

# Music Arrangement via Quantum Annealing

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30 January 2025

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# Overview

Theory

Music arrangement

Quantum annealing

Methods

Results

Conclusions

# Theory

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# Music arrangement

The image displays a musical score for Beethoven's String Quartet No. 10, Op. 106. The score is written for four string instruments: Violin I, Violin II, Viola, and Violoncello. The key signature is three flats (B-flat, E-flat, A-flat), and the time signature is 4/4. The tempo marking is "Poco Adagio". The score is divided into two systems. The first system shows the initial measures of the piece, with each instrument part. The second system shows a more complex passage, featuring dynamic markings such as "cresc." (crescendo), "p" (piano), "f" (forte), and "espress." (espressivo). The notation includes various musical symbols like notes, rests, and slurs, indicating the melodic and harmonic development of the quartet.

Beethoven's String Quartet No. 10

# Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons

The image displays the first system of Beethoven's String Quartet No. 10, Op. 10, No. 1. The score is written for four instruments: Violin I, Violin II, Viola, and Violoncello. The key signature is two flats (B-flat and E-flat), and the time signature is 4/4. The tempo marking is 'Poco Adagio'. The first system shows the initial measures of the piece, with each instrument part clearly delineated. The Violoncello part includes the instruction 'sotto voce'.

Beethoven's String Quartet No. 10

# Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming

The image displays the first system of Beethoven's String Quartet No. 10, Op. 10, No. 1. The score is written for four parts: Violin I, Violin II, Viola, and Violoncello. The key signature is two flats (B-flat and E-flat), and the time signature is 4/4. The tempo marking is 'Poco Adagio'. The first system shows the initial measures of the piece, with each instrument part clearly delineated. The second system continues the music, featuring dynamic markings such as 'cresc.' (crescendo) and 'p' (piano). The third system includes the marking 'espress.' (espressivo) and 'f' (forte). The fourth system concludes the first system with a 'p' marking.

Beethoven's String Quartet No. 10

# Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

The image displays the first system of a musical score for Beethoven's String Quartet No. 10. The score is written for four instruments: Violin I, Violin II, Viola, and Violoncello. The key signature is three flats (B-flat, E-flat, A-flat) and the time signature is 4/4. The tempo marking is 'Poco Adagio'. The first system shows the initial measures of the piece, with each instrument part clearly delineated. The second system continues the music, featuring dynamic markings such as 'cresc.' (crescendo) and 'p' (piano). The third system includes the marking 'espress.' (espressivo) and continues the musical development. The fourth system shows further musical notation with dynamic markings like 'p' and 'f' (forte).

Beethoven's String Quartet No. 10

# Adiabatic quantum computing (AQC)



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$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

[Lucas, 2014]

# Quantum annealing

## Ising model

$$H_p(\sigma^z) = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

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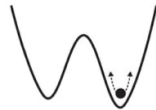
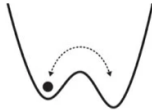
## Initial state

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

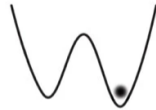
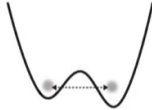
[Lucas, 2014]

# Quantum annealing

Classical  
thermal  
annealing



Quantum  
annealing



[Johnson et al., 2011]





## Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

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$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

**How to combine them?**

# Methods

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## Problem formulation



# Problem formulation

1. Split parts into phrases

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2. Arrange phrases into a graph

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3. Solve graph problem using QPU
4. Construct arrangement from solution

## 1. Split parts

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## Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

[Cambouropoulos, 2011]

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$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

[Cambouropoulos, 2011]

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a sequence of red notes: a quarter rest, an eighth note, a quarter note, a half note, a quarter note, and a half note, followed by a blue quarter note and a whole rest. The Violin part (treble clef) has a whole rest, followed by two green quarter notes, and an orange quarter note with a whole rest. The Bassoon part (bass clef) has a purple whole note with a slur underneath it. The notes are color-coded to represent different segments or boundaries in the music.

