Quantum Annealing for Music Arrangement

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Overview

Motivations

Motivations

- Small lit review[1]
- Quantum computer music
- My own novel adaption of the method
- THIS IS MY OWN IDEA

[1] cambouropoulos˙lbdm˙2011.

Theory

Adiabatic quantum computing

Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.[2]

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

[2] born˙beweis˙1928.

Quantum annealing

- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic

Model

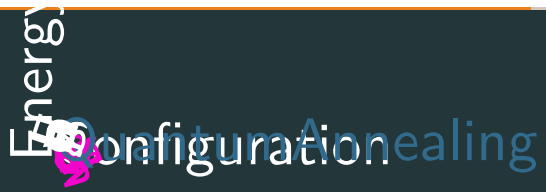
Initial state

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

Ising model

$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^{N} h_i \sigma_i^z$$

Uses



QUBO

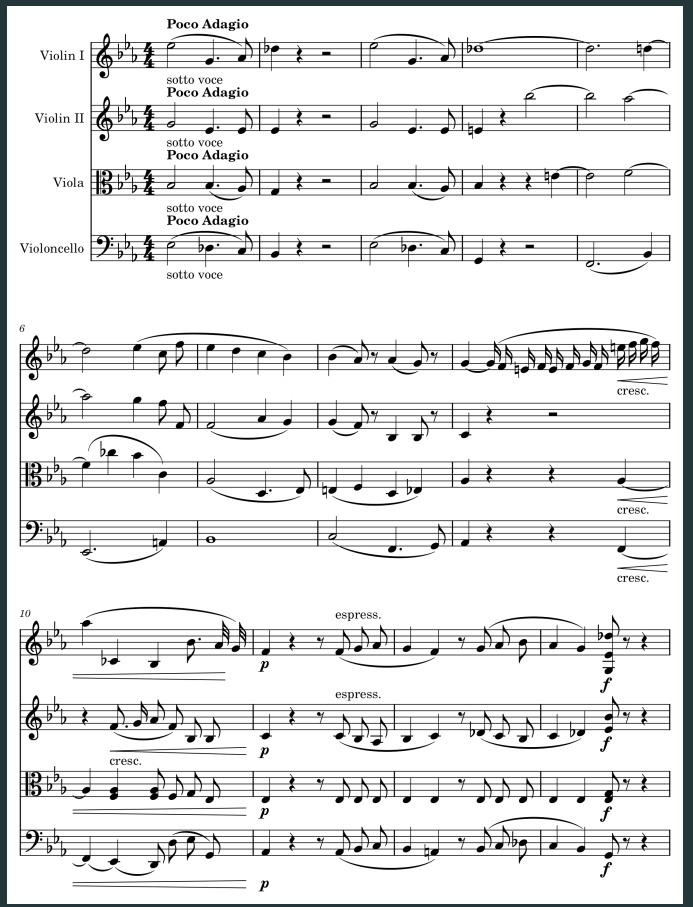
Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Sent to the QPU for optimisation

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**



Beethoven's String Quartet No. 10

How to combine them?

Methods

Problem formulation

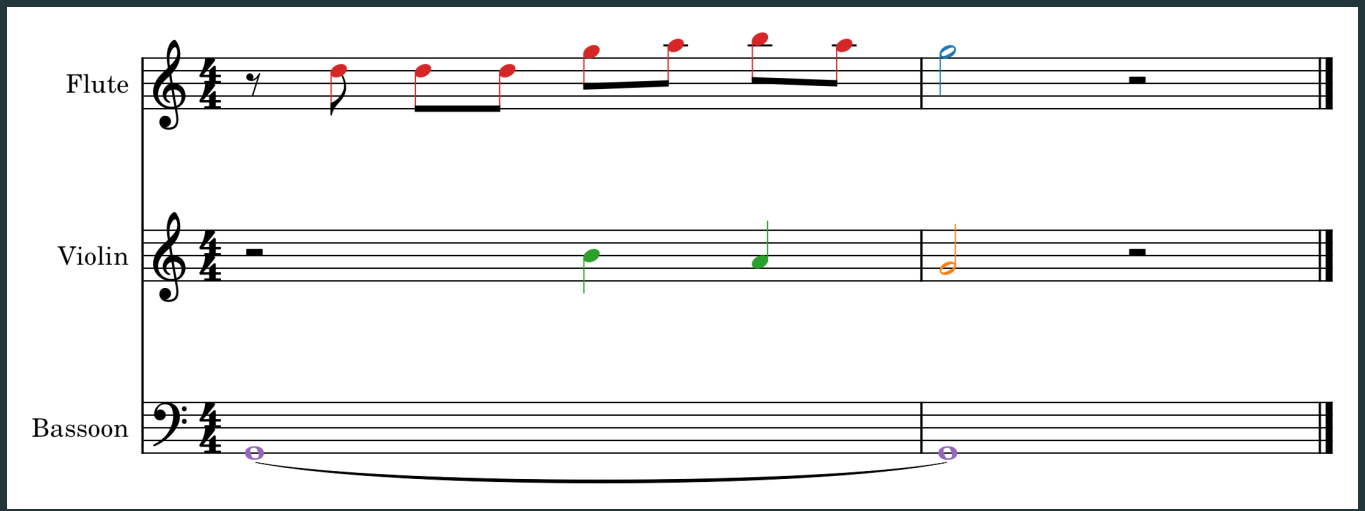
1. Split score into musical phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

