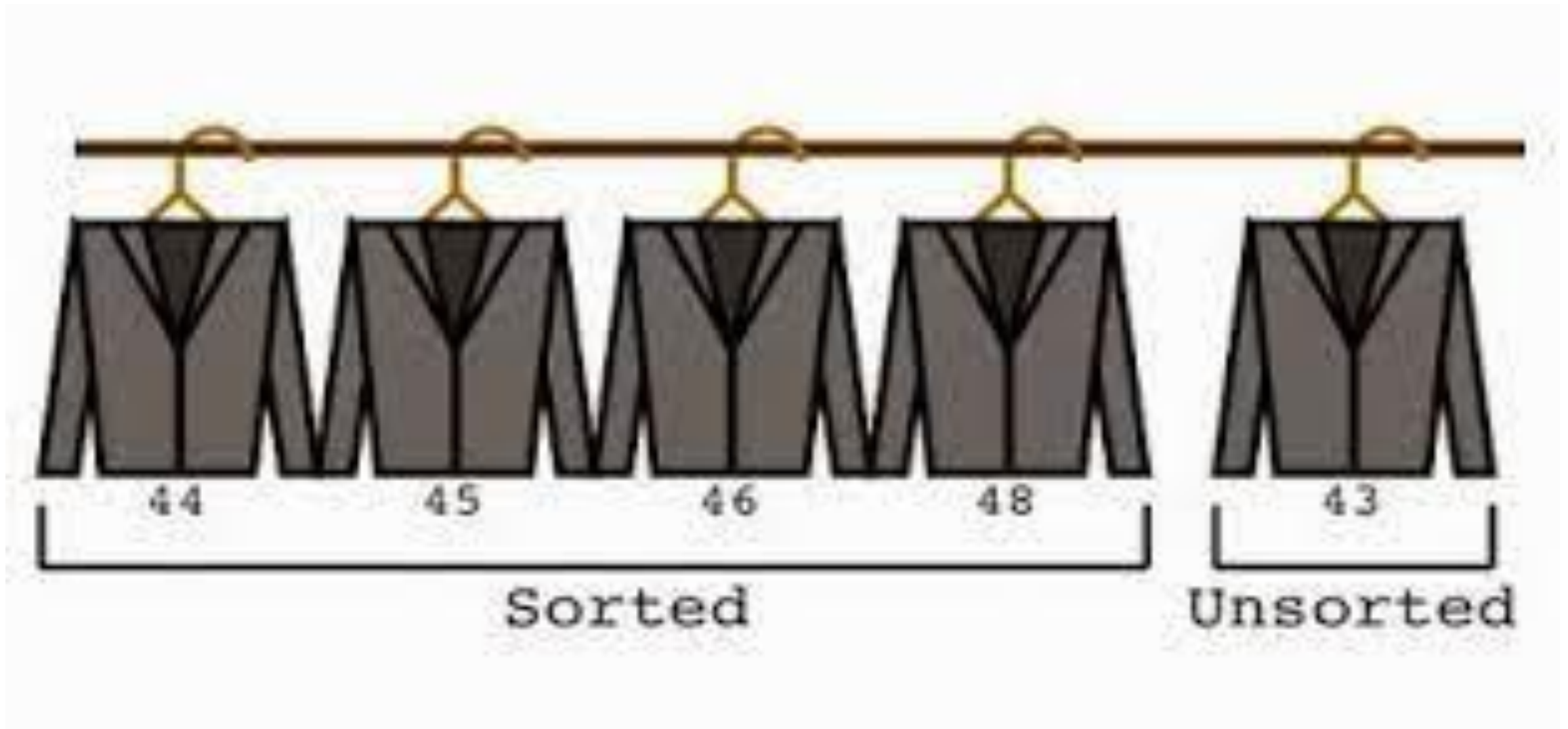
Worst case : $O(n^2)$

Best Case : Sorted array

1 2 3 4 5
 $O(n)$

space: $O(n)$