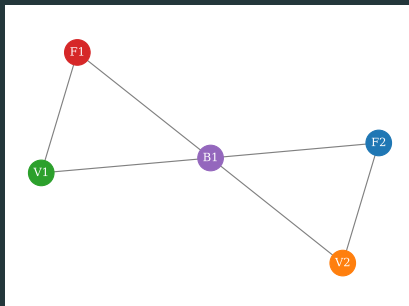
## 2. Create graph

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) consists of a quarter rest, followed by a quarter note G4, an eighth note A4, an eighth note B4, a quarter note C5, a quarter note D5, a quarter note E5, and a quarter note F5. The Violin part (treble clef) consists of a half rest, followed by a quarter note G4, a quarter note A4, and a quarter note B4. The Bassoon part (bass clef) consists of a half rest, followed by a quarter note G2, a quarter note A2, and a quarter note B2. The score is written on three staves, with the Flute staff at the top, the Violin staff in the middle, and the Bassoon staff at the bottom. The time signature is 4/4 for all parts.

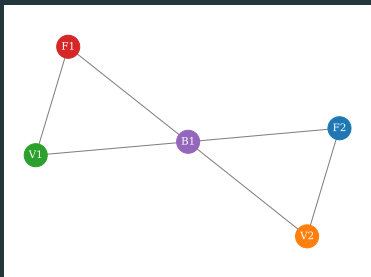


## 2. Create graph

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (top staff) begins with a quarter rest, followed by a sequence of eighth notes: G4, A4, B4, C5, D5, E5, and F5. The Violin part (middle staff) has a whole rest in the first measure, followed by two eighth notes (G4 and A4) in the second measure, and a quarter note (B4) in the third measure. The Bassoon part (bottom staff) has a whole rest in the first measure, followed by a whole note (G3) in the second measure. A purple line connects the start of the Bassoon part to the end of the Flute part, indicating a measure rest.



### 3. Solve graph



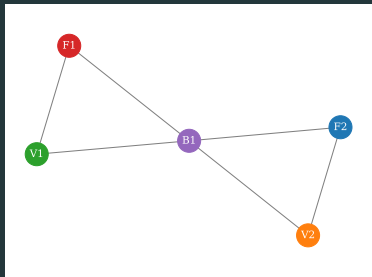
### 3. Solve graph

#### Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$

[Lucas, 2014]



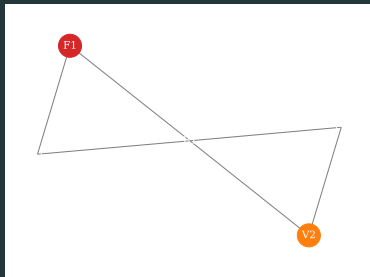
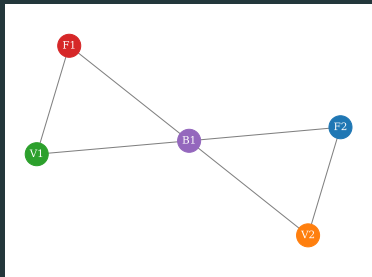
### 3. Solve graph

#### Maximal independent set (MIS)

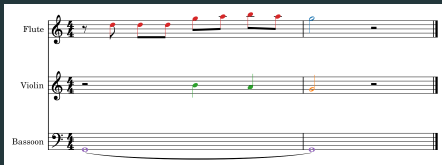
Largest subset of nodes such that no nodes within the subset are connected by an edge

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$

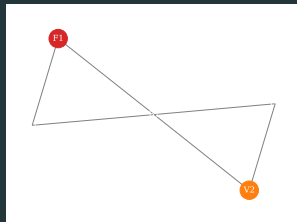
[Lucas, 2014]



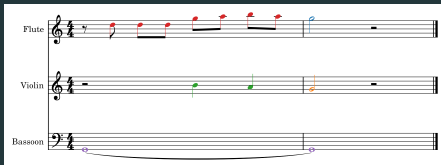
## 4. Construct arrangement



A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) starts with a quarter rest, followed by a sequence of eighth notes: G4 (red), A4 (red), B4 (red), C5 (red), B4 (red), A4 (red), G4 (blue), and a final quarter rest. The Violin part (treble clef) has a whole rest in the first measure, followed by a half note G4 (green) in the second measure, a half note A4 (green) in the third measure, and a quarter rest in the fourth measure. The Bassoon part (bass clef) has a whole note G3 (purple) in the first measure, which is tied to a whole note G3 (purple) in the second measure, followed by a quarter rest in the third measure.



