

Quantum Annealing for Music Arrangement

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4 March 2025

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Overview

Theory

Adiabatic quantum computing

Quantum annealing

Motivations

Music arrangement

Method

Results

Conclusions

Theory

Adiabatic quantum computing

Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.¹

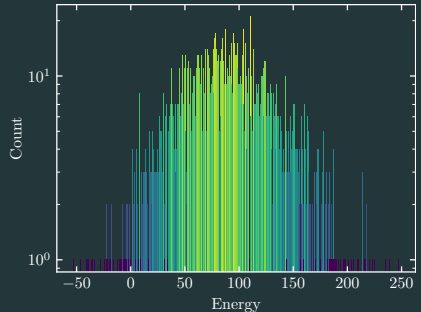
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

¹Born and Fock, 'Beweis des Adiabatenatzes'.

Quantum annealing

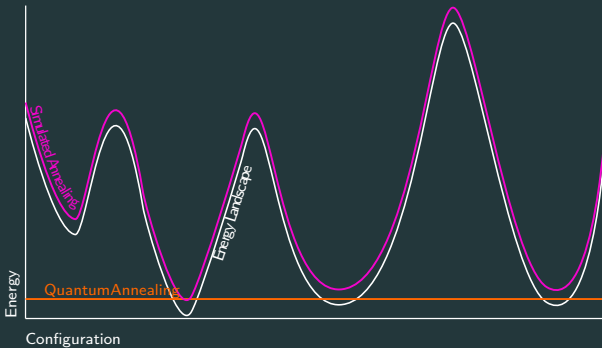
- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic



Distribution of 2000 solution energies

Advantages

- Find the ground state of complicated Hamiltonians
- Quantum tunneling avoids local minima



By Brianlechthaler - Own work, CC BY-SA 4.0,
<https://commons.wikimedia.org/w/index.php?curid=112382195>

Ising model

Lattice of variables with two discrete values

Initial Hamiltonian

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

Problem Hamiltonian

$$H_p = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

Quadratic Unconstrained Binary Optimisation

Vector x of qubits, matrix Q of weights

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Aim to minimise this function
- Difficult to solve analytically
- Mapped to H_p using simple change of variable
- Encodes problem solution into Hamiltonian's ground state

Motivations

What problems can we solve?

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally difficult and time-consuming
- *Reduction* can be shown to be computationally complex²



www.freepik.com

²Moses and Demaine, 'Computational Complexity of Arranging Music'.

Motivations

- Already exist classical methods of automatic arrangement³
- Quantum annealing used to generate music⁴
- Field of quantum computer music is very new⁵
- Novel adaption of this method to a new problem
- *This has never been done before!*

³Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'; Nakamura and Yoshii, 'Statistical piano reduction controlling performance difficulty'; Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

⁴Freedline, 'Algorhythms'; Arya et al., 'Music Composition Using Quantum Annealing'.

⁵Miranda, *Quantum Computer Music*.

Method

Aims

- Arrange a musical score for a smaller ensemble
- All notes are taken from the original score
- Each instrument can only play one note at a time



Joseph Haydn playing in a string quartet,
painting from the StaatsMuseum,
Vienna

Method

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate optimisation problem
4. Solve problem using QPU
5. Construct arrangement from solution

1. Split score

- Musical phrases chosen as smallest unit of music
- Preserve melody and structure when rearranged

Local boundary detection model (LBDM)⁶

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

The diagram shows a musical score for three instruments: Flute, Violin, and Bassoon, all in 4/4 time. The Flute part has a red box around a five-note ascending phrase and a blue box around a whole rest. The Violin part has a green box around a whole rest followed by two eighth notes, and an orange box around a whole rest. The Bassoon part has a purple box around a whole rest. A long purple slur connects the end of the Bassoon part to the beginning of the Flute part, indicating a cross-instrument phrase boundary.

