Music Arrangement via Quantum Annealing

Lucas Kirby

18th January 2025

Durham University

Music Arrangement via Quantum Annealing

Lucas Kirby
18th January 2025
Durham University

Overview

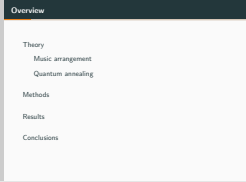
Theory

- Music arrangement
- Quantum annealing

Methods

Results

Conclusions

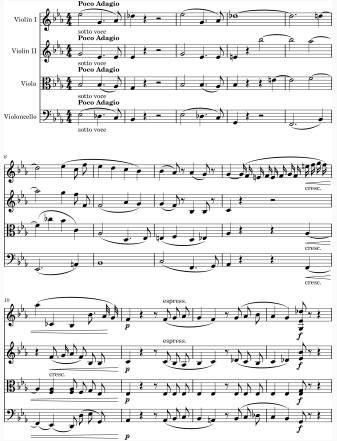


Theory

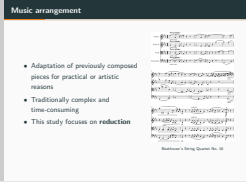
Theory

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**



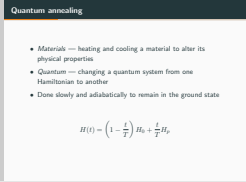
Beethoven's String Quartet No. 10



Quantum annealing

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$



QUBO

Encodes problem solution into Hamiltonian's ground state

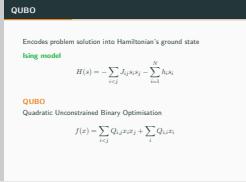
Ising model

$$H(s) = -\sum_{i<j} J_{ij} s_i s_j - \sum_{i=1}^{N} h_i s_i$$

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j} Q_{i,j} x_i x_j + \sum_i Q_{i,i} x_i$$



1. Lattice of spins with two discrete values
2. Coupling strengths J_{ij} and field strengths h_i
3. Function to be minimised

How to combine them?

Methods

Methods

Problem formulation

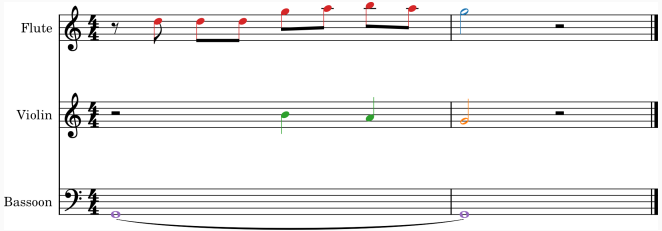
1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

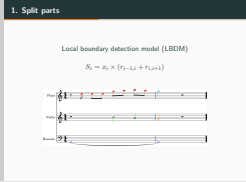
Problem formulation
1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

1. Split parts

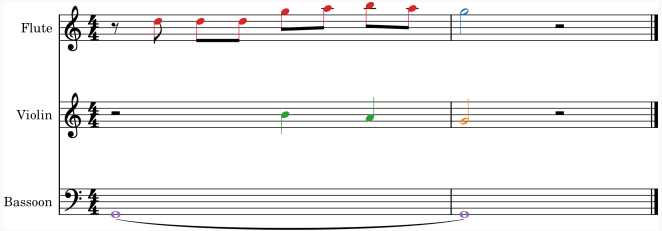
Local boundary detection model (LBDM)

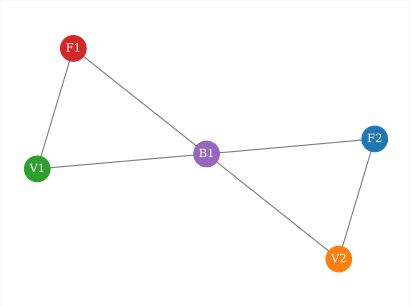
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

