Quantum Annealing for Music Arrangement

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4 March 2025

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Overview

Motivations

Motivations

- Small lit review[1]
- Quantum computer music
- My own novel adaption of the method
- THIS IS MY OWN IDEA

[1] Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

Theory

Adiabatic quantum computing

Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.[2]

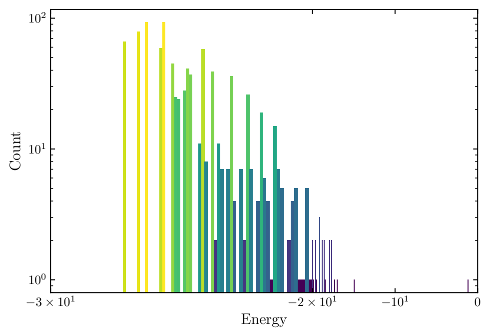
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

[2] Born and Fock, 'Beweis des Adiabatensatzes'.

Quantum annealing

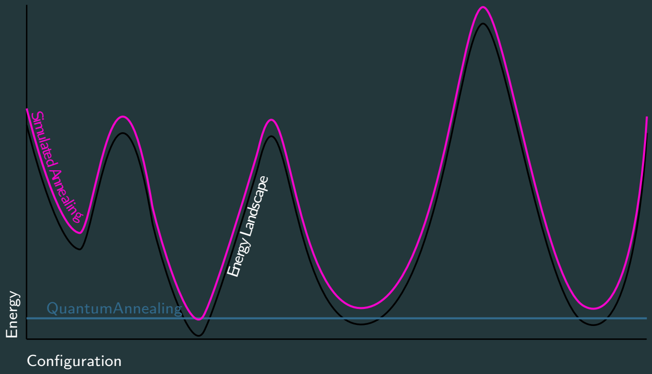
- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic



Own work

Advantages

- Find the ground state of a complicated Hamiltonian
- Quantum tunneling overcomes local minima



Ising model

Initial Hamiltonian

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

Problem Hamiltonian

$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^{N} h_i \sigma_i^z$$

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Difficult to solve analytically
- Mapped to H_p using simple change of variable
- Encodes problem solution into Hamiltonian's ground state

What problems can we solve?

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- *Reduction* can be shown to be computationally complex[3]

REPLACE WITH HAYDN

[3] Moses and Demaine, 'Computational Complexity of Arranging Music'

Methods

Problem formulation

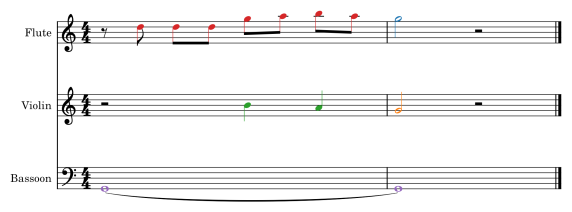
1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate graph optimisation problem
4. Solve problem using QPU
5. Construct arrangement from solution

1. Split score

- Musical phrases chosen as smallest unit of music
- Preserve melody and structure when arranged

Local boundary detection model (LBDM)[4]

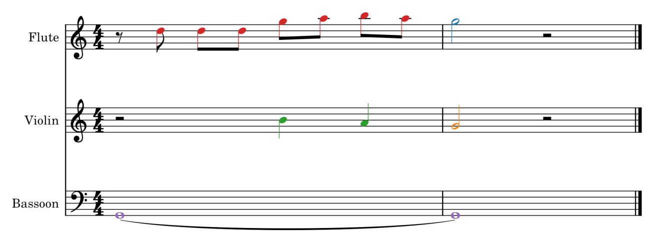
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

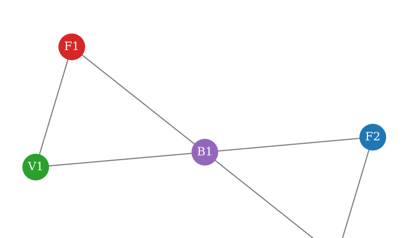


[4] Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

2. Create graph

- Vertices (nodes) connected by edges
- Models pairwise relations between objects





