# Quantum Annealing for Music Arrangement

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for Music Arrangement

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# Overview

- Theory
  - Adiabatic quantum computing
  - Quantum annealing
- Motivations
  - Music arrangement
- Method
- Results
- Conclusions





Overview
Theory
Adiabatic quantum computing
Quantum annealing
Motivations
Music arrangement
Method
Results
Conclusions

- AQC, more general umbrella term for the technique
- Quantum annealing as a subset of AQC and what that involves
- Music arrangement and why we're looking at this problem
- How the problem is solved, and the following results
- Conclusions and future work

# Theory



Theory

# Adiabatic quantum computing

## Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.[1]

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

[1] Born and Fock, 'Beweis des Adiabatensatzes'.





Adiabatic quantum computing
Adiabatic principle
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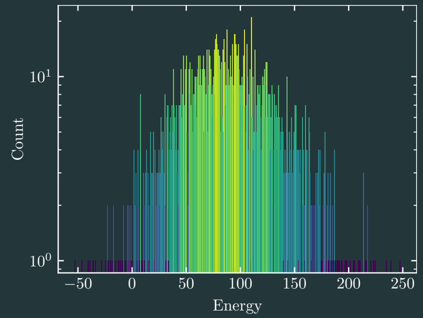
- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice
[1] Born and Fock, 'Beweis des Adiabatensatzes'.

- Adiabatic principle — system remains in the same eigenstate if perturbed slowly enough (without transferring heat)
- Equation shows evolution from initial Hamiltonian $H_0$ to final $H_p$ over time $T$
- Importantly, if the system starts in the ground state, it will end in the ground state
- Impossible in practice as true adiabatic evolution would take infinite time, infinitely many steps

# Quantum annealing

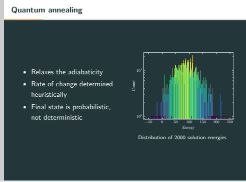
- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic




Distribution of 2000 solution energies



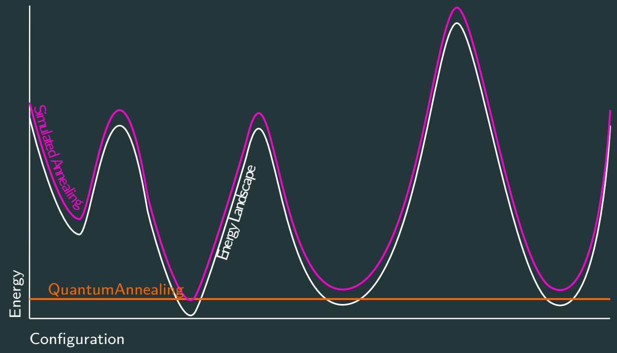





- Subset of AQC, relaxes the adiabaticity condition
- Annealing — slow heating of a material to change its properties
- Evolution time shortened (order of a few $\mu$s)
- End state no longer guaranteed, if started in ground state could end in excited state
- Able to run the evolution many times
- Probabilistic distribution of outcomes, sometimes will get lucky

## Advantages

- Find the ground state of complicated Hamiltonians
- Quantum tunneling avoids local minima




By Brianlechthaler - Own work, CC BY-SA 4.0,
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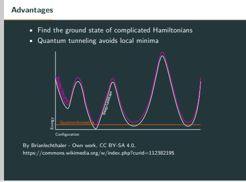

- Why is this technique useful?
- Allows us to find the ground state of complicated Hamiltonians by starting from an easy one
- Diagram — energy against configuration space, simulated annealing (classical) traverses the "energy landscape" whereas quantum annealing tunnels through it
- As opposed to classical methods, does not get affected by local minima
- Technique very good for solving optimisation problems e.g. travelling salesman, with complicated energy landscapes