⁶Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

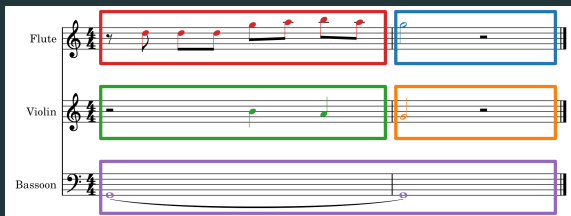
2. Create graph

- Vertices (nodes) connected by edges
- Models pairwise relations between objects

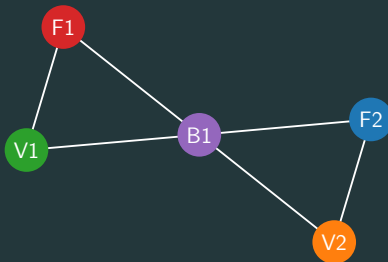
The image displays a musical score for three instruments: Flute, Violin, and Bassoon, all in 4/4 time. The score is divided into two measures. The first measure contains musical notation for each instrument, while the second measure contains rests. Colored rectangular boxes highlight specific phrases: a red box for the first measure of the Flute, a blue box for the second measure of the Flute, a green box for the first measure of the Violin, an orange box for the second measure of the Violin, and a purple box for the first measure of the Bassoon. A curved line with arrows at both ends connects the end of the purple box to the start of the blue box, indicating an edge between these two phrases.

- Nodes — phrases
- Edges — overlap between phrases

2. Create graph



A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a red box around the first four measures and a blue box around the last measure. The Violin part (treble clef) has a green box around the first four measures and an orange box around the last measure. The Bassoon part (bass clef) has a purple box around the entire staff, which contains a single note in the first measure and a whole rest for the rest of the piece.



3. Create optimisation problem

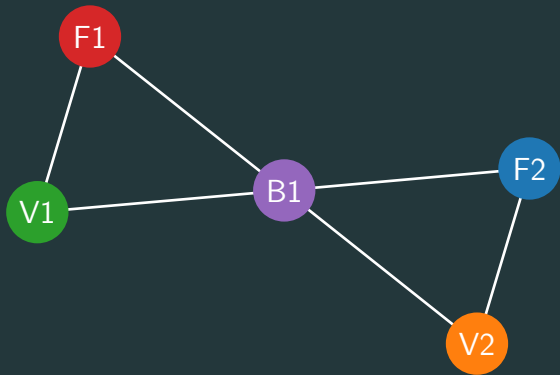
Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

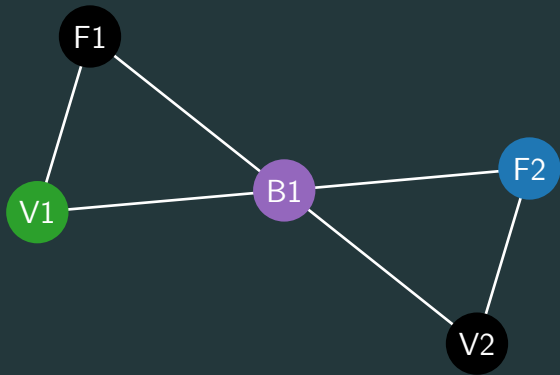
$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} f(x) = & +A \sum_{v \in V} \left(1 - \sum_{i=1}^n x_{v,i} \right)^2 + B \sum_{(u,v) \in E} \sum_{i=1}^n x_{u,i} x_{v,i} \\ & - C \sum_{v \in V} \sum_{i=1}^n W_v x_{v,i} - D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^n \sum_{j=1}^n x_{u,i} x_{v,j} \end{aligned}$$

3. Create optimisation problem

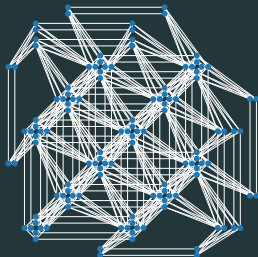


3. Create optimisation problem



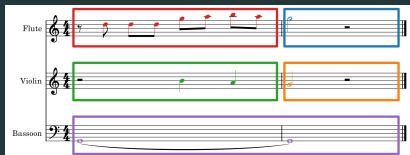
4. Solve problem

- Problem embedded onto D-Wave quantum hardware
- Quantum annealer optimises QUBO formulation
- Returns a sample set of results
- Run many times to find lowest-energy solution

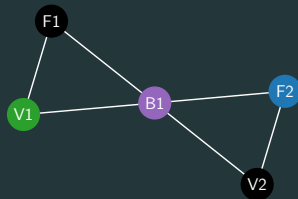


D-Wave Advantage QPU topology. Own work.

5. Construct arrangement



A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (top staff) has a red box around the first measure and a blue box around the second measure. The Violin part (middle staff) has a green box around the first measure and an orange box around the second measure. The Bassoon part (bottom staff) has a purple box around the first measure. The measures are connected by a horizontal line, indicating they are part of the same musical phrase.



