

Quantum Annealing for Music Arrangement

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Overview

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Adiabatic quantum computing

Quantum annealing

Music arrangement

Motivations

Method

Results

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Theory

Adiabatic quantum computing

Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.¹

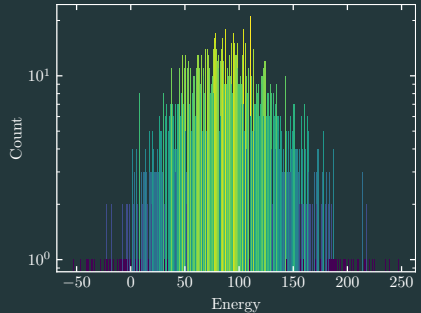
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

¹Born and Fock, 'Beweis des Adiabatenatzes'.

Quantum annealing

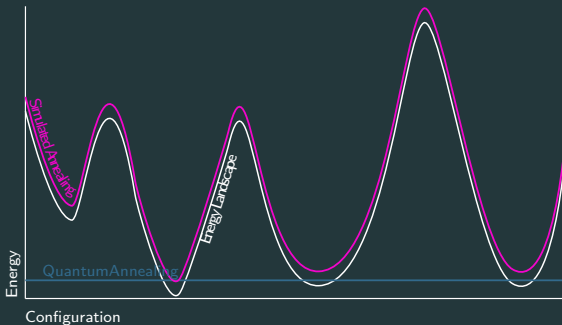
- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic



Distribution of 2000 solution energies

Advantages

- Find the ground state of complicated Hamiltonians
- Quantum tunneling avoids local minima



By Brianlechthaler - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=112382195>

Ising model

Initial Hamiltonian

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

Problem Hamiltonian

$$H_p(\sigma^z) = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

Quadratic Unconstrained Binary Optimisation

Vector x of qubits, matrix Q of weights

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Aim to minimise this function
- Difficult to solve analytically
- Mapped to H_p using simple change of variable
- Encodes problem solution into Hamiltonian's ground state

What problems can we solve?

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- *Reduction* can be shown to be computationally complex²

Quartet No. 1 in B \flat major

Joseph Haydn

Allegro

The image shows a musical score for a string quartet, measures 1 through 12. The title is 'Quartet No. 1 in B-flat major' by Joseph Haydn, with the tempo marking 'Allegro'. The score is for four parts: Violin I, Violin II, Viola, and Cello. The key signature has two flats (B-flat major), and the time signature is 4/4. The notation includes various musical symbols such as notes, rests, and bar lines. The first system covers measures 1-4, the second system covers measures 5-8, and the third system covers measures 9-12. The Cello part is written in the bass clef, while the other three parts are in the treble clef.

Quartet No. 1 in B \flat major by Joseph Haydn

²Moses and Demaine, 'Computational Complexity of Arranging Music'

Motivations

Motivations

- Classical methods of automatic arrangement³
- Field of quantum computer music is very new⁴
- Quantum annealing used to generate music⁵
- Novel adaption of this method to a new problem
- *This has never been done before!*

³Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'; Nakamura and Yoshii, 'Statistical piano reduction controlling performance difficulty'; Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

⁴Miranda, *Quantum Computer Music*.

⁵Freedline, 'Algorhythms'; Arya et al., 'Music Composition Using Quantum Annealing'.

