Quantum Annealing for Music Arrangement

Lucas Kirby

4 March 2025

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Overview

Theory

Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.[1]

[1] Born and Fock, 'Beweis des Adiabatensatzes'.

Adiabatic quantum computing

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- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state

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- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

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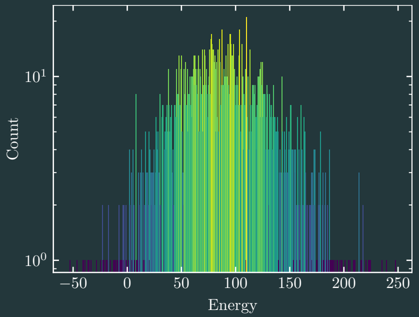
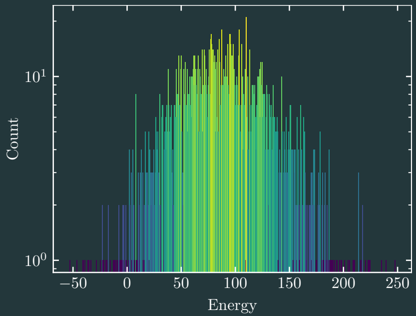
Quantum annealing



Distribution of 2000 solution energies

Quantum annealing

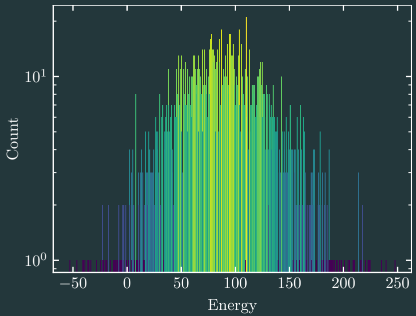
- Relaxes the adiabaticity



Distribution of 2000 solution energies

Quantum annealing

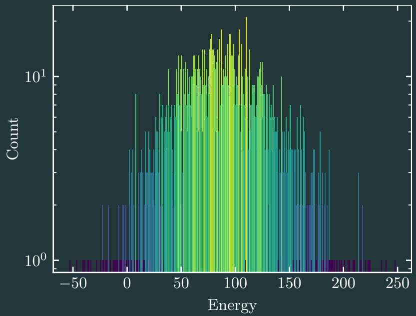
- Relaxes the adiabaticity
- Rate of change determined heuristically



Distribution of 2000 solution energies

Quantum annealing

- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic



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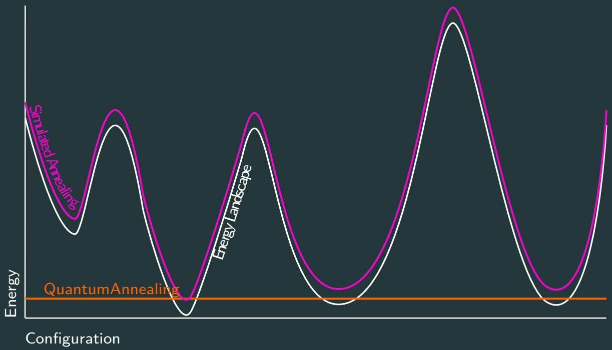
Advantages



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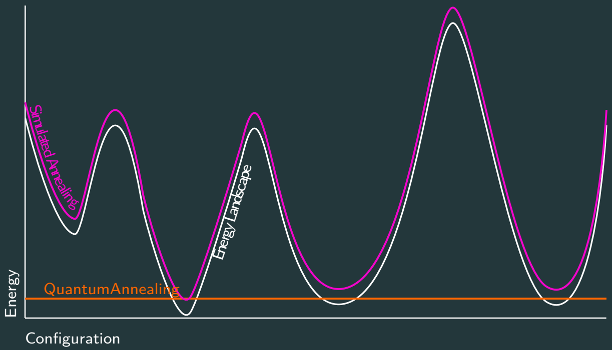
Advantages

- Find the ground state of complicated Hamiltonians



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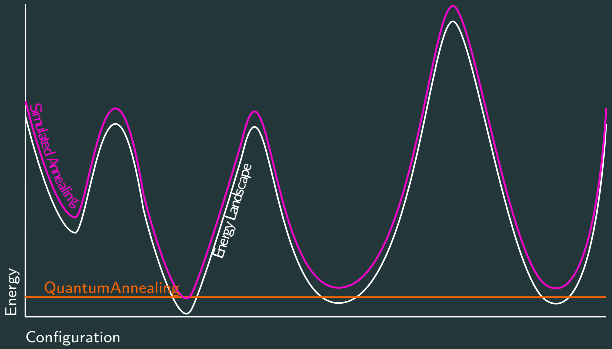
Advantages