A musical score for two instruments: Flute and Violin, in 4/4 time. The Flute part (top staff) has a red box around the first measure. The Violin part (bottom staff) has an orange box around the second measure. The measures are connected by a horizontal line, indicating they are part of the same musical phrase.

Results

Score

- Smaller ensemble chosen for problem size
- Well-defined musical structure
- Reduction to three instruments

Quartet No. 1 in B \flat major

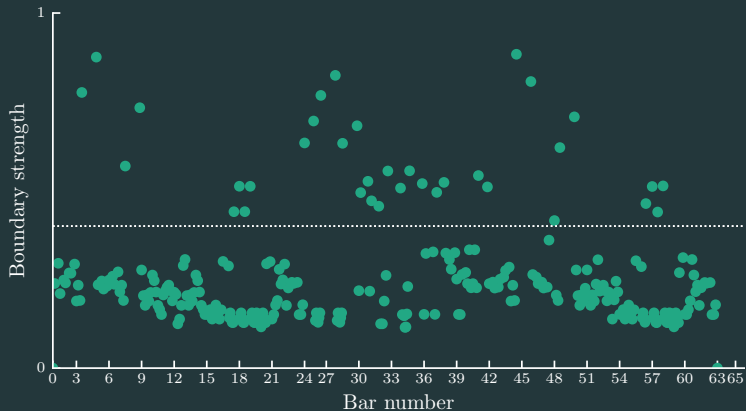
Joseph Haydn

Prezio

The image displays a musical score for a string quartet. The title is 'Quartet No. 1 in B \flat major' by Joseph Haydn. The score is written for four instruments: Violin I, Violin II, Viola, and Cello. The key signature is one flat (B \flat major), and the time signature is 4/4. The score is divided into three systems. The first system is marked 'Prezio' and the second system is marked 'f'. The third system is marked 'ff'. The notation includes various musical symbols such as notes, rests, and dynamic markings.

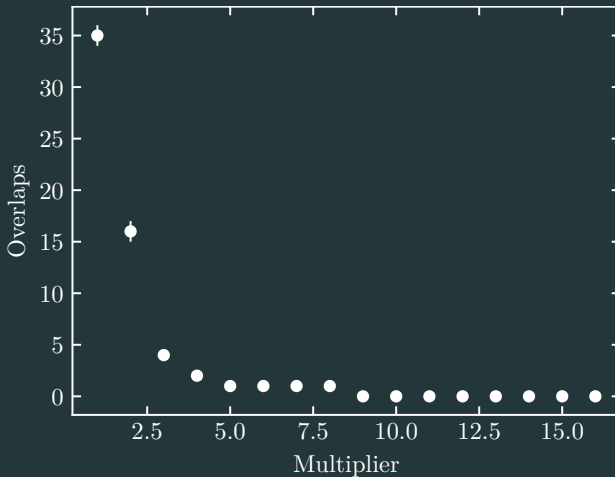
Quartet No. 1 in B \flat major by Joseph Haydn

Phrase detection



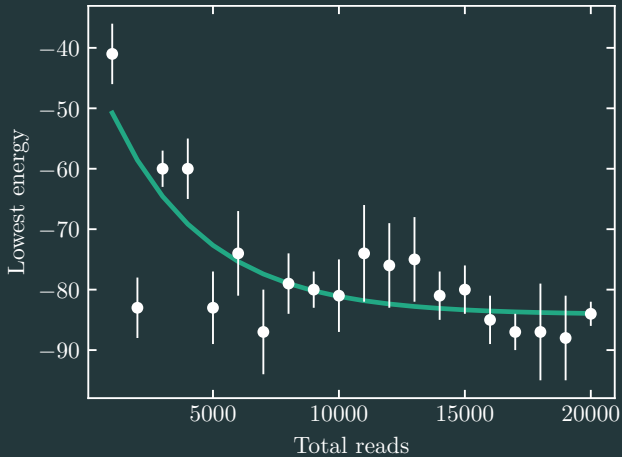
Boundary strengths for the Violin I part

QUBO parameter variation



Variation of the edge constraint Lagrange parameter B

Optimisation



Variation of the number of QPU reads, with the lowest-energy solution found

Conclusions

Conclusions

- Successful novel application of quantum annealing
- QPU returns low-energy samples
- Necessary constraints for a valid arrangement fulfilled
- Still very new technology, does not show quantum advantage (yet)

Future work

- Variation in problem size
- Comparison to classical methods
- Lagrange parameter tuning
- Qualitative judgement of computer arrangements⁷

⁷Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

Thank you!