Method

Method

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate optimisation problem
4. Solve problem using QPU
5. Construct arrangement from solution

1. Split score

- Musical phrases chosen as smallest unit of music
- Preserve melody and structure when arranged

Local boundary detection model (LBDM)⁶

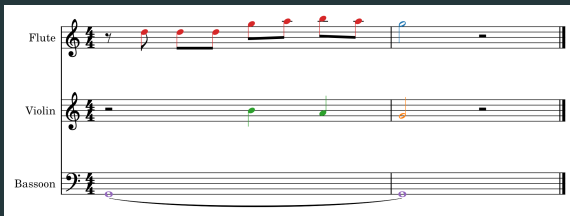
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

The image shows a musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (top staff) has a melody starting with a quarter rest, followed by eighth notes (red), quarter notes (red), and a half note (blue) at the end of the first measure. The Violin part (middle staff) has a half rest, followed by quarter notes (green) and a half note (orange) at the end of the first measure. The Bassoon part (bottom staff) has a half rest, followed by a half note (purple) at the end of the first measure. A purple line connects the two purple notes in the Bassoon part, indicating a boundary. The score is divided into two measures by a bar line.

⁶Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

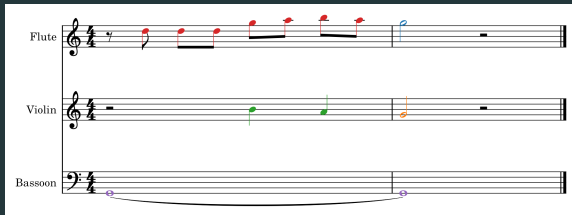
2. Create graph

- Vertices (nodes) connected by edges
- Models pairwise relations between objects

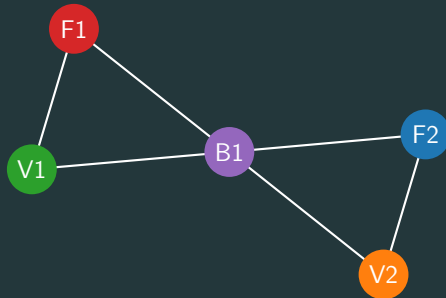


- Nodes — phrases
- Edges — overlap between phrases

2. Create graph



A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) starts with a quarter rest, followed by four eighth notes (G4, A4, B4, C5) with red stems, and ends with a quarter rest. The Violin part (treble clef) starts with a quarter rest, followed by two eighth notes (G4, A4) with green stems, and ends with a quarter rest. The Bassoon part (bass clef) starts with a quarter rest, followed by a half note (G3) with a purple stem, and ends with a quarter rest. A purple line connects the two Bassoon notes.



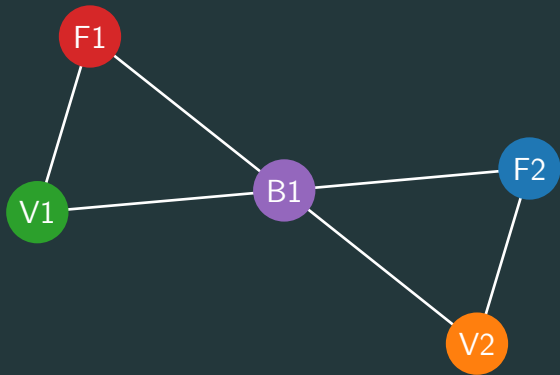
3. Create optimisation problem

Proper vertex colouring

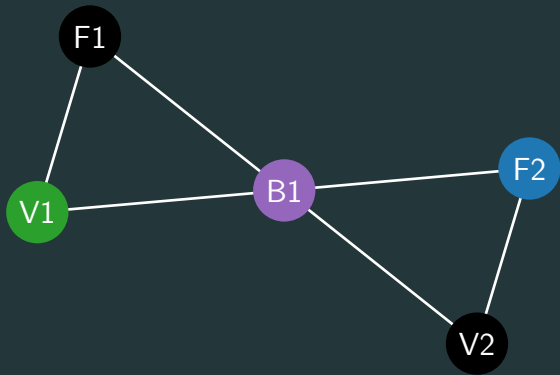
Colour each vertex such that no edge connects two vertices of the same colour

$$\begin{aligned} f(x) = & +A \sum_{v \in V} \left(1 - \sum_{i=1}^n x_{v,i} \right)^2 & +B \sum_{(u,v) \in E} \sum_{i=1}^n x_{u,i} x_{v,i} \\ & -C \sum_{v \in V} \sum_{i=1}^n W_v x_{v,i} & -D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^n \sum_{j=1}^n x_{u,i} x_{v,j} \end{aligned}$$