- Find the ground state of complicated Hamiltonians
- Quantum tunneling avoids local minima



Ising model

Lattice of variables with two discrete values

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Initial Hamiltonian

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

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Problem Hamiltonian

$$H_p = \sum_{i<j}^{N} J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^{N} h_i \sigma_i^z$$

QUBO

QUBO

Quadratic Unconstrained Binary Optimisation

Vector x of qubits, matrix Q of weights

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

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- Aim to minimise this function
- Difficult to solve analytically
- Mapped to H_p using simple change of variable
- Encodes problem solution into Hamiltonian's ground state

Motivations

What problems can we solve?

Music arrangement

www.freepik.com

[2] Moses and Demaine, ‘Computational Complexity of Arranging Music’.

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons

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Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally difficult and time-consuming
- *Reduction* can be shown to be computationally complex[2]

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Motivations

[3] Huang, Chiu and Shan, ‘Towards an automatic music arrangement framework using score reduction’; Nakamura and Yoshii, ‘Statistical piano reduction controlling performance difficulty’; Li et al., ‘Automatic Piano Reduction of Orchestral Music Based on Musical Entropy’.

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Motivations

- Already exist classical methods of automatic arrangement[3]

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- Field of quantum computer music is very new[5]
- Novel adaption of this method to a new problem
- *This has never been done before!*

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Method

Aims

Joseph Haydn playing in a string quartet,
painting from the StaatsMuseum, Vienna

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- All notes are taken from the original score
- Each instrument can only play one note at a time

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5. Construct arrangement from solution

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Local boundary detection model (LBDM)[6]

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

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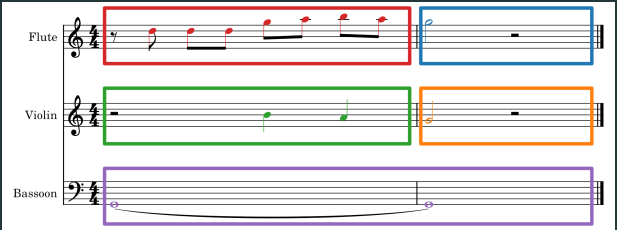

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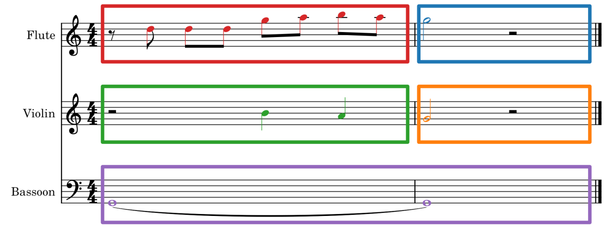

- Nodes — phrases
- Edges — overlap between phrases

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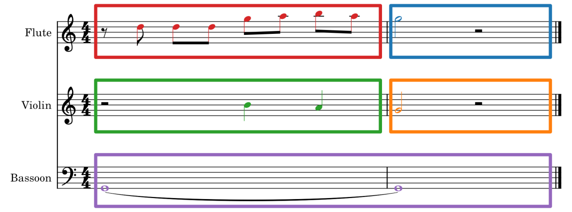

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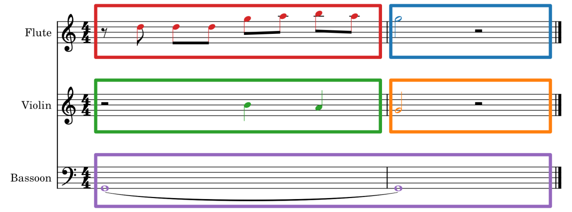

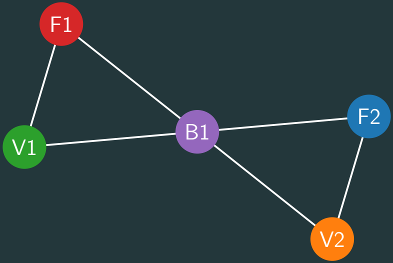

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Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

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$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

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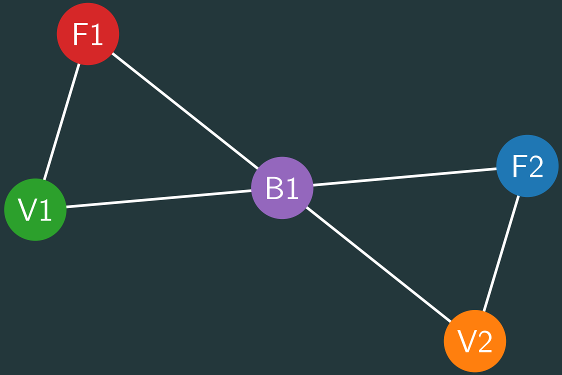