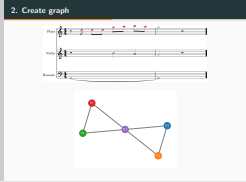




2. Create graph





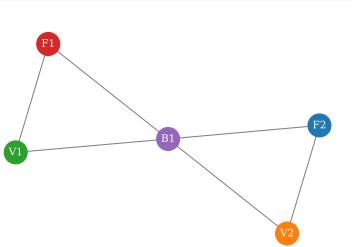


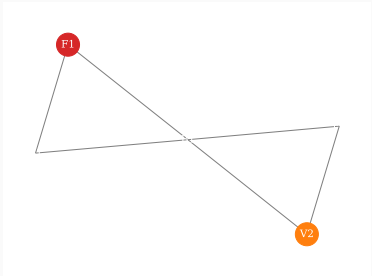
Each phrase becomes a nodeEdges between nodes if phrases overlap

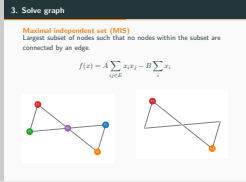
3. Solve graph

Maximal independent set (MIS)
Largest subset of nodes such that no nodes within the subset are connected by an edge.

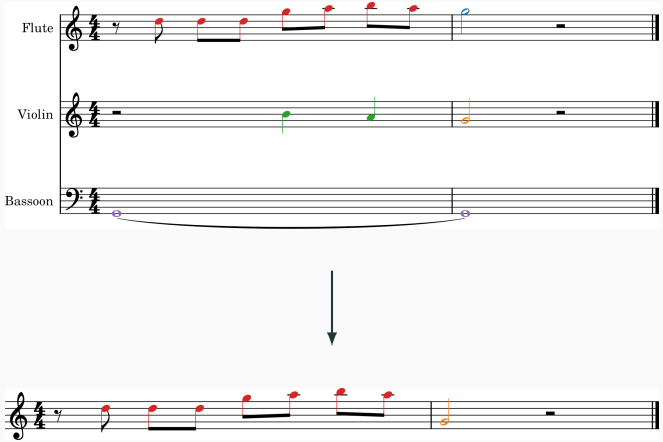
$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$