3. Create optimisation problem

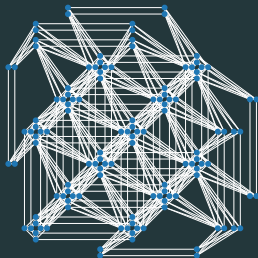


3. Create optimisation problem



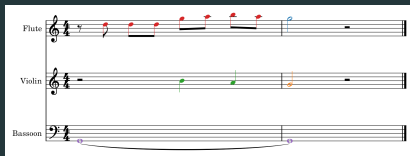
4. Solve problem

- Problem embedded onto D-Wave quantum hardware
- Quantum annealer optimises QUBO formulation
- Returns a sample set of results

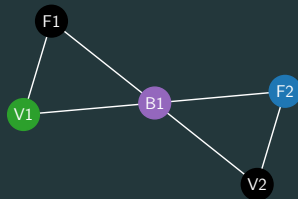


D-Wave Advantage QPU topology. Own work.

5. Construct arrangement



A musical score in 4/4 time for three instruments: Flute, Violin, and Bassoon. The Flute part (treble clef) starts with a quarter rest, followed by a sequence of eighth notes: G4 (red), A4 (red), B4 (red), C5 (red), D5 (blue), and E5 (blue), ending with a quarter rest. The Violin part (treble clef) has a whole rest for the first measure, followed by a half note G4 (green) and a half note A4 (orange) in the second measure, ending with a whole rest. The Bassoon part (bass clef) has a whole note G3 (purple) spanning both measures.



A single-instrument musical score in 4/4 time, likely for the Flute. It starts with a quarter rest, followed by a sequence of eighth notes: G4 (red), A4 (red), B4 (red), C5 (red), D5 (red), and E5 (red), ending with a half note G4 (orange) and a whole rest.

Results

Score

- Smaller ensemble chosen for problem size
- Well-defined musical structure
- Reduction to three instruments

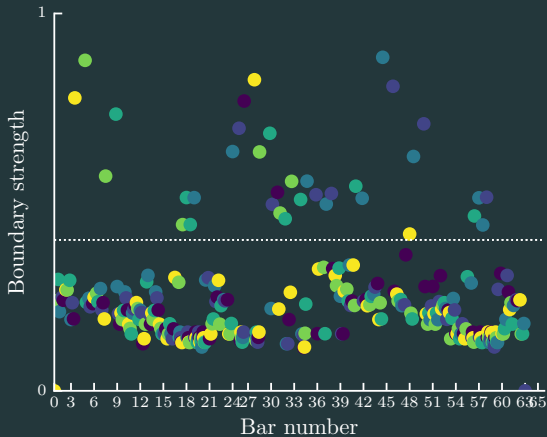
Quartet No. 1 in B \flat major

Joseph Haydn

Presto

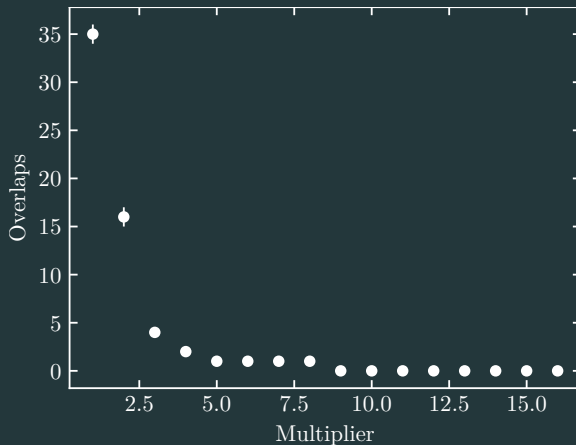
The image displays the first 12 measures of the 'Presto' movement from Haydn's Quartet No. 1 in B-flat major. The score is arranged in four systems, each containing staves for Violin I, Violin II, Viola, and Cello. The key signature has two flats (B-flat and E-flat), and the time signature is 2/4. The tempo is marked 'Presto'. The notation includes various musical symbols such as notes, rests, and bar lines. The first system covers measures 1-5, the second system covers measures 6-10, and the third system covers measures 11-12. The Cello part is notably more active than the other instruments in the first system.

