Music Arrangement via Quantum Annealing

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30 January 2025

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Overview

Theory

- Music arrangement
- Quantum annealing

Methods

Results

Conclusions

Theory

Music arrangement

Beethoven's String Quartet No. 10

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons

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- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming

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Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

Beethoven's String Quartet No. 10

Adiabatic quantum computing (AQC)

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$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

[Lucas, 2014]

Quantum annealing

Quantum annealing

Ising model

$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$

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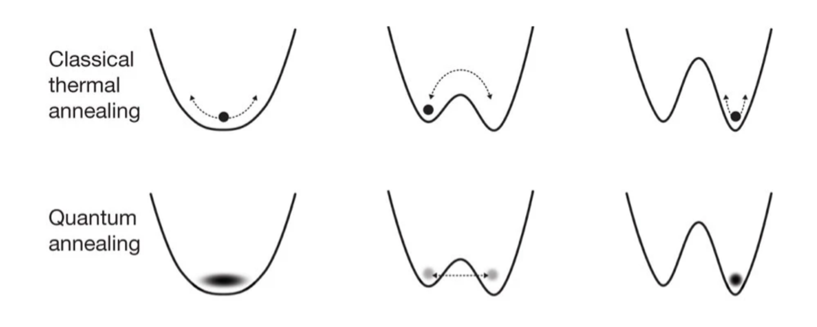
Initial state

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

[Lucas, 2014]

Quantum annealing



[Johnson et al., 2011]

QUBO

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Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

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$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Sent to the QPU for optimisation

How to combine them?

Methods

Problem formulation

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1. Split score into musical phrases

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2. Arrange phrases into a graph

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2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

1. Split score

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Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

[Cambouropoulos, 2011]

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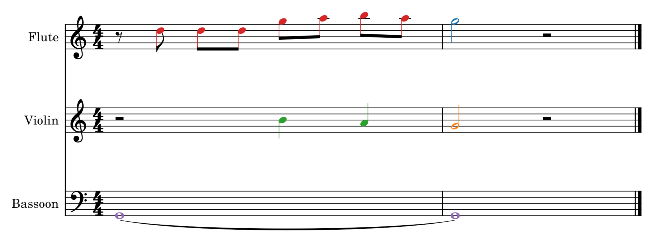
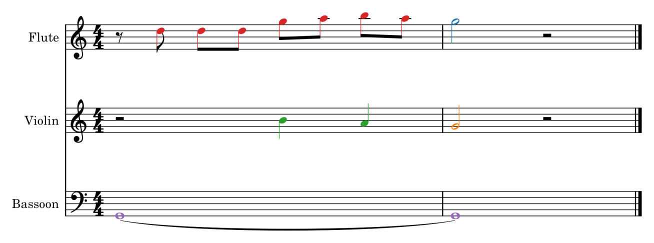


2. Create graph



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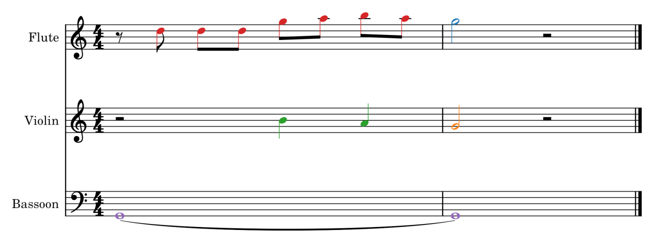

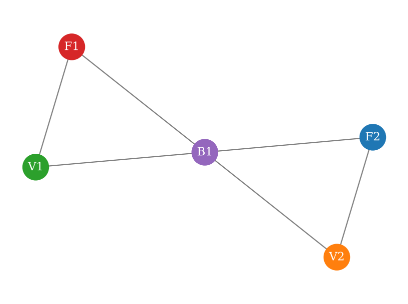

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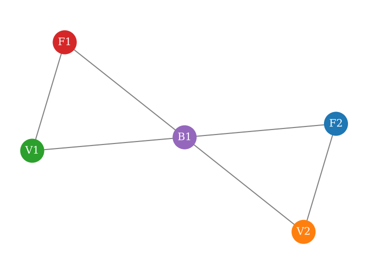
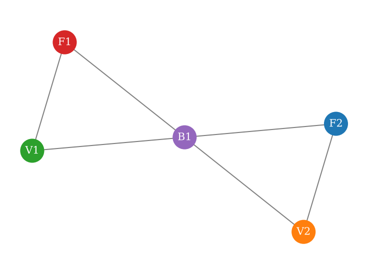


Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge



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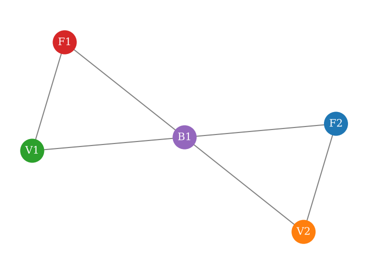
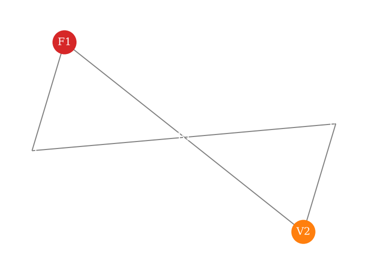
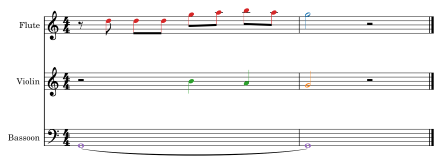
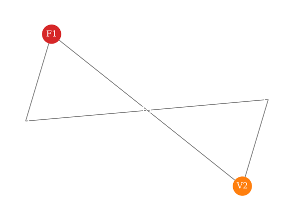