1. Split score

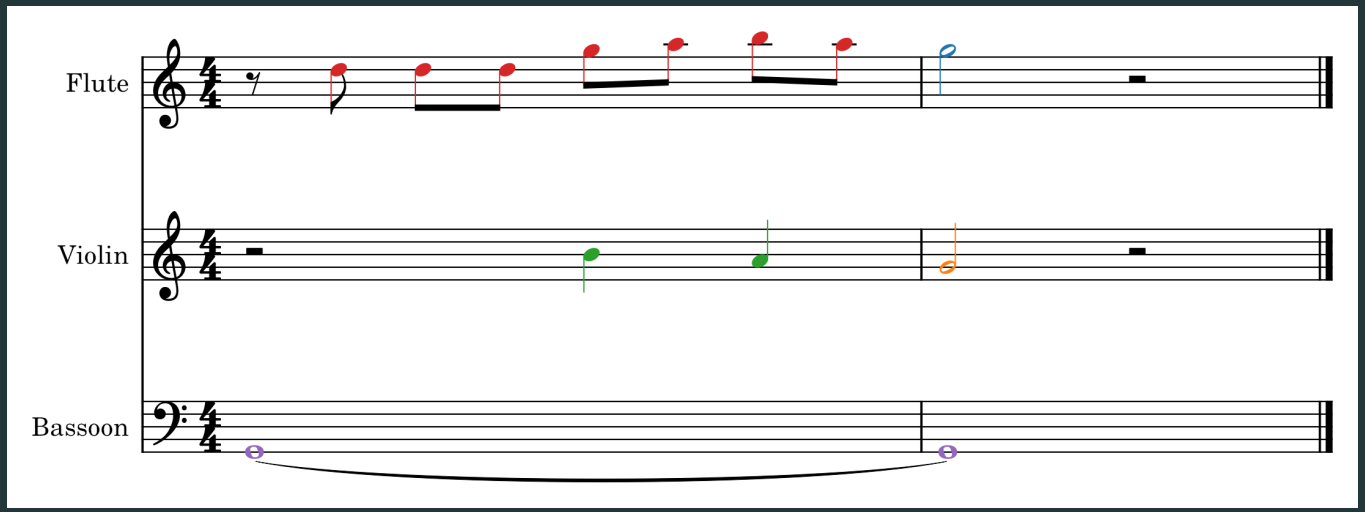
Local boundary detection model (LBDM)

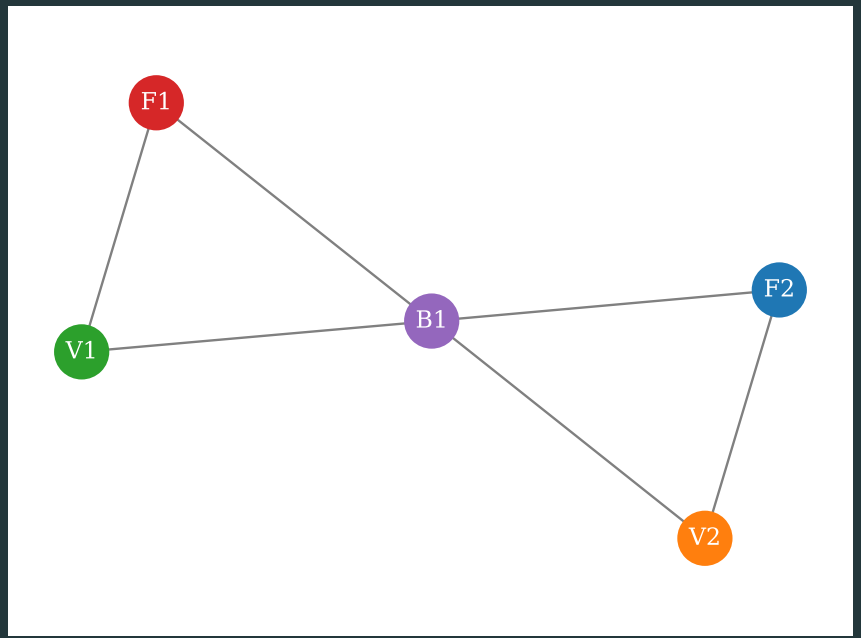
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

