

# **Quantum Annealing for Music Arrangement**

---

Lucas Kirby

4 March 2025

Department of Physics, Durham University

# Overview

---

Theory

- Adiabatic quantum computing

- Quantum annealing

Motivations

- Music arrangement

Method

Results

Conclusions

# Theory

---

# Adiabatic quantum computing

## Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.<sup>1</sup>

---

<sup>1</sup>Born and Fock, 'Beweis des Adiabatensatzes'.

# Adiabatic quantum computing

## Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.<sup>1</sup>

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

---

<sup>1</sup>Born and Fock, 'Beweis des Adiabatensatzes'.

# Adiabatic quantum computing

## Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.<sup>1</sup>

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed

---

<sup>1</sup>Born and Fock, 'Beweis des Adiabatensatzes'.

# Adiabatic quantum computing

## Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.<sup>1</sup>

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state

---

<sup>1</sup>Born and Fock, 'Beweis des Adiabatensatzes'.

# Adiabatic quantum computing

## Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.<sup>1</sup>

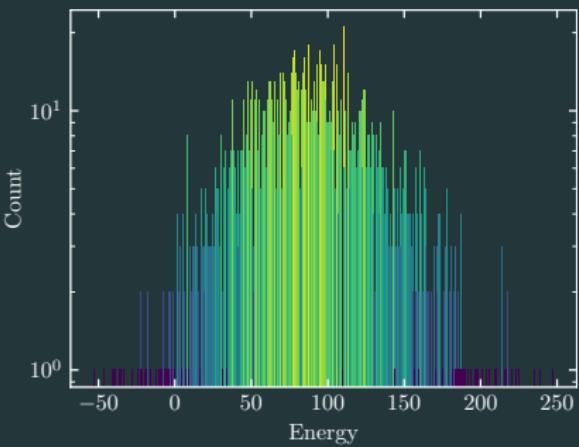
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

---

<sup>1</sup>Born and Fock, 'Beweis des Adiabatensatzes'.

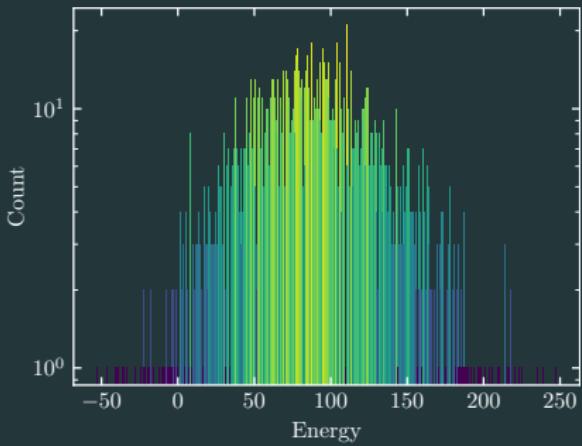
# Quantum annealing



Distribution of 2000 solution energies

# Quantum annealing

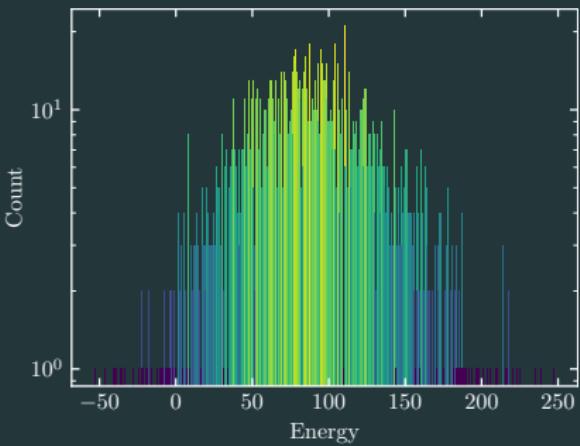
- Relaxes the adiabaticity



Distribution of 2000 solution energies

# Quantum annealing

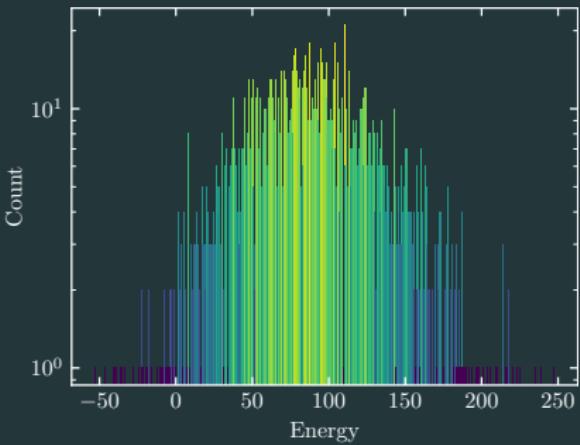
- Relaxes the adiabaticity
- Rate of change determined heuristically



