# Music Arrangement via Quantum Annealing

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30 January 2025

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# Overview








Overview

Theory
Music arrangement
Quantum annealing
Methods
Results
Conclusions


- What is music arrangement? What is quantum annealing?
- Methods used to solve the music arrangement problem
- Preliminary results from application of the method
- Concluding thoughts about this process

# Theory



Theory

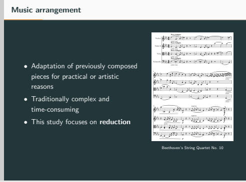
# Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**


Beethoven's String Quartet No. 10








- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet to solo)

# Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$





Adiabatic quantum computing (AQC)

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$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

# Quantum annealing

**Ising model**

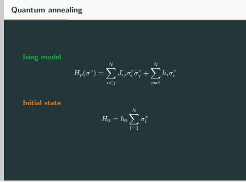
$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$

**Initial state**

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

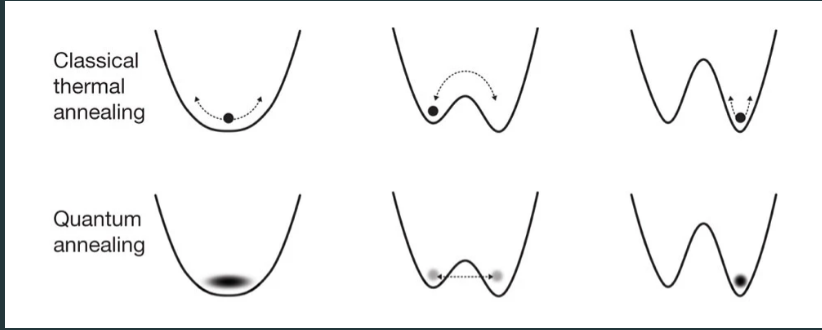






- How is this used to solve problems?
- Ising model, create a lattice of variables with two discrete values (spin up/down)
- Problem Hamiltonian, qubits $\sigma^z$, coupling strengths $J_{ij}$ and field strengths $h_i$
- Initial state is a superposition of all possible states
- If problem solution is encoded within the ground state, system will give solution after evolution

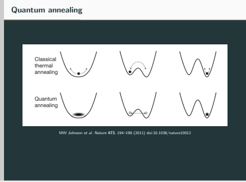
# Quantum annealing




MW Johnson *et al.* *Nature* **473**, 194–198 (2011) doi:10.1038/nature10012







- What does this look like?
- Evolution of superposition to a particular state
- More efficiently escape from local minima via quantum tunneling
- Can solve harder problems with a more turbulent energy landscape

# QUBO

**Quadratic Unconstrained Binary Optimisation**

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling





QUBO
Quadratic Unconstrained Binary Optimisation
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- How to encode a problem into a Hamiltonian?
- QUBO is a function to be minimised
- Set of binary variables $x$, matrix $Q$ of real weights that describes interactions between variables

# How to combine them?





- How to apply quantum annealing to the problem of music arrangement?

# Methods

# Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution





Problem formulation

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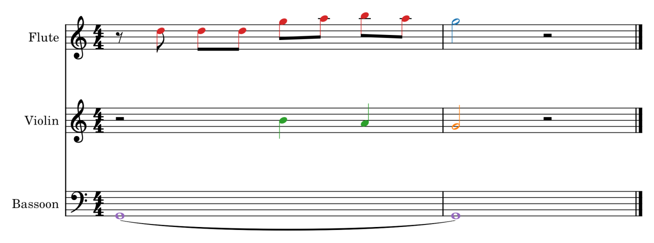
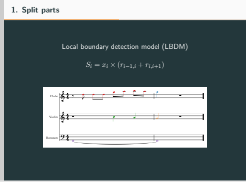
Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