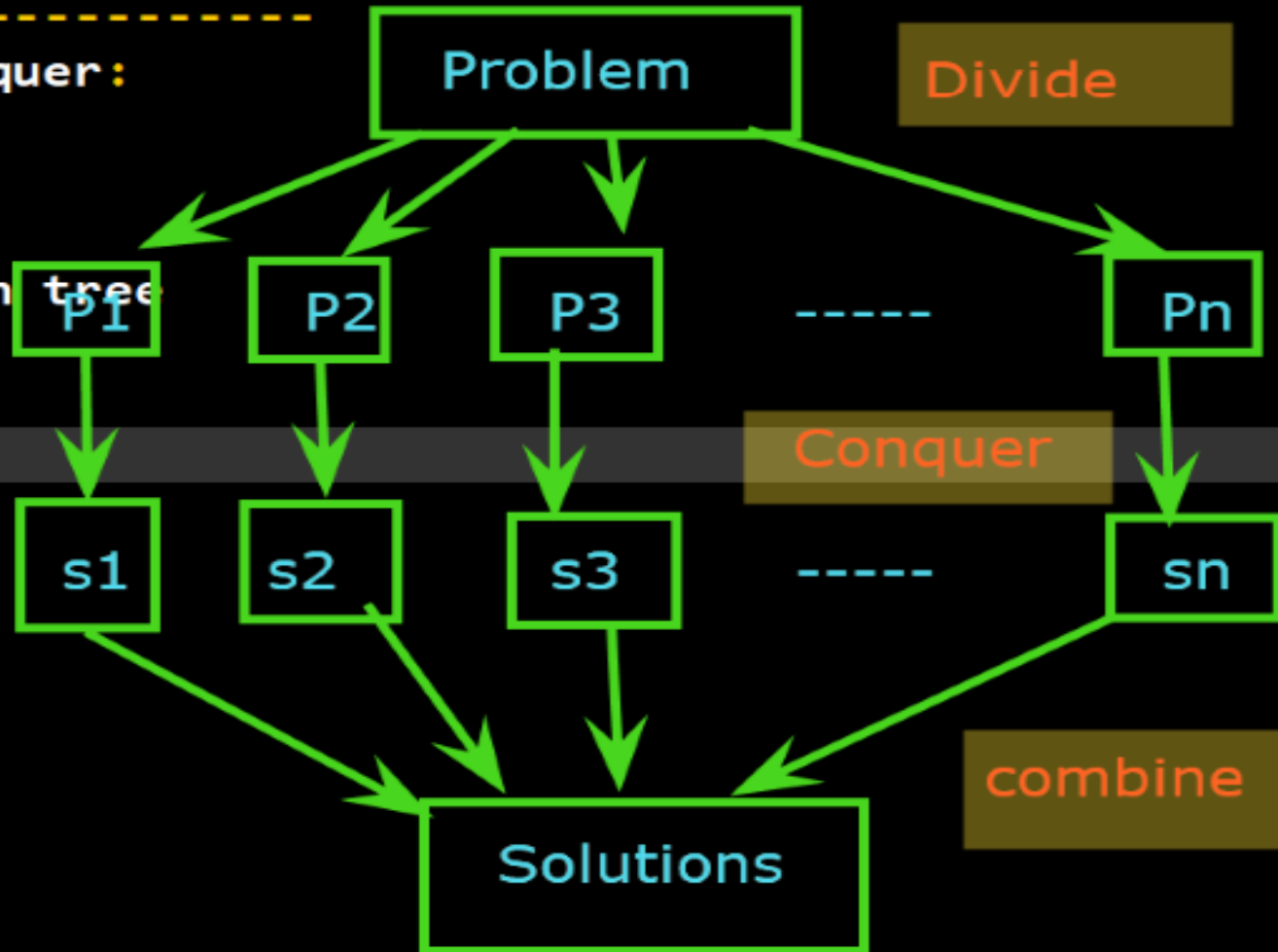




$$k = \log n$$

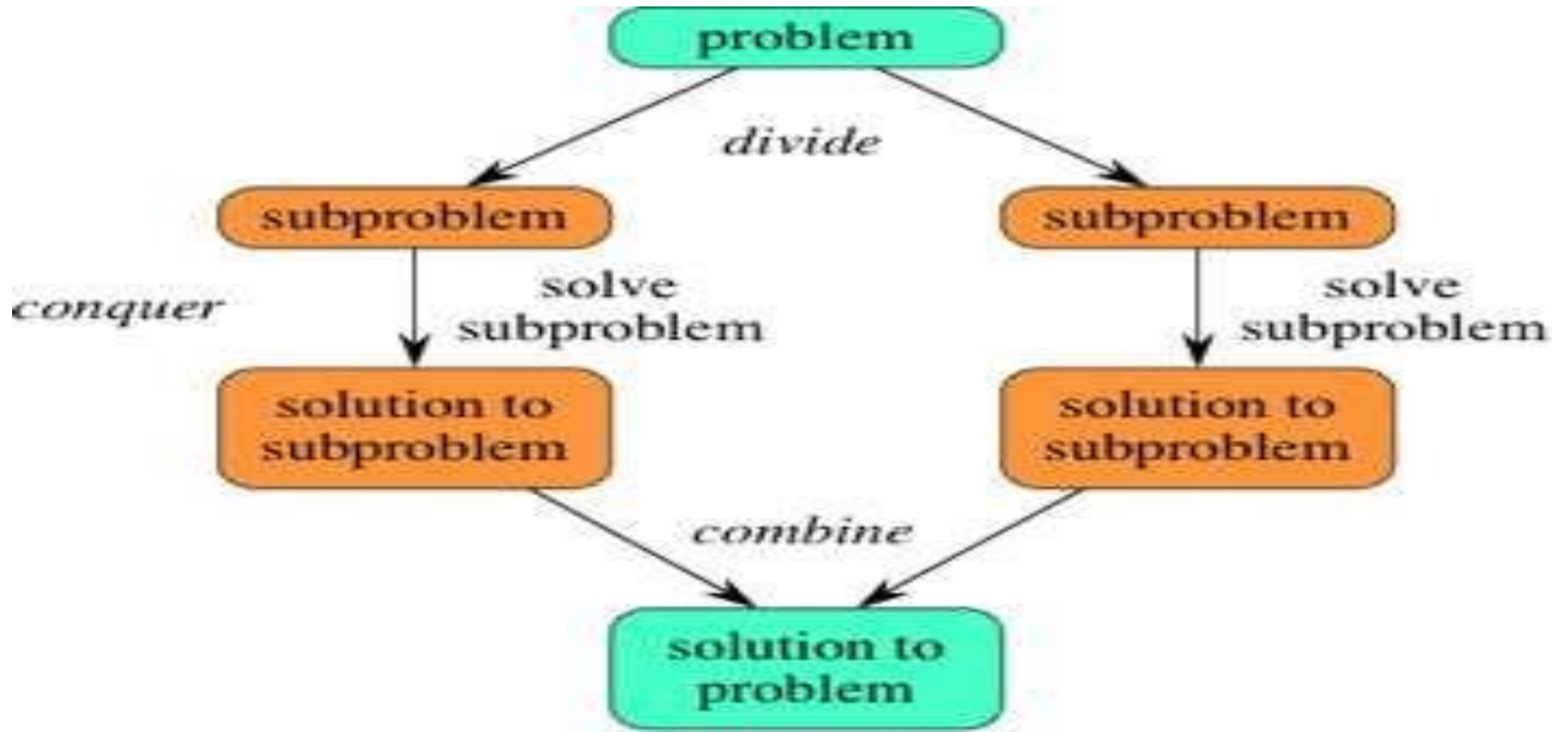
Divide and Conquer:

- Binary search
- Merge sort
- Quick sort
- Binary search tree



Divide and Conquer

Divide-and-conquer



Merge sort:

