Music Arrangement via Quantum Annealing

Lucas Kirby
30 January 2025
Department of Physics, Durham University

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Overview

Theory

Music arrangement

Quantum annealing

Methods

Results

Conclusions

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Conclusions

- What is music arrangement? What is quantum annealing?
- Methods used to solve the music arrangement problem
- Preliminary results from application of the method
- Concluding thoughts about this process

Theory

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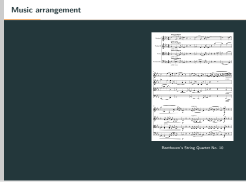
Music arrangement

Beethoven's String Quartet No. 10



- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet)

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons

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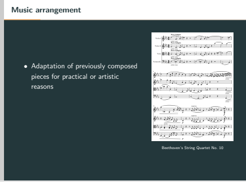

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- Traditionally complex and time-consuming

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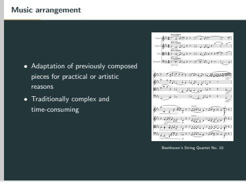

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Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

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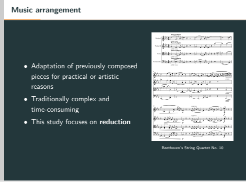

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- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

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[Lucas, 2014]

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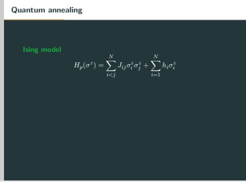
Quantum annealing

Quantum annealing

- How is this used to solve problems?
- Ising model, create a lattice of variables with two discrete values (spin up/down)
- Problem Hamiltonian, qubits σ^z, coupling strengths J_{ij} and field strengths h_i
- Initial state is a superposition of all possible states
- If problem solution is encoded within the ground state, system will give solution after evolution

Ising model

$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$



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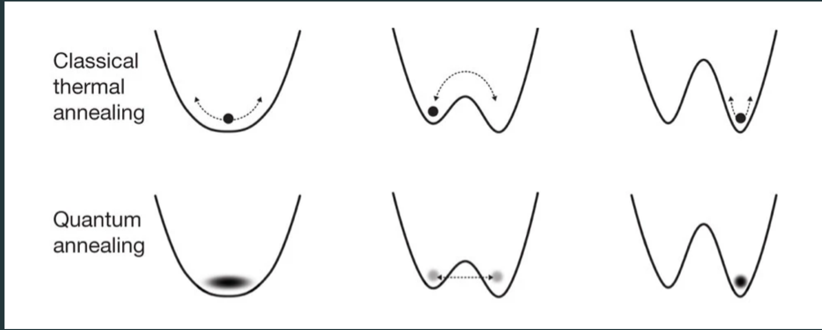
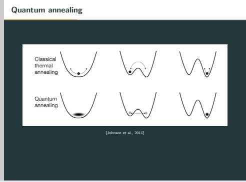


[Johnson et al., 2011]



- What does this look like?
- Evolution of superposition to a particular state
- More efficiently escape from local minima via quantum tunneling
- Can solve harder problems with a more turbulent energy landscape

QUBO

- How to encode a problem into a Hamiltonian?
- Similar form to the Ising model, but with binary variables (0 or 1)
- QUBO is a function to be minimised
- Set of binary variables x, matrix Q of real weights that describes interactions between variables
- Can read out variable values after evolution to give solution

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Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

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How to combine them?

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- How to apply quantum annealing to the problem of music arrangement?

Methods

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Problem formulation

Problem formulation

- Formulating arrangement as a problem to be solved via annealing, four-step process
- Split parts into musical phrases
- Arrange phrases into a graph (nodes and edges)
- Solve corresponding graph problem using quantum computing
- Construct final arrangement from the solution returned

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- First stage to separate each part of original score into phrases
- Phrases — smallest unit of music that preserves melody and structure
- Boundaries between phrases found using LBDM
- Measures the degree of change of a certain parameter between notes (explain equation)
- Check pitch and IOI
- Strengths above a threshold value are considered phrases

1. Split score

Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

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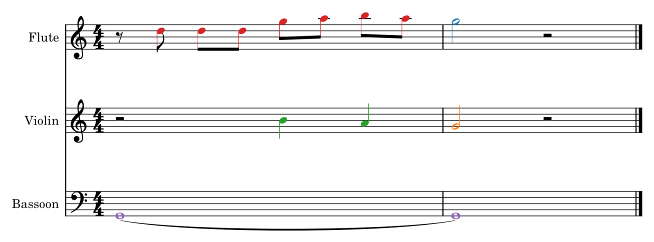