# Ising model

Lattice of variables with two discrete values

**Initial Hamiltonian**

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

**Problem Hamiltonian**

$$H_p = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$





Ising model
Lattice of variables with two discrete values
Initial Hamiltonian
$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$
Problem Hamiltonian
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- How can we model the Hamiltonians?
- Ising model, a lattice of variables with two discrete values (+1/-1), acted on by spin operators $\sigma$
- Start with initial Hamiltonian, superposition of all possible states, easy to prepare and find the ground state
- Problem Hamiltonian, coupling strengths $J_{ij}$ and field strengths $h_i$, describe interactions (biases) of the spins
- Want to encode the problem solution into the ground state of this Hamiltonian so that the system will give the solution after evolution

# QUBO

**Quadratic Unconstrained Binary Optimisation**

Vector $x$ of qubits, matrix $Q$ of weights

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Aim to minimise this function
- Difficult to solve analytically
- Mapped to $H_p$ using simple change of variable
- Encodes problem solution into Hamiltonian's ground state





QUBO

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- How to encode a problem into a Hamiltonian?
- Similar form to the Ising model, but with binary variables (0/1)
- Minimisation of this function should be the problem solution
- Set of binary variables $x$, matrix $Q$ of real weights that describes interactions between variables
- After evolution, can read out the values of $x$ to give solution

# Motivations

# What problems can we solve?



What problems can we solve?

# Music arrangement

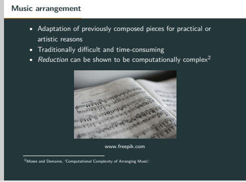
- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally difficult and time-consuming
- *Reduction* can be shown to be computationally complex[2]

www.freepik.com

[2] Moses and Demaine, 'Computational Complexity of Arranging Music'.








- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet)
- Very large configuration space, many different combinations of notes that could produce the final arrangement
- For those interested, NP-hard in computational complexity theory, cannot be solved in polynomial time
- NB: all scores shown are own reproductions from public domain files

## Motivations

- Already exist classical methods of automatic arrangement[3]
- Quantum annealing used to generate music[4]
- Field of quantum computer music is very new[5]
- Novel adaption of this method to a new problem
- *This has never been done before!*

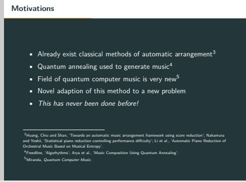
[3] Huang, Chiu and Shan, ‘Towards an automatic music arrangement framework using score reduction’; Nakamura and Yoshii, ‘Statistical piano reduction controlling performance difficulty’; Li et al., ‘Automatic Piano Reduction of Orchestral Music Based on Musical Entropy’.

[4] Freedline, ‘Algorhythms’; Arya et al., ‘Music Composition Using Quantum Annealing’.

[5] Miranda, *Quantum Computer Music*.







- Context of previous work
- Classical methods — machine learning, statistical analysis, rule-based systems, iterative and slow
- Applying quantum computing to music in the last five years, still a very young technology with limitations
- Has been used to generate music, not arrange it
- Methods shown here have not been found in the literature

# Method



Method

# Aims

- Arrange a musical score for a smaller ensemble
- All notes are taken from the original score
- Each instrument can only play one note at a time

Joseph Haydn playing in a string quartet,
painting from the StaatsMuseum, Vienna





Aims
- Arrange a musical score for a smaller ensemble
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Joseph Haydn playing in a string quartet,
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- What are we trying to do? What are the constraints to our problem?
- Take a musical score and reduce it to a smaller ensemble
- All notes must be taken from the original score, no new notes can be added
- Each instrument can only take notes from one part at a time

# Method

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate optimisation problem
4. Solve problem using QPU
5. Construct arrangement from solution





Method
1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate optimisation problem
4. Solve problem using QPU
5. Construct arrangement from solution

- Formulating arrangement as a problem to be solved via annealing, five-step process
- Split parts into musical phrases
- Arrange phrases into a graph (will explain later)
- Formulate the optimisation problem
- Solve corresponding graph problem using a quantum computer
- Construct final arrangement from the solution returned