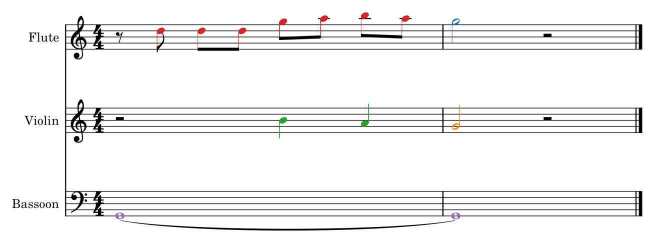


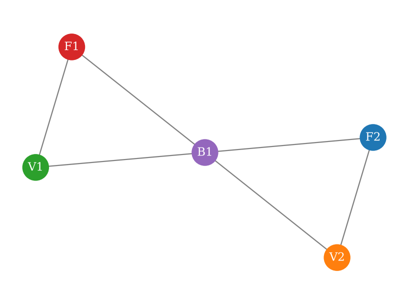




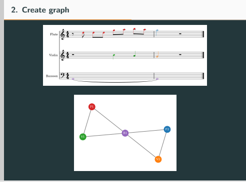
# 2. Create graph







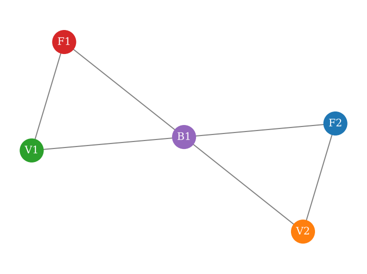




Each phrase becomes a nodeEdges between nodes if phrases overlap

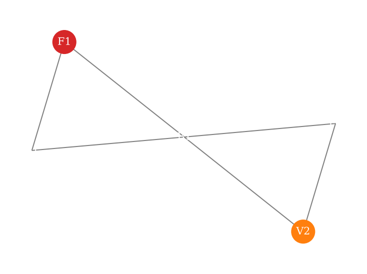
## Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge.

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$



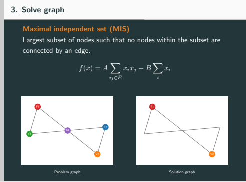

Problem graph



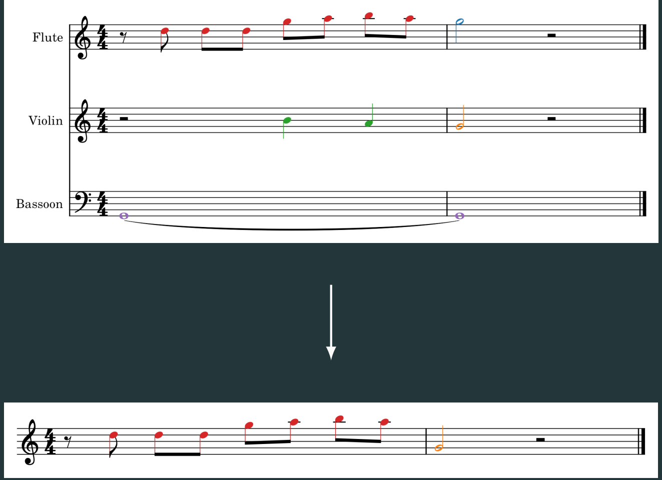

Solution graph





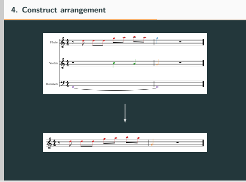


# 4. Construct arrangement








Take selected nodes and combine to create final arrangement

# Results



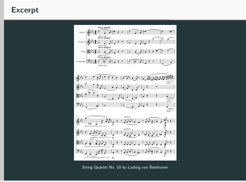
Results

# Excerpt


String Quartet No. 10 by Ludwig van Beethoven

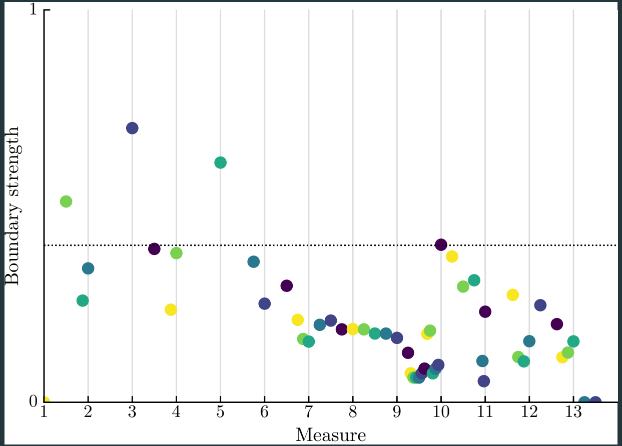






# Phrase detection

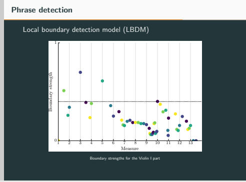
## Local boundary detection model (LBDM)