cambouropoulos˙lbdm˙2011



2. Create graph

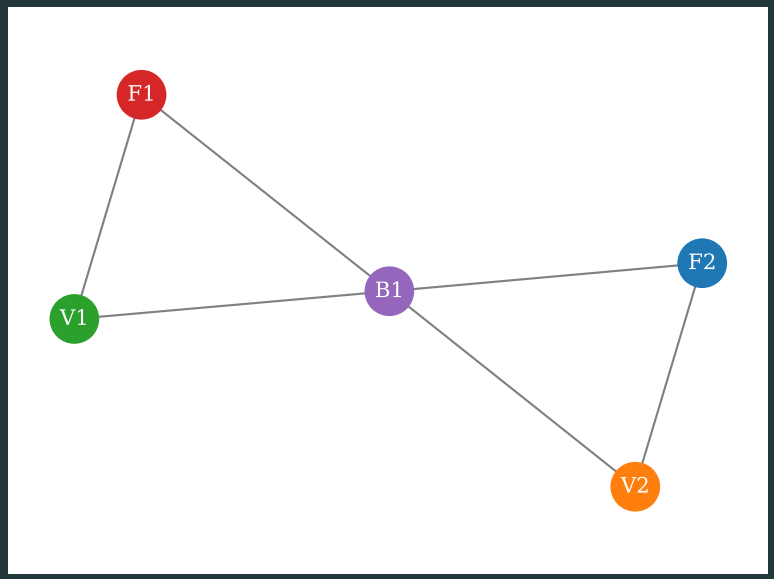


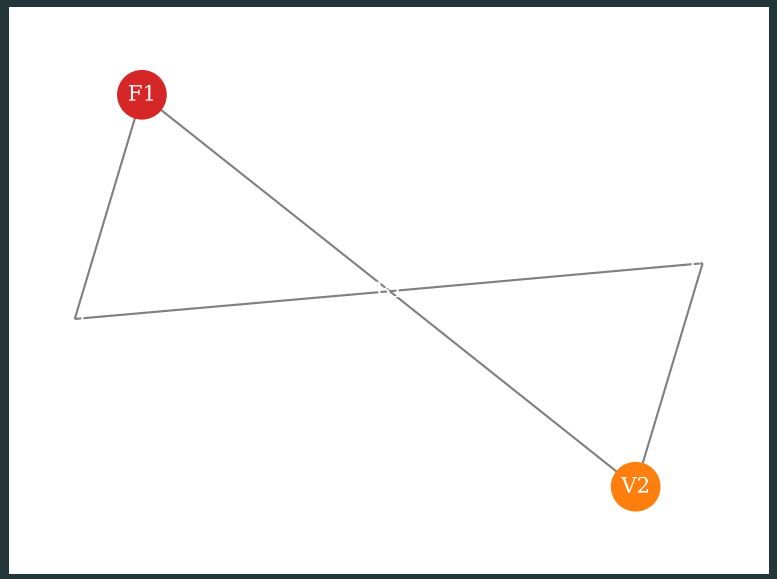


3. Solve graph

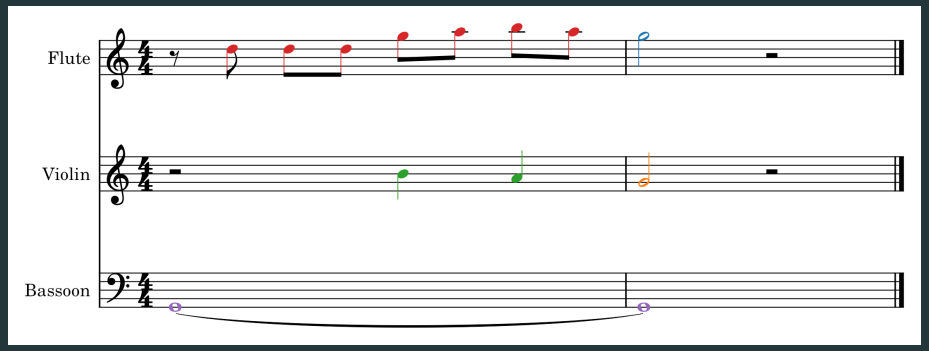
Maximal independent set (MIS)

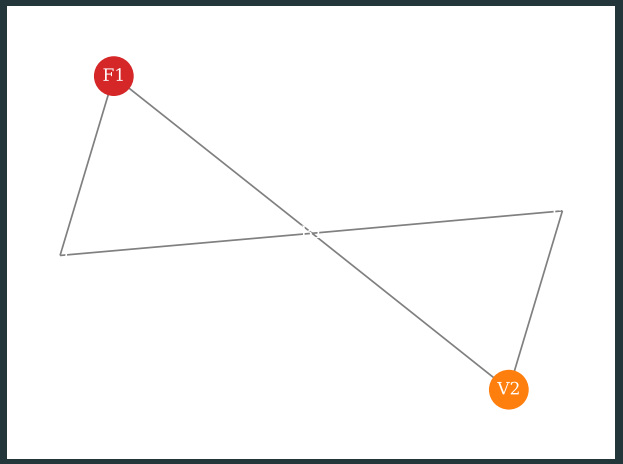
Largest subset of nodes such that no nodes within the subset are connected by an edge

