Music Arrangement via Quantum Annealing

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30 January 2025

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Overview

- What is music arrangement? What is quantum annealing?
- Methods used to solve the music arrangement problem
- Preliminary results from application of the method
- Concluding thoughts about this process

Theory

2025-01-27

Beethoven's String Quartet No. 10

- Adaptation of music in terms of instrumentation, medium, or style

- Traditionally a complex process that requires a deep understanding of musical theory and structure

- Musically interesting whilst still remaining faithful to the source material

- Interest in automating this process

- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet)

3

- Adaptation of previously composed pieces for practical or artistic reasons

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Music arrangement

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- Traditionally complex and time-consuming

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Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

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Adiabatic quantum computing (AQC)

Music Arrangement via Quantum Annealing
└─Theory
  └─Quantum annealing
    └─Adiabatic quantum computing (AQC)

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Adiabatic quantum computing (AQC)

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle

- Quantum computing, slow evolution of a system between Hamiltonians

- Done adiabatically (closed system), system remains in ground state

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$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

[Lucas, 2014]

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Quantum annealing

Music Arrangement via Quantum Annealing
└─Theory
  └─Quantum annealing
    └─Quantum annealing

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- How is this used to solve problems?

- Ising model, create a lattice of variables with two discrete values (spin up/down)

- Problem Hamiltonian, qubits σ^z, coupling strengths J_{ij} and field strengths h_i

- Initial state is a superposition of all possible states

- If problem solution is encoded within the ground state, system will give solution after evolution

Ising model

$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$

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Initial state

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

[Lucas, 2014]

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Quantum annealing
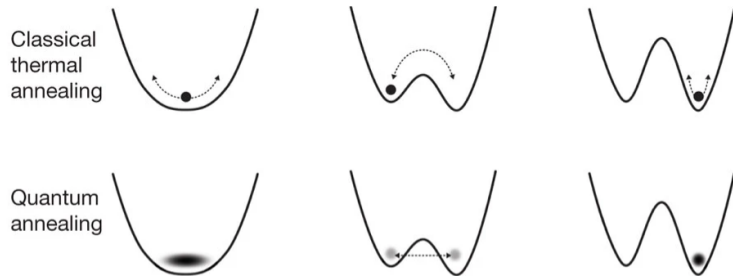
[Johnson et al., 2011]

- What does this look like?

- Evolution of superposition to a particular state

- More efficiently escape from local minima via quantum tunneling

- Can solve harder problems with a more turbulent energy landscape

QUBO

Music Arrangement via Quantum Annealing
└─Theory
  └─Quantum annealing
    └─QUBO

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QUBO

- How to encode a problem into a Hamiltonian?

- QUBO is a function to be minimised

- Set of binary variables x, matrix Q of real weights that describes interactions between variables

Quadratic Unconstrained Binary Optimisation

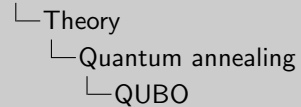$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

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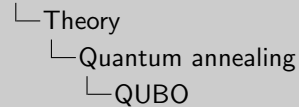- Set of binary variables x, matrix Q of real weights that describes interactions between variables

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j}x_ix_j + \sum_{i}^{N} Q_{i,i}x_i$$

• Encodes problem solution into Hamiltonian's ground state

• Remains in low-energy state via quantum tunneling

• How to encode a problem into a Hamiltonian?

• QUBO is a function to be minimised

• Set of binary variables x, matrix Q of real weights that describes interactions between variables

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How to combine them?