Phrase detection



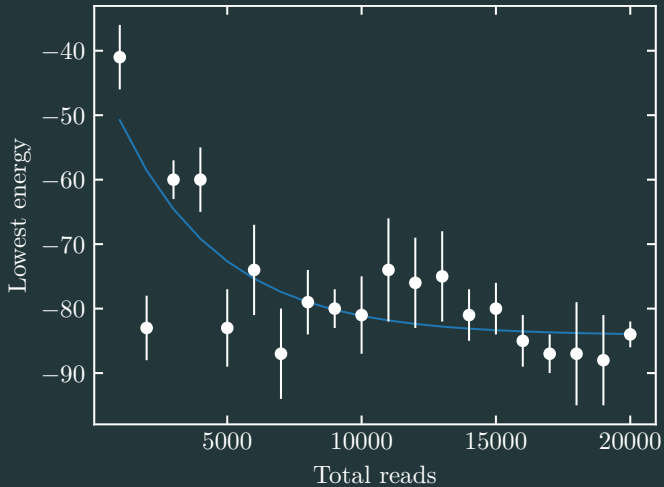
Boundary strengths for the Violin I part

QUBO parameter variation

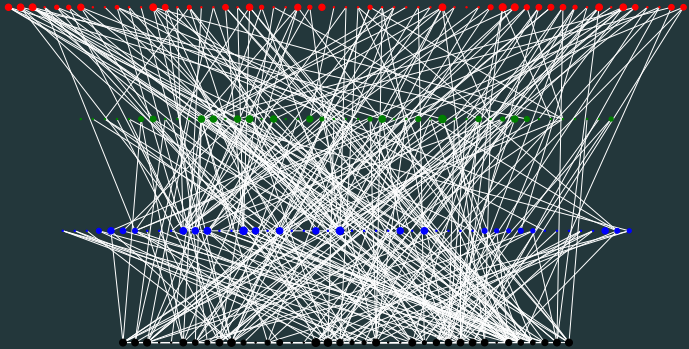


Variation of with edge weight Lagrange parameter, in multiples of the maximum node weight.

Optimisation



Example solution



Conclusions

Conclusions

- Novel application of quantum annealing
- QPU successfully returns low-energy states
- Necessary constraints for a valid arrangement fulfilled

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Qualitative judgement of computer arrangements⁷

⁷Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

Thank you!

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Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3} (S'_{\text{pitch}} + 2S'_{\text{IOI}})$$

Phrase entropy

x_i — parameter x (pitch, IOI) of note i

Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

⁹Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.



Arya, Ashish et al. **‘Music Composition Using Quantum Annealing’**. In: *arXiv* (Jan. 2022). DOI:

10.48550/arXiv.2201.10557. (Visited on 26/10/2024).



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-  Freedline, Alex. **‘Algorhythms: Generating Music with D-Wave’s Quantum Annealer’**. en. In: *MIT 6.s089—Intro to Quantum Computing* (Feb. 2021).
-  Huang, Jiun-Long, Shih-Chuan Chiu and Man-Kwan Shan. **‘Towards an automatic music arrangement framework using score reduction’**. In: *ACM Trans. Multimedia Comput. Commun. Appl.* 8.1 (Feb. 2012), 8:1–8:23. ISSN: 1551-6857. DOI: 10.1145/2071396.2071404. (Visited on 05/12/2024).



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