## 4. Construct arrangement


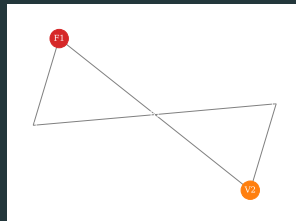


Flute

Violin

Bassoon

The score is in 4/4 time. The Flute part (treble clef) has a quarter rest followed by a quarter note G4, then eighth notes A4, B4, C5, D5, E5, and F5. The Violin part (treble clef) has a whole rest, then a quarter note G4, then a quarter note A4, and a half note B4. The Bassoon part (bass clef) has a half note G2, then a half note A2, and a whole note B2.



Flute

The score is in 4/4 time. The Flute part (treble clef) has a quarter rest followed by a quarter note G4, then eighth notes A4, B4, C5, D5, E5, and F5. The Violin part (treble clef) has a whole rest, then a quarter note G4, then a quarter note A4, and a half note B4. The Bassoon part (bass clef) has a half note G2, then a half note A2, and a whole note B2.

# Results

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# Excerpt

**Poco Adagio**

Violin I  
sotto voce  
**Poco Adagio**

Violin II  
sotto voce  
**Poco Adagio**

Viola  
sotto voce  
**Poco Adagio**

Violoncello  
sotto voce  
**Poco Adagio**

6

cresc.

cresc.

cresc.

10

espress.

*p*

*f*

espress.

*p*

*f*

cresc.

*p*

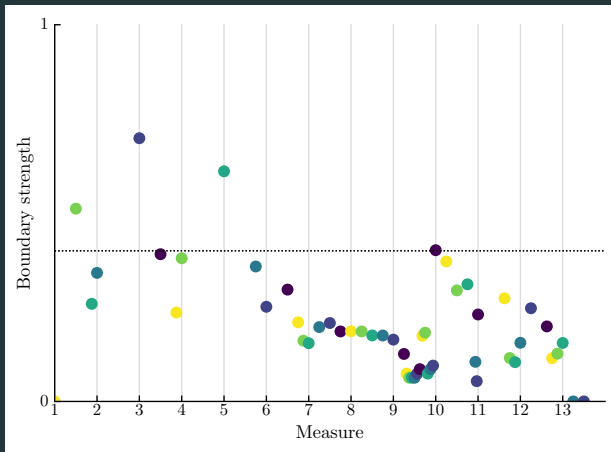
*f*

*p*

String Quartet No. 10 by Ludwig van Beethoven

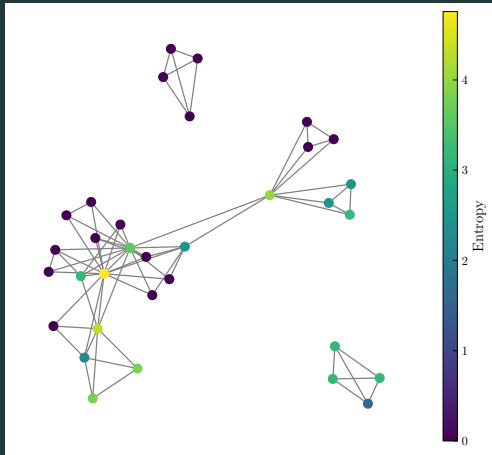


# Phrase detection



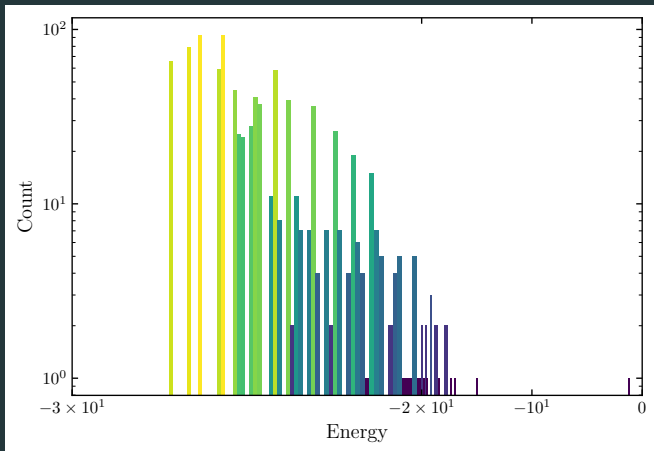
Boundary strengths for the Violin I part

# Problem graph



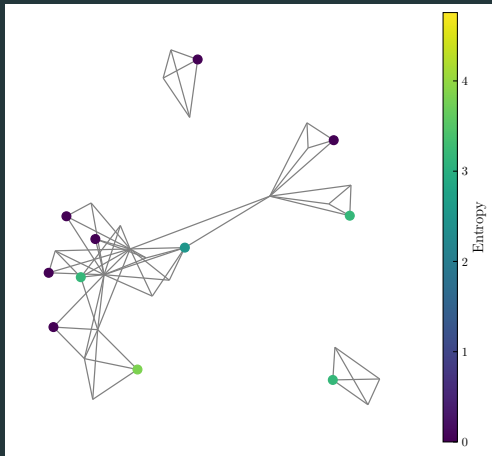
Problem graph with 33 nodes and 70 edges

# Solutions



Returned solutions for 1000 reads

## Example solution



Solution graph returning a subset of 11 nodes

# Final arrangement

**Poco Adagio**

Violin I  
sotto voce

Violin II  
sotto voce

Viola  
sotto voce

Violoncello  
sotto voce

6

cresc.

cresc.

cresc.

10

espress.

espress.

cresc.

*p*

*f*

*p*

*f*

*p*

*f*

Selected phrases

**Poco Adagio**

sotto voce

7

espress.

cresc.

*p*

12

*f*

Final arrangement

# Conclusions

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# Conclusions

Poco Adagio  $\flat$

sotto voce

7

cresc.

espress.

$p$

12

$f$

The musical score is written for a vocal line in 4/4 time with a key signature of one flat (B-flat). The tempo is marked 'Poco Adagio'. The first staff begins with the instruction 'sotto voce'. The score is divided into measures, with measure numbers 7 and 12 indicated. The notation includes various note values, rests, and dynamic markings: 'cresc.' (crescendo), 'espress.' (espressivo), 'p' (piano), and 'f' (forte). The piece concludes with a double bar line.

# Conclusions

- Successful in creating a valid single-part reduction

The image displays a musical score for a piece titled "Poco Adagio" in B-flat major (two flats) and 4/4 time. The score is presented as a single-part reduction across three staves. The first staff begins with the tempo marking "Poco Adagio" and the instruction "sotto voce". The second staff includes the measure number "7" at the start, a "cresc." (crescendo) marking, and an "espress." (expressive) marking. The third staff starts with the measure number "12" and features a forte "f" dynamic marking. The notation includes various musical symbols such as notes, rests, and dynamic markings.



# Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms [Huang et al., 2012]



# Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms [Huang et al., 2012]
- Removes skill barrier for music arrangement



## Future work

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- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements  
[Pearce and Wiggins, 2001]



**Thank you!**

# Music Arrangement via Quantum Annealing

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**The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing.**

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**Towards an automatic music arrangement framework using score reduction.**

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**Automatic Piano Reduction of Orchestral Music Based on Musical Entropy.**

*In 2019 53rd Annual Conference on Information Sciences and Systems (CISS)*, pages 1–5.



Lucas, A. (2014).

**Ising formulations of many NP problems.**

*Frontiers in Physics*, 2.

Publisher: Frontiers.



Pearce, M. and Wiggins, G. A. (2001).

**Towards A Framework for the Evaluation of Machine Compositions.**

*In Proceedings of the AISB'01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences.*

## Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

## Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

## Weighting

$$S = \frac{1}{3} (S'_{\text{pitch}} + 2S'_{\text{IOI}})$$

# Phrase entropy

## Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

## Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

[Li et al., 2019]