

Quantum Annealing for Music Arrangement

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Overview

Motivations

Motivations

- Small lit review¹
- Quantum computer music
- My own novel adaption of the method
- THIS IS MY OWN IDEA

¹cambouropoulos'ibdm'2011.

Theory

Adiabatic quantum computing

Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.²

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

²born`beweis`1928.

Quantum annealing

- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic

Initial state

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

Ising model

$$H_p(\sigma^z) = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

Uses

Energy

Configuration

Healing

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Sent to the QPU for optimisation

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

The image displays the first system of a musical score for Beethoven's String Quartet No. 10. The score is written for four instruments: Violin I, Violin II, Viola, and Violoncello. The key signature is three flats (B-flat, E-flat, A-flat) and the time signature is 4/4. The tempo marking is 'Poco Adagio'. The first system shows the initial measures of the piece, with each instrument part clearly delineated. The second system continues the music, featuring dynamic markings such as 'cresc.' (crescendo) and 'p' (piano). The third system includes the marking 'espress.' (espressivo) and continues the musical development. The fourth system shows further musical notation, including a 'p' marking. The score is presented in a clear, professional layout with standard musical notation.

Beethoven's String Quartet No. 10

How to combine them?

Methods

Problem formulation

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

1. Split score

Local boundary detection model (LBDM)

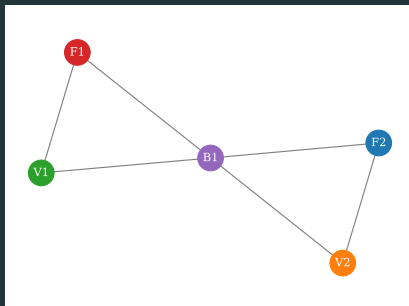
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

cambouropoulos'lbdbm'2011

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a melody of eighth notes: G4 (red), A4 (red), B4 (red), C5 (red), D5 (red), E5 (blue), followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two eighth notes: G4 (green), A4 (green), and an eighth rest, followed by a whole rest. The Bassoon part (bass clef) has a whole rest, followed by a whole note: G3 (purple), and a whole rest. A purple slur connects the two whole notes in the Bassoon part.

2. Create graph

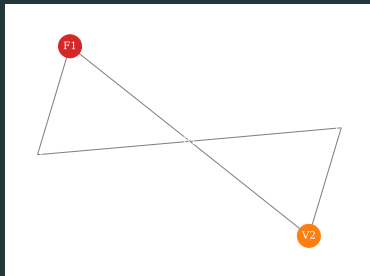
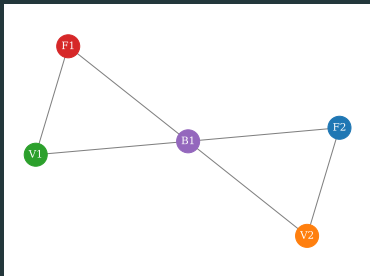
A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (top staff) begins with a quarter rest, followed by a sequence of eighth notes: G4, A4, B4, C5, D5, E5, and F5. The Violin part (middle staff) has a whole rest in the first measure, followed by two eighth notes (G4 and A4) in the second measure, and a half note (B4) in the third measure. The Bassoon part (bottom staff) has a whole rest in the first measure, followed by a half note (G3) in the second measure, and a whole note (F3) in the third measure. A purple line connects the Bassoon staff to the Violin staff, indicating a relationship between the two parts.



3. Solve graph

Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge



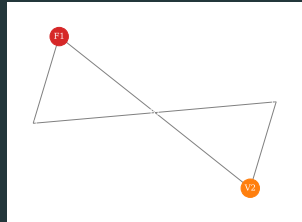
4. Construct arrangement

Flute

Violin

Bassoon

The score shows three staves in 4/4 time. The Flute staff has a sequence of eighth notes (G4, A4, B4, C5, D5, E5) with red stems, followed by a quarter rest. The Violin staff has a sequence of eighth notes (G3, A3, B3, C4, D4, E4) with green stems, followed by a quarter rest. The Bassoon staff has a sequence of eighth notes (G2, A2, B2, C3, D3, E3) with purple stems, followed by a quarter rest.



Flute

The score shows a single staff in 4/4 time. It contains a sequence of eighth notes (G4, A4, B4, C5, D5, E5) with red stems, followed by a quarter rest.

Results

Excerpt

Poco Adagio

Violin I
sotto voce
Poco Adagio

Violin II
sotto voce
Poco Adagio

Viola
sotto voce
Poco Adagio

Violoncello
sotto voce
Poco Adagio

6

cresc.

cresc.

cresc.