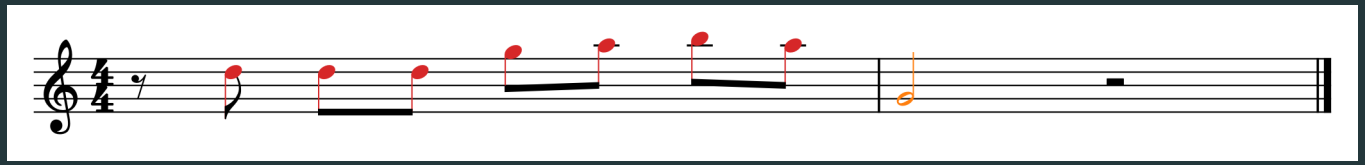




4. Construct arrangement

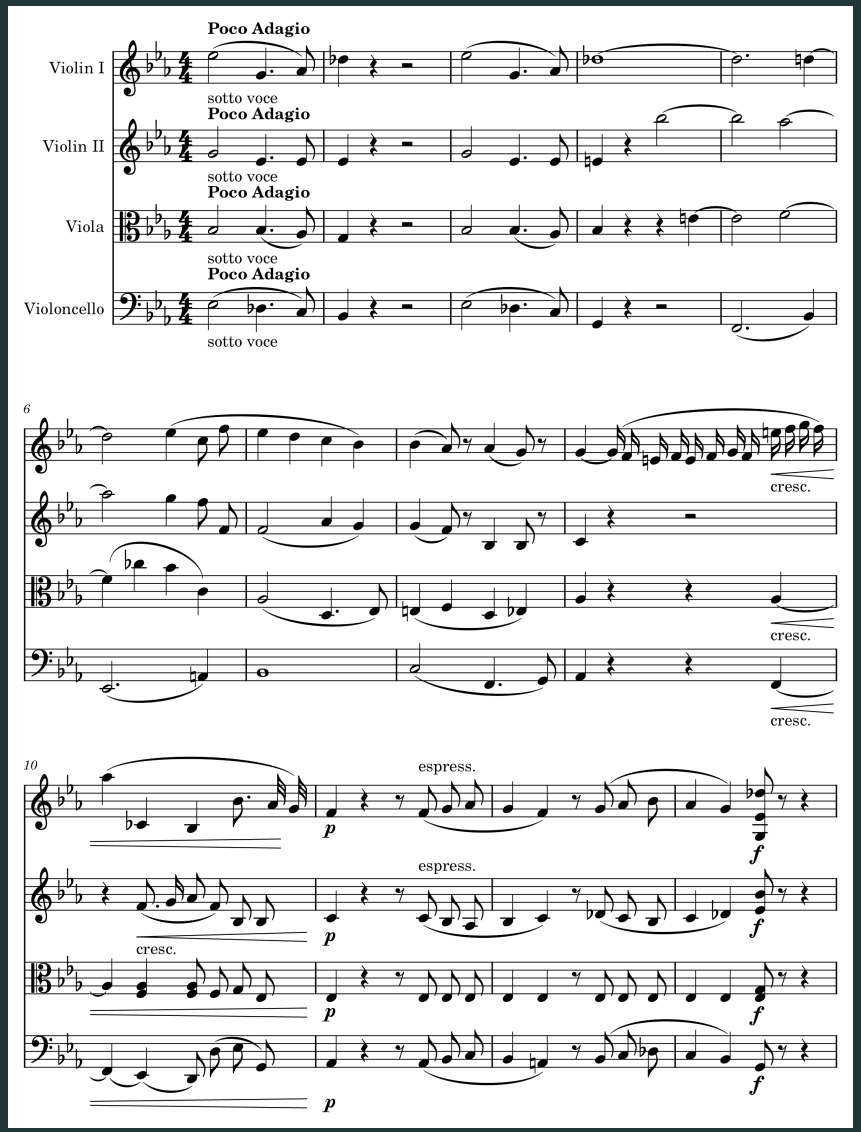