# 1. Split score

- Musical phrases chosen as smallest unit of music
- Preserve melody and structure when rearranged

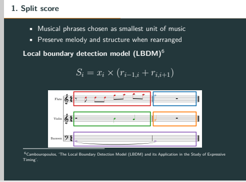
**Local boundary detection model (LBDM)**[6]

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

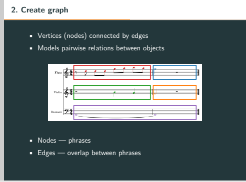

[6] Cambouropoulos, ‘The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing’.







- First stage to separate each part of original score into phrases
- Phrases — smallest unit of music that preserves melody and structure
- Boundaries between phrases found using LBDM
- Measures the degree of change of a certain parameter $(x)$ between notes $(i)$ (explain equation)
- Strength calculated for both pitch and IOI, weighted and summed to give the final strength
- Strengths above a threshold value are considered phrases



# 2. Create graph

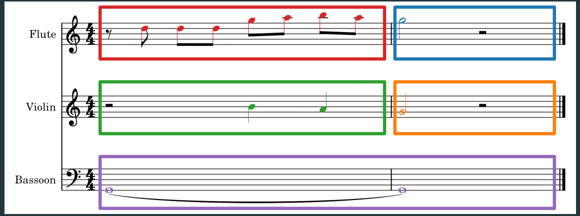
- Vertices (nodes) connected by edges
- Models pairwise relations between objects

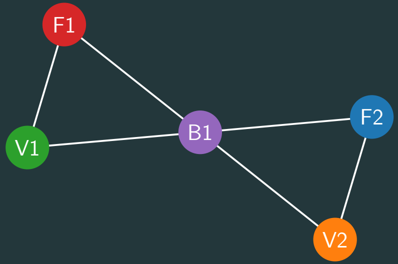

- Nodes — phrases
- Edges — overlap between phrases



- What is a graph? Nodes connected by edges, useful to model pairwise relations between objects
- Each phrase becomes a node, edges between nodes if phrases overlap (play at the same time)

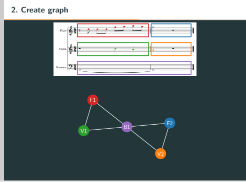
# 2. Create graph







- Score on top becomes graph on bottom

## Proper vertex colouring

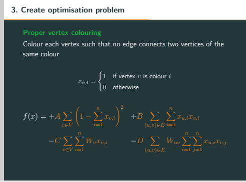
Colour each vertex such that no edge connects two vertices of the same colour

$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} f(x) = &+A \sum_{v \in V} \left(1 - \sum_{i=1}^{n} x_{v,i}\right)^2 &&+B \sum_{(u,v) \in E} \sum_{i=1}^{n} x_{u,i} x_{v,i} \\ &-C \sum_{v \in V} \sum_{i=1}^{n} W_v x_{v,i} &&-D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{u,i} x_{v,j} \end{aligned}$$

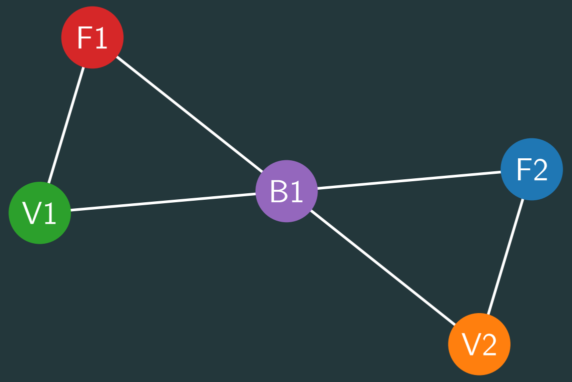






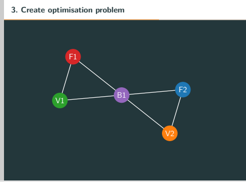
- Use a graph theory problem to create the optimisation problem that matches our constraints
- Here each colour represents an instrument we are arranging for
- QUBO, set of $n$ colours, $x_{v,i}$ is 1 if node $v$ is colour $i$
- $A$ — each node is only coloured once, sum over colours is one
- $B$ — penalise adjacent nodes with the same colour
- $C$ — weight of each node, preference for certain nodes
- $D$ — weight of each edge, preference for certain edges
- Weights here are musical entropy i.e. how interesting the phrase is musically

