Quantum Annealing for Music Arrangement

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Overview

Theory

Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.[1]

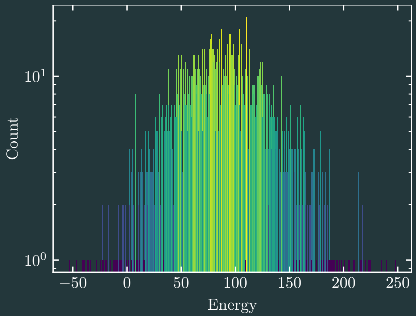
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

[1] Born and Fock, 'Beweis des Adiabatensatzes'.

Quantum annealing

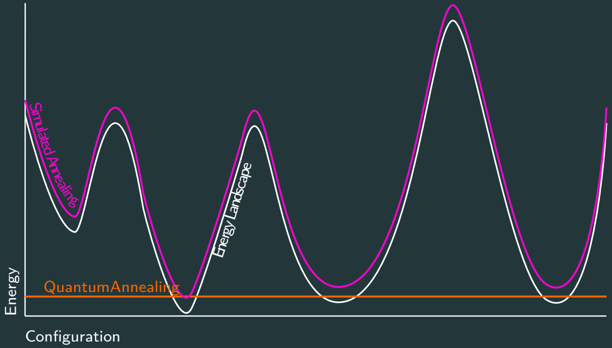
- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic



Distribution of 2000 solution energies

Advantages

- Find the ground state of complicated Hamiltonians
- Quantum tunneling avoids local minima



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Ising model

Lattice of variables with two discrete values

Initial Hamiltonian

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

Problem Hamiltonian

$$H_p = \sum_{i<j}^{N} J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^{N} h_i \sigma_i^z$$

QUBO

Quadratic Unconstrained Binary Optimisation

Vector x of qubits, matrix Q of weights

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Aim to minimise this function
- Difficult to solve analytically
- Mapped to H_p using simple change of variable
- Encodes problem solution into Hamiltonian's ground state

Motivations

What problems can we solve?

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally difficult and time-consuming
- *Reduction* can be shown to be computationally complex[2]

www.freepik.com

[2] Moses and Demaine, 'Computational Complexity of Arranging Music'.

Motivations

- Already exist classical methods of automatic arrangement[3]
- Quantum annealing used to generate music[4]
- Field of quantum computer music is very new[5]
- Novel adaption of this method to a new problem
- *This has never been done before!*

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[3] Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'; Nakamura and Yoshii, 'Statistical piano reduction controlling performance difficulty'; Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

[4] Freedline, 'Algorhythms'; Arya et al., 'Music Composition Using Quantum Annealing'.

[5] Miranda, *Quantum Computer Music*.

Method

Aims

- Arrange a musical score for a smaller ensemble
- All notes are taken from the original score
- Each instrument can only play one note at a time

Joseph Haydn playing in a string quartet,
painting from the StaatsMuseum, Vienna

Method

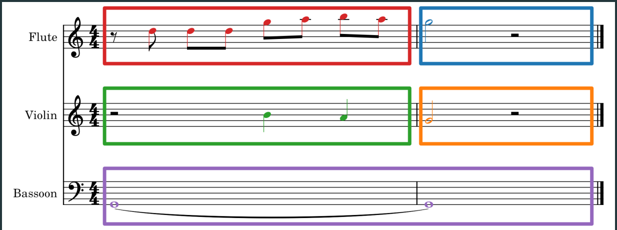
1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate optimisation problem
4. Solve problem using QPU
5. Construct arrangement from solution

1. Split score

- Musical phrases chosen as smallest unit of music
- Preserve melody and structure when rearranged

Local boundary detection model (LBDM)[6]

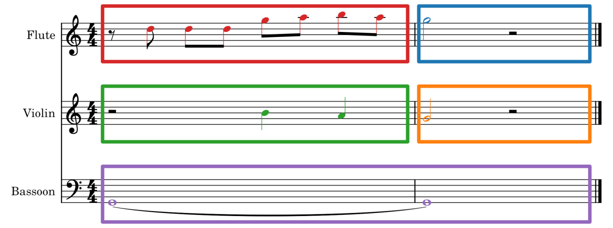
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$