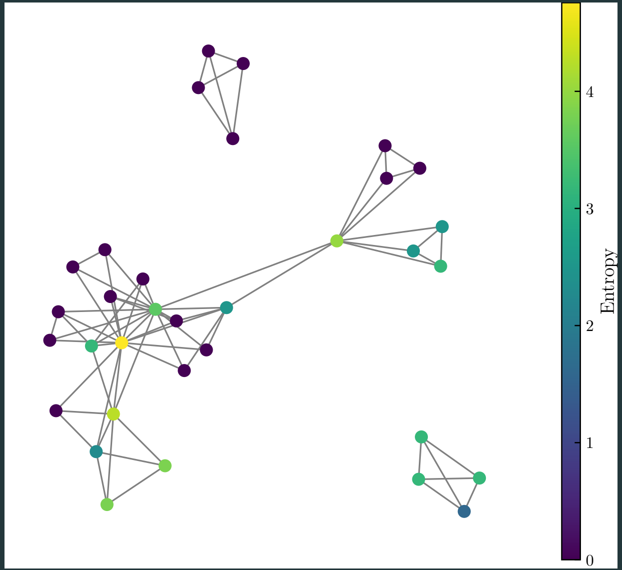

Boundary strengths for the Violin I part





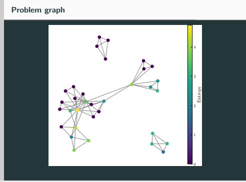


# Problem graph

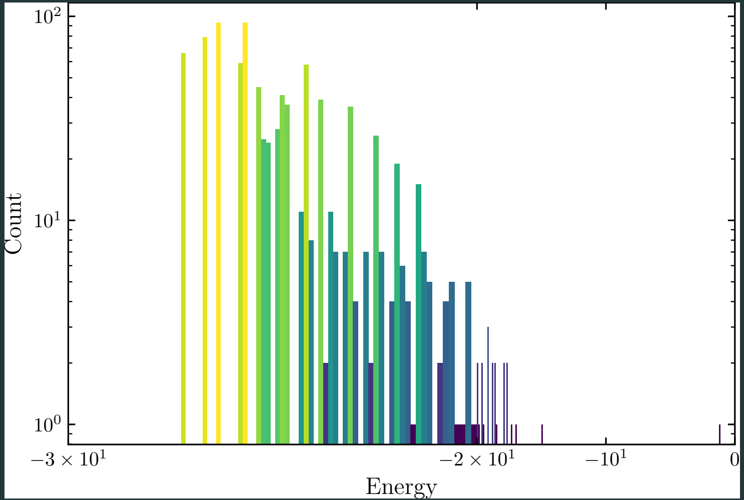






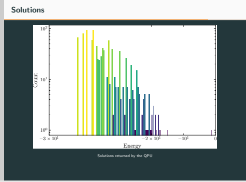
# Solutions




Solutions returned by the QPU







Lowest energy solution was -26.8 with a degeneracy of 34

# Example solution







Example solution

# Blocks

$$\oiint_A E \cdot dA = \frac{Q}{\varepsilon_0}$$

The *net electric flux* through any closed surface is proportional to the **enclosed charge**.

**Alert**
This is an alert.

**Example**
This is an example.





Blocks

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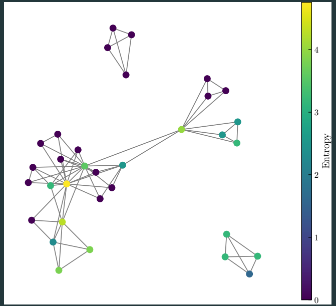
## Apperance sync

- Volume rate of flow equal to divergence
- Summed over entire volume
- Equal to net flow across the boundary

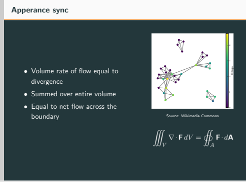



Source: Wikimedia Commons

$$\iiint_V \nabla \cdot \mathbf{F} \, dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$$







# Conclusions



Conclusions

# Equation gather

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_0 I$$



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