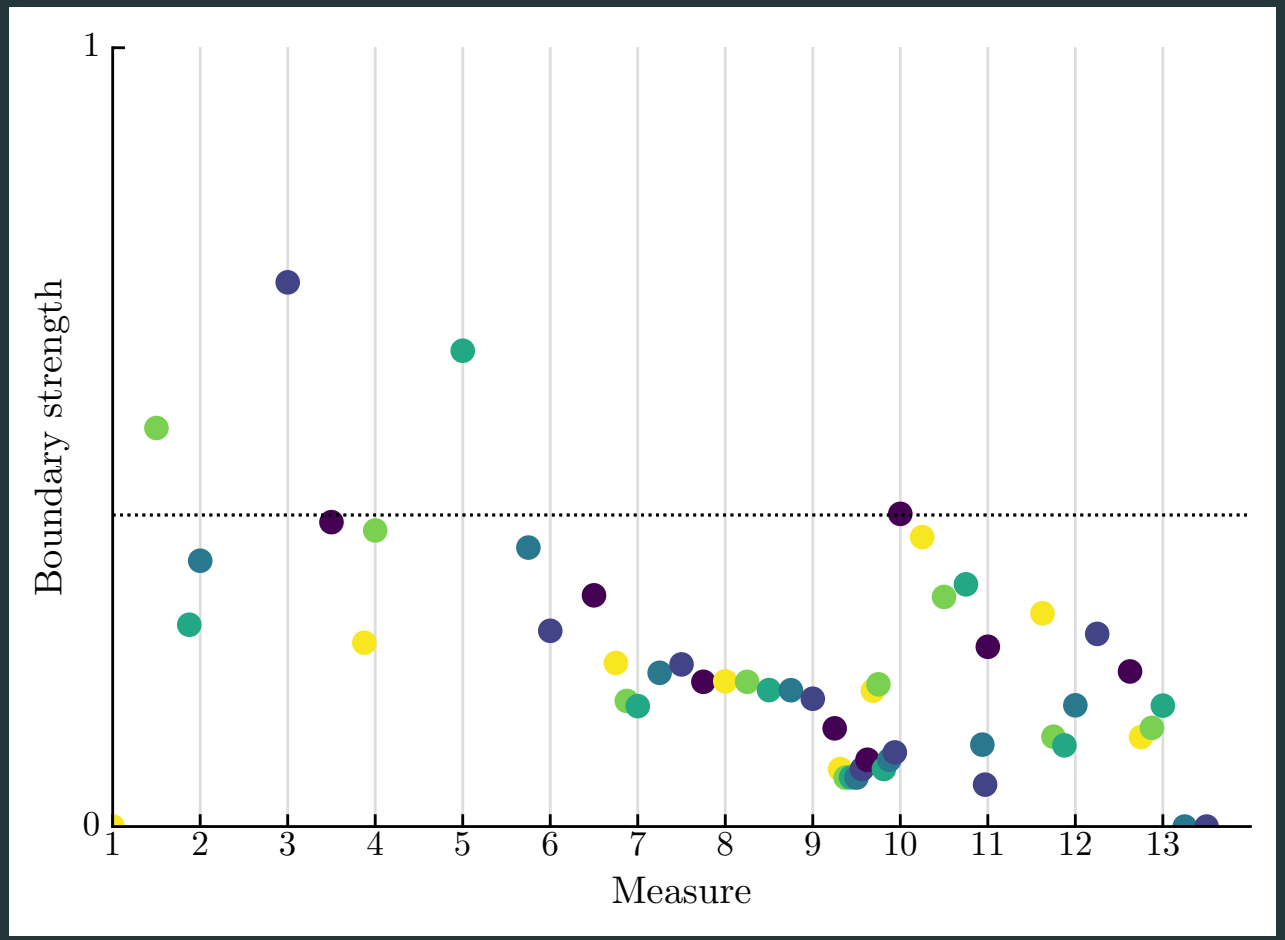
Results

Excerpt



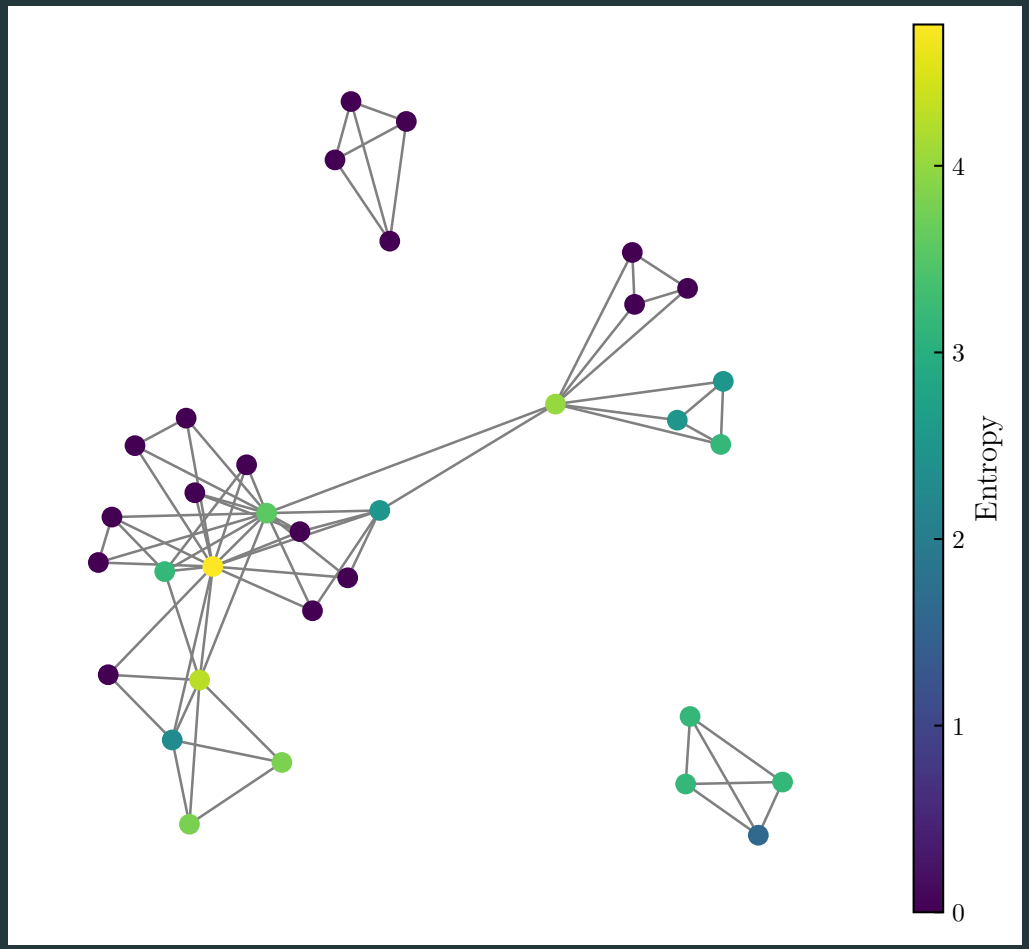
String Quartet No. 10 by Ludwig van Beethoven

