

Music Arrangement via Quantum Annealing

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18th January 2025

Durham University

2025-01-18

Music Arrangement via Quantum Annealing

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- Theory
 - Music arrangement
 - Quantum annealing
- Methods
- Results
- Conclusions

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Music Arrangement via Quantum Annealing

└ Overview

Overview

- Theory
 - Music arrangement
 - Quantum annealing
- Methods
- Results
- Conclusions

Theory

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Music Arrangement via Quantum Annealing
└ Theory

Theory

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

The image displays a page from a musical score for Beethoven's String Quartet No. 10. It features four staves: Violin I, Violin II, Viola, and Violoncello. The tempo is marked 'Poco Adagio'. The key signature has two flats (B-flat and E-flat). The score includes various musical notations such as notes, rests, and dynamic markings like 'cresc.', 'espress.', 'p' (piano), and 'f' (forte). The text 'sotto voce' is also present.

Beethoven's String Quartet No. 10

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Music Arrangement via Quantum Annealing

- └ Theory
 - └ Music arrangement
 - └ Music arrangement

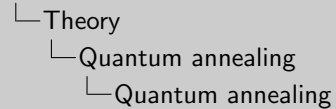
Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

A small snippet of a musical score, showing a single staff with several notes and rests, likely representing a reduction of a more complex piece.

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$



- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

Encodes problem solution into Hamiltonian's ground state

Ising model

$$H(s) = - \sum_{i < j} J_{ij} s_i s_j - \sum_{i=1}^N h_i s_i$$

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j} Q_{i,j} x_i x_j + \sum_i Q_{i,i} x_i$$

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- └ Theory
 - └ Quantum annealing
 - └ QUBO

1. Lattice of spins with two discrete values
2. Coupling strengths J_{ij} and field strengths h_i
3. Function to be minimised

Encodes problem solution into Hamiltonian's ground state

Ising model

$$H(s) = - \sum_{i < j} J_{ij} s_i s_j - \sum_{i=1}^N h_i s_i$$

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j} Q_{i,j} x_i x_j + \sum_i Q_{i,i} x_i$$

How to combine them?

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└ Theory

└ Quantum annealing

How to combine them?

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Methods

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

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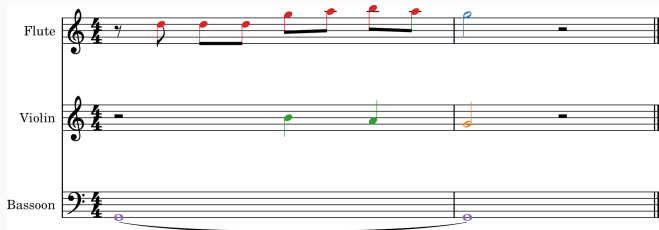
└─Methods

└─Problem formulation

1. Split parts

Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$



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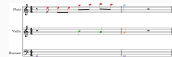
Methods

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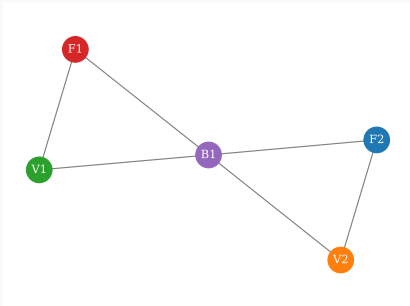
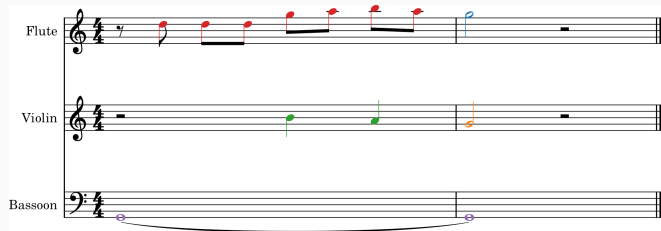


2. Create graph

Flute

Violin

Bassoon



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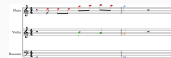
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└ Methods