Distribution of 2000 solution energies

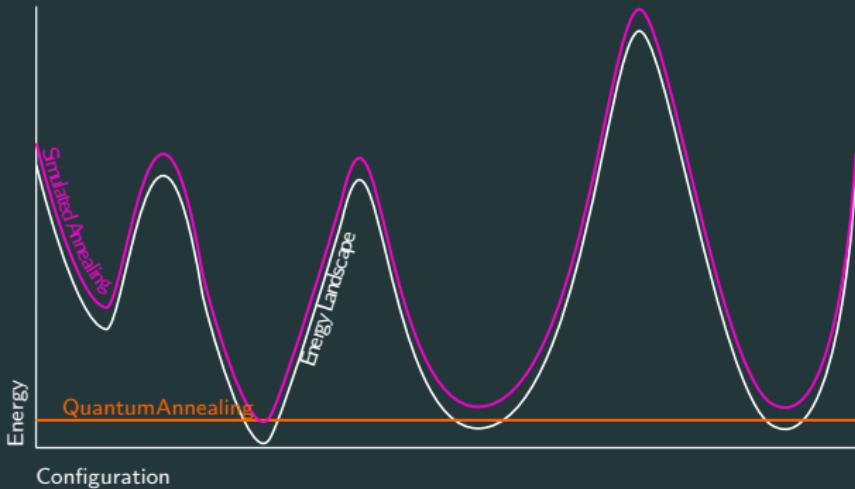
# Quantum annealing

- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic



Distribution of 2000 solution energies

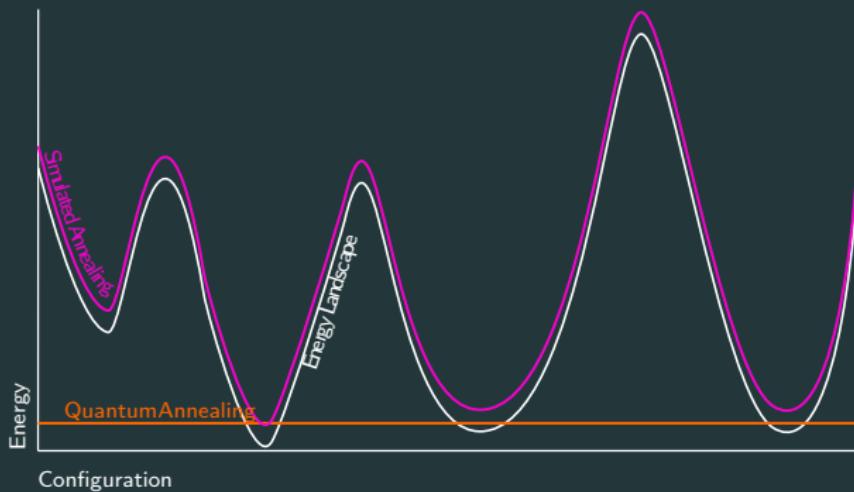
# Advantages



By Brianlechthaler - Own work, CC BY-SA 4.0,  
<https://commons.wikimedia.org/w/index.php?curid=112382195>

# Advantages

- Find the ground state of complicated Hamiltonians

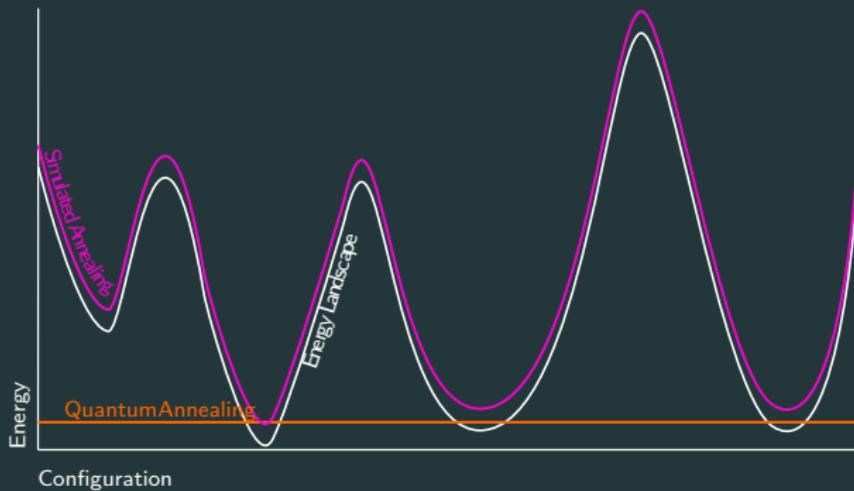


By Brianlechthaler - Own work, CC BY-SA 4.0,

<https://commons.wikimedia.org/w/index.php?curid=112382195>

# Advantages

- Find the ground state of complicated Hamiltonians
- Quantum tunneling avoids local minima



By Brianlechthaler - Own work, CC BY-SA 4.0,  
<https://commons.wikimedia.org/w/index.php?curid=112382195>

# Ising model

---

Lattice of variables with two discrete values

# Ising model

---

Lattice of variables with two discrete values

## Initial Hamiltonian

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

# Ising model

Lattice of variables with two discrete values

## Initial Hamiltonian

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

## Problem Hamiltonian

$$H_p = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

# QUBO

---

## Quadratic Unconstrained Binary Optimisation

Vector  $x$  of qubits, matrix  $Q$  of weights

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

## Quadratic Unconstrained Binary Optimisation

Vector  $x$  of qubits, matrix  $Q$  of weights

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

## Quadratic Unconstrained Binary Optimisation

Vector  $x$  of qubits, matrix  $Q$  of weights

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

## Quadratic Unconstrained Binary Optimisation

Vector  $x$  of qubits, matrix  $Q$  of weights

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

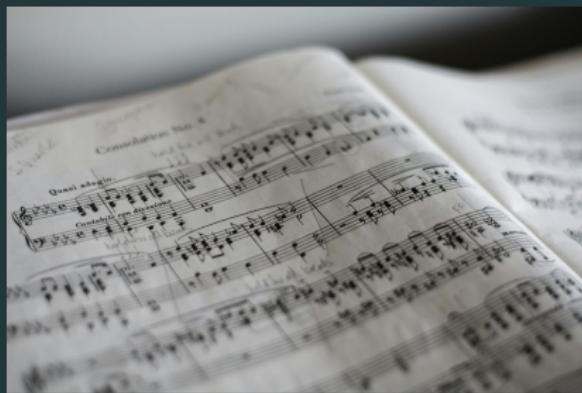
- Aim to minimise this function
- Difficult to solve analytically
- Mapped to  $H_p$  using simple change of variable
- Encodes problem solution into Hamiltonian's ground state

# Motivations

---

**What problems can we solve?**

# Music arrangement



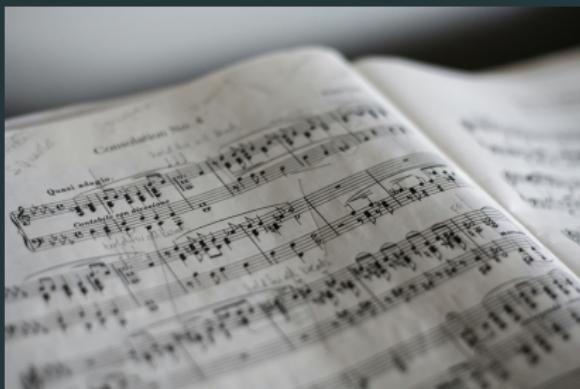
[www.freepik.com](http://www.freepik.com)

---

<sup>2</sup>Moses and Demaine, 'Computational Complexity of Arranging Music'.

# Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons



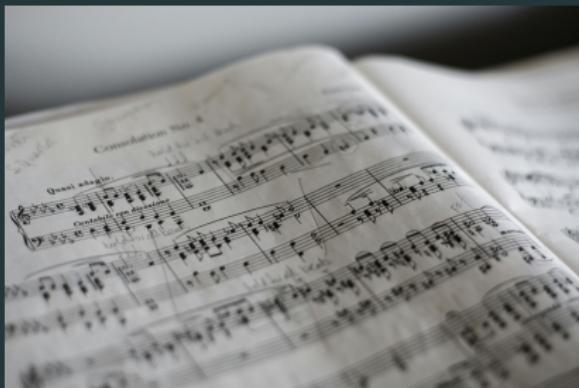
[www.freepik.com](http://www.freepik.com)

---

<sup>2</sup>Moses and Demaine, 'Computational Complexity of Arranging Music'.

# Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally difficult and time-consuming



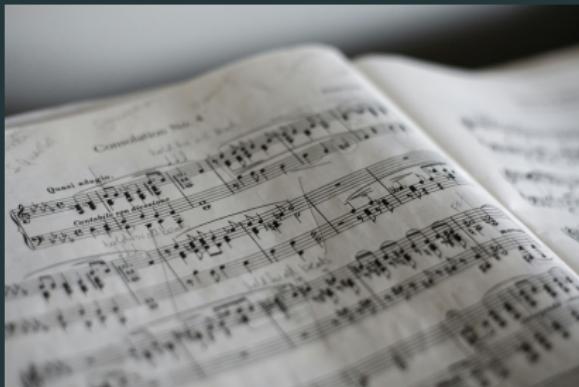
[www.freepik.com](http://www.freepik.com)

---

<sup>2</sup>Moses and Demaine, 'Computational Complexity of Arranging Music'.

# Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally difficult and time-consuming
- *Reduction* can be shown to be computationally complex<sup>2</sup>



[www.freepik.com](http://www.freepik.com)

---

<sup>2</sup>Moses and Demaine, 'Computational Complexity of Arranging Music'.

# Motivations

---

<sup>3</sup>Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'; Nakamura and Yoshii, 'Statistical piano reduction controlling performance difficulty'; Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

<sup>4</sup>Freedline, 'Algorhythms'; Arya et al., 'Music Composition Using Quantum Annealing'.

<sup>5</sup>Miranda, *Quantum Computer Music*.

# Motivations

---

- Already exist classical methods of automatic arrangement<sup>3</sup>

---

<sup>3</sup>Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'; Nakamura and Yoshii, 'Statistical piano reduction controlling performance difficulty'; Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

<sup>4</sup>Freedline, 'Algorhythms'; Arya et al., 'Music Composition Using Quantum Annealing'.

<sup>5</sup>Miranda, *Quantum Computer Music*.

# Motivations

---

- Already exist classical methods of automatic arrangement<sup>3</sup>
- Quantum annealing used to generate music<sup>4</sup>

---

<sup>3</sup>Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'; Nakamura and Yoshii, 'Statistical piano reduction controlling performance difficulty'; Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

<sup>4</sup>Freedline, 'Algorhythms'; Arya et al., 'Music Composition Using Quantum Annealing'.

<sup>5</sup>Miranda, *Quantum Computer Music*.

# Motivations

---

- Already exist classical methods of automatic arrangement<sup>3</sup>
- Quantum annealing used to generate music<sup>4</sup>
- Field of quantum computer music is very new<sup>5</sup>

---

<sup>3</sup>Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'; Nakamura and Yoshii, 'Statistical piano reduction controlling performance difficulty'; Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

<sup>4</sup>Freedline, 'Algorhythms'; Arya et al., 'Music Composition Using Quantum Annealing'.

<sup>5</sup>Miranda, *Quantum Computer Music*.

# Motivations

---

- Already exist classical methods of automatic arrangement<sup>3</sup>
- Quantum annealing used to generate music<sup>4</sup>
- Field of quantum computer music is very new<sup>5</sup>
- Novel adaption of this method to a new problem

---

<sup>3</sup>Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'; Nakamura and Yoshii, 'Statistical piano reduction controlling performance difficulty'; Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

<sup>4</sup>Freedline, 'Algorhythms'; Arya et al., 'Music Composition Using Quantum Annealing'.

<sup>5</sup>Miranda, *Quantum Computer Music*.

# Motivations

---

- Already exist classical methods of automatic arrangement<sup>3</sup>
- Quantum annealing used to generate music<sup>4</sup>
- Field of quantum computer music is very new<sup>5</sup>
- Novel adaption of this method to a new problem
- *This has never been done before!*

---

<sup>3</sup>Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'; Nakamura and Yoshii, 'Statistical piano reduction controlling performance difficulty'; Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

<sup>4</sup>Freedline, 'Algorhythms'; Arya et al., 'Music Composition Using Quantum Annealing'.

<sup>5</sup>Miranda, *Quantum Computer Music*.

# Method

---

# Aims

---



Joseph Haydn playing in a string quartet,  
painting from the StaatsMuseum,  
Vienna

# Aims

- Arrange a musical score for a smaller ensemble



Joseph Haydn playing in a string quartet,  
painting from the StaatsMuseum,  
Vienna

# Aims

- Arrange a musical score for a smaller ensemble
- All notes are taken from the original score



Joseph Haydn playing in a string quartet,  
painting from the StaatsMuseum,  
Vienna

# Aims

- Arrange a musical score for a smaller ensemble
- All notes are taken from the original score
- Each instrument can only play one note at a time



Joseph Haydn playing in a string quartet,  
painting from the StaatsMuseum,  
Vienna

# Method

---

# Method

---

1. Split score into musical phrases

# Method

---

1. Split score into musical phrases
2. Arrange phrases into a graph

# Method

---

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate optimisation problem

# Method

---

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate optimisation problem
4. Solve problem using QPU

# Method

---

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate optimisation problem
4. Solve problem using QPU
5. Construct arrangement from solution

## 1. Split score

---

<sup>6</sup>Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

## 1. Split score

- Musical phrases chosen as smallest unit of music
- Preserve melody and structure when rearranged