[6] Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

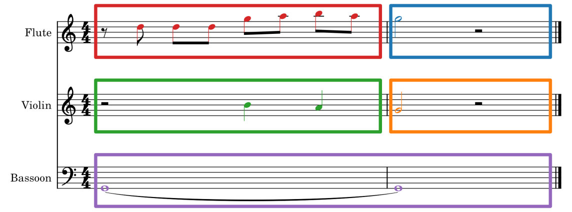
2. Create graph

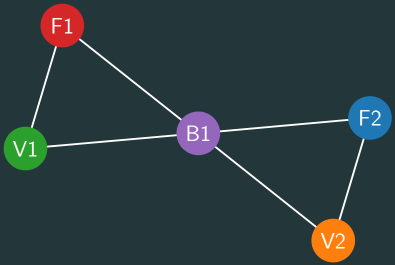
- Vertices (nodes) connected by edges
- Models pairwise relations between objects



- Nodes — phrases
- Edges — overlap between phrases

2. Create graph





3. Create optimisation problem

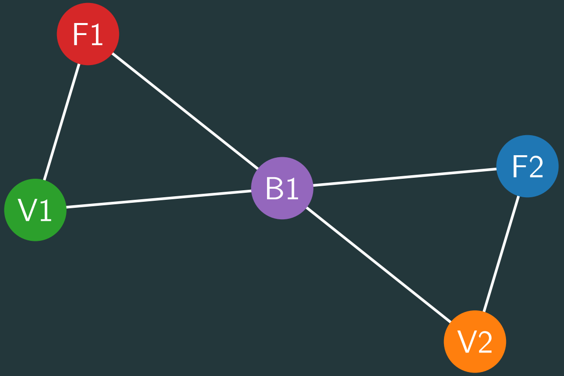
Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

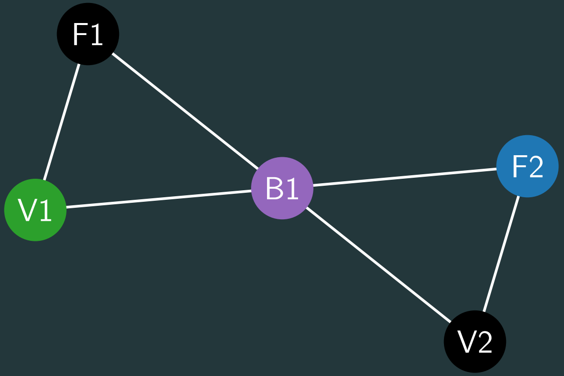
$$x_{v,i} = \begin{cases} 1 & \text{if vertex } v \text{ is colour } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} f(x) = &+A \sum_{v \in V} \left(1 - \sum_{i=1}^{n} x_{v,i}\right)^2 &&+B \sum_{(u,v) \in E} \sum_{i=1}^{n} x_{u,i} x_{v,i} \\ &-C \sum_{v \in V} \sum_{i=1}^{n} W_v x_{v,i} &&-D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{u,i} x_{v,j} \end{aligned}$$

3. Create optimisation problem

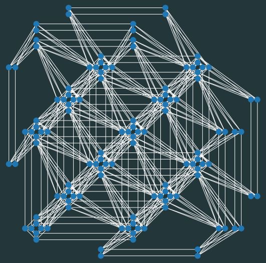


3. Create optimisation problem



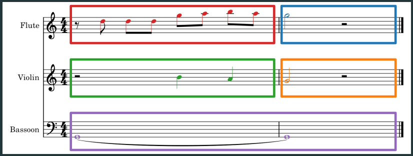
4. Solve problem

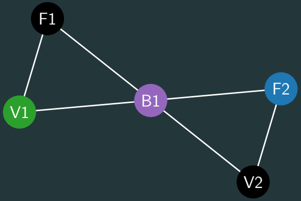
- Problem embedded onto D-Wave quantum hardware
- Quantum annealer optimises QUBO formulation
- Returns a sampleset of results
- Run many times to find lowest-energy solution

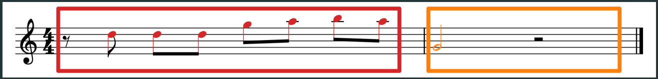


D-Wave Advantage QPU topology. Own work.