# 3. Create optimisation problem

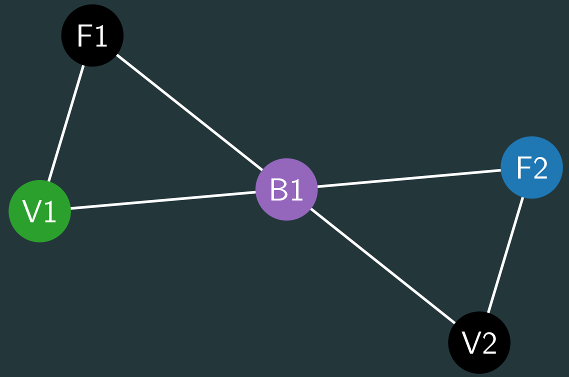






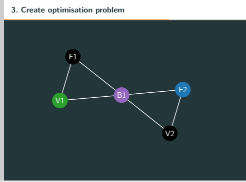



# 3. Create optimisation problem





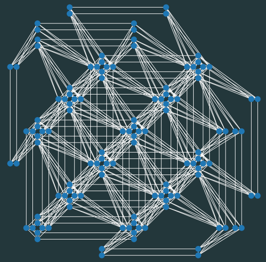



- $n = 1$
- One of many possible solutions

# 4. Solve problem

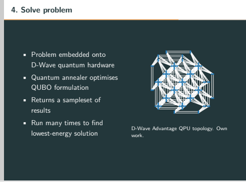
- Problem embedded onto D-Wave quantum hardware
- Quantum annealer optimises QUBO formulation
- Returns a sampleset of results
- Run many times to find lowest-energy solution



D-Wave Advantage QPU topology. Own work.

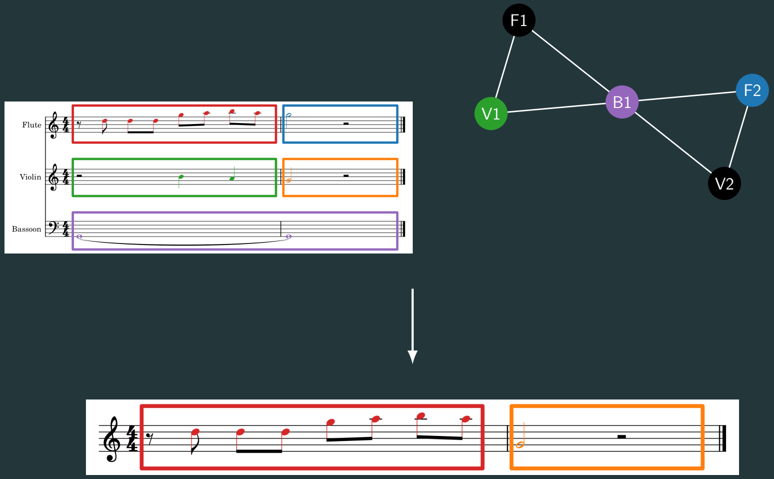






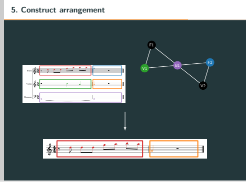

- D-Wave Systems is a company that gives access to true quantum annealers, normally for business applications
- Interact via a Python SDK, submit problems to the QPU
- Returns a distribution of results, each with an associated energy
- Run the problem thousands of times to find the lowest-energy solutions