10

espress.

p

f

espress.

p

f

cresc.

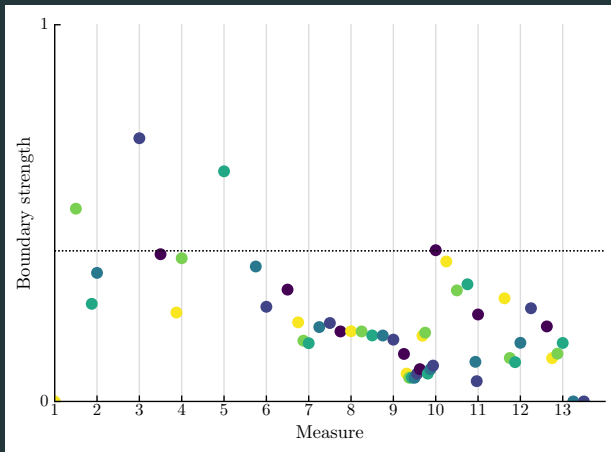
p

f

p

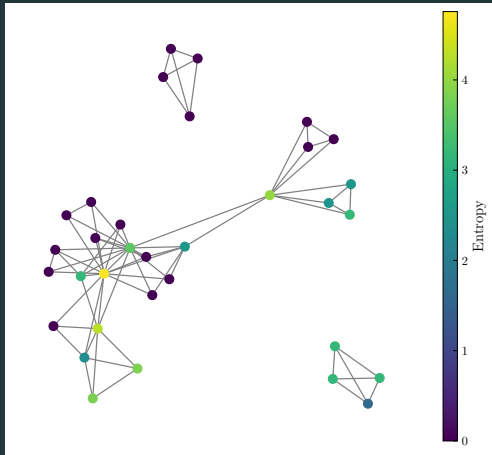
String Quartet No. 10 by Ludwig van Beethoven

Phrase detection



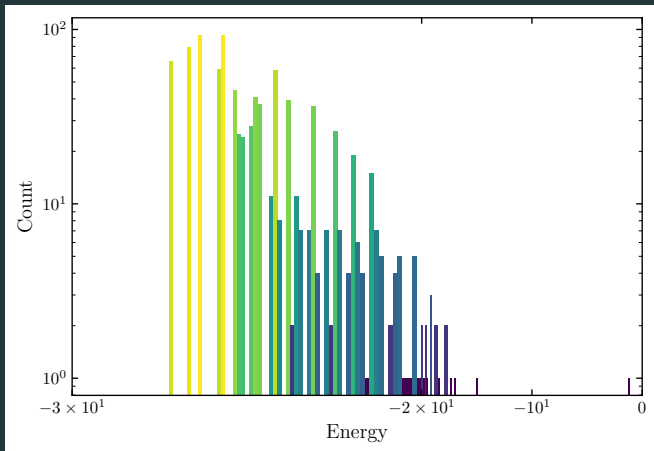
Boundary strengths for the Violin I part

Problem graph



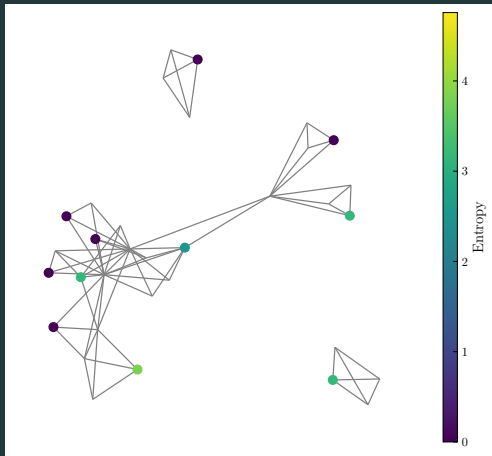
Problem graph with 33 nodes and 70 edges

Solutions



Returned solutions for 1000 reads

Example solution



Solution graph returning a subset of 11 nodes

Final arrangement

Poco Adagio
sotto voce

Violin I

Violin II

Viola

Violoncello

6

cresc.

cresc.

cresc.

10

espress.

p

f

cresc.

p

f

p

Selected phrases

Poco Adagio
sotto voce

7

espress.

cresc.

p

12

f

Final arrangement

Conclusions

Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms **huang'towards'2012**
- Removes skill barrier for music arrangement



Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements

pearce'towards'2001

Thank you!

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Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3} (S'_{\text{pitch}} + 2S'_{\text{IOI}})$$

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

lucas'ising'2014

$A/B \geq 2 \max(W)$ to weight the constraint term more heavily than any objective term

Phrase entropy

Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

li`automatic`2019