5. Construct arrangement







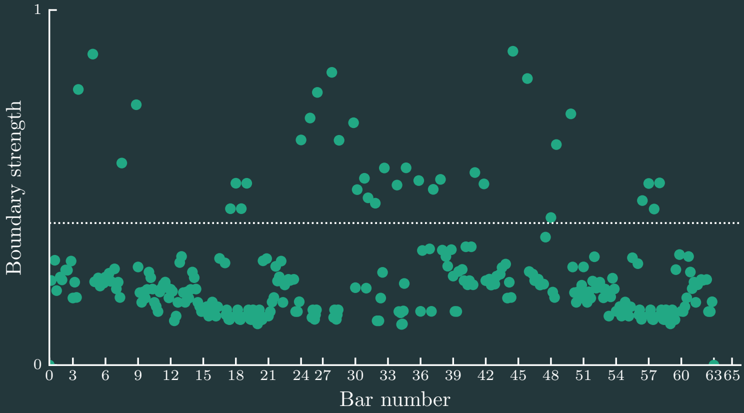
Results

Score

- Smaller ensemble chosen for problem size
- Well-defined musical structure
- Reduction to three instruments

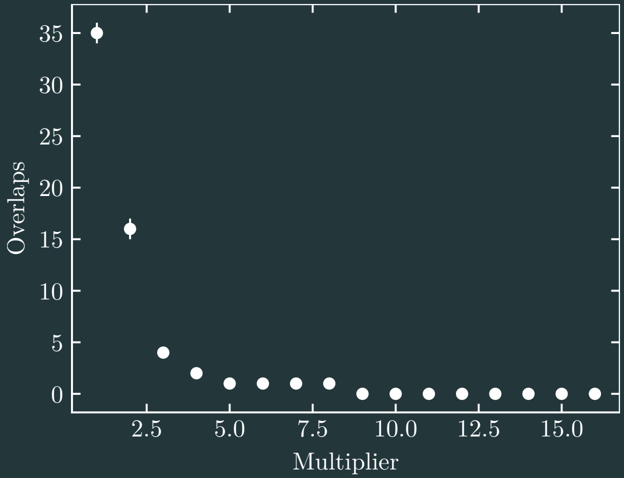
Quartet No. 1 in Bb major by Joseph Haydn

Phrase detection



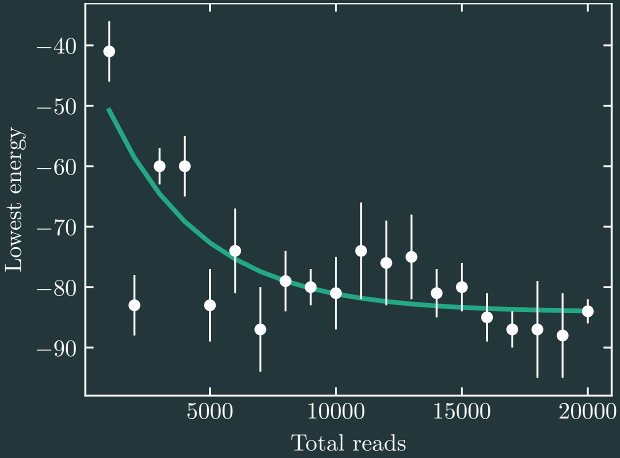
Boundary strengths for the Violin I part

QUBO parameter variation



Variation of the edge constraint Lagrange parameter B

Optimisation



Variation of the number of QPU reads, with the lowest-energy solution found

Conclusions

Conclusions

- Successful novel application of quantum annealing
- QPU returns low-energy samples
- Necessary constraints for a valid arrangement fulfilled
- Still very new technology, does not show quantum advantage (yet)

Future work

- Variation in problem size
- Comparison to classical methods
- Lagrange parameter tuning
- Qualitative judgement of computer arrangements[7]

[7] Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'.

