

Music Arrangement via Quantum Annealing

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30 January 2025

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Theory

Music arrangement

Quantum annealing

Methods

Results

Conclusions

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Music Arrangement via Quantum Annealing

└ Overview

- What is music arrangement? What is quantum annealing?
- Methods used to solve the music arrangement problem
- Preliminary results from application of the method
- Concluding thoughts about this process

Overview

Theory

Music arrangement

Quantum annealing

Methods

Results

Conclusions

Theory

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Music Arrangement via Quantum Annealing

└ Theory

Theory

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**



Beethoven's String Quartet No. 10

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Music Arrangement via Quantum Annealing

- └ Theory
 - └ Music arrangement
 - └ Music arrangement

- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet to solo)

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**



Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

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- └ Theory
 - └ Quantum annealing
 - └ Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

Ising model

$$H_p(\sigma^z) = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

Initial state

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

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└ Theory

└ Quantum annealing

└ Quantum annealing

Quantum annealing

Ising model

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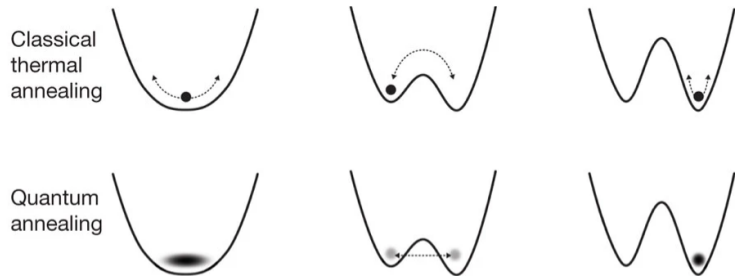
- How is this used to solve problems?
- Ising model, create a lattice of variables with two discrete values (spin up/down)
- Problem Hamiltonian, qubits σ^z , coupling strengths J_{ij} and field strengths h_i
- Initial state is a superposition of all possible states
- If problem solution is encoded within the ground state, system will give solution after evolution

Quantum annealing

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- └ Theory
 - └ Quantum annealing
 - └ Quantum annealing



MW Johnson *et al. Nature* **473**, 194–198 (2011) doi:10.1038/nature10012

- What does this look like?
- Evolution of superposition to a particular state
- More efficiently escape from local minima via quantum tunneling
- Can solve harder problems with a more turbulent energy landscape

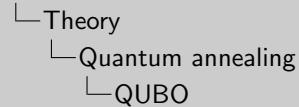
Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

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- How to encode a problem into a Hamiltonian?
- QUBO is a function to be minimised
- Set of binary variables x , matrix Q of real weights that describes interactions between variables

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

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└ Theory

└ Quantum annealing

How to combine them?

How to combine them?

- How to apply quantum annealing to the problem of music arrangement?

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Methods



1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

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Music Arrangement via Quantum Annealing

└─Methods

└─Problem formulation

1. Split parts

Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

Flute

Violin

Bassoon

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Music Arrangement via Quantum Annealing

└ Methods

└ 1. Split parts

1. Split parts

Local boundary detection model (LBDM)

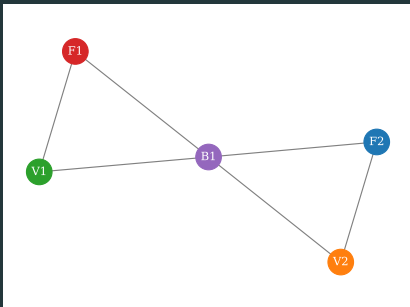
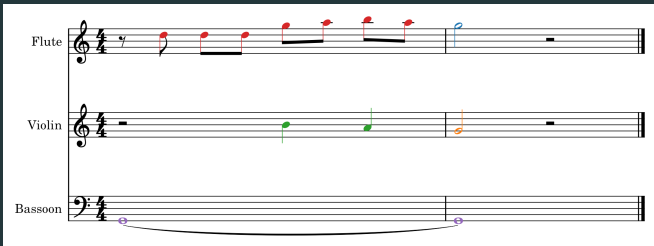
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