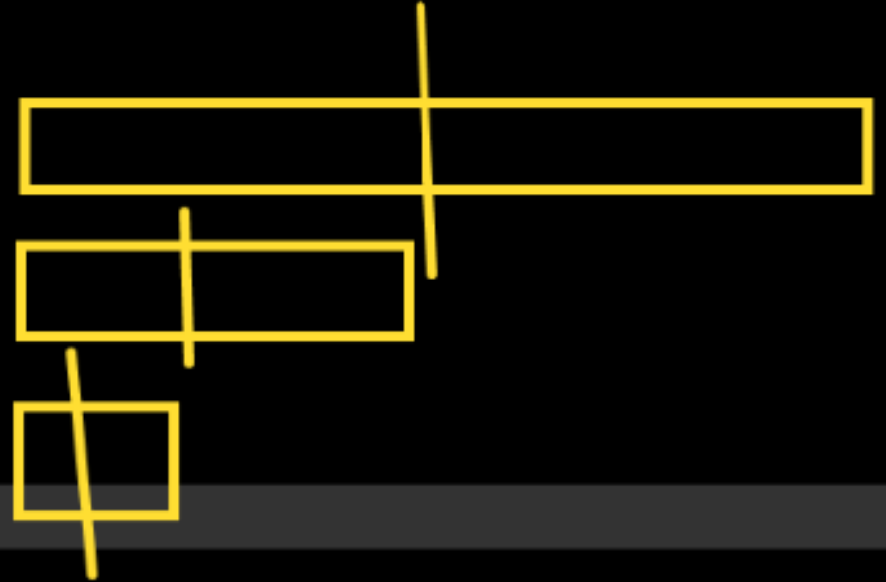
- follows divide and conquer strategy
- divide an array in 2 parts
- recursive order

n---->elements in the array

A = 2 8 15 18 \xrightarrow{i} m elements

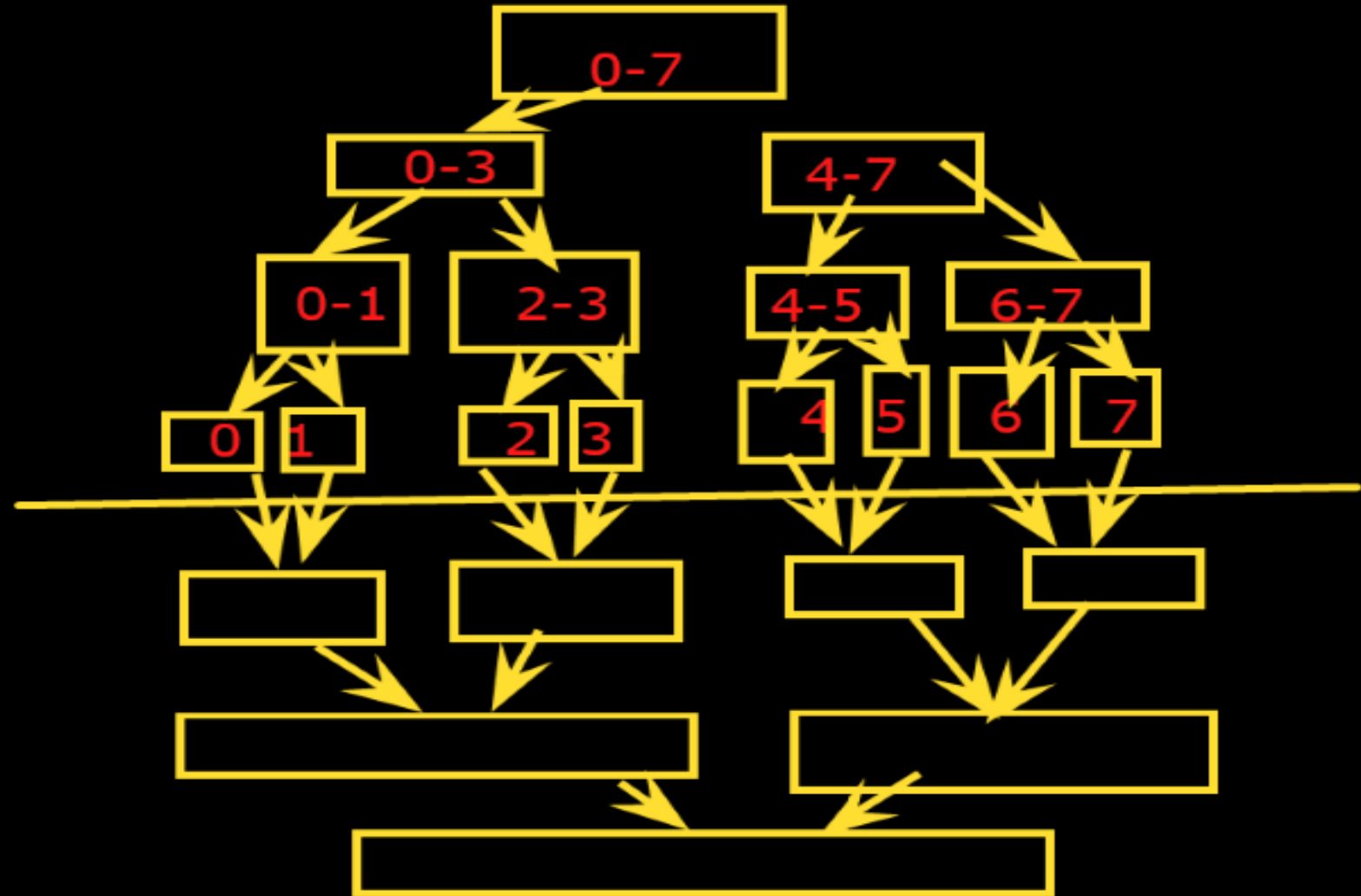
B = 5 9 12 17 \xrightarrow{j} n elements