Thank you!

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Arya, Ashish et al. **'Music Composition Using Quantum Annealing'.** In: *arXiv* (Jan. 2022). DOI: 10.48550/arXiv.2201.10557. (Visited on 26/10/2024).

Born, M. and V. Fock. **'Beweis des Adiabatensatzes'.** de. In: *Zeitschrift für Physik* 51.3 (Mar. 1928), pp. 165–180. ISSN: 0044-3328. DOI: 10.1007/BF01343193. URL: https://doi.org/10.1007/BF01343193 (visited on 01/03/2025).

Cambouropoulos, Emilios. **'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.** In: *International Computer Music Association* (2011). ISSN: 2223-3881.

Freedline, Alex. **'Algorhythms: Generating Music with D-Wave's Quantum Annealer'.** en. In: *MIT 6.s089—Intro to Quantum Computing* (Feb. 2021).

Huang, Jiun-Long, Shih-Chuan Chiu and Man-Kwan Shan. **'Towards an automatic music arrangement framework using score reduction'.** In: *ACM Trans. Multimedia Comput. Commun. Appl.* 8.1 (Feb. 2012), 8:1–8:23. ISSN: 1551-6857. DOI: 10.1145/2071396.2071404. (Visited on 05/12/2024).

- Li, You et al. **'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.** In: *2019 53rd Annual Conference on Information Sciences and Systems (CISS)*. Mar. 2019, pp. 1–5. DOI: 10.1109/CISS.2019.8693036. URL: https://ieeexplore.ieee.org/document/8693036 (visited on 27/12/2024).
- Miranda, Eduardo Reck, ed. ***Quantum Computer Music: Foundations, Methods and Advanced Concepts*.** en. Springer International Publishing, 2022. ISBN: 978-3-031-13908-6 978-3-031-13909-3. DOI: 10.1007/978-3-031-13909-3. (Visited on 28/12/2024).

- Moses, William S. and Erik D. Demaine. **‘Computational Complexity of Arranging Music’.** In: *arXiv* (July 2016). arXiv:1607.04220. DOI: 10.48550/arXiv.1607.04220. (Visited on 09/11/2024).
- Nakamura, Eita and Kazuyoshi Yoshii. **‘Statistical piano reduction controlling performance difficulty’.** en. In: *APSIPA Transactions on Signal and Information Processing* 7 (Jan. 2018), e13. ISSN: 2048-7703. DOI: 10.1017/ATSIP.2018.18. (Visited on 17/12/2024).
- Pearce, M. and Geraint A. Wiggins. **‘Towards A Framework for the Evaluation of Machine Compositions’.** In: *Proceedings of the AISB’01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences*. 2001.

LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

[8]

[8] Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

Phrase entropy

x_i — parameter x of note i

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

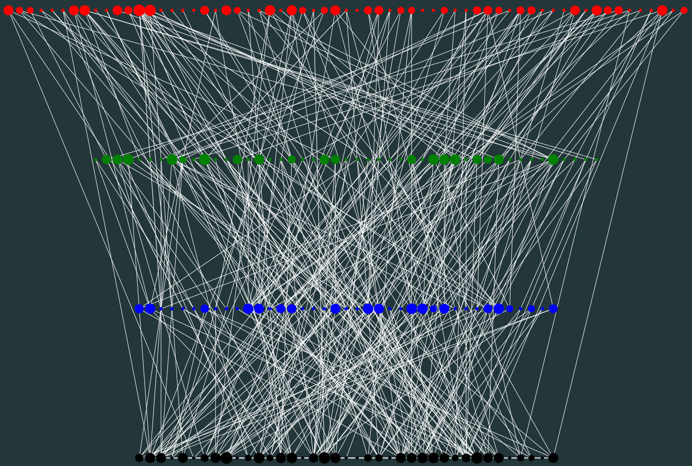
Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

[9]

[9] Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.

Solution graph



Solution score