

Quantum Annealing for Music Arrangement

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Overview

Motivations

Theory

Adiabatic quantum computing

Quantum annealing

Music arrangement

Methods

Results

Conclusions

Motivations

Motivations

- Small lit review¹
- Quantum computer music
- My own novel adaption of the method
- THIS IS MY OWN IDEA

¹Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

Theory

Adiabatic quantum computing

Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.²

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

²Born and Fock, 'Beweis des Adiabatenatzes'.

Quantum annealing

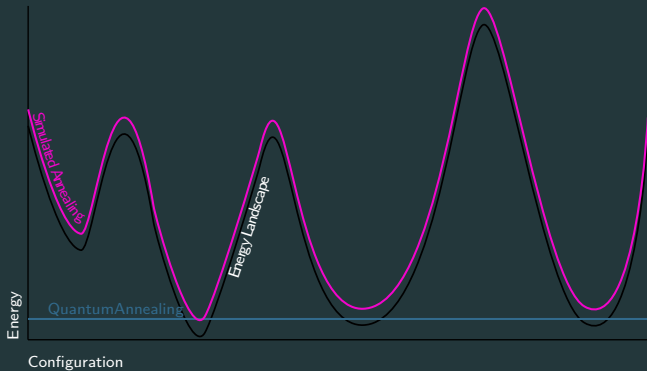
- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic

Initial state

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

Ising model

$$H_p(\sigma^z) = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$



By Brianlechthaler - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=112382195>

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Sent to the QPU for optimisation

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

The image displays the first system of a musical score for Beethoven's String Quartet No. 10. The score is written for four instruments: Violin I, Violin II, Viola, and Violoncello. The key signature is three flats (B-flat, E-flat, A-flat) and the time signature is 4/4. The tempo marking is 'Poco Adagio'. The first system shows the initial measures of the piece, with each instrument part clearly delineated. The second system continues the music, featuring dynamic markings such as 'cresc.' (crescendo) and 'p' (piano). The third system includes the marking 'espress.' (espressivo) and continues the musical development. The fourth system concludes the excerpt with a 'p' marking. The notation includes various musical symbols such as notes, rests, and dynamic markings.

Beethoven's String Quartet No. 10

How to combine them?

Methods

Problem formulation

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

1. Split score

Local boundary detection model (LBDM)

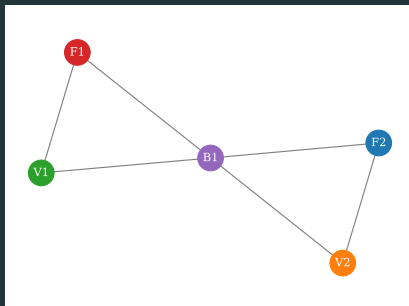
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'

The image displays a musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) features a sequence of eighth notes with red stems and flags, indicating a specific rhythmic pattern. The Violin part (treble clef) shows a few notes with green stems, suggesting a different rhythmic or expressive timing. The Bassoon part (bass clef) has a long, sustained note with a purple stem and a curved line underneath, indicating a long note or a specific expressive timing. The score is presented in a split format, with each instrument on its own staff.

2. Create graph

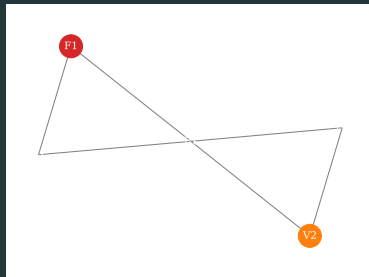
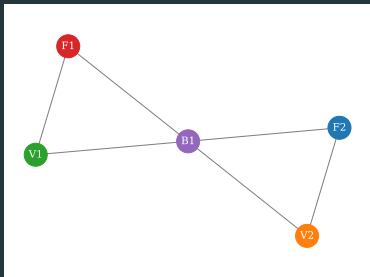
A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (top staff) begins with a quarter rest, followed by a sequence of eighth notes: G4, A4, B4, C5, D5, E5, and F5. The Violin part (middle staff) has a whole rest for the first two measures, then plays a half note G4 in the third measure and a half note A4 in the fourth measure. The Bassoon part (bottom staff) plays a whole note G2 in the first measure and a whole note A2 in the second measure, with a slur connecting the two notes. The score ends with a double bar line.