---

<sup>6</sup>Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

## 1. Split score

- Musical phrases chosen as smallest unit of music
- Preserve melody and structure when rearranged

### Local boundary detection model (LBDM)<sup>6</sup>

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

---

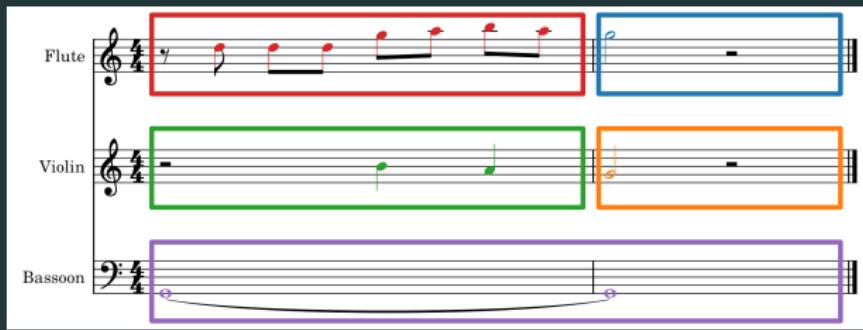
<sup>6</sup>Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

# 1. Split score

- Musical phrases chosen as smallest unit of music
- Preserve melody and structure when rearranged

## Local boundary detection model (LBDM)<sup>6</sup>

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$



<sup>6</sup>Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

## 2. Create graph

---

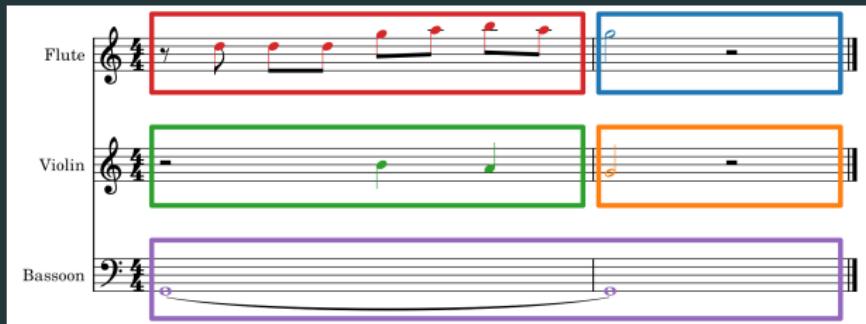
## 2. Create graph

---

- Vertices (nodes) connected by edges
- Models pairwise relations between objects

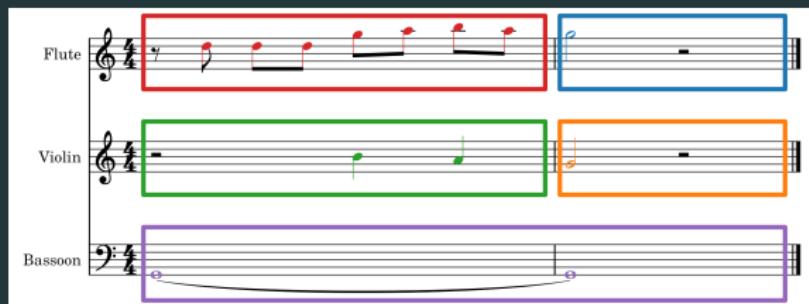
## 2. Create graph

- Vertices (nodes) connected by edges
- Models pairwise relations between objects

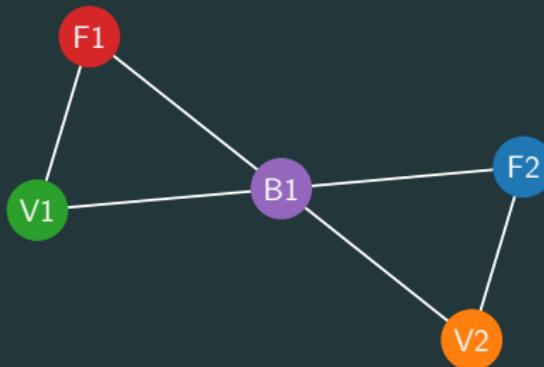
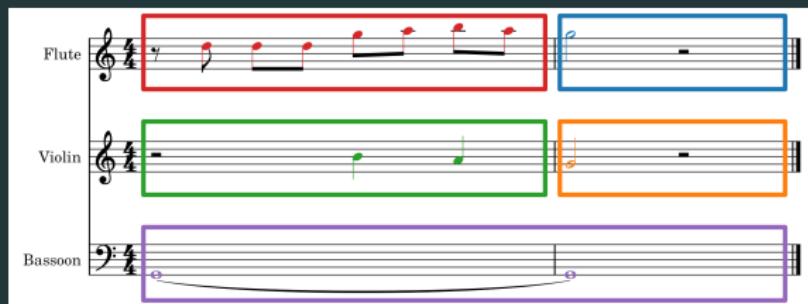


- Nodes — phrases
- Edges — overlap between phrases

## 2. Create graph



## 2. Create graph



### 3. Create optimisation problem

---

### 3. Create optimisation problem

---

#### Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

### 3. Create optimisation problem

---

#### Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

### 3. Create optimisation problem

#### Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} f(x) = & +A \sum_{v \in V} \left( 1 - \sum_{i=1}^n x_{v,i} \right)^2 + B \sum_{(u,v) \in E} \sum_{i=1}^n x_{u,i} x_{v,i} \\ & -C \sum_{v \in V} \sum_{i=1}^n W_v x_{v,i} \quad \quad \quad -D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^n \sum_{j=1}^n x_{u,i} x_{v,j} \end{aligned}$$

### 3. Create optimisation problem

#### Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} f(x) = & +A \sum_{v \in V} \left( 1 - \sum_{i=1}^n x_{v,i} \right)^2 + B \sum_{(u,v) \in E} \sum_{i=1}^n x_{u,i} x_{v,i} \\ & -C \sum_{v \in V} \sum_{i=1}^n W_v x_{v,i} \quad \quad \quad -D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^n \sum_{j=1}^n x_{u,i} x_{v,j} \end{aligned}$$

