Music Arrangement via Quantum Annealing

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Overview

Theory

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

Beethoven's String Quartet No. 10

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another (AQC)
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

Quantum annealing

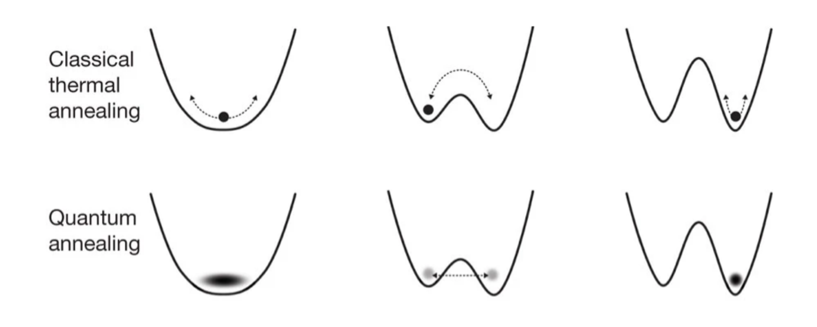
Ising model

$$H_p(\sigma^z) = \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$

Initial state

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

Quantum annealing



MW Johnson *et al.* *Nature* **473**, 194–198 (2011) doi:10.1038/nature10012

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j} Q_{i,j} x_i x_j + \sum_i Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

How to combine them?

Methods

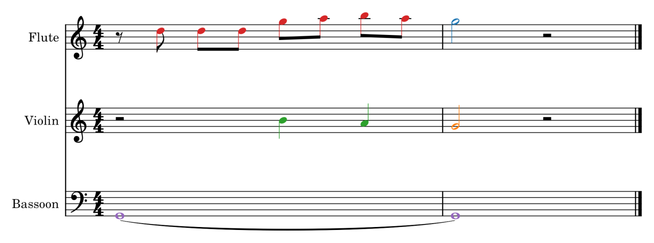
Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

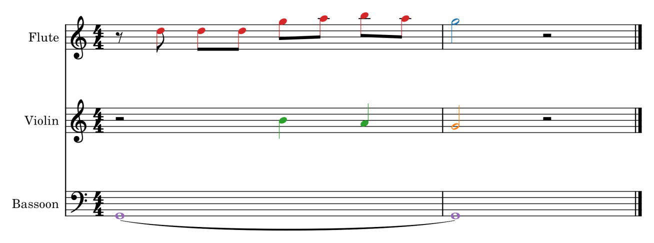
1. Split parts

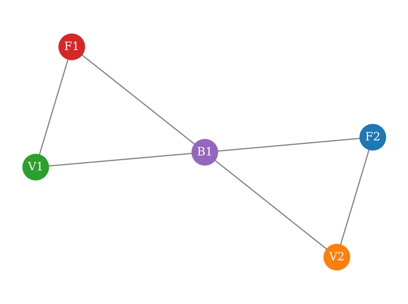
Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$



2. Create graph

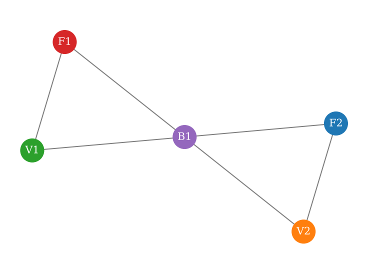




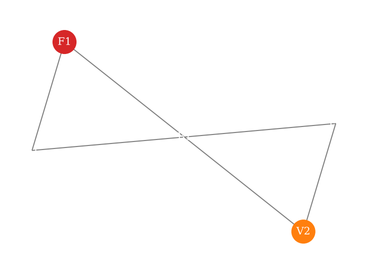
Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge.