3. Solve graph

Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge



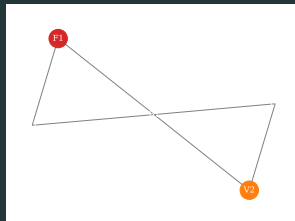
4. Construct arrangement

Flute

Violin

Bassoon

The score shows three staves in 4/4 time. The Flute staff has a sequence of eighth notes (G4, A4, B4, C5, D5, E5) with red stems, followed by a whole note (F5) with a blue stem. The Violin staff has a sequence of eighth notes (G3, A3, B3, C4, D4, E4) with green stems, followed by a whole note (F4) with an orange stem. The Bassoon staff has a whole note (G2) with a purple stem, followed by a whole note (F2) with a purple stem.



Flute

The score shows a single staff in 4/4 time. It contains a sequence of eighth notes (G4, A4, B4, C5, D5, E5) with red stems, followed by a whole note (F5) with an orange stem.

Results

Excerpt

Poco Adagio

Violin I
sotto voce
Poco Adagio

Violin II
sotto voce
Poco Adagio

Viola
sotto voce
Poco Adagio

Violoncello
sotto voce
Poco Adagio

6

cresc.

cresc.

cresc.

10

espress.

p

f

espress.

p

f

cresc.

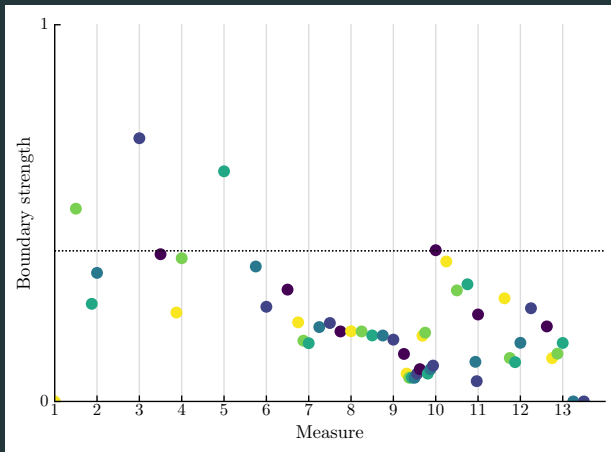
p

f

p

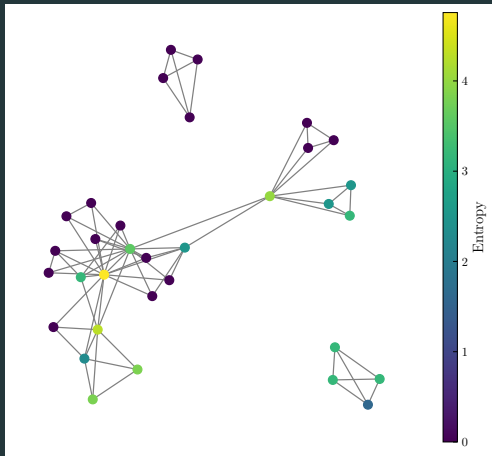
String Quartet No. 10 by Ludwig van Beethoven

Phrase detection



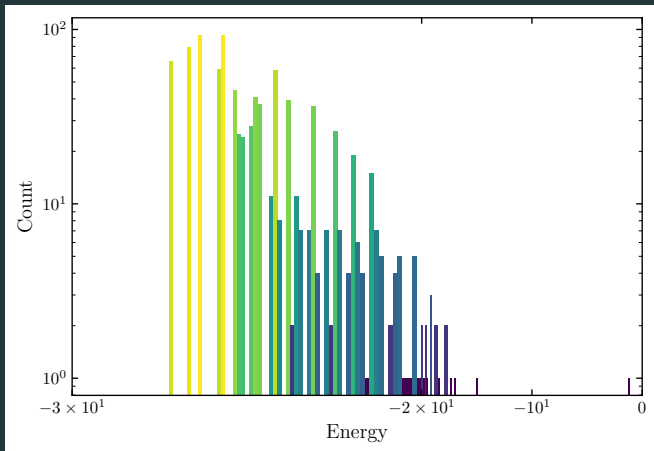
Boundary strengths for the Violin I part