└ 2. Create graph

Each phrase becomes a node
Edges between nodes if phrases overlap

2. Create graph

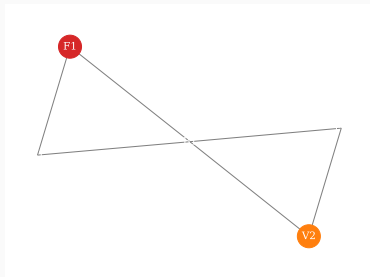
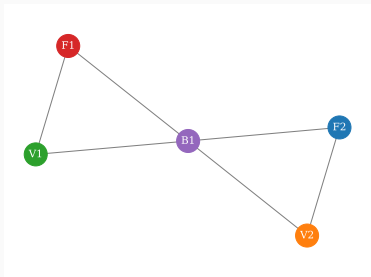


3. Solve graph

Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge.

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$



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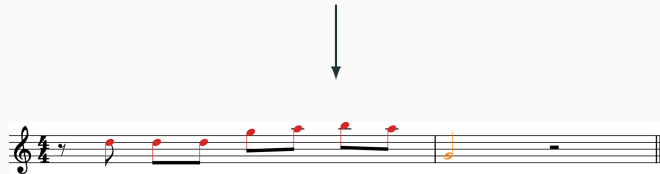
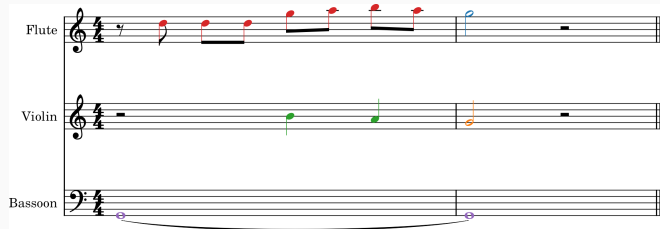


4. Construct arrangement

Flute

Violin

Bassoon



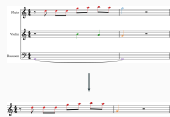
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Music Arrangement via Quantum Annealing

└ Methods

└ 4. Construct arrangement

4. Construct arrangement



Take selected nodes and combine to create final arrangement

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Results



Violin I

Poco Adagio

sotto voce

Violin II

Poco Adagio

sotto voce

Viola

Poco Adagio

sotto voce

Violoncello

Poco Adagio

sotto voce

6

cresc.

10

espress.

p

f

cresc.

p

f

p

String Quartet No. 10 by Ludwig van Beethoven

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Music Arrangement via Quantum Annealing

└ Results

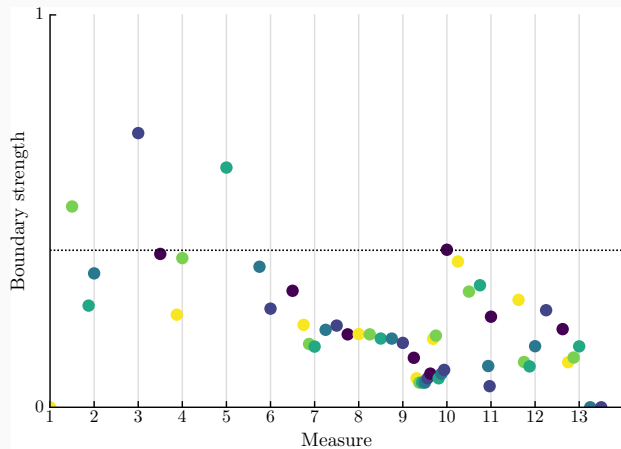
└ Excerpt

Excerpt

String Quartet No. 10 by Ludwig van Beethoven

Phrase detection

Local boundary detection model (LBDM)



Boundary strengths for the Violin I part

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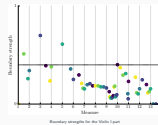
Music Arrangement via Quantum Annealing