- **How to apply quantum annealing to the problem of music arrangement?**

Methods

- Formulating arrangement as a problem to be solved via annealing, four-step process

- Split parts into musical phrases

- Arrange phrases into a graph (nodes and edges)

- Solve corresponding graph problem using quantum computing

- Construct final arrangement from the solution returned

Problem formulation

1. Split parts into phrases

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Music Arrangement via Quantum Annealing

└─Methods

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└─Problem formulation

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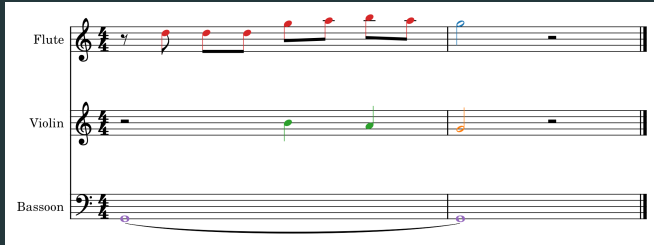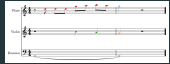- Construct final arrangement from the solution returned

- First stage to separate each part of original score into phrases

- Phrases — smallest unit of music that preserves melody and structure

- Boundaries between phrases found using LBDM

- Measures the degree of change of a certain parameter between notes (explain equation)

- Check pitch and IOI

- Strengths above a threshold value are considered phrases

Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

[Cambouropoulos, 2011]

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2. Create graph

Music Arrangement via Quantum Annealing
└─Methods

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└─2. Create graph

- Construct problem graph

- Each phrase becomes a node, edges between nodes if phrases overlap

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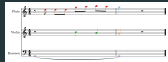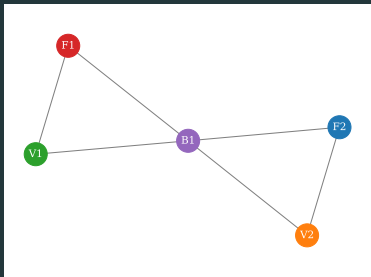
3. Solve graph

- Special case for reducing to a single monophonic part (one phrase played at a time)

- Solve problem graph using a graph theory problem called MIS

- Find largest subset of nodes such that no nodes within the subset are connected by an edge

- Enforces that only one simultaneous phrase can be played at once

- Constraint term enforces no edges

- Objective term is quantity to be minimised

- Lagrange paramters A/B determine the balance between constraint and objective

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3. Solve graph

Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge

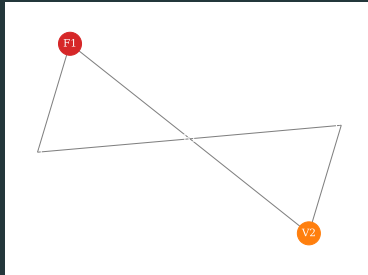$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$
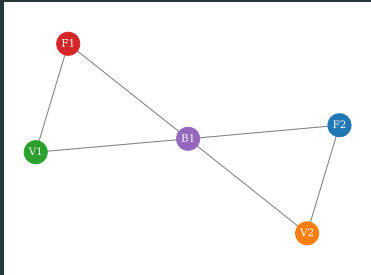
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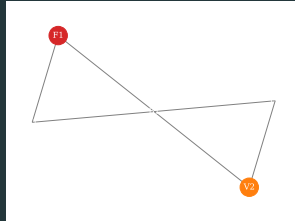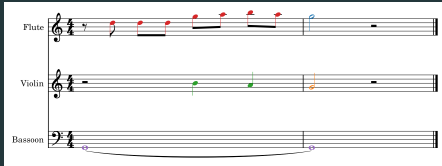
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- Take solution graph and combine selected nodes to create final arrangement
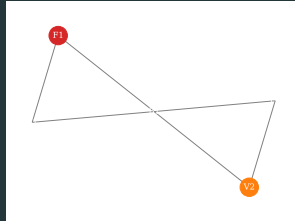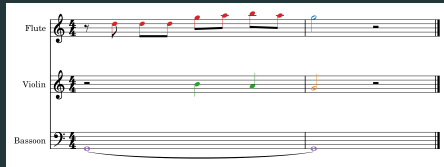
4. Construct arrangement