Problem graph



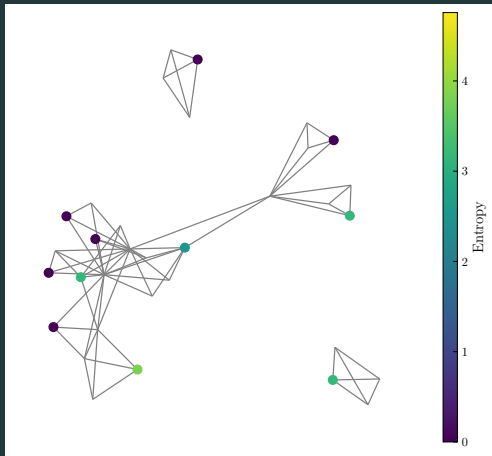
Problem graph with 33 nodes and 70 edges

Solutions



Returned solutions for 1000 reads

Example solution



Solution graph returning a subset of 11 nodes

Final arrangement

Poco Adagio
sotto voce

Violin I

Violin II

Viola

Violoncello

6

cresc.

cresc.

cresc.

10

espress.

p

f

cresc.

p

f

p

Selected phrases

Poco Adagio
sotto voce

7

espress.

cresc.

p

12

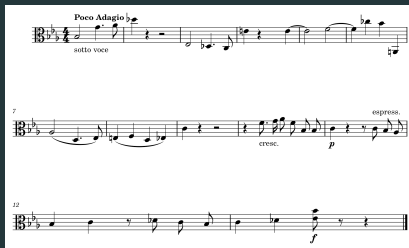
f

Final arrangement

Conclusions

Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'
- Removes skill barrier for music arrangement



Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'

Thank you!

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Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3} (S'_{\text{pitch}} + 2S'_{\text{IOI}})$$

Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

Lucas, 'Ising formulations of many NP problems'

$A/B \geq 2 \max(W)$ to weight the constraint term more heavily than any objective term

Phrase entropy



Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'

-  Born, M. and V. Fock. **‘Beweis des Adiabatenatzes’**. de. In: *Zeitschrift für Physik* 51.3 (Mar. 1928), pp. 165–180. ISSN: 0044-3328. DOI: 10.1007/BF01343193. URL: <https://doi.org/10.1007/BF01343193> (visited on 01/03/2025).
-  Cambouropoulos, Emilios. **‘The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing’**. In: *International Computer Music Association* (2011). ISSN: 2223-3881.
-  Huang, Jiun-Long, Shih-Chuan Chiu and Man-Kwan Shan. **‘Towards an automatic music arrangement framework using score reduction’**. In: *ACM Trans. Multimedia Comput. Commun. Appl.* 8.1 (Feb. 2012), 8:1–8:23. ISSN: 1551-6857. DOI: 10.1145/2071396.2071404. (Visited on 05/12/2024).



Li, You et al. **‘Automatic Piano Reduction of Orchestral Music Based on Musical Entropy’**. In: *2019 53rd Annual Conference on Information Sciences and Systems (CISS)*. Mar. 2019, pp. 1–5. DOI: 10.1109/CISS.2019.8693036. URL: <https://ieeexplore.ieee.org/document/8693036> (visited on 27/12/2024).



Lucas, Andrew. **‘Ising formulations of many NP problems’**. English. In: *Frontiers in Physics* 2 (Feb. 2014). Publisher: Frontiers. ISSN: 2296-424X. DOI: 10.3389/fphy.2014.00005. (Visited on 14/10/2024).



Pearce, M. and Geraint A. Wiggins. **'Towards A Framework for the Evaluation of Machine Compositions'**. In:

Proceedings of the AISB'01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences. 2001.