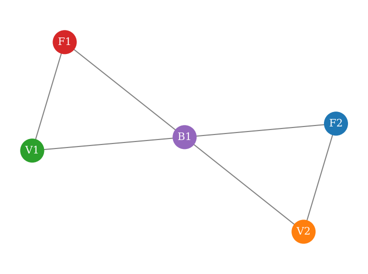
3. Create optimisation problem

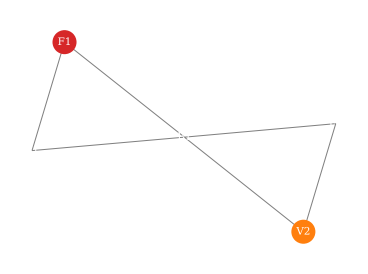
Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

$$
\begin{aligned}
f(x) = & A \sum_{v \in V} \left(1 - \sum_{i=1}^{n} x_{v,i} \right)^2 + B \sum_{(u,v) \in E} \sum_{i=1}^{n} x_{u,i} x_{v,i} \\
& - C \sum_{v \in V} \sum_{i=1}^{n} W_v x_{v,i} - D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{u,i} x_{v,j}
\end{aligned}
$$

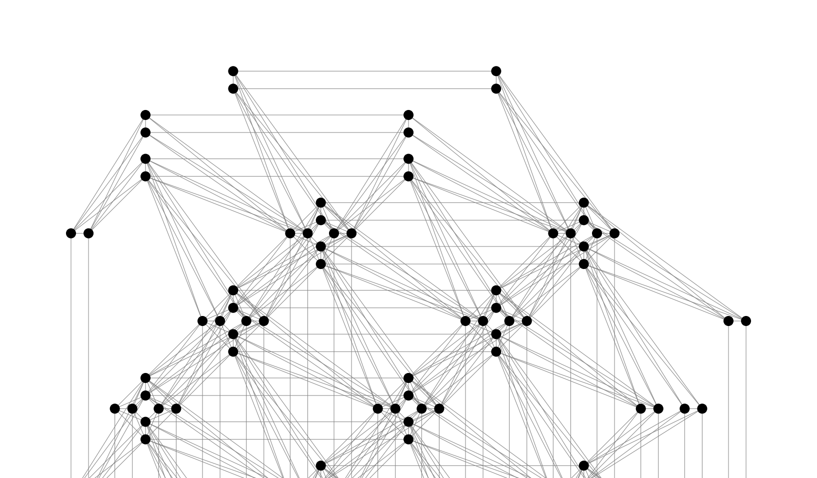
3. Create optimisation problem



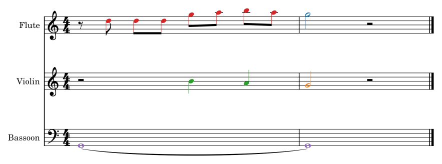


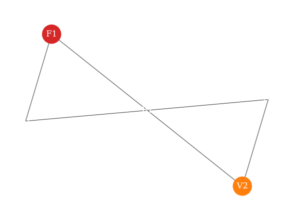
4. Solve problem

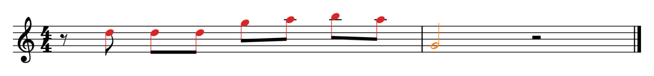
- Problem embedded onto D-Wave quantum hardware
- True quantum annealer optimises QUBO formulation
- Returns a sampleset of results