Phrase detection



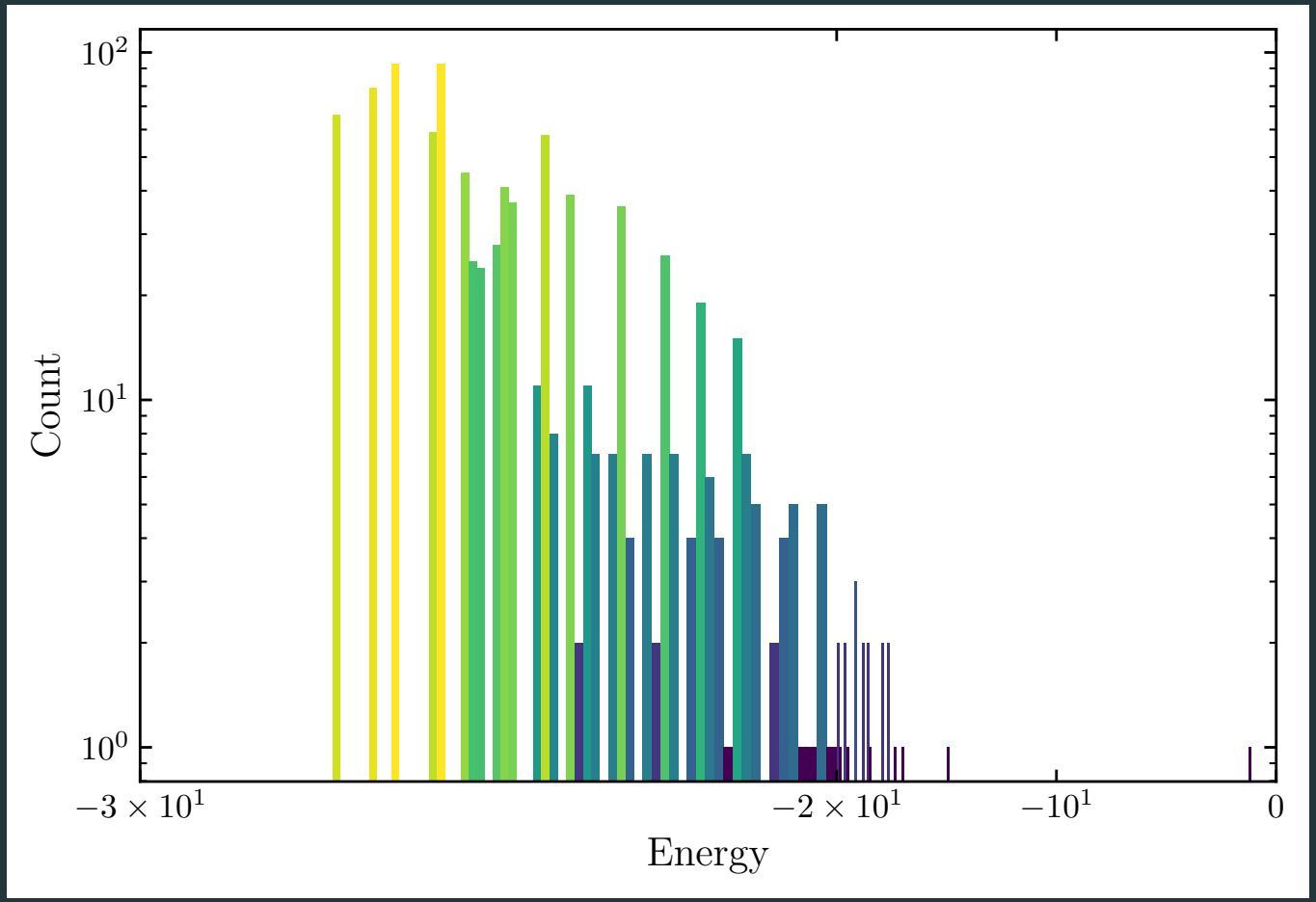
Boundary strengths for the Violin I part