### 3. Create optimisation problem

#### Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} f(x) = & +A \sum_{v \in V} \left( 1 - \sum_{i=1}^n x_{v,i} \right)^2 + B \sum_{(u,v) \in E} \sum_{i=1}^n x_{u,i} x_{v,i} \\ & -C \sum_{v \in V} \sum_{i=1}^n W_v x_{v,i} \quad -D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^n \sum_{j=1}^n x_{u,i} x_{v,j} \end{aligned}$$

### 3. Create optimisation problem

#### Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} f(x) = & +A \sum_{v \in V} \left( 1 - \sum_{i=1}^n x_{v,i} \right)^2 + B \sum_{(u,v) \in E} \sum_{i=1}^n x_{u,i} x_{v,i} \\ & -C \sum_{v \in V} \sum_{i=1}^n W_v x_{v,i} \quad -D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^n \sum_{j=1}^n x_{u,i} x_{v,j} \end{aligned}$$

### 3. Create optimisation problem

#### Proper vertex colouring

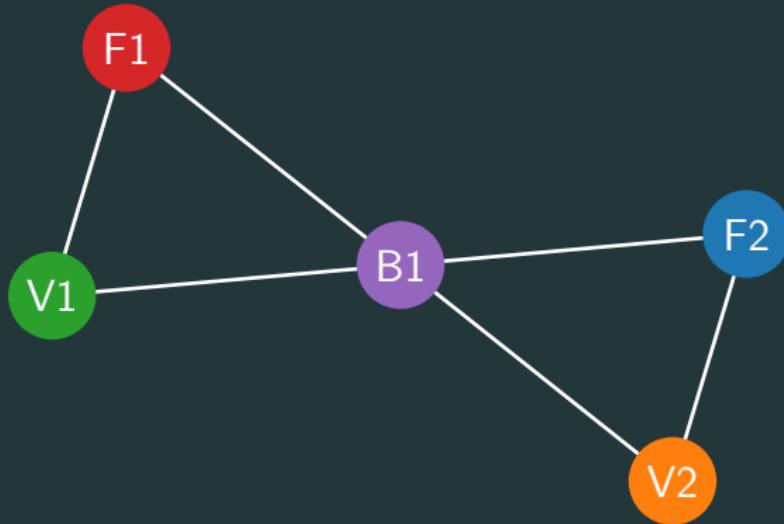
Colour each vertex such that no edge connects two vertices of the same colour

$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} f(x) = & +A \sum_{v \in V} \left( 1 - \sum_{i=1}^n x_{v,i} \right)^2 + B \sum_{(u,v) \in E} \sum_{i=1}^n x_{u,i} x_{v,i} \\ & - C \sum_{v \in V} \sum_{i=1}^n W_v x_{v,i} \quad - D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^n \sum_{j=1}^n x_{u,i} x_{v,j} \end{aligned}$$

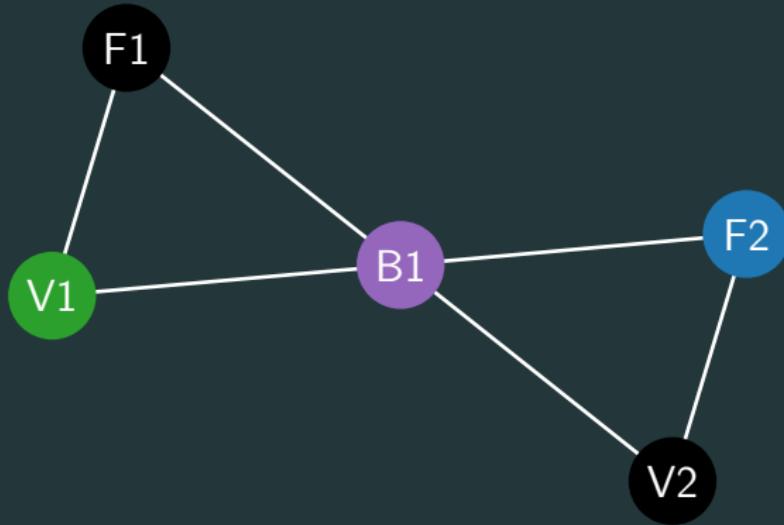
### 3. Create optimisation problem

---



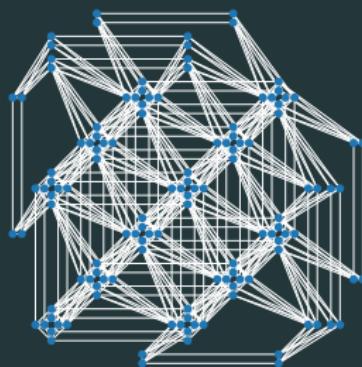
### 3. Create optimisation problem

---



## 4. Solve problem

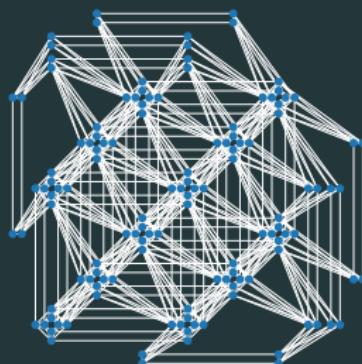
---



D-Wave Advantage QPU topology. Own work.

## 4. Solve problem

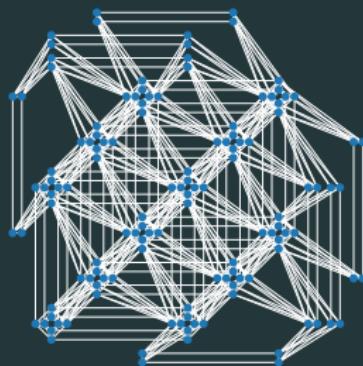
- Problem embedded onto D-Wave quantum hardware



D-Wave Advantage QPU topology. Own work.

## 4. Solve problem

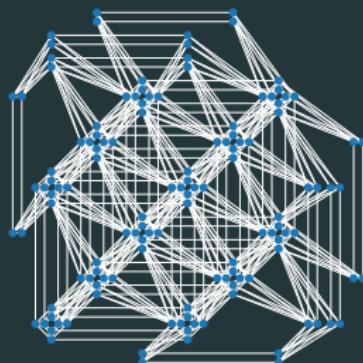
- Problem embedded onto D-Wave quantum hardware
- Quantum annealer optimises QUBO formulation



D-Wave Advantage QPU topology. Own work.