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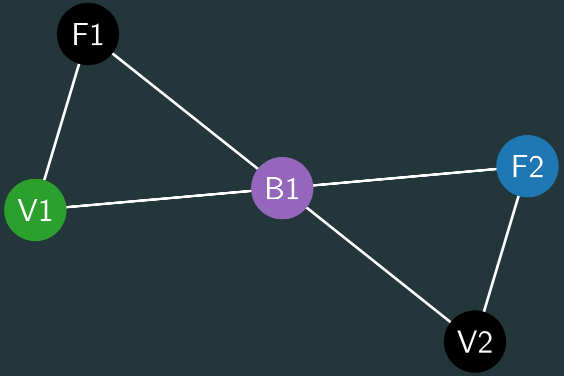

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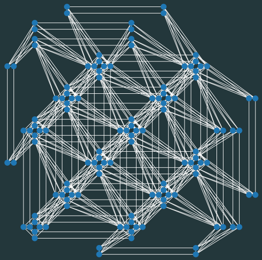
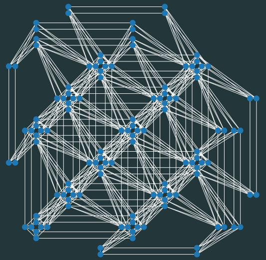


D-Wave Advantage QPU topology. Own work.

4. Solve problem

- Problem embedded onto D-Wave quantum hardware



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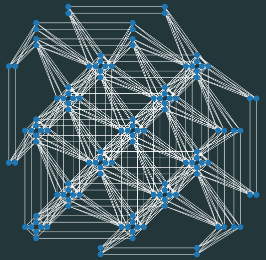

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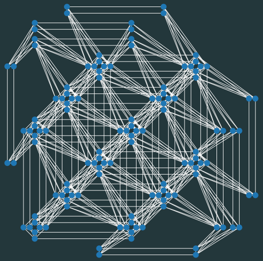
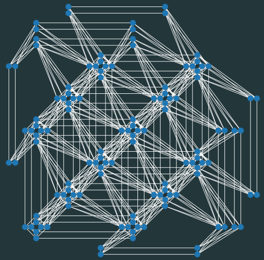


D-Wave Advantage QPU topology. Own work.

4. Solve problem

- Problem embedded onto D-Wave quantum hardware
- Quantum annealer optimises QUBO formulation
- Returns a sampleset of results
- Run many times to find lowest-energy solution



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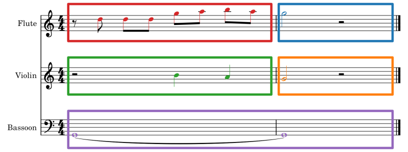

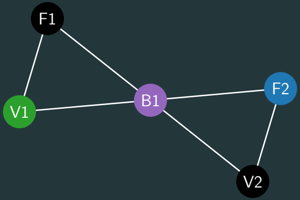

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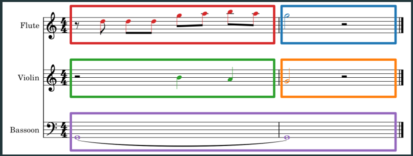
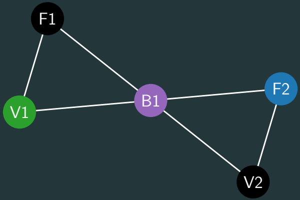
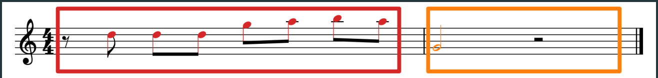






Results

Score

Quartet No. 1 in Bb major by Joseph Haydn

Score

- Smaller ensemble chosen for problem size

Quartet No. 1 in Bb major by Joseph Haydn

Score

- Smaller ensemble chosen for problem size
- Well-defined musical structure

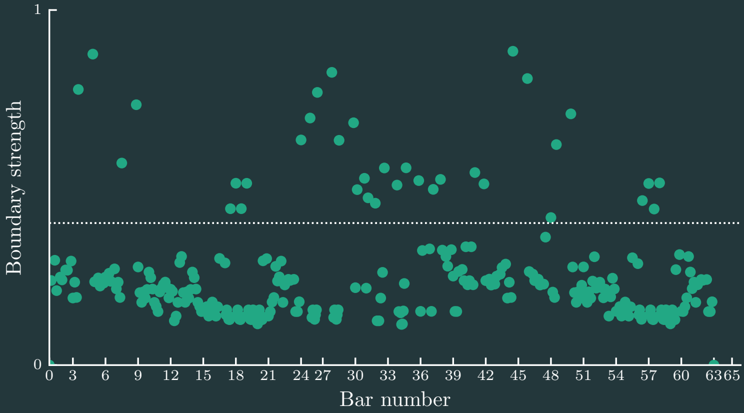
Quartet No. 1 in Bb major by Joseph Haydn