2. Create graph

Flute

Violin

Bassoon



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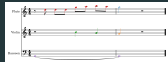
Music Arrangement via Quantum Annealing

└ Methods

└ 2. Create graph

Each phrase becomes a node
Edges between nodes if phrases overlap

2. Create graph

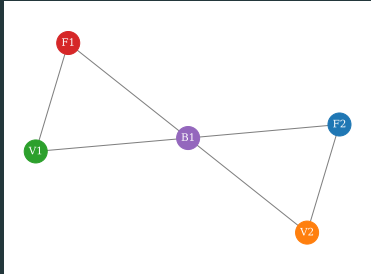


3. Solve graph

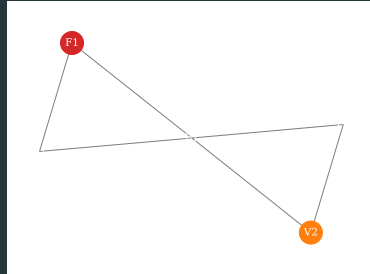
Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge.

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$



Problem graph



Solution graph

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$



4. Construct arrangement

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a melody of eighth notes: G4, A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two eighth notes: G4 and A4, and then a whole rest. The Bassoon part (bass clef) has a whole note G3, followed by a whole rest. A curved line connects the two G3 notes in the Bassoon part.



The final constructed arrangement, showing the Flute and Violin parts combined in a single staff. The Flute part (treble clef) has a melody of eighth notes: G4, A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two eighth notes: G4 and A4, and then a whole rest.

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└ Methods

└ 4. Construct arrangement

4. Construct arrangement

A musical score for three instruments: Piano, Violin, and Bassoon, in 4/4 time. The Piano part (treble clef) has a melody of eighth notes: G4, A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two eighth notes: G4 and A4, and then a whole rest. The Bassoon part (bass clef) has a whole note G3, followed by a whole rest.



The final constructed arrangement, showing the Piano and Violin parts combined in a single staff. The Piano part (treble clef) has a melody of eighth notes: G4, A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two eighth notes: G4 and A4, and then a whole rest.

Take selected nodes and combine to create final arrangement

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Results



Excerpt

Poco Adagio

Violin I

sotto voce

Poco Adagio

Violin II

sotto voce

Poco Adagio

Viola

sotto voce

Poco Adagio

Violoncello

sotto voce

6

cresc.

cresc.

cresc.

10

espress.

p

f

espress.

p

f

cresc.

p

f

p

String Quartet No. 10 by Ludwig van Beethoven

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Results

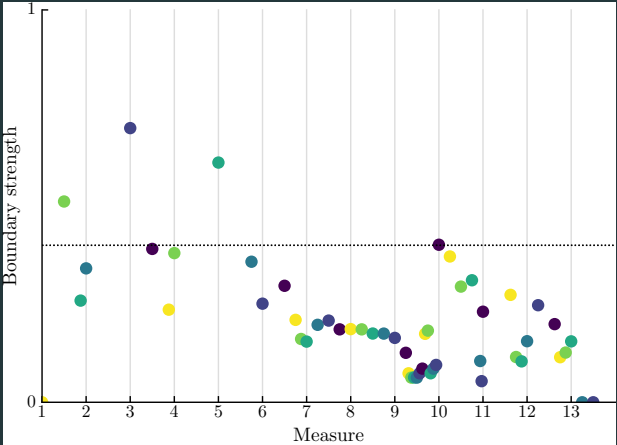
Excerpt

Excerpt

String Quartet No. 10 by Ludwig van Beethoven

Phrase detection

Local boundary detection model (LBDM)