5. Construct arrangement





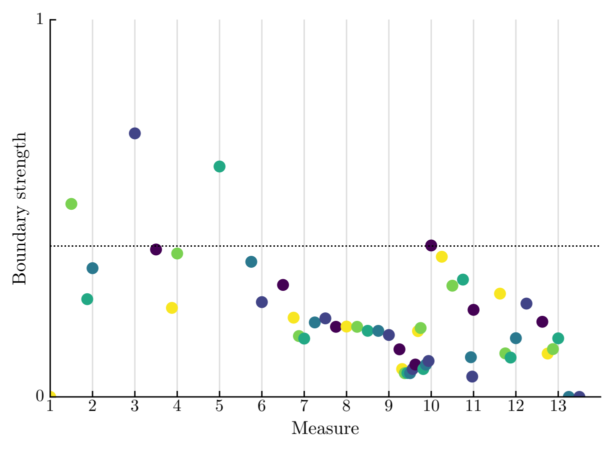


Results

Excerpt

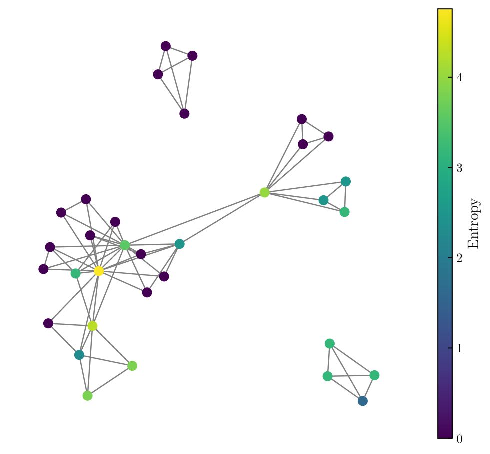
String Quartet No. 10 by Ludwig van Beethoven

Phrase detection



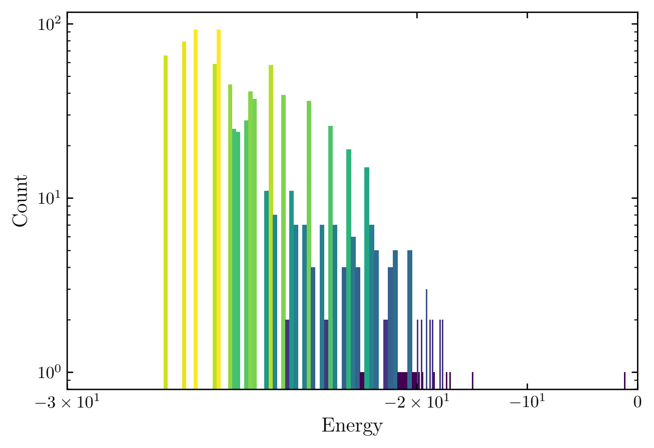
Boundary strengths for the Violin I part

Problem graph



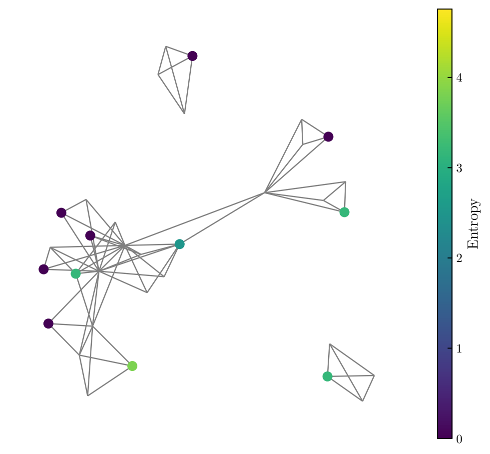
Problem graph with 33 nodes and 70 edges

Solutions



Returned solutions for 1000 reads

Example solution



Solution graph returning a subset of 11 nodes

Final arrangement

Selected phrases

Final arrangement

Conclusions

Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms Huang, Chiu and Shan, ‘Towards an automatic music arrangement framework using score reduction’
- Removes skill barrier for music arrangement

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'

Thank you!

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LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$
$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'

MIS

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

Lucas, 'Ising formulations of many NP problems'

$A/B >= 2\max(W)$ to weight the constraint term more heavily than any objective term

Phrase entropy

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

Li et al., ‘Automatic Piano Reduction of Orchestral Music Based on Musical Entropy’

References i

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