4. Construct arrangement





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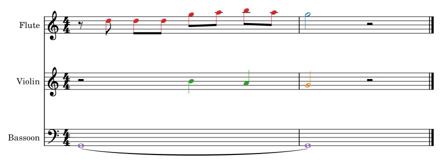
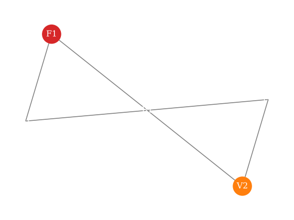
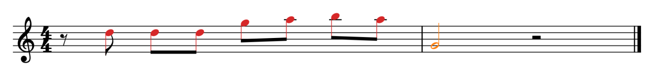




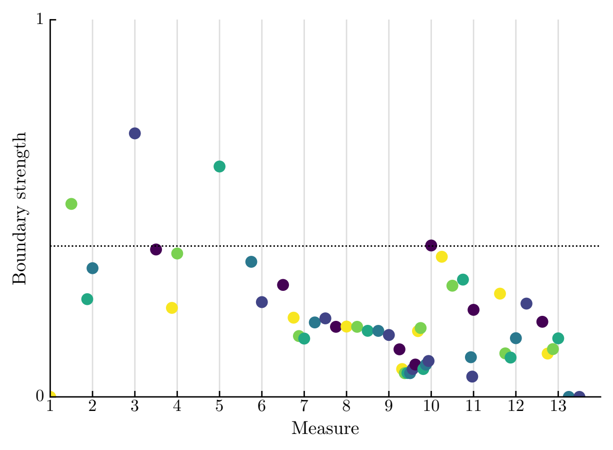


Results

Excerpt

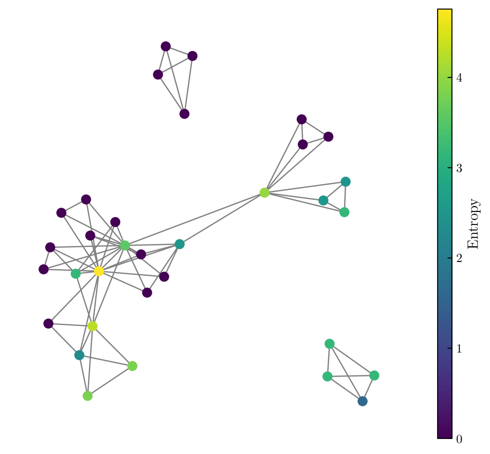
String Quartet No. 10 by Ludwig van Beethoven

Phrase detection



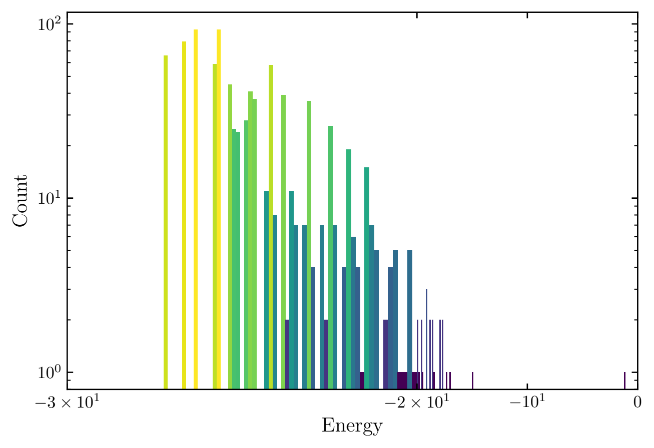
Boundary strengths for the Violin I part

Problem graph



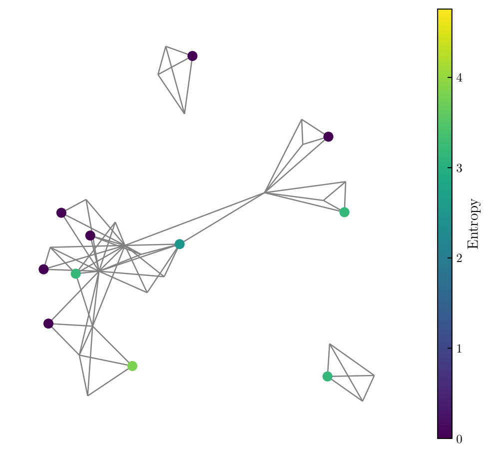
Problem graph with 33 nodes and 70 edges

Solutions



Returned solutions for 1000 reads

Example solution



Solution graph returning a subset of 11 nodes

Final arrangement

Selected phrases

Final arrangement

Conclusions

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- Successful in creating a valid single-part reduction

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- Advantage over classical algorithms [Huang et al., 2012]

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- Successful in creating a valid single-part reduction
- Advantage over classical algorithms [Huang et al., 2012]
- Removes skill barrier for music arrangement

Future work

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- Increased problem size

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- Parametric variation of LBDM

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- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements [Pearce and Wiggins, 2001]

Thank you!

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LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$
$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

[Cambouropoulos, 2011]

MIS

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

[Lucas, 2014]

$A/B >= 2\max(W)$ to weight the constraint term more heavily than any objective term

Phrase entropy

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

[Li et al., 2019]