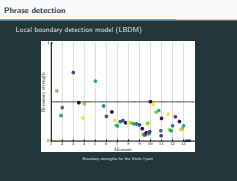


Boundary strengths for the Violin I part

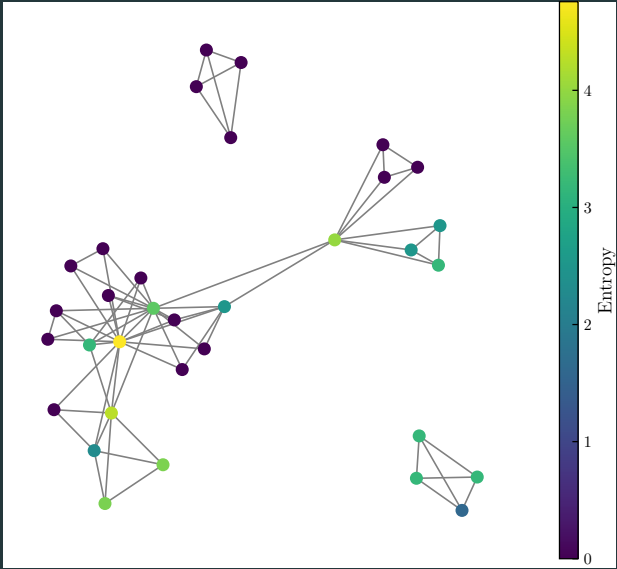
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- Results
 - Phrase detection



Problem graph



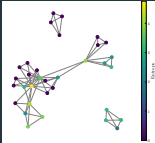
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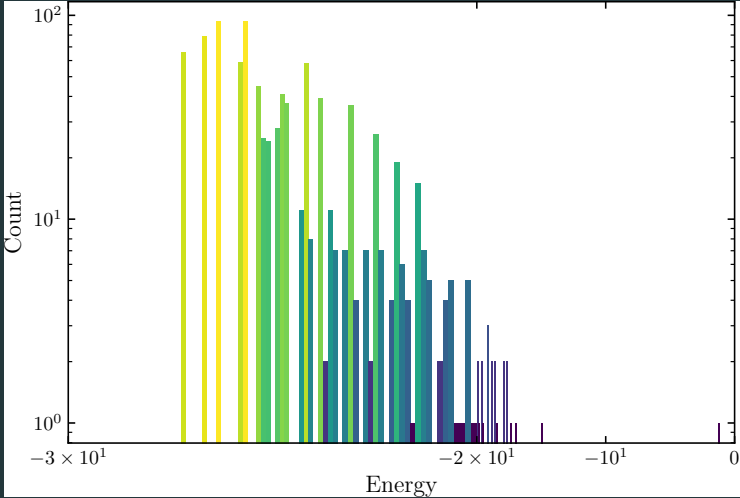
Music Arrangement via Quantum Annealing

└ Results

└ Problem graph

Problem graph





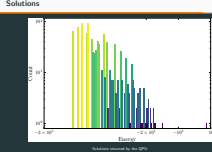
Solutions returned by the QPU

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Music Arrangement via Quantum Annealing

└ Results

└ Solutions



Lowest energy solution was -26.8 with a degeneracy of 34

Example solution

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Music Arrangement via Quantum Annealing

└─ Results

└─ Example solution

$$\oiint_A \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

The *net electric flux* through any **closed** surface is proportional to the **enclosed charge**.

Alert

This is an alert.

Example

This is an example.

└ Results

└ Blocks

$$\oiint_A \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

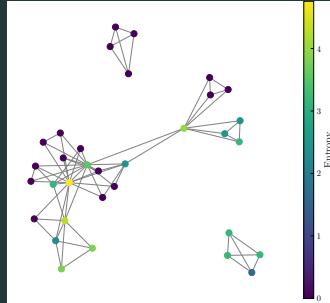
The net electric flux through any **closed** surface is proportional to the **enclosed charge**.

Alert
This is an alert.

Example
This is an example.

Apperance sync

- Volume rate of flow equal to divergence
- Summed over entire volume
- Equal to net flow across the boundary



Source: Wikimedia Commons

$$\iiint_V \nabla \cdot \mathbf{F} dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$$

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Music Arrangement via Quantum Annealing

└ Results

└ Apperance sync

Apperance sync

- Volume rate of flow equal to divergence
- Summed over entire volume
- Equal to net flow across the boundary



Source: Wikimedia Commons

$$\iiint_V \nabla \cdot \mathbf{F} dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$$

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Music Arrangement via Quantum Annealing
└─ Conclusions

Conclusions

Conclusions

Equation gather

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 I\end{aligned}$$

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└─Equation gather

Equation gather

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