Problem graph



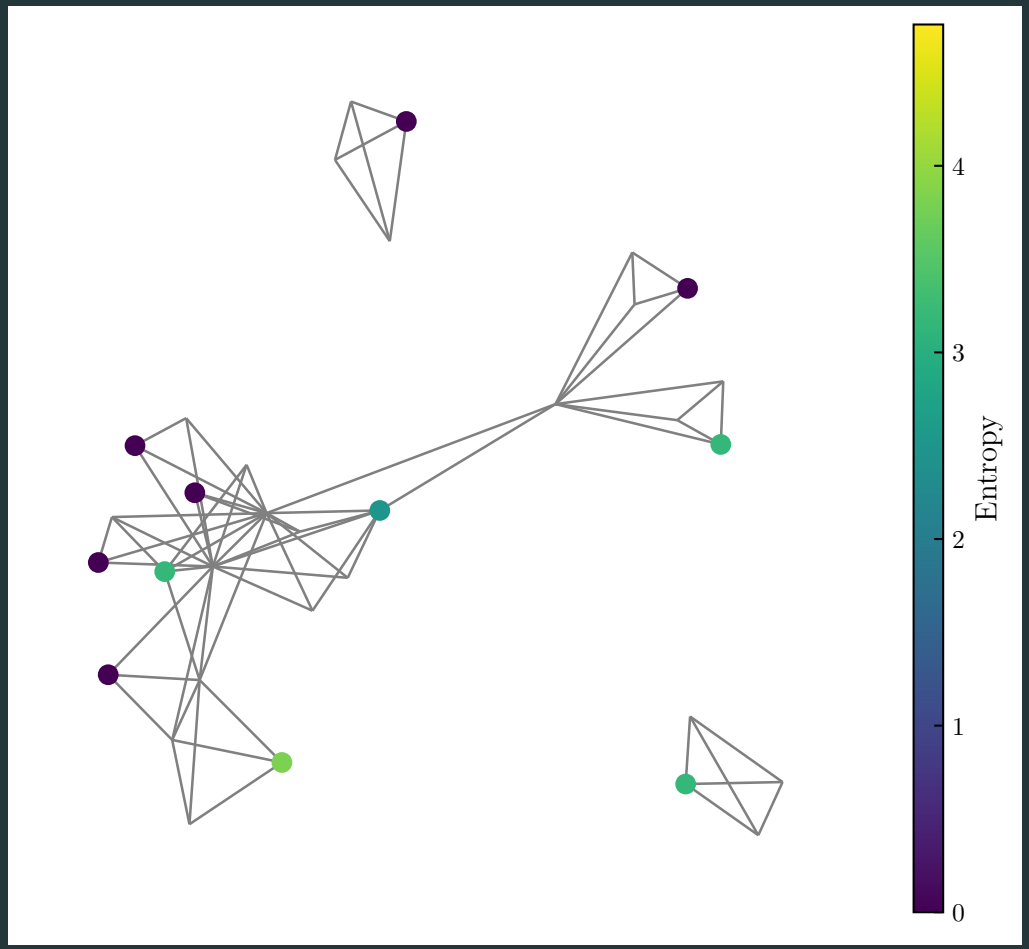
Problem graph with 33 nodes and 70 edges

