

Music Arrangement via Quantum Annealing

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30 January 2025

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Theory

Music arrangement

Quantum annealing

Methods

Results

Conclusions

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Music Arrangement via Quantum Annealing

└ Overview

- What is music arrangement? What is quantum annealing?
- Methods used to solve the music arrangement problem
- Preliminary results from application of the method
- Concluding thoughts about this process

Overview

Theory

Music arrangement

Quantum annealing

Methods

Results

Conclusions

Theory

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Music Arrangement via Quantum Annealing
└ Theory

Theory

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**



Beethoven's String Quartet No. 10

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Music Arrangement via Quantum Annealing

- └ Theory
 - └ Music arrangement
 - └ Music arrangement

- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet to solo)

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**



Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

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Music Arrangement via Quantum Annealing

- └ Theory
 - └ Quantum annealing
 - └ Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

Ising model

$$H_p(\sigma^z) = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

Initial state

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

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└ Theory

└ Quantum annealing

└ Quantum annealing

- How is this used to solve problems?
- Ising model, create a lattice of variables with two discrete values (spin up/down)
- Problem Hamiltonian, qubits σ^z , coupling strengths J_{ij} and field strengths h_i
- Initial state is a superposition of all possible states
- If problem solution is encoded within the ground state, system will give solution after evolution

Ising model

$$H_p(\sigma^z) = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

Initial state

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

Quantum annealing

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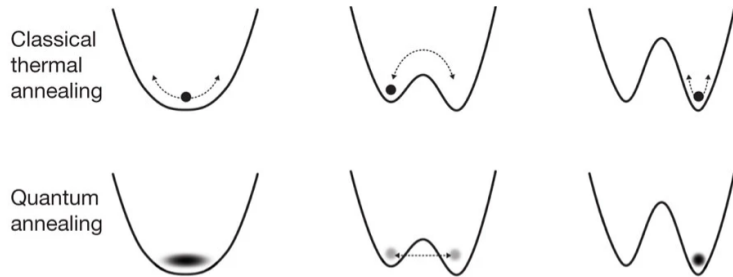
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└ Theory

└ Quantum annealing

└ Quantum annealing

Quantum annealing



MW Johnson *et al. Nature* **473**, 194–198 (2011) doi:10.1038/nature10012

- What does this look like?
- Evolution of superposition to a particular state
- More efficiently escape from local minima via quantum tunneling
- Can solve harder problems with a more turbulent energy landscape

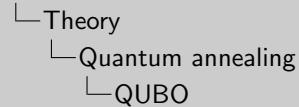
Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

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- How to encode a problem into a Hamiltonian?
- QUBO is a function to be minimised
- Set of binary variables x , matrix Q of real weights that describes interactions between variables

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

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└ Theory

└ Quantum annealing

How to combine them?

How to combine them?

- How to apply quantum annealing to the problem of music arrangement?

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Methods

Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

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Music Arrangement via Quantum Annealing

└─Methods

└─Problem formulation

- Formulating arrangement as a problem to be solved via annealing
- Four-step process
- Split parts into musical phrases
- Arrange phrases into a graph (nodes and edges)
- Solve corresponding graph problem using quantum computing
- Construct final arrangement from the solution returned

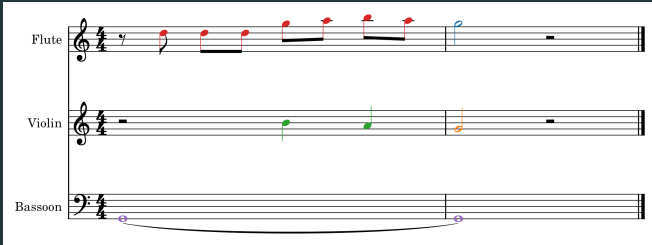
Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
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4. Construct arrangement from solution

1. Split parts

Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$



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Methods

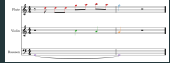
1. Split parts

- First stage to separate each part of original score into phrases
- Phrases — smallest unit of music that preserves melody and structure
- Boundaries between phrases found using LBDM
- Measures the degree of change of a certain parameter between notes (explain equation)
- Check pitch and IOI
- Strengths above a threshold value are considered phrases

1. Split parts

Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

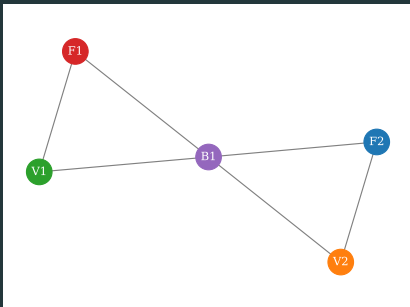


2. Create graph

Flute

Violin

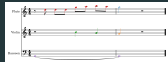
Bassoon



└ Methods

└ 2. Create graph

- Construct problem graph
- Each phrase becomes a node, edges between nodes if phrases overlap

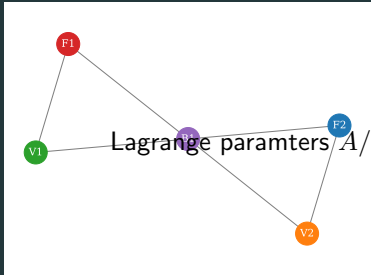


3. Solve graph

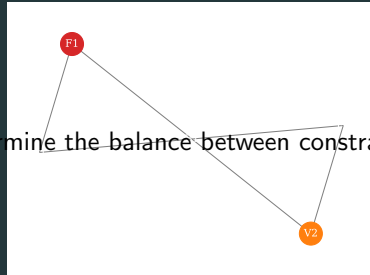
Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge.