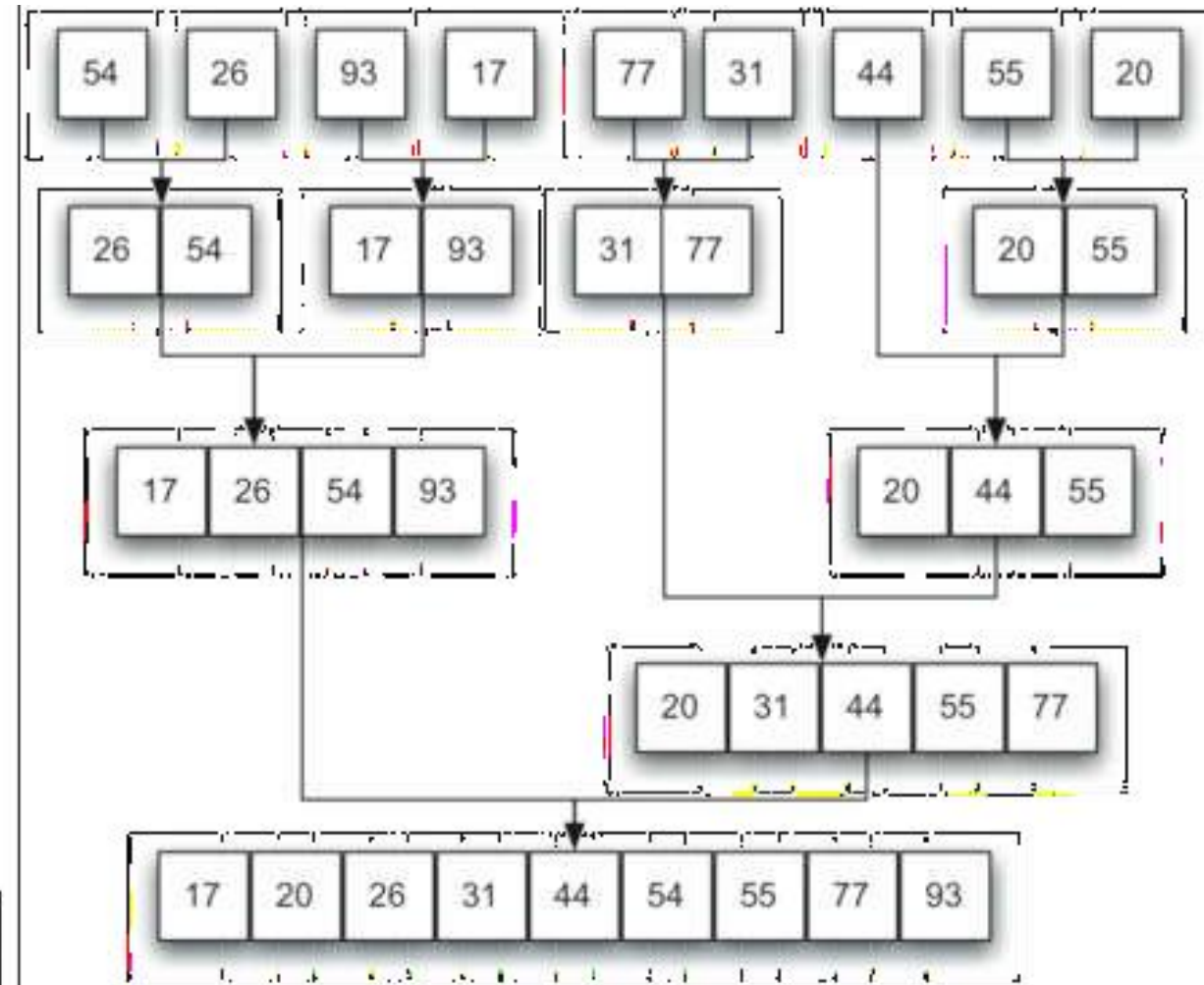