Solutions



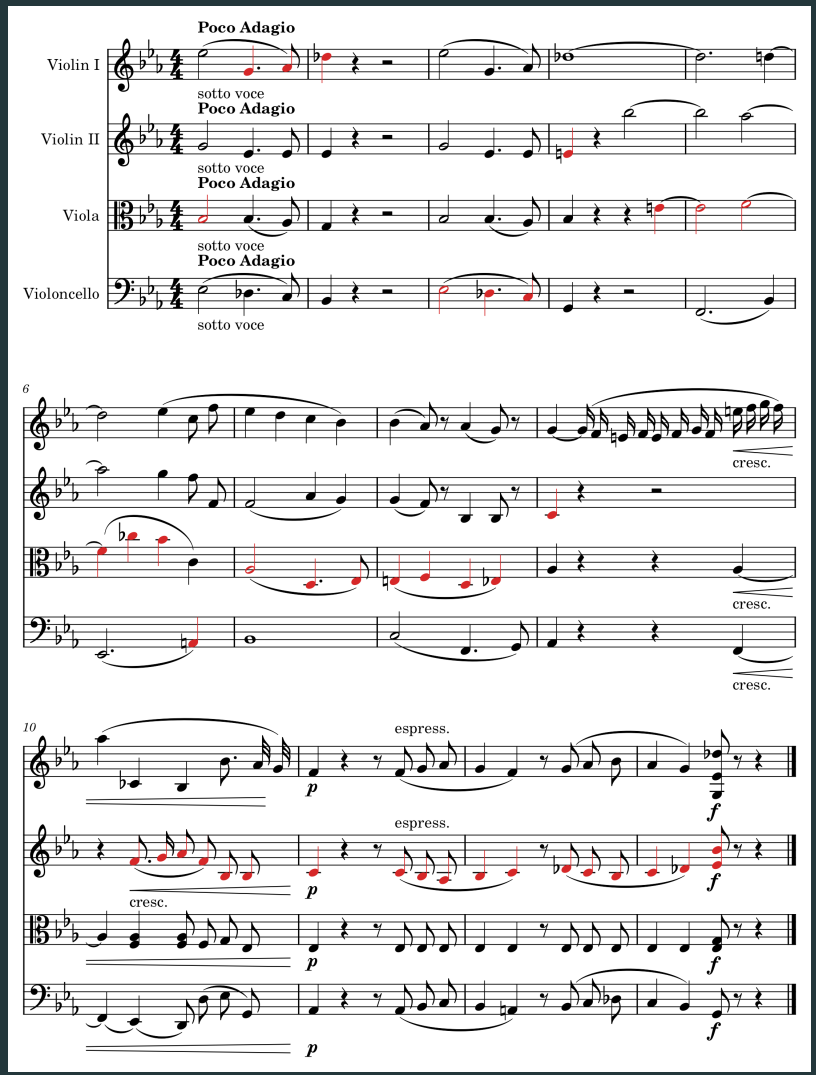
Returned solutions for 1000 reads

Example solution

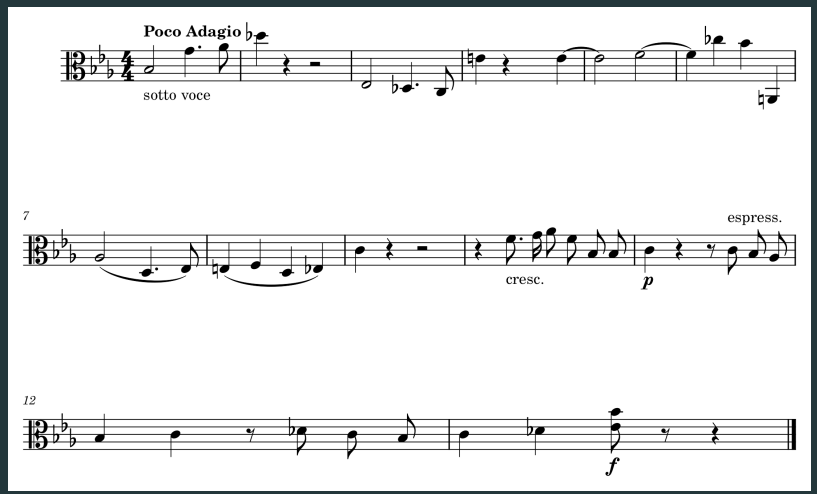


Solution graph returning a subset of 11 nodes

Final arrangement



Selected phrases

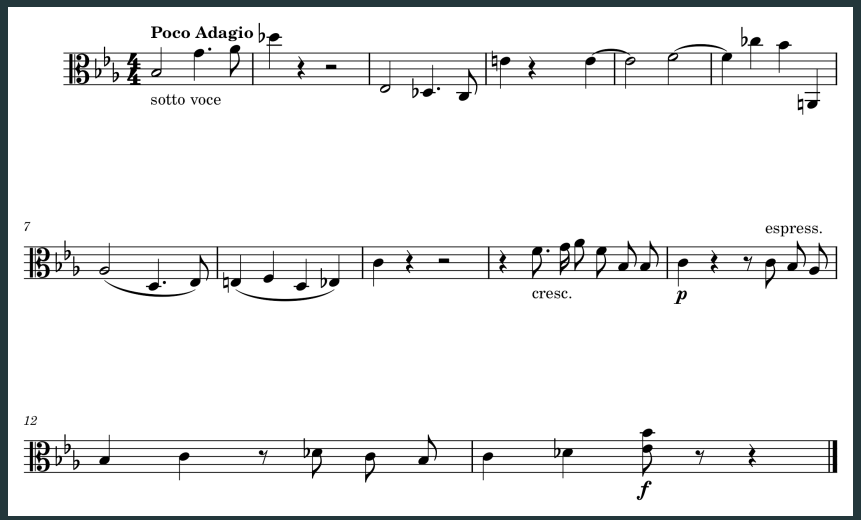


Final arrangement

Conclusions

Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms **huang˙towards˙2012**
- Removes skill barrier for music arrangement



Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements
  pearce˙towards˙2001

Thank you!

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4 March 2025

