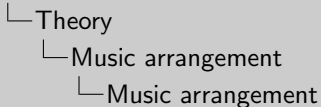


# Music Arrangement via Quantum Annealing

## └ Overview

- What is music arrangement? What is quantum annealing?
- Methods used to solve the music arrangement problem
- Preliminary results from application of the method
- Concluding thoughts about this process

# Music Arrangement via Quantum Annealing



- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**



- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet)

# Music Arrangement via Quantum Annealing

## └ Theory

### └ Quantum annealing

### └ Adiabatic quantum computing (AQC)

- Materials — heating and cooling a material to alter its physical properties
- Quantum — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

[Lucas, 2014]

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

# Music Arrangement via Quantum Annealing

## └ Theory

### └ Quantum annealing

### └ Quantum annealing

Ising model

$$H_p(\sigma^z) = \sum_{i,j=1}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

Initial state

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

[Lucas, 2014]

- How is this used to solve problems?
- Ising model, create a lattice of variables with two discrete values (spin up/down)
- Problem Hamiltonian, qubits  $\sigma^z$ , coupling strengths  $J_{ij}$  and field strengths  $h_i$
- Initial state is a superposition of all possible states
- If problem solution is encoded within the ground state, system will give solution after evolution

# Music Arrangement via Quantum Annealing

└ Theory

└ Quantum annealing

└ Quantum annealing



- What does this look like?
- Evolution of superposition to a particular state
- More efficiently escape from local minima via quantum tunneling
- Can solve harder problems with a more turbulent energy landscape

# Music Arrangement via Quantum Annealing

└ Theory

└ Quantum annealing

└ QUBO

## Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i,j} Q_{ij} x_i x_j + \sum_i Q_i x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

- How to encode a problem into a Hamiltonian?
- Similar form to the Ising model, but with binary variables (0 or 1)
- QUBO is a function to be minimised
- Set of binary variables  $x$ , matrix  $Q$  of real weights that describes interactions between variables
- Can read out variable values after evolution to give solution

# Music Arrangement via Quantum Annealing

└ Theory

└ Quantum annealing

How to combine them?

- How to apply quantum annealing to the problem of music arrangement?

# Music Arrangement via Quantum Annealing

## └ Methods

### └ Problem formulation

#### Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

- Formulating arrangement as a problem to be solved via annealing, four-step process
- Split parts into musical phrases
- Arrange phrases into a graph (nodes and edges)
- Solve corresponding graph problem using quantum computing
- Construct final arrangement from the solution returned



# Music Arrangement via Quantum Annealing

## └ Methods

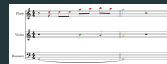
### └ 1. Split parts

#### 1. Split parts

##### Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

[Cambouropoulos, 2011]



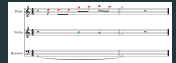
- First stage to separate each part of original score into phrases
- Phrases — smallest unit of music that preserves melody and structure
- Boundaries between phrases found using LBDM
- Measures the degree of change of a certain parameter between notes (explain equation)
- Check pitch and IOI
- Strengths above a threshold value are considered phrases

# Music Arrangement via Quantum Annealing

## └ Methods

### └ 2. Create graph

- Construct problem graph
- Each phrase becomes a node, edges between nodes if phrases overlap



# Music Arrangement via Quantum Annealing

## └ Methods

### └ 3. Solve graph

- Special case for reducing to a single monophonic part (one phrase played at a time)
- Solve problem graph using a graph theory problem called MIS
- Find largest subset of nodes such that no nodes within the subset are connected by an edge
- Enforces that only one simultaneous phrase can be played at once
- Constraint term enforces no edges
- Objective term is quantity to be minimised
- Lagrange parameters  $A/B$  determine the balance between constraint and objective

#### 3. Solve graph

##### Maximal Independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge

$$f(x) = A \sum_{i,j \in E} x_i x_j - B \sum_i x_i$$

[Lucas, 2014]

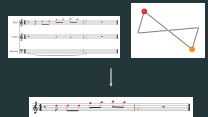


# Music Arrangement via Quantum Annealing

## └ Methods

### └ 4. Construct arrangement

- Take solution graph and combine selected nodes to create final arrangement

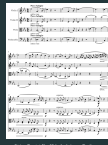


# Music Arrangement via Quantum Annealing

## Results

### Excerpt

Excerpt

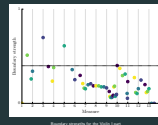


- Excerpt of String Quartet No. 10 in E-flat major, Op. 74, by Ludwig van Beethoven
- Chosen due to its relatively simple structure and smaller instrumentation, keeping the problem graph small

# Music Arrangement via Quantum Annealing

## └ Results

### └ Phrase detection



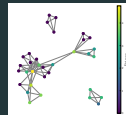
- Example of the LBDM finding suitable boundaries for phrases
- Threshold value of 0.4 chosen, finds five phrases

# Music Arrangement via Quantum Annealing

## └ Results

### └ Problem graph

Problem graph



Problem graph with 33 nodes and 70 edges

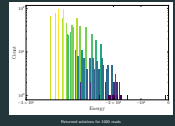
- 33 identified phrases (nodes) with 70 overlaps (edges)
- Nodes are weighted by the phrase entropy, how musically interesting the distribution of notes is

# Music Arrangement via Quantum Annealing

└ Results

└ Solutions

Solutions



- Histogram of the returned solutions, only energies below zero shown
- Distribution of solutions due to the stochastic nature of annealing
- Not always guaranteed the ground state during evolution
- Lowest energy solution  $-26.8$  with a degeneracy of 34



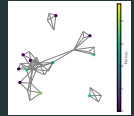
# Music Arrangement via Quantum Annealing

## └ Results

### └ Example solution

- Selected nodes from one of the lowest energy solutions
- Note in the cliques only one node could be selected

Example solution




# Music Arrangement via Quantum Annealing

└ Results

└ Final arrangement

Final arrangement



Selected phrases

Final arrangement

- Selected phrases from solution graph highlighted
- Phrases concatenated to create the final arrangement

# Music Arrangement via Quantum Annealing

## └ Conclusions

## └ Conclusions

### Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms [Huang et al., 2012]
- Removes skill barrier for music arrangement



- Valid reduction gained from implemented method
- Monophonic lowest-energy solution
- Phrase identification and selection result in musically interesting final arrangement
- Displays some advantages over classical algorithms
- Does not require training data, costly in both time and resources
- Faster solve time compared to similarly-sized problems
- Method and more advanced versions of it removes the skill barrier for music arrangement

# Music Arrangement via Quantum Annealing

## └ Conclusions

## └ Future work

### Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements [Pearce and Wiggins, 2001]

- Problem size — full piece has upwards of 1000 phrases, compare solve times with classical algorithms
- LBDM parameters — phrases should be short and similar in length, prevents important notes being hidden in long phrases with low entropy
- Physical limitations — note ranges, note change speed
- Multiple parts — need different problem formulation, colouring problem with each colour representing a reduced part
- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

# Music Arrangement via Quantum Annealing

└ LBDM

- Boundaries always taken at beginning/end of piece
- Weightings derived by trial and error

LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3} (S'_{\text{pitch}} + 2S'_{\text{ch}})$$

[Cambouroupoulos, 2011]

# Music Arrangement via Quantum Annealing

└ Phrase entropy

- Shannon entropy units in bits due to  $\log_2$
- Distribution calculated for pitch and duration

Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

[Li et al., 2019]