## 4. Solve problem

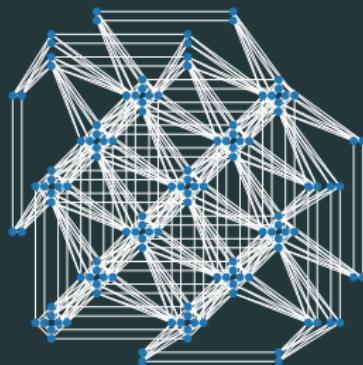
- Problem embedded onto D-Wave quantum hardware
- Quantum annealer optimises QUBO formulation
- Returns a sampleset of results



D-Wave Advantage QPU topology. Own work.

## 4. Solve problem

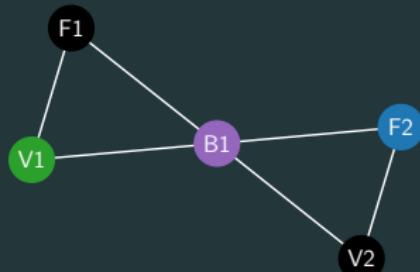
- Problem embedded onto D-Wave quantum hardware
- Quantum annealer optimises QUBO formulation
- Returns a sampleset of results
- Run many times to find lowest-energy solution



D-Wave Advantage QPU topology. Own work.

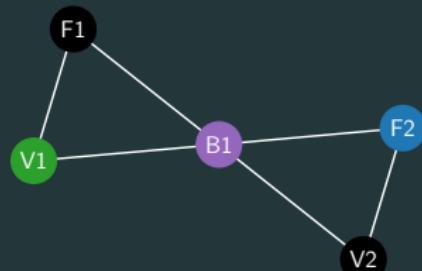
## 5. Construct arrangement

A musical score for three instruments: Flute, Violin, and Bassoon. The score consists of two measures. In the first measure, the Flute plays a sixteenth-note pattern (red box), the Violin plays eighth notes (green box), and the Bassoon has a sustained note (purple box). In the second measure, the Flute rests (blue box), the Violin rests (orange box), and the Bassoon rests (black box).



## 5. Construct arrangement

Musical score showing three staves: Flute, Violin, and Bassoon. The Flute staff has two measures: the first measure contains notes highlighted by a red box, and the second measure contains notes highlighted by a blue box. The Violin staff has two measures: the first measure contains notes highlighted by a green box, and the second measure contains notes highlighted by an orange box. The Bassoon staff has one measure with notes highlighted by a purple box.



Revised musical score showing two staves: Flute and Bassoon. The Flute staff has two measures: the first measure contains notes highlighted by a red box, and the second measure contains notes highlighted by an orange box. The Bassoon staff has one measure with notes highlighted by a purple box.

# Results

---

# Score

Quartet No. I in B $\flat$  major  
Joseph Haydn

The musical score consists of three staves of string quartet music. The top staff is for Violin I, the middle for Violin II, and the bottom for Cello. The first page begins with a Presto tempo, featuring eighth-note patterns and sixteenth-note figures. The second page continues with similar rhythmic patterns, with the violins playing eighth-note pairs and the cello providing harmonic support. The third page concludes with a dynamic section, indicated by a crescendo symbol followed by a decrescendo symbol.

Quartet No. 1 in Bb major by  
Joseph Haydn

# Score

- Smaller ensemble chosen for problem size

Quartet No. 1 in B $\flat$  major  
Joseph Haydn

The image shows three staves of musical notation for a string quartet. The top staff is for Violin I, the middle for Violin II, and the bottom for Cello. The music is in common time, with a key signature of one sharp (F#). The first section, labeled "Presto", consists of six measures of eighth-note patterns. The second section, labeled "Adagio", begins at measure 7 with eighth-note patterns. The third section, labeled "Allegro", begins at measure 13 with eighth-note patterns.

Quartet No. 1 in Bb major by  
Joseph Haydn

# Score

- Smaller ensemble chosen for problem size
- Well-defined musical structure

Quartet No. 1 in B $\flat$  major

Joseph Haydn

The image shows three staves of musical notation for a string quartet. The top staff is for Violin I, the middle for Violin II, and the bottom for Cello. The notation consists of vertical stems with horizontal dashes indicating pitch and rhythm. The first staff begins with a treble clef, the second with an alto clef, and the third with a bass clef. The key signature is B-flat major, indicated by two flats in the circle of fifths. The tempo is marked 'Presto' at the beginning of the first staff. The score is divided into measures by vertical bar lines.

Quartet No. 1 in Bb major by  
Joseph Haydn

# Score

- Smaller ensemble chosen for problem size
- Well-defined musical structure
- Reduction to three instruments

Quartet No. I in B $\flat$  major

Joseph Haydn

Presto

Violin I  
Violin II  
Viola  
Cello

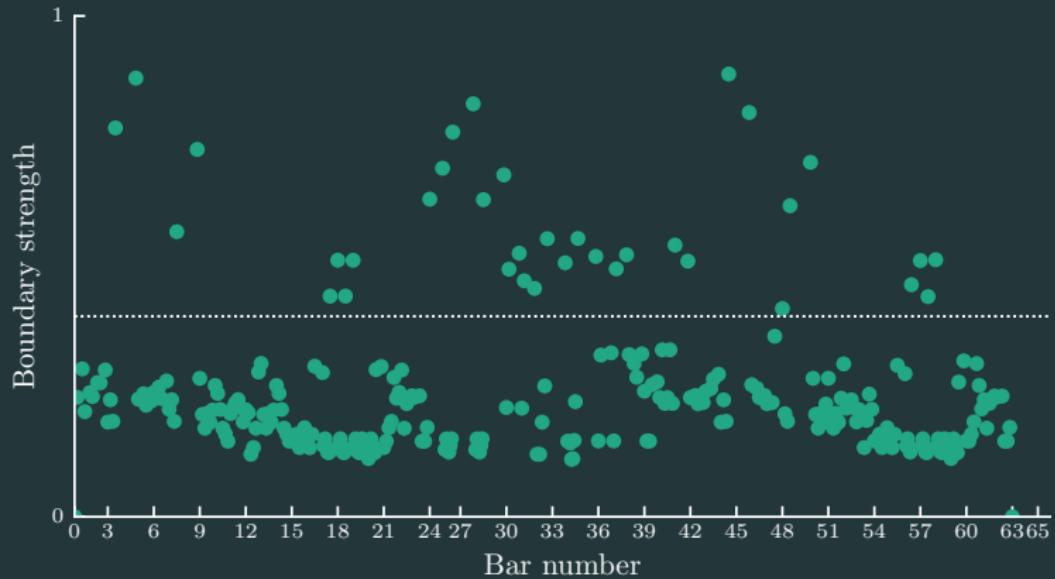
Vln. I  
Vln. II  
Vla.  
Vc.