# 5. Construct arrangement








- Take chosen low-energy solution and construct the final arrangement
- Map each node back to its phrase, with colour corresponding to the instrument

# Results



Results

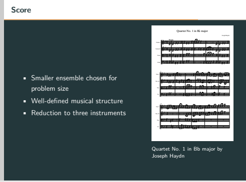
## Score

- Smaller ensemble chosen for problem size
- Well-defined musical structure
- Reduction to three instruments


Quartet No. 1 in Bb major by Joseph Haydn







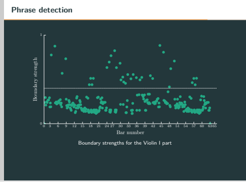

- Quartet No. 1 in Bb major by Joseph Haydn
- Smaller instrumentation and length (about 3 min), keeping the problem graph small and manageable
- Musical style has well-defined structure and phrases

# Phrase detection


Boundary strengths for the Violin I part

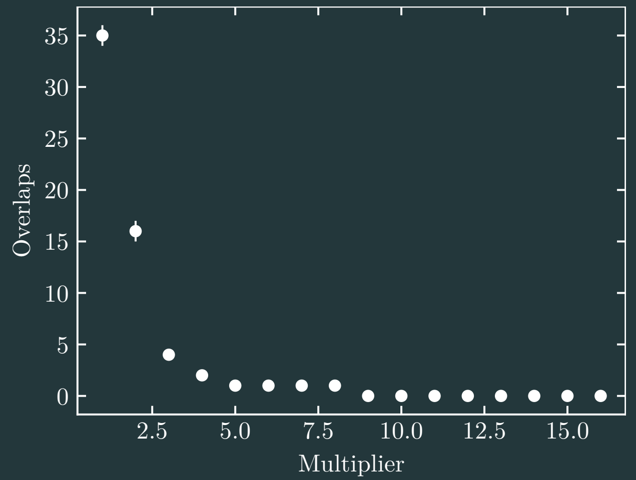






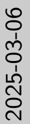
- Example of the LBDM finding suitable boundaries for phrases
- Threshold value of $0.4$ chosen manually

# QUBO parameter variation

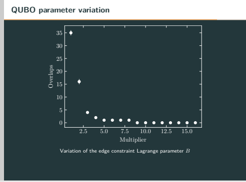



Variation of the edge constraint Lagrange parameter $B$



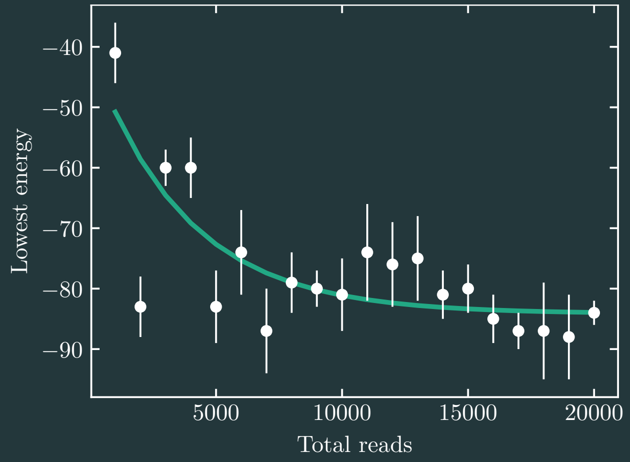


Quantum Annealing for Music Arrangement
└─Results
└─QUBO parameter variation




- Each QUBO problem submitted five times with different edge constraint Lagrange parameter
- Checking against fulfillment of the desired constraint
- Lagrange parameters taken as multipliers of the maximum node weight for normalisation
- $12.0$ chosen as the best parameter, with all others equal to one