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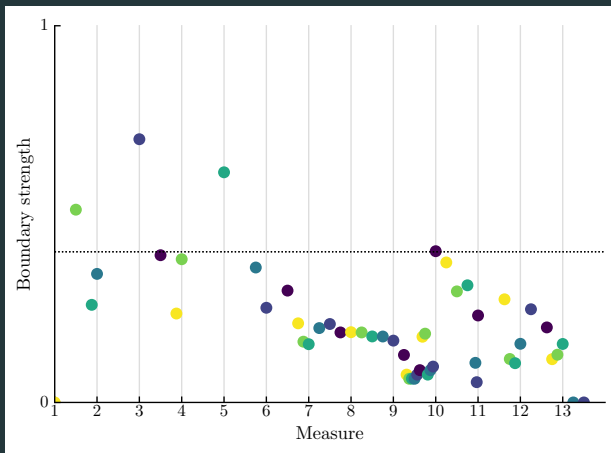
Results

Excerpt
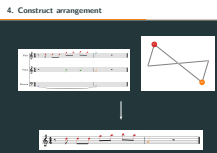
String Quartet No. 10 by Ludwig van Beethoven

- Excerpt of String Quartet No. 10 in E-flat major, Op. 74, by Ludwig van Beethoven

- Chosen due to its relatively simple structure and smaller instrumentation, keeping the problem graph small

Phrase detection

Boundary strengths for the Violin I part
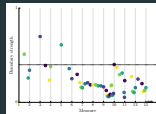
- Example of the LBDM finding suitable boundaries for phrases

- Threshold value of 0.4 chosen, finds five phrases
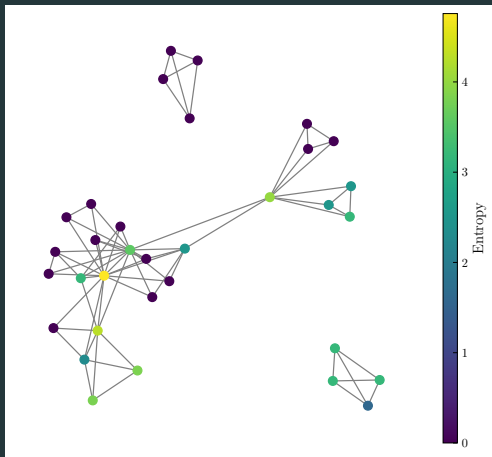
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Problem graph with 33 nodes and 70 edges

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└─Results

└─Problem graph

Problem graph

Problem graph with 33 nodes and 70 edges

- 33 identified phrases (nodes) with 70 overlaps (edges)

- Nodes are weighted by the phrase entropy, how musically interesting the distribution of notes is

15
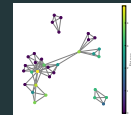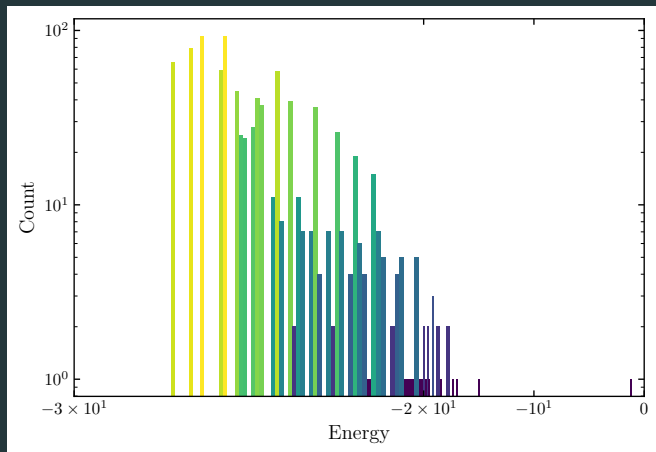
Solutions

Returned solutions for 1000 reads
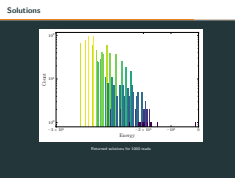
- Histogram of the returned solutions, only energies below zero shown
- Distribution of solutions due to the stochastic nature of annealing
- Not always guaranteed the ground state during evolution
- Lowest energy solution -26.8 with a degeneracy of 34