Vln. I  
Vln. II  
Vla.  
Vc.

Vln. I  
Vln. II  
Vla.  
Vc.

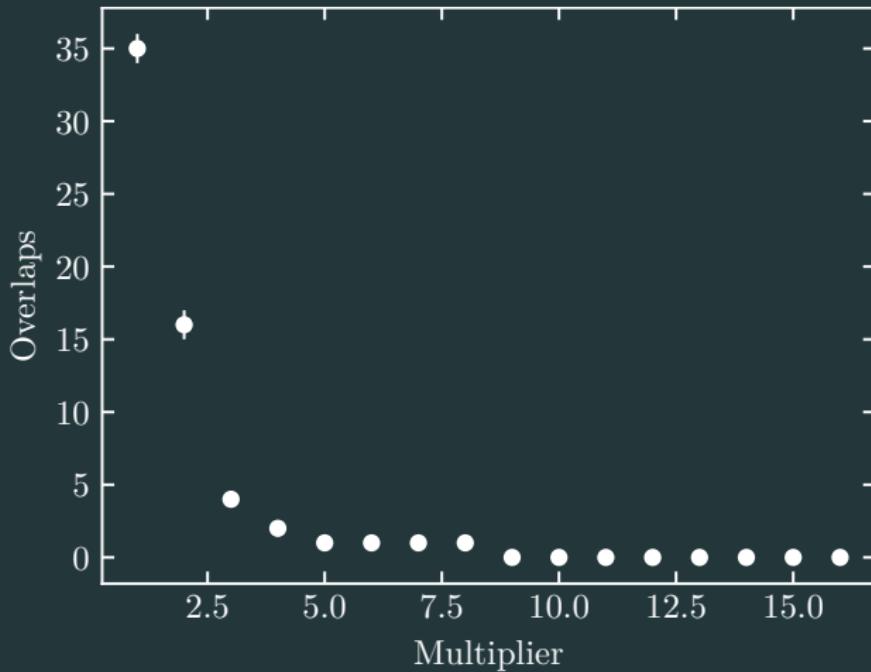
Quartet No. 1 in Bb major by Joseph Haydn

# Phrase detection



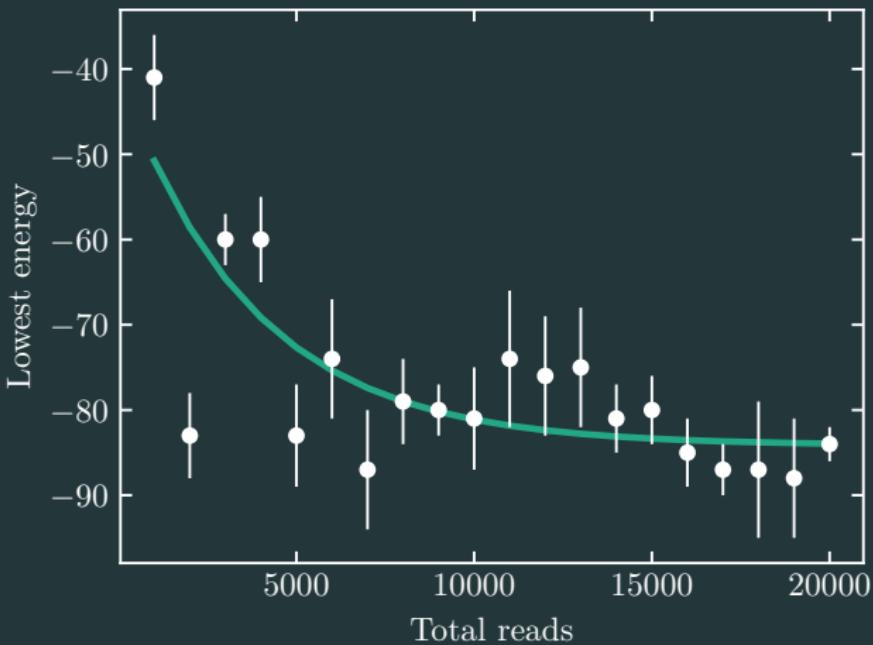
Boundary strengths for the Violin I part

## QUBO parameter variation



Variation of the edge constraint Lagrange parameter  $B$

# Optimisation



Variation of the number of QPU reads, with the lowest-energy solution found

## Conclusions

---

# Conclusions

---

## Conclusions

---

- Successful novel application of quantum annealing

## Conclusions

---

- Successful novel application of quantum annealing
- QPU returns low-energy samples

## Conclusions

---

- Successful novel application of quantum annealing
- QPU returns low-energy samples
- Necessary constraints for a valid arrangement fulfilled

## Conclusions

---

- Successful novel application of quantum annealing
- QPU returns low-energy samples
- Necessary constraints for a valid arrangement fulfilled
- Still very new technology, does not show quantum advantage (yet)

## Future work

---

<sup>7</sup>Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

## Future work

- Variation in problem size

---

<sup>7</sup>Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

## Future work

- Variation in problem size
- Comparison to classical methods

---

<sup>7</sup>Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

## Future work

- Variation in problem size
- Comparison to classical methods
- Lagrange parameter tuning

---

<sup>7</sup>Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

## Future work

- Variation in problem size
- Comparison to classical methods
- Lagrange parameter tuning
- Qualitative judgement of computer arrangements<sup>7</sup>

---

<sup>7</sup>Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

**Thank you!**

# **Quantum Annealing for Music Arrangement**

---

Lucas Kirby

4 March 2025

Department of Physics, Durham University

## References i

-  Arya, Ashish et al. '**Music Composition Using Quantum Annealing**'. In: *arXiv* (Jan. 2022). DOI: 10.48550/arXiv.2201.10557. (Visited on 26/10/2024).
-  Born, M. and V. Fock. '**Beweis des Adiabatensatzes**'. In: *Zeitschrift für Physik* 51.3 (Mar. 1928), pp. 165–180. ISSN: 0044-3328. DOI: 10.1007/BF01343193. URL: <https://doi.org/10.1007/BF01343193> (visited on 01/03/2025).
-  Cambouropoulos, Emilios. '**The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing**'. In: *International Computer Music Association* (2011). ISSN: 2223-3881.