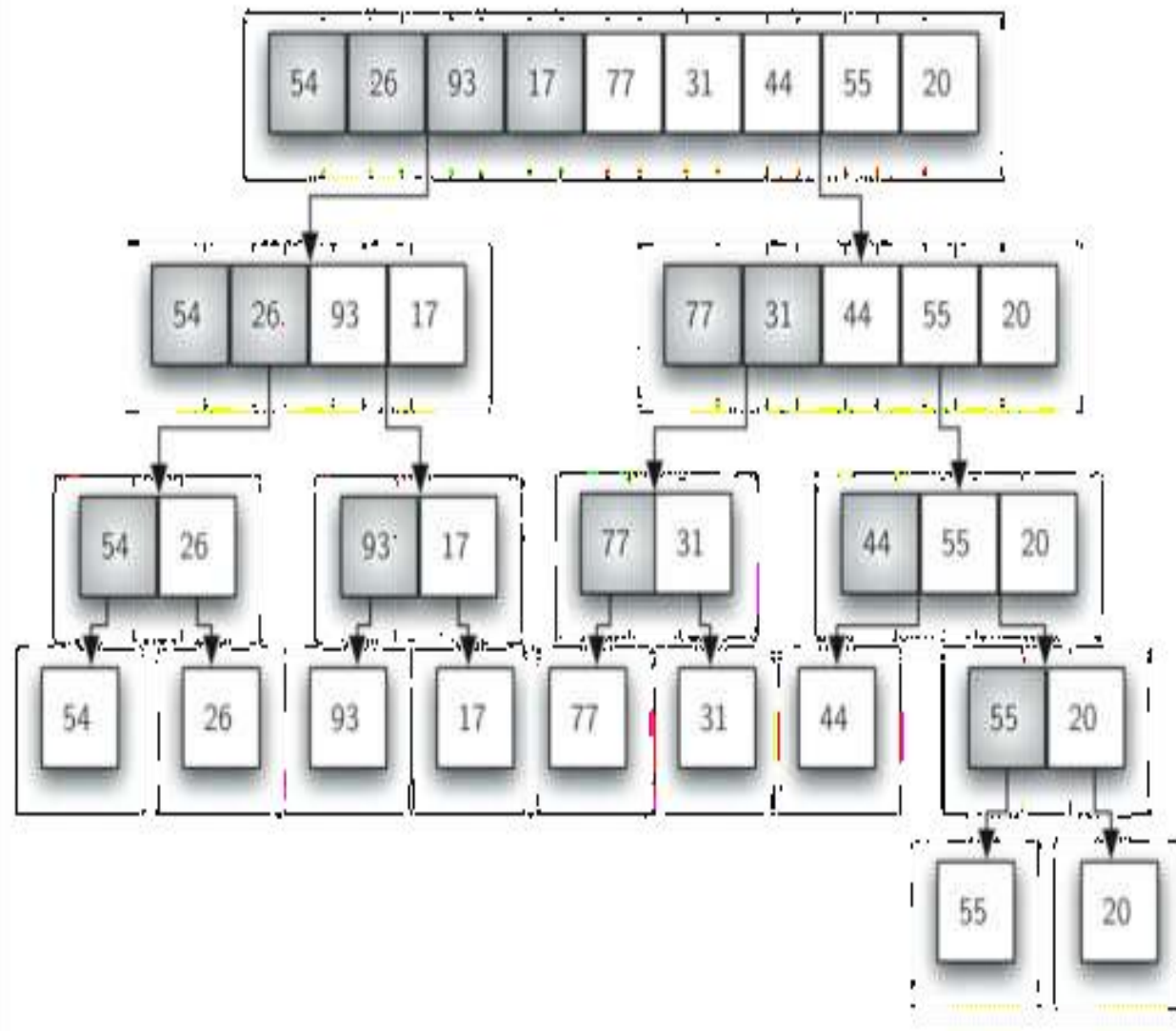
C = 2 5 8 9 12 15 17 18 (m+n)