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$



Problem graph



Solution graph

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Methods

3. Solve graph

- Solve problem graph using a graph theory problem called MIS
- Special case for reducing to a single monophonic part (one phrase played at a time)
- Find largest subset of nodes such that no nodes within the subset are connected by an edge
- Enforces that only one simultaneous phrase can be played at once
- Constraint term enforces no edges
- Objective term is quantity to be minimised

3. Solve graph

Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge.

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$



4. Construct arrangement

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a melody of eighth notes: G4, A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two eighth notes: G4 and A4, and then a whole rest. The Bassoon part (bass clef) has a whole rest, followed by a half note: G3, and then a whole rest. A curved line connects the two eighth notes in the Bassoon part.



The final constructed arrangement, showing the Flute and Violin parts combined. The Flute part (treble clef) has a melody of eighth notes: G4, A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two eighth notes: G4 and A4, and then a whole rest.

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Methods

4. Construct arrangement

- Take solution graph and combine selected nodes to create final arrangement

4. Construct arrangement

A musical score for three instruments: Piano, Violin, and Bassoon, in 4/4 time. The Piano part (treble clef) has a melody of eighth notes: G4, A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two eighth notes: G4 and A4, and then a whole rest. The Bassoon part (bass clef) has a whole rest, followed by a half note: G3, and then a whole rest.

A musical score for two instruments: Flute and Violin, in 4/4 time. The Flute part (treble clef) has a melody of eighth notes: G4, A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two eighth notes: G4 and A4, and then a whole rest.

Results

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Music Arrangement via Quantum Annealing
└ Results

Results

The image shows a musical score for String Quartet No. 10 by Ludwig van Beethoven. The score is for four instruments: Violin I, Violin II, Viola, and Violoncello. The tempo is marked 'Poco Adagio'. The key signature has two flats (B-flat and E-flat), and the time signature is 4/4. The score includes dynamic markings such as 'cresc.' (crescendo) and 'espress.' (espressivo). The first system shows the initial measures with 'sotto voce' (softly) markings. The second system starts at measure 6 and includes a 'cresc.' marking. The third system starts at measure 10 and includes 'espress.' and 'p' (piano) markings.

String Quartet No. 10 by Ludwig van Beethoven

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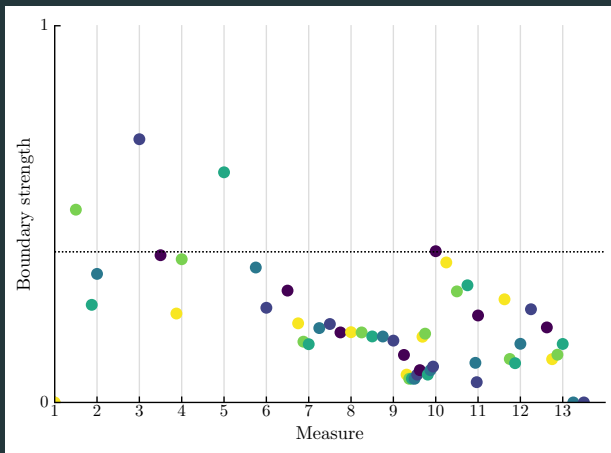
└ Results

└ Excerpt

- Excerpt of String Quartet No. 10 in E-flat major, Op. 74, by Ludwig van Beethoven
- Chosen due to its relatively simple structure and smaller instrumentation, keeping the problem graph small

This is a small thumbnail version of the musical score shown in the main image, featuring the same four staves and musical notation.

Phrase detection



Boundary strengths for the Violin I part

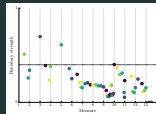
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└ Results

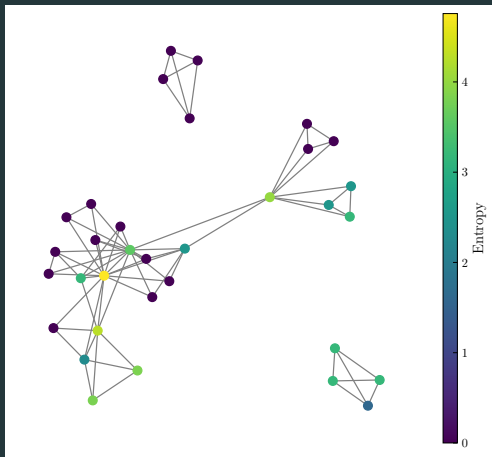
└ Phrase detection

Phrase detection



- Example of the LBDM finding suitable boundaries for phrases
- Threshold value of 0.4 chosen, finds five phrases