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$

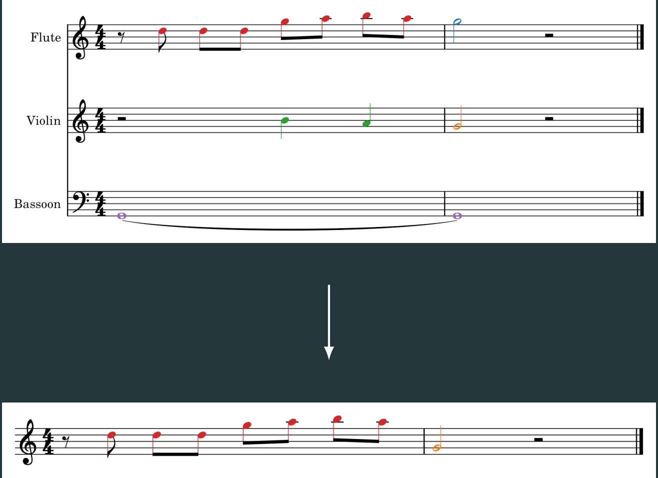


Problem graph



Solution graph

4. Construct arrangement



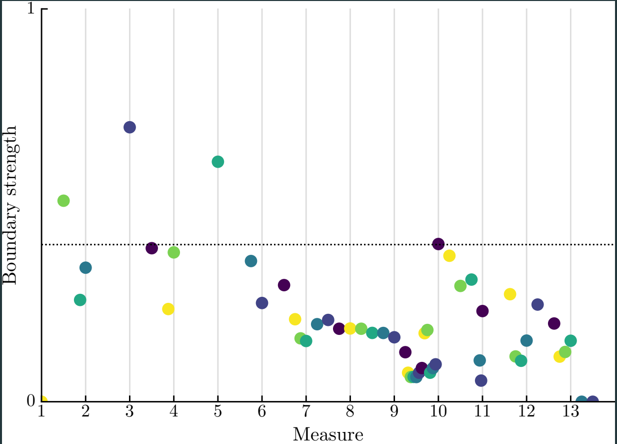
Results

Excerpt

String Quartet No. 10 by Ludwig van Beethoven

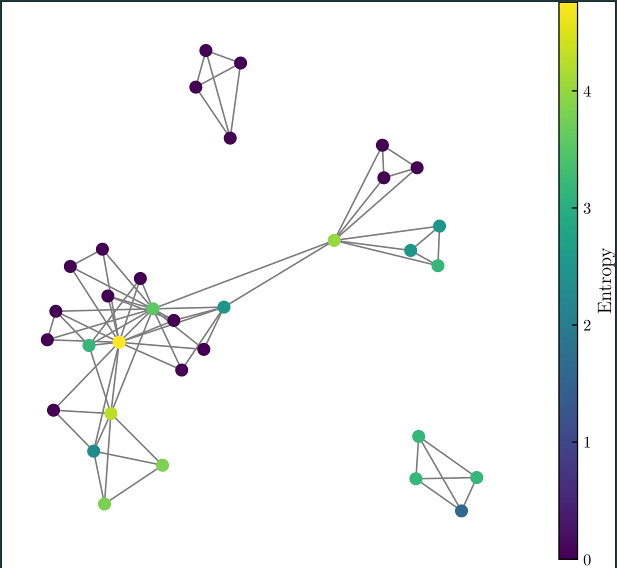
Phrase detection

Local boundary detection model (LBDM)

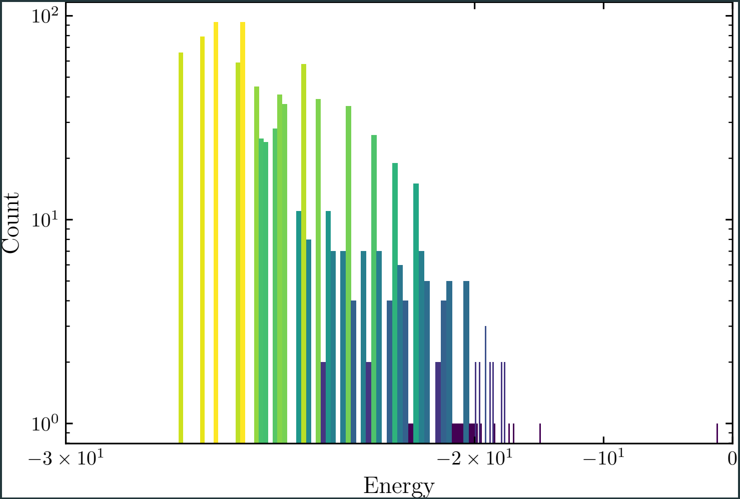


Boundary strengths for the Violin I part

Problem graph



Solutions



Solutions returned by the QPU

Example solution

Blocks

$$\oiint_A E \cdot dA = \frac{Q}{\varepsilon_0}$$

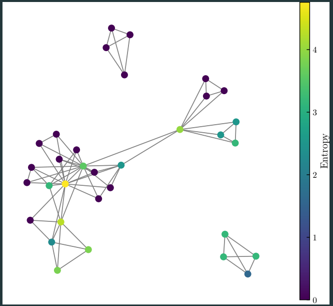
The *net electric flux* through any closed surface is proportional to the **enclosed charge**.

Alert
This is an alert.

Example
This is an example.

Apperance sync

- Volume rate of flow equal to divergence
- Summed over entire volume
- Equal to net flow across the boundary



Source: Wikimedia Commons

$$\iiint_V \nabla \cdot \mathbf{F} \, dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$$

Conclusions

Equation gather

$$
\begin{gather}
\nabla \cdot \mathsf{E} = \frac{\rho}{\varepsilon_0} \\
\nabla \cdot \mathsf{B} = 0 \\
\nabla \times \mathsf{E} = -\frac{\partial \mathsf{B}}{\partial t} \\
\nabla \times \mathsf{B} = \frac{1}{c^2}\frac{\partial \mathsf{E}}{\partial t} + \mu_0 I
\end{gather}
$$