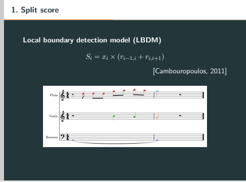

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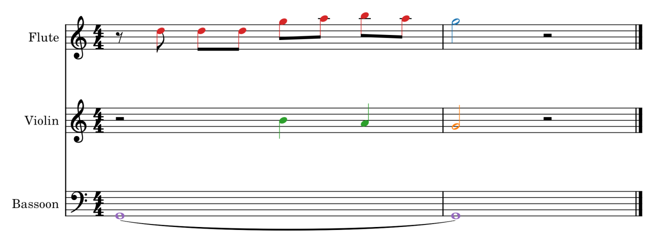
2. Create graph



- Construct problem graph
- Each phrase becomes a node, edges between nodes if phrases overlap

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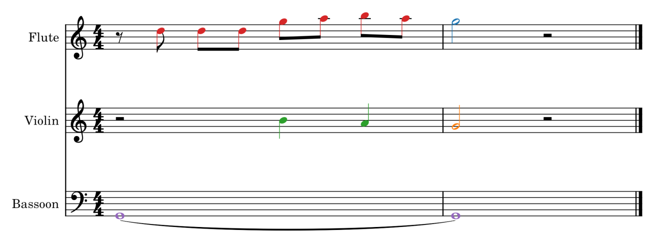

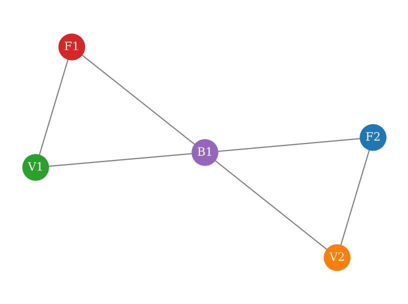

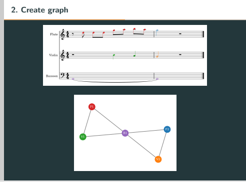

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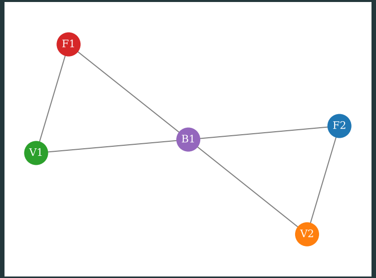
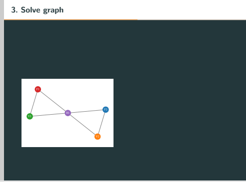


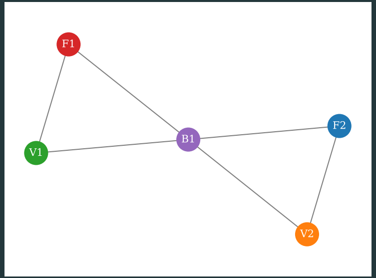


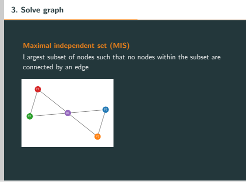
- Special case for reducing to a single monophonic part (one phrase played at a time)
- Solve problem graph using a graph theory problem called MIS
- Enforces that only one simultaneous phrase can be played at once
- Nodes can also be weighted to introduce preference for certain nodes e.g. musicality

3. Solve graph

Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge





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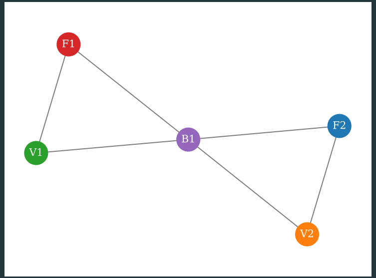