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Arya, Ashish et al. **‘Music Composition Using Quantum Annealing’**. In: *arXiv* (Jan. 2022). DOI:

10.48550/arXiv.2201.10557. (Visited on 26/10/2024).



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-  Freedline, Alex. **‘Algorhythms: Generating Music with D-Wave’s Quantum Annealer’**. en. In: *MIT 6.s089—Intro to Quantum Computing* (Feb. 2021).
-  Huang, Jiun-Long, Shih-Chuan Chiu and Man-Kwan Shan. **‘Towards an automatic music arrangement framework using score reduction’**. In: *ACM Trans. Multimedia Comput. Commun. Appl.* 8.1 (Feb. 2012), 8:1–8:23. ISSN: 1551-6857. DOI: 10.1145/2071396.2071404. (Visited on 05/12/2024).



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Moses, William S. and Erik D. Demaine. **‘Computational Complexity of Arranging Music’**. In: *arXiv* (July 2016).

arXiv:1607.04220. DOI: 10.48550/arXiv.1607.04220. (Visited on 09/11/2024).



Nakamura, Eita and Kazuyoshi Yoshii. **‘Statistical piano reduction controlling performance difficulty’**. en. In: *APSIPA Transactions on Signal and Information Processing* 7 (Jan. 2018),

e13. ISSN: 2048-7703. DOI: 10.1017/ATSIP.2018.18. (Visited on 17/12/2024).



Pearce, M. and Geraint A. Wiggins. **‘Towards A Framework for the Evaluation of Machine Compositions’**. In:

Proceedings of the AISB’01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences. 2001.

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3} (S'_{\text{pitch}} + 2S'_{\text{IOI}})$$

Phrase entropy

x_i — parameter x of note i

Shannon entropy

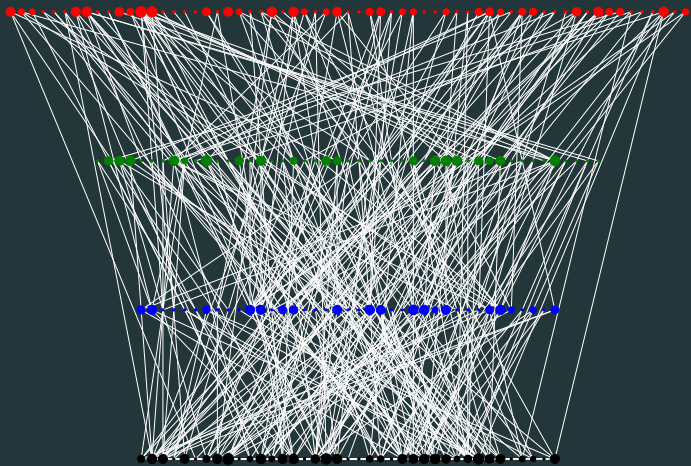
$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

⁹Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

Solution graph



This musical score is for the piece 'The Rose Tree' from the 'The Rose Tree' album. It is a 4/4 piece in G major, with a tempo of 120 beats per minute. The score is arranged for a woodwind ensemble consisting of Flute, Oboe, Bassoon, Flute I, Oboe II, and Bassoon II. The score is divided into three systems, each with a rehearsal mark (5, 20, and 24). The first system (measures 1-4) features the Flute, Oboe, and Bassoon. The second system (measures 5-8) features the Flute I, Oboe II, and Bassoon II. The third system (measures 9-12) features the Flute I, Oboe II, and Bassoon II. The score is written in a standard musical notation style, with a key signature of one sharp (F#) and a time signature of 4/4. The music is in a major key, and the tempo is marked as 120 beats per minute. The score is divided into three systems, each with a rehearsal mark (5, 20, and 24). The first system (measures 1-4) features the Flute, Oboe, and Bassoon. The second system (measures 5-8) features the Flute I, Oboe II, and Bassoon II. The third system (measures 9-12) features the Flute I, Oboe II, and Bassoon II. The score is written in a standard musical notation style, with a key signature of one sharp (F#) and a time signature of 4/4. The music is in a major key, and the tempo is marked as 120 beats per minute.