└ Results

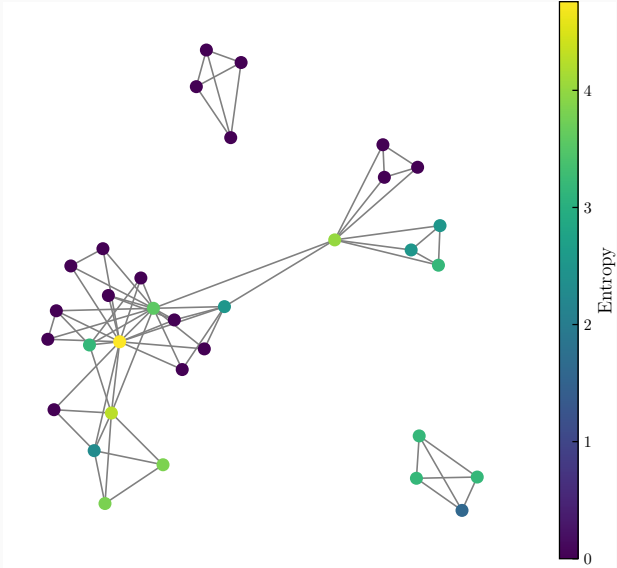
└ Phrase detection

Phrase detection

Local boundary detection model (LBDM)



Problem graph

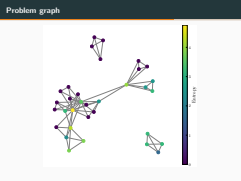


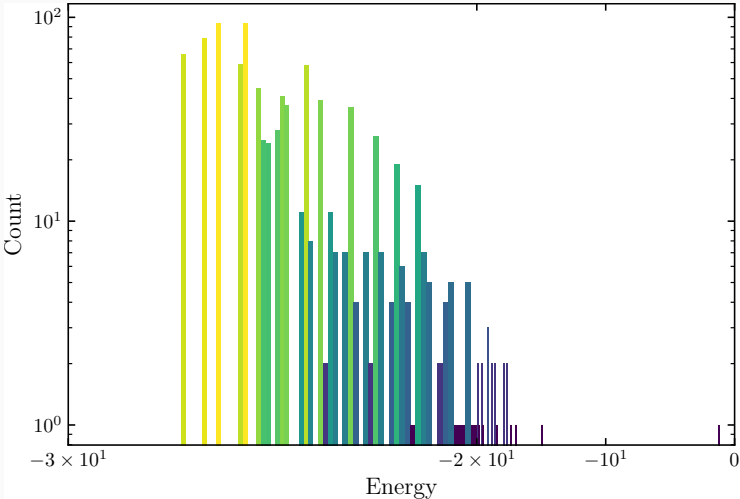
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Music Arrangement via Quantum Annealing

└ Results

└ Problem graph





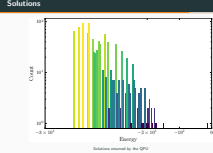
Solutions returned by the QPU

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Music Arrangement via Quantum Annealing

Results

Solutions



Lowest energy solution was -26.8 with a degeneracy of 34

$$\oiint_A \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

The *net electric flux* through any **closed** surface is proportional to the **enclosed charge**.

Alert

This is an alert.

Example

This is an example.

$$\oiint_A \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

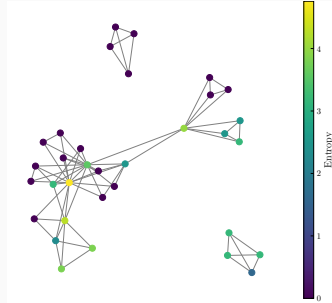
The net electric flux through any **closed** surface is proportional to the **enclosed charge**.

Alert
This is an alert.

Example
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Apperance sync

- Volume rate of flow equal to divergence
- Summed over entire volume
- Equal to net flow across the boundary



Source: Wikimedia Commons

$$\iiint_V \nabla \cdot \mathbf{F} dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$$

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
Music Arrangement via Quantum Annealing

└ Results

└ Apperance sync

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Source: Wikimedia Commons

$$\iiint_V \nabla \cdot \mathbf{F} dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$$

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Conclusions

Equation gather

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{I}\end{aligned}$$

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└─ Conclusions

└─ Equation gather

Equation gather

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