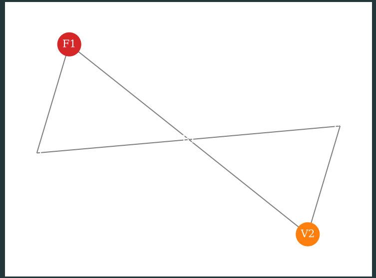

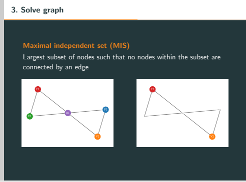

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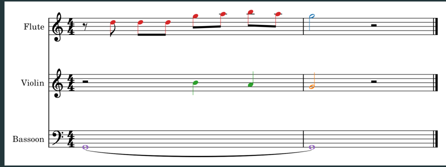
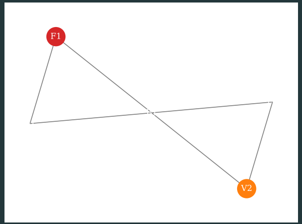
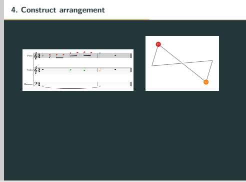
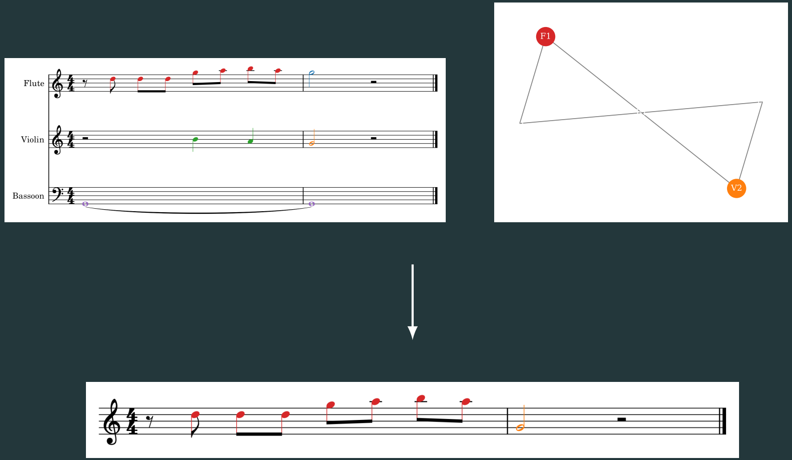


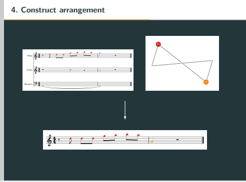




- Take solution graph and combine selected nodes to create final arrangement

4. Construct arrangement





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Results

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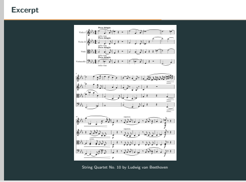
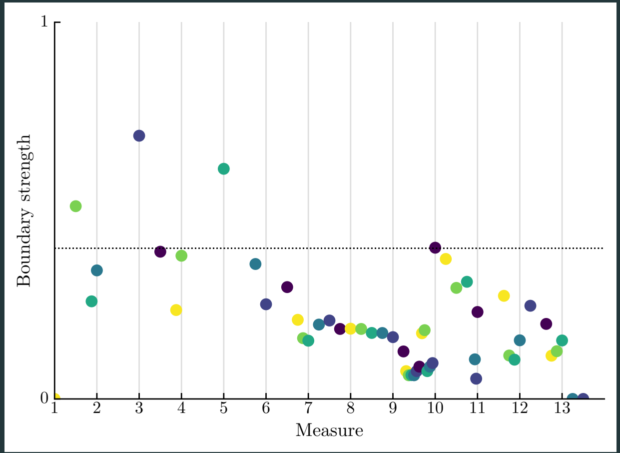
Excerpt

String Quartet No. 10 by Ludwig van Beethoven

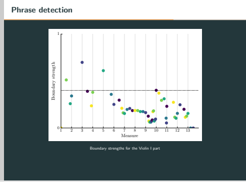


- Excerpt of String Quartet No. 10 in E-flat major, Op. 74, by Ludwig van Beethoven
- Chosen due to its relatively simple structure and smaller instrumentation, keeping the problem graph small

Phrase detection

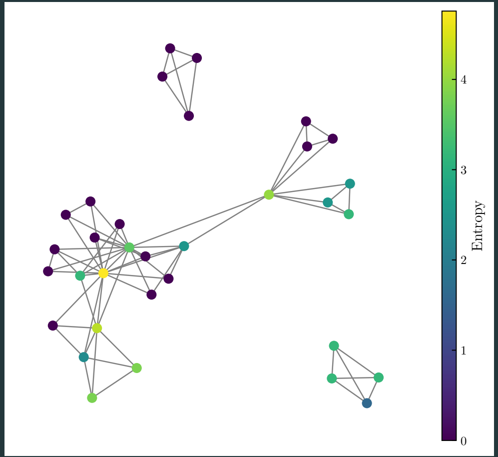


Boundary strengths for the Violin I part

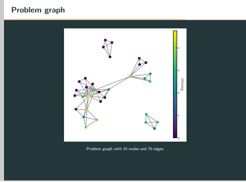


- Example of the LBDM finding suitable boundaries for phrases
- Threshold value of 0.4 chosen, finds five phrases