4. Construct arrangement

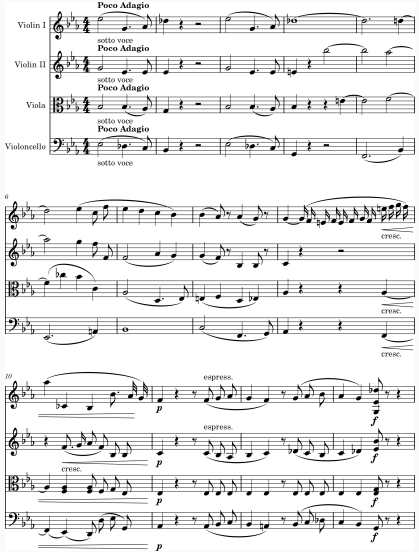


Take selected nodes and combine to create final arrangement

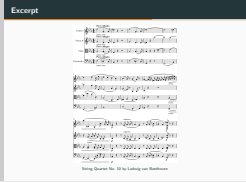
Results

Results

Excerpt

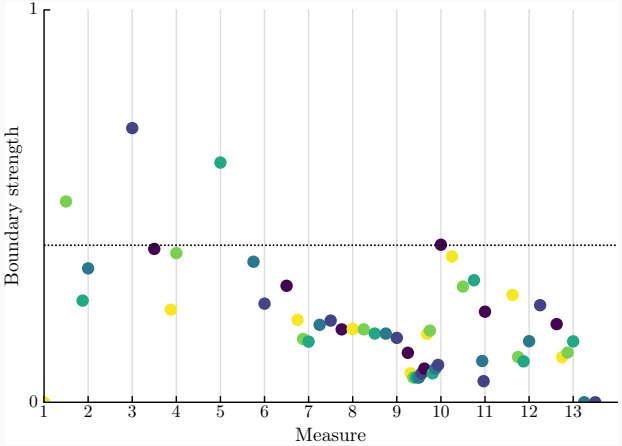


String Quartet No. 10 by Ludwig van Beethoven

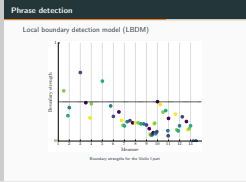


Phrase detection

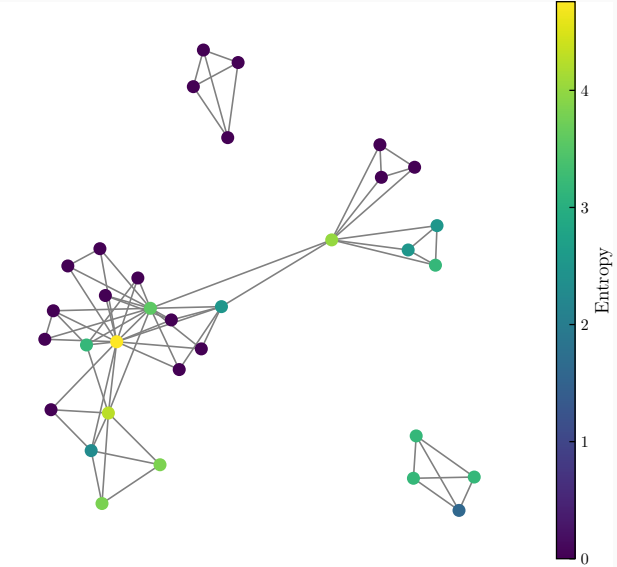
Local boundary detection model (LBDM)

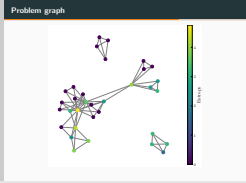


Boundary strengths for the Violin I part

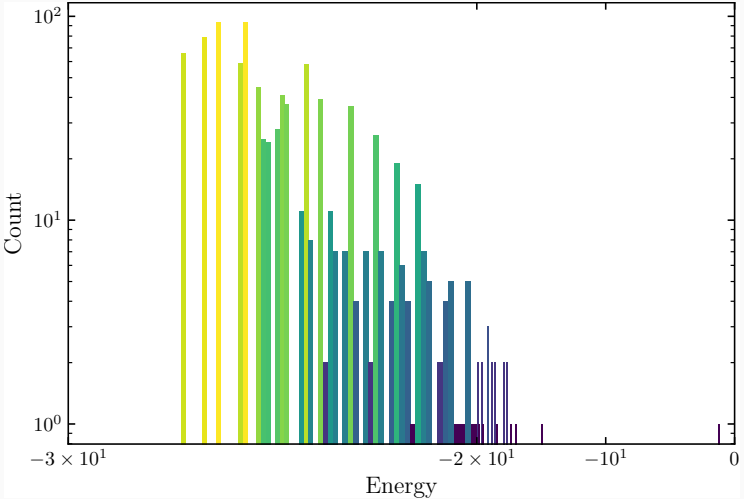


Problem graph

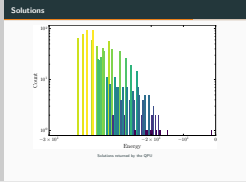




Solutions



Solutions returned by the QPU



Lowest energy solution was -26.8 with a degeneracy of 34

Example solution

Example solution

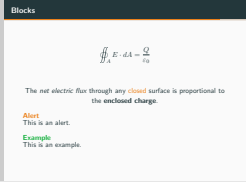
Blocks

$$\oiint_A E \cdot dA = \frac{Q}{\varepsilon_0}$$

The *net electric flux* through any closed surface is proportional to the **enclosed charge**.

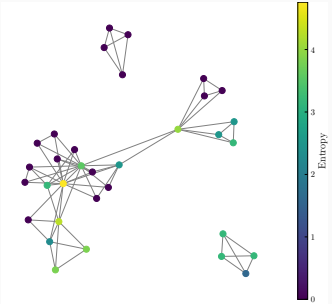
Alert
This is an alert.

Example
This is an example.



Apperance sync

- Volume rate of flow equal to divergence
- Summed over entire volume
- Equal to net flow across the boundary



Source: Wikimedia Commons

$$\iiint_V \nabla \cdot \mathbf{F}\, dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$$

Apperance sync
• Volume rate of flow equal to divergence
• Summed over entire volume
• Equal to net flow across the boundary
Source: Wikimedia Commons
$\iiint_V \nabla \cdot \mathbf{F}\, dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$

Conclusions

Equation gather

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_0 I$$