0	1	2	3	4	5	6	7	: index
9	3	7	5	6	4	8	2	array
1							h	

$$\text{mid} = (\text{low} + \text{high}) / 2$$





Merge Sort:

Here is the pseudocode for Merge Sort, modified to include a counter:

```

count ← 0
Merge_Sort(A, p, r)
1   if p < r
2       then q ← ⌊(p + r)/2⌋
3           Merge-Sort (A, p, q)
4           Merge-Sort (A, q+1, r)
5           merge (A, p, q, r)

```

And here is the modified algorithm for the Merge function used by Merge Sort:

```

Merge (A, p, q, r)
1   n1 ← (q - p) + 1
2   n2 ← (r - q)
3   create arrays L[1..n1+1] and R[1..n2+1]
4   for i ← 1 to n1 do
5       L[i] ← A[(p + i) - 1]
6   for j ← 1 to n2 do
7       R[j] ← A[q + j]
8   L[n1 + 1] ← ∞
9   R[n2 + 1] ← ∞
10  i ← 1
11  j ← 1
12  for k ← p to r do
12.5  count ← count + 1
13      if L[i] ≤ R[j]
14          then A[k] ← L[i]
15              i ← i + 1
16          else A[k] ← R[j]
17              j ← j + 1

```



Merge Sort:

Here is the pseudocode for Merge Sort, modified to include a counter:

```
count ← 0
Merge_Sort(A, p, r)
1   if p < r
2       then q ← ⌊(p + r)/2⌋
3           Merge-Sort (A, p, q)
4           Merge-Sort (A, q+1, r)
5           Merge (A, p, q, r)
```

And here is the modified algorithm for the Merge function used by Merge Sort:

```
Merge (A, p, q, r)
1   n1 ← (q - p) + 1
2   n2 ← (r - q)
3   create arrays L[1..n1+1] and R[1..n2+1]
4   for i ← 1 to n1 do
5       L[i] ← A[(p + i) - 1]
6   for j ← 1 to n2 do
7       R[j] ← A[q + j]
8   L[n1 + 1] ← ∞
9   R[n2 + 1] ← ∞
10  i ← 1
11  j ← 1
12  for k ← p to r do
12.5    count ← count + 1
13      if L[i] ≤ R[j]
14          then A[k] ← L[i]
15              i ← i + 1
16      else A[k] ← R[j]
17          j ← j + 1
```



```

}
while(j<n2)
{
    a1[k] = R[j];
    j++;
    k++;
}

```

n

n

```

static void display(int a1[])
{
    int n = a1.length;
    for(int i=0;i<n;i++)
    {
        System.out.print(a1[i]+" ");
    }
}