Problem graph

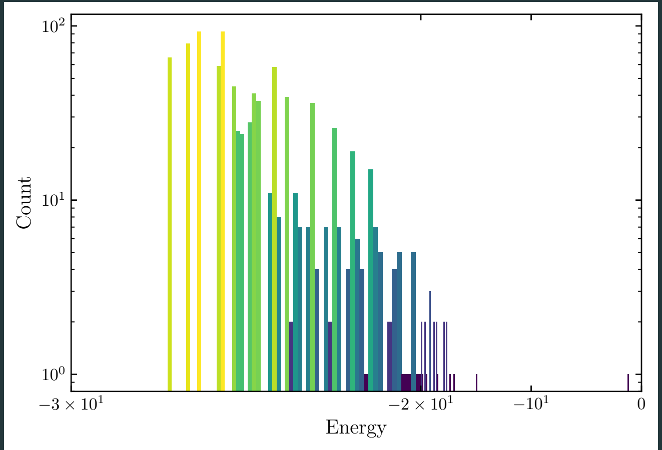


Problem graph with 33 nodes and 70 edges

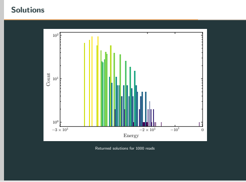


- 33 identified phrases (nodes) with 70 overlaps (edges)
- Nodes are weighted by the phrase entropy, how musically interesting the distribution of notes is

Solutions

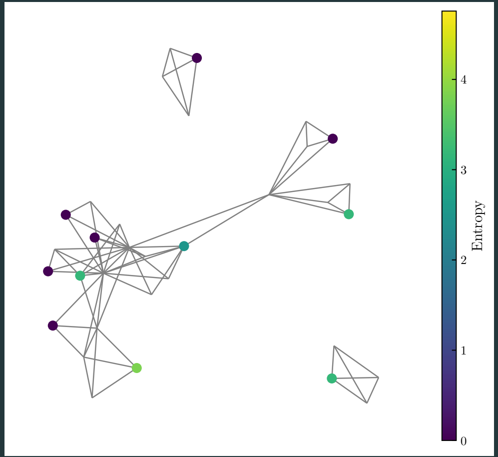


Returned solutions for 1000 reads

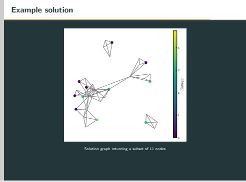


- Histogram of the returned solutions, only energies below zero shown
- Distribution of solutions due to the stochastic nature of annealing
- Lowest energy solution most musically interesting due to construction of weighted MIS QUBO
- (Lowest energy solution -26.8 with a degeneracy of 34)

Example solution



Solution graph returning a subset of 11 nodes

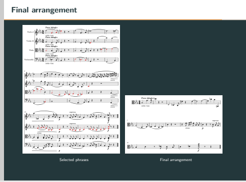


- Selected nodes from one of the lowest energy solutions
- Note in the cliques only one node could be selected

Final arrangement

Selected phrases

Final arrangement



- Selected phrases from solution graph highlighted
- Phrases concatenated to create the final arrangement

Conclusions

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- Valid reduction gained from implemented method
- Monophonic lowest-energy solution
- Phrase identification and selection result in musically interesting final arrangement
- Displays some advantages over classical algorithms
- Does not require training data, costly in both time and resources
- Faster solve time compared to similarly-sized problems
- Method and more advanced versions of it removes the skill barrier for music arrangement

Conclusions

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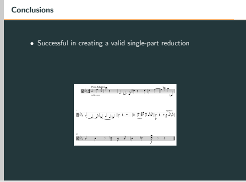

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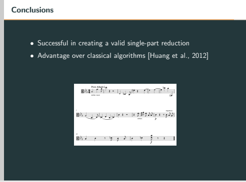

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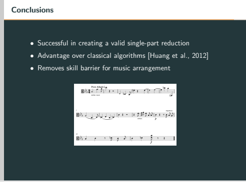

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Future work

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- Problem size — full piece has upwards of 1000 phrases, compare solve times with classical algorithms
- LBDM parameters — phrases should be short and similar in length, prevents important notes being hidden in long phrases with low entropy
- Physical limitations — note ranges, note change speed
- Multiple parts — need different problem formulation, colouring problem with each colour representing a reduced part
- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

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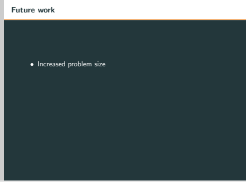
- Increased problem size



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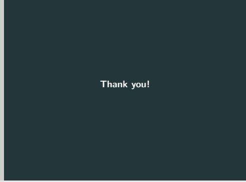

Music Arrangement via Quantum Annealing

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30 January 2025
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LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$
$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

[Cambouropoulos, 2011]

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Weighting
$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$
[Cambouropoulos, 2011]

- Boundaries always taken at beginning/end of piece
- Weightings derived by trial and error

MIS

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

[Lucas, 2014]

$A/B >= 2\max(W)$ to weight the constraint term more heavily than any objective term

MIS

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

[Lucas, 2014]

$A/B >= 2\max(W)$ to weight the constraint term more heavily than any objective term

- Weighted MIS QUBO
- Lagrange paramters A/B determine the balance between constraint and objective

Phrase entropy

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

[Li et al., 2019]

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- Shannon entropy units in bits due to $\log_2$
- Distribution calculated for pitch and duration