Problem graph



Problem graph with 33 nodes and 70 edges

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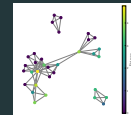
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└ Results

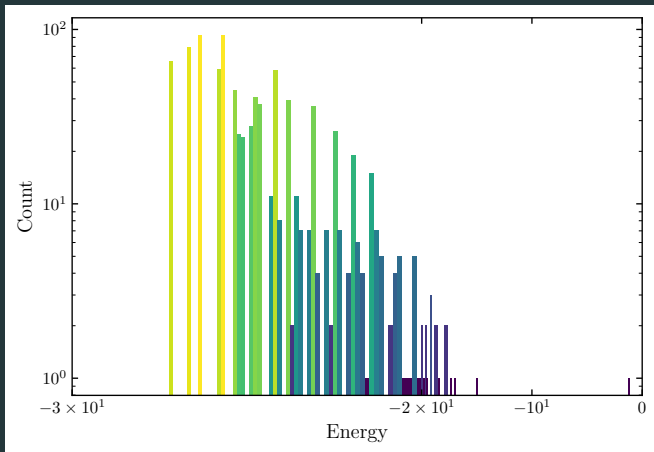
└ Problem graph

- 33 identified phrases (nodes) with 70 overlaps (edges)
- Nodes are weighted by the phrase entropy, how musically interesting the distribution of notes is

Problem graph



Problem graph with 33 nodes and 70 edges



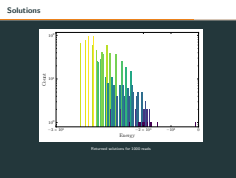
Returned solutions for 1000 reads

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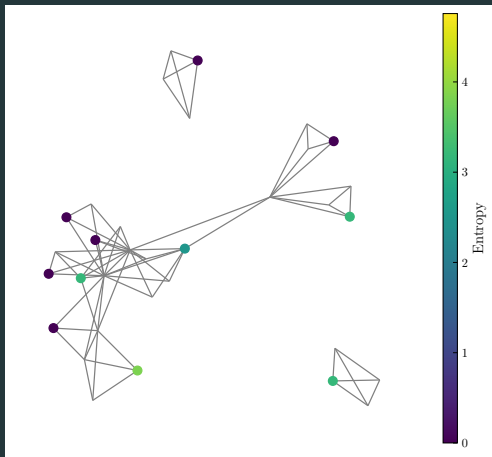
└ Results

└ Solutions



- Histogram of the returned solutions, only energies below zero shown
- Distribution of solutions due to the stochastic nature of annealing
- Not always guaranteed the ground state during evolution
- Lowest energy solution -26.8 with a degeneracy of 34

Example solution



Solution graph returning a subset of 11 nodes

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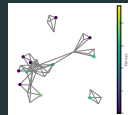
Music Arrangement via Quantum Annealing

└ Results

└ Example solution

- Selected nodes from one of the lowest energy solutions
- Note in the cliques only one node could be selected

Example solution



Solution graph returning a subset of 11 nodes

Final arrangement

Poco Adagio

Violin I
sotto voce

Violin II
sotto voce

Viola
sotto voce

Violoncello
sotto voce

6

cresc.

cresc.

cresc.

10

espress.

espress.

cresc.

p

f

f

p

Selected phrases

Poco Adagio

sotto voce

7

espress.

cresc.

p

12

f

Final arrangement

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└ Results

└ Final arrangement

- Selected phrases from solution graph highlighted
- Phrases concatenated to create the final arrangement

Final arrangement

Selected phrases

Final arrangement

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└─ Conclusions

Conclusions

Conclusions

Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms (deep learning, genetic)
 - No training data needed
 - Faster solve time

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└─ Conclusions

└─ Conclusions

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Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms (deep learning, genetic)
 - No training data needed
 - Faster solve time

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements

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└─ Conclusions

└─ Future work

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Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements

Thank you!

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└ **Conclusions**

Thank you!

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Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

Degree of change

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S)}{\max(S) - \min(S)}$$

Weighting

$$S = \frac{1}{3} (S_{\text{pitch}} + 2S_{\text{IOI}})$$

LBDM	
Boundary strength	$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$
Degree of change	$r_{i,i+1} = \frac{ x_i - x_{i+1} }{x_i + x_{i+1}}$
Normalisation	$S'_i = \frac{S_i - \min(S)}{\max(S) - \min(S)}$
Weighting	$S = \frac{1}{3} (S_{\text{pitch}} + 2S_{\text{IOI}})$

Phrase entropy

Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

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└ Phrase entropy

Phrase entropy

Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$