```

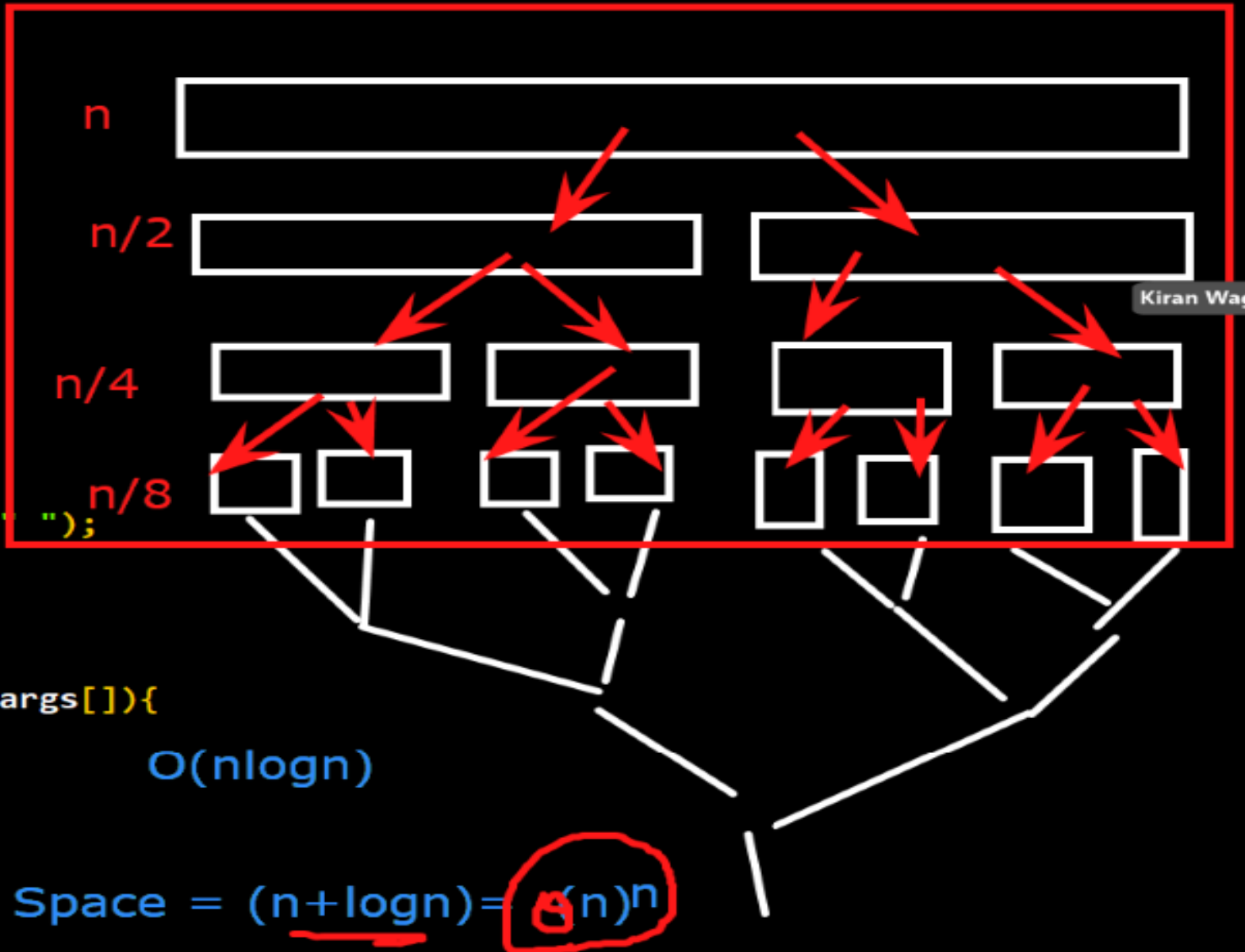
n

n

```

public static void main(String args[]){
    //int a1[] = new int[10];
    int a1[]={2,13,24,45,9,30};
    int n = a1.length;
    display(a1);
    System.out.println();
    //bsort(a1);
    //ssort(a1);
}

```

 $O(n \log n)$ Space = $(n + \log n) = \Theta(n)$ 

The following procedure implements quicksort:

QUICKSORT(A, p, r)

```
1  if  $p < r$ 
2       $q = \text{PARTITION}(A, p, r)$ 
3      QUICKSORT( $A, p, q - 1$ )
4      QUICKSORT( $A, q + 1, r$ )
```

To sort an entire array A , the initial call is QUICKSORT($A, 1, A.length$).

Partitioning the array

The key to the algorithm is the PARTITION procedure, which rearranges the subarray $A[p..r]$ in place.

PARTITION(A, p, r)

```
1   $x = A[r]$ 
2   $i = p - 1$ 
3  for  $j = p$  to  $r - 1$ 
4      if  $A[j] \leq x$ 
5           $i = i + 1$ 
6          exchange  $A[i]$  with  $A[j]$ 
7  exchange  $A[i + 1]$  with  $A[r]$ 
8  return  $i + 1$ 
```