## References ii

-  Freedline, Alex. '**Algorhythms: Generating Music with D-Wave's Quantum Annealer**'. en. In: *MIT 6.s089—Intro to Quantum Computing* (Feb. 2021).
-  Huang, Jiun-Long, Shih-Chuan Chiu and Man-Kwan Shan. '**Towards an automatic music arrangement framework using score reduction**'. In: *ACM Trans. Multimedia Comput. Commun. Appl.* 8.1 (Feb. 2012), 8:1–8:23. ISSN: 1551-6857. DOI: 10.1145/2071396.2071404. (Visited on 05/12/2024).

## References iii

-  Li, You et al. '**Automatic Piano Reduction of Orchestral Music Based on Musical Entropy**'. In: *2019 53rd Annual Conference on Information Sciences and Systems (CISS)*. Mar. 2019, pp. 1–5. DOI: 10.1109/CISS.2019.8693036. URL: <https://ieeexplore.ieee.org/document/8693036> (visited on 27/12/2024).
-  Miranda, Eduardo Reck, ed. ***Quantum Computer Music: Foundations, Methods and Advanced Concepts***. en. Springer International Publishing, 2022. ISBN: 978-3-031-13908-6 978-3-031-13909-3. DOI: 10.1007/978-3-031-13909-3. (Visited on 28/12/2024).

## References iv

-  Moses, William S. and Erik D. Demaine. '**Computational Complexity of Arranging Music**'. In: *arXiv* (July 2016). arXiv:1607.04220. DOI: 10.48550/arXiv.1607.04220. (Visited on 09/11/2024).
-  Nakamura, Eita and Kazuyoshi Yoshii. '**Statistical piano reduction controlling performance difficulty**'. en. In: *APSIPA Transactions on Signal and Information Processing* 7 (Jan. 2018), e13. ISSN: 2048-7703. DOI: 10.1017/ATSIP.2018.18. (Visited on 17/12/2024).
-  Pearce, M. and Geraint A. Wiggins. '**Towards A Framework for the Evaluation of Machine Compositions**'. In: *Proceedings of the AISB'01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences*. 2001.

## Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

## Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

## Weighting

$$S = \frac{1}{3} \left( S'_{\text{pitch}} + 2S'_{\text{IOI}} \right)$$

---

<sup>8</sup>Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

## Phrase entropy

$x_i$  — parameter  $x$  of note  $i$

## Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

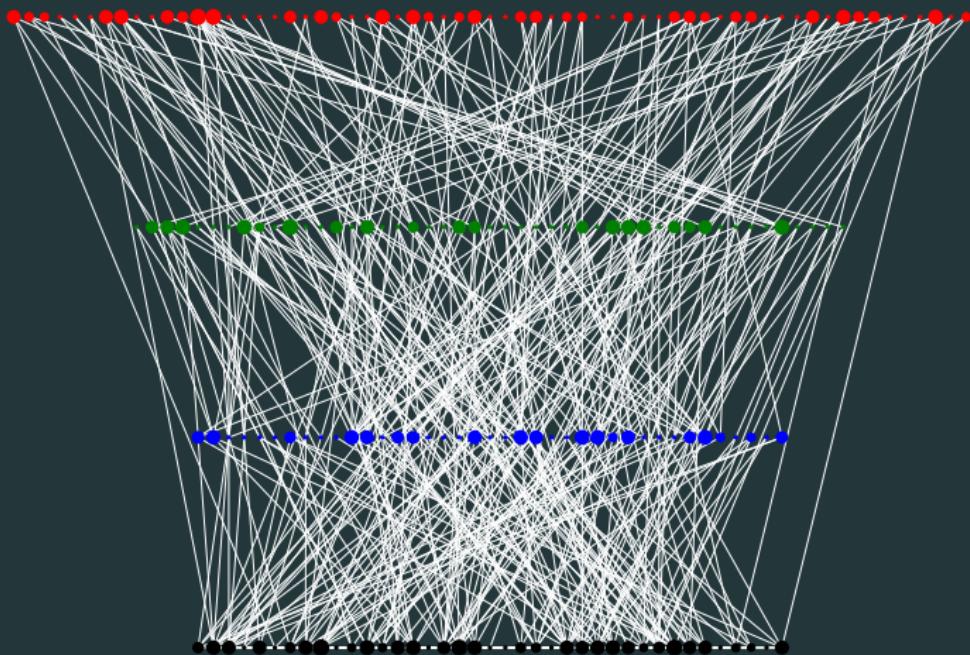
## Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

---

<sup>9</sup>Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

# Solution graph



# Solution score

A musical score for three woodwind instruments: Flute, Oboe, and Bassoon. The score consists of four staves, each with a treble clef and a key signature of one flat. The time signature is 4/4 throughout.

The score is divided into four systems:

- System 1 (Measures 1-4):** The Flute has eighth-note patterns starting with a grace note. The Oboe and Bassoon provide harmonic support with sustained notes and eighth-note chords.
- System 2 (Measures 5-8):** The Flute continues its eighth-note patterns. The Oboe and Bassoon play eighth-note chords.
- System 3 (Measures 9-12):** The Flute has eighth-note patterns. The Oboe and Bassoon play eighth-note chords.
- System 4 (Measures 13-16):** The Flute has eighth-note patterns. The Oboe and Bassoon play eighth-note chords.

Measure numbers 5, 9, and 13 are explicitly marked above the staves.