# Morphological Operations Applied to Crack Detection in Digital Art Restoration

M. Kirbie Dramdahl
Division of Science and Mathematics
University of Minnesota, Morris
Morris, Minnesota, USA 56267
dramd002@morris.umn.edu

## **ABSTRACT**

## 1. INTRODUCTION

# 2. EDGE DETECTION

## 3. MORPHOLOGICAL OPERATIONS

Mathematical morphology is an area of set theory, and a method of image processing. Typically, it is used in processing binary (black and white) images, although there are also variations used for gray scale images. Here, we will focus on binary images. Morphological functions take two inputs. The first input is the image to be processed, divided into foreground (typically white) and background (typically black) regions. The second input is a structuring element, a (typically small, in comparison to the image) set of coordinate points. The structuring element is then used to modify the input image. The image that results from the morphological operation is determined by the shape, size, and point of origin of the structuring element [1, 2, 3, 4]. Subsections 3.1, 3.2, 3.3, and 3.4 explain the fundamental operations of mathematical morphology.

#### 3.1 Erosion

Erosion of an image strips away a layer of pixels from the boundaries of foreground regions, and is denoted by the equation

$$g = f \ominus s$$

where g is the resulting image, f is the original image, and s is the structuring element [1, 2]. This is accomplished by placing the origin of the structuring element over every pixel of the foreground regions in turn. If every point within the structuring element is in line with a foreground pixel, the foreground pixel lined up with the origin of the structuring element is left unchanged. If at least one point within the structuring element is in line with a background pixel, then the pixel lined up with the origin of the structuring element

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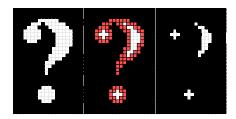


Figure 1: Erosion: Left: Original Image. Center: Erosion Marked in Red. Right: Results of Erosion.

is converted to a background pixel [3]. An example of erosion using a 3x3 square structuring element with the origin located at the center is presented in Figure 1. The erosion of foreground regions is equivalent to the dilation (discussed in subsection 3.2) of background regions [3].

## 3.2 Dilation

Dilation of an image adds a layer of pixels to the boundaries of foreground regions, and is denoted by the equation

$$g = f \oplus s$$

where g is the resulting image, f is the original image, and s is the structuring element  $[1,\,2]$ . This is accomplished by placing the origin of the structuring element over every pixel of the background regions in turn. If every point within the structuring element is in line with a background pixel, the background pixel lined up with the origin of the structuring element is left unchanged. If at least one point within the structuring element is in line with a foreground pixel, then the pixel lined up with the origin of the structuring element is converted to a foreground pixel [3]. An example of dilation using a 3x3 square structuring element with the origin located at the center is presented in Figure 2. The dilation of foreground regions is equivalent to the erosion (discussed in subsection 3.1) of background regions [3].

# 3.3 Opening

Opening of an image is an erosion followed by a dilation, and is denoted by the equation

$$g = f \circ s = (f \ominus s) \oplus s$$

where g is the resulting image, f is the original image, and s is the structuring element [1, 2]. Similar to erosion, open-

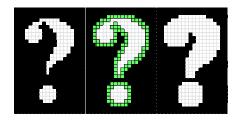


Figure 2: Dilation: Left: Original Image. Center: Dilation Marked in Green. Right: Results of Dilation.



Figure 3: Opening: Left: Original Image. Second from Left: Erosion Marked in Red. Center: Results of Erosion. Second from Right: Dilation Marked in Green. Right: Results of Dilation (Opening Complete).

ing strips away foreground pixels at the boundaries of foreground regions, but is less destructive of the initial foreground regions than erosion. Opening is therefore typically used to preserve foreground regions with a similar size and shape to the structuring element, while removing or reducing other foreground regions [3]. An example of opening using a 3x3 square structuring element with the origin located at the center is presented in Figure 3. Additionally, opening is idempotent, meaning that once an image has been opened, additional openings with the same structuring element will have no further effect on the image [3, 4]. The idempotence of opening is denoted by the equation

$$q = (f \circ s) \circ s = f \circ s$$

where, as before, g is the resulting image, f is the original image, and s is the structuring element [1]. The opening of foreground regions is equivalent to the closing of background regions [3].

# 3.4 Closing

Closing of an image is a dilation followed by an erosion, and is denoted by the equation

$$g=f\bullet s=(f\oplus s)\ominus s$$

where g is the resulting image, f is the original image, and s is the structuring element [1, 2]. Similar to dilation, closing adds foreground pixels at the boundaries of foreground regions, but is less destructive of the initial background regions than dilation. Closing is therefore typically used to preserve background regions with a similar size and shape to the structuring element, while removing or reducing other background regions [3]. An example of closing using a 3x3 square structuring element with the origin located at the



Figure 4: Closing: Left: Original Image. Second from Left: Dilation Marked in Green. Center: Results of Dilation. Second from Right: Erosion Marked in Red. Right: Results of Erosion (Closing Complete).

center is presented in Figure 4. Additionally, closing is idempotent, meaning that once and image has been closed, additional closings with the same structuring element will have no further effect on the image [3, 4]. The idempotence of closing is denoted by the equation

$$g = (f \bullet s) \bullet s = f \bullet s$$

where, as before, g is the resulting image, f is the original image, and s is the structuring element [1]. The closing of foreground regions is equivalent to the opening of background regions [3].

# 4. METHODS OF CRACK DETECTION

# 4.1 Top-Hat Transform

- 4.1.1 Black Top-Hat
- 4.1.2 White Top-Hat
- 4.1.3 Multiscale Top-Hat

# 4.2 Alternative Methods

#### 5. RESULTS

## 6. CONCLUSIONS

## 7. ACKNOWLEDGMENTS

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