Score

- Smaller ensemble chosen for problem size
- Well-defined musical structure
- Reduction to three instruments

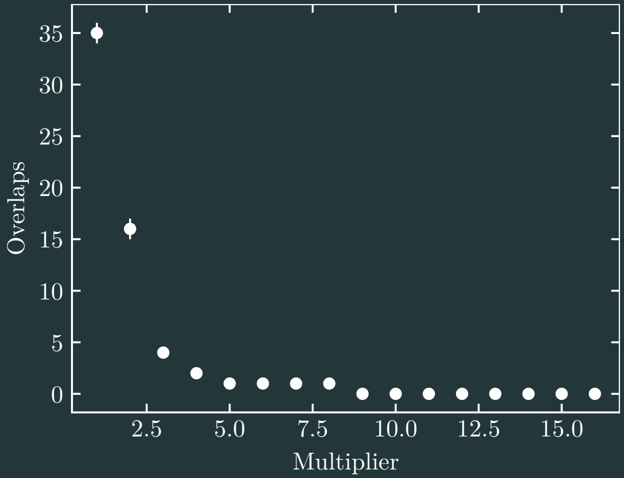
Quartet No. 1 in Bb major by Joseph Haydn

Phrase detection



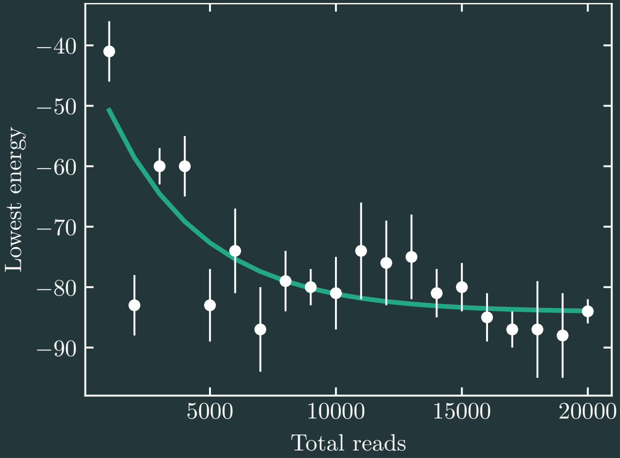
Boundary strengths for the Violin I part

QUBO parameter variation



Variation of the edge constraint Lagrange parameter B

Optimisation



Variation of the number of QPU reads, with the lowest-energy solution found

Conclusions

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Conclusions

- Successful novel application of quantum annealing
- QPU returns low-energy samples
- Necessary constraints for a valid arrangement fulfilled
- Still very new technology, does not show quantum advantage (yet)

Future work

[7] Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

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- Variation in problem size

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Future work

- Variation in problem size
- Comparison to classical methods
- Lagrange parameter tuning
- Qualitative judgement of computer arrangements[7]

[7] Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

Thank you!

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4 March 2025

Department of Physics, Durham University

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LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$
$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

[8] Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

Phrase entropy

x_i — parameter x of note i

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

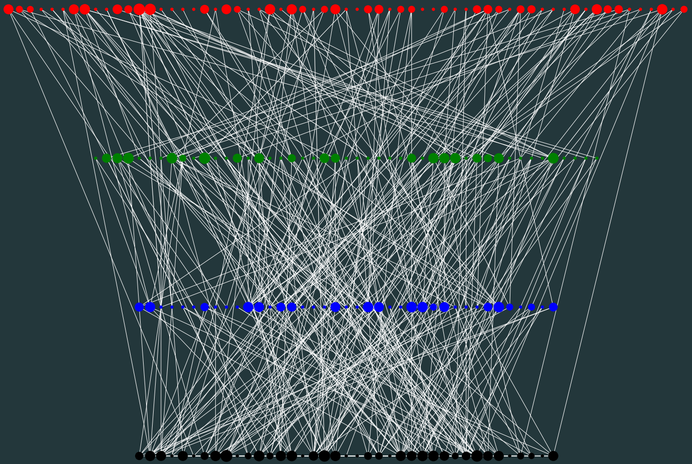
Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

[9]

[9] Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

Solution graph



Solution score