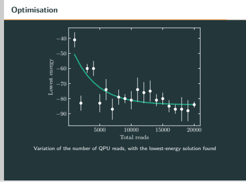
## Optimisation




Variation of the number of QPU reads, with the lowest-energy solution found







- Once Lagrange parameters chosen, can check how well the annealer optmises the problem
- In general, more reads is more likely to find lower-energy solutions
- Sometimes the annealer gets lucky (see 2000 reads)
- Each number of reads repeated five times, exponential decay fitted

# Conclusions

## Conclusions

- Successful novel application of quantum annealing
- QPU returns low-energy samples
- Necessary constraints for a valid arrangement fulfilled
- Still very new technology, does not show quantum advantage (yet)





Conclusions

- Successful novel application of quantum annealing
- QPU returns low-energy samples
- Necessary constraints for a valid arrangement fulfilled
- Still very new technology, does not show quantum advantage (yet)

- Successful application of this method on a new problem
- QPU returns samples that fulfill the constraints of the problem, creating a valid arrangement
- New technology, limited in power
- What would it take for quantum to show advantage?

# Future work

- Variation in problem size
- Comparison to classical methods
- Lagrange parameter tuning
- Qualitative judgement of computer arrangements[7]

[7] Pearce and Wiggins, ‘Towards A Framework for the Evaluation of Machine Compositions’.





Future work

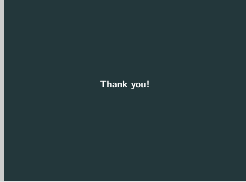
- Variation in problem size
- Comparison to classical methods
- Lagrange parameter tuning
- Qualitative judgement of computer arrangements[7]

[7] Pearce and Wiggins, ‘Towards A Framework for the Evaluation of Machine Compositions’.

- How well does the method scale with larger scores? How well can it find low energies with smaller problems?
- Compare to classical optimisation methods, time to solution, energy of solutions
- Only tuned one parameter by hand, could use a more systematic approach to find lower-energy solutions
- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

# Thank you!





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└─References

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### Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$
$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

### Normalisation

$$S_i' = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

### Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

[8]

[8] Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.



- Boundaries always taken at beginning/end of piece
- Weightings derived by trial and error

# Phrase entropy

$x_i$ — parameter $x$ of note $i$

**Shannon entropy**

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

**Probability distribution**

$$P(x_i) = \frac{n_i}{N}$$



[9] Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.



Phrase entropy
$x_i$ — parameter $x$ of note $i$
Shannon entropy
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Probability distribution
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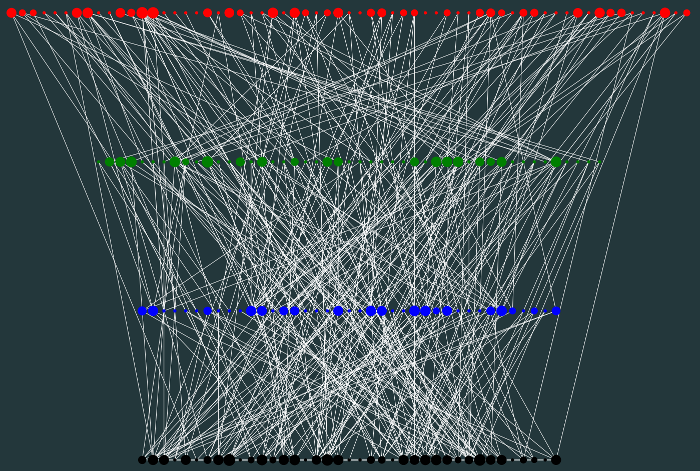
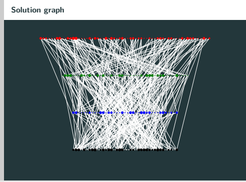
9
[9] Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

- Shannon entropy units in bits due to $\log_2$
- Distribution calculated for pitch and duration

# Solution graph







# Solution score


2025-03-06

Quantum Annealing for Music Arrangement

└Solution score