Example solution
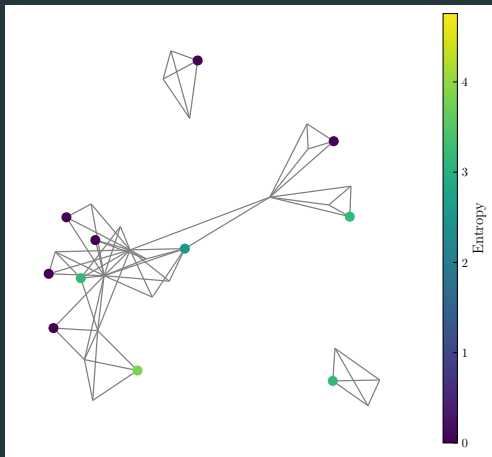
Solution graph returning a subset of 11 nodes

- Selected nodes from one of the lowest energy solutions
- Note in the cliques only one node could be selected

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Final arrangement

Selected phrases

Final arrangement

- Selected phrases from solution graph highlighted

- Phrases concatenated to create the final arrangement

Conclusions

- Valid reduction gained from implemented method

- Monophonic lowest-energy solution

- Phrase identification and selection result in musically interesting final arrangement

- Displays some advantages over classical algorithms

- Does not require training data, costly in both time and resources

- Faster solve time compared to similarly-sized problems

- Method and more advanced versions of it removes the skill barrier for music arrangement

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• Successful in creating a valid single-part reduction

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Future work

- Problem size — full piece has upwards of 1000 phrases, compare solve times with classical algorithms

- LBDM parameters — phrases should be short and similar in length, prevents important notes being hidden in long phrases with low entropy

- Physical limitations — note ranges, note change speed

- Multiple parts — need different problem formulation, colouring problem with each colour representing a reduced part

- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

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- Parametric variation of LBDM
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- Quality comparison of computer arrangements
  [Pearce and Wiggins, 2001]

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Thank you!

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References i

📄 Cambouropoulos, E. (2011).
The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing.
International Computer Music Association.

📄 Huang, J.-L., Chiu, S.-C., and Shan, M.-K. (2012).
Towards an automatic music arrangement framework using score reduction.
ACM Trans. Multimedia Comput. Commun. Appl.,
8(1):8:1–8:23.

📄 Johnson, M. W., Amin, M. H. S., Gildert, S., Lanting, T., Hamze, F., Dickson, N., Harris, R., Berkley, A. J., Johansson, J., Bunyk, P., Chapple, E. M., Enderud, C., Hilton, J. P., Karimi, K., Ladizinsky, E., Ladizinsky, N., Oh, T., Perminov, I., Rich, C., Thom, M. C., Tolkacheva, E., Truncik, C. J. S., Uchaikin, S., Wang, J., Wilson, B., and Rose, G. (2011).
Quantum annealing with manufactured spins.
Nature, 473(7346):194–198.
Publisher: Nature Publishing Group.

📄 Li, Y., Wilk, C. M., Hori, T., and Sagayama, S. (2019).
Automatic Piano Reduction of Orchestral Music Based on Musical Entropy.
In *2019 53rd Annual Conference on Information Sciences and Systems (CISS)*, pages 1–5.

📄 Lucas, A. (2014).
Ising formulations of many NP problems.
Frontiers in Physics, 2.
Publisher: Frontiers.

📑 Pearce, M. and Wiggins, G. A. (2001).
Towards A Framework for the Evaluation of Machine Compositions.
In *Proceedings of the AISB'01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences.*

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S_i' = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S_{\text{pitch}}' + 2S_{\text{IOI}}'\right)$$

[Cambouropoulos, 2011]

- Boundaries always taken at beginning/end of piece

- Weightings derived by trial and error

Phrase entropy

Shannon entropy
$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution
$$P(x_i) = \frac{n_i}{N}$$

[Li et al., 2019]

- Shannon entropy units in bits due to $\log_2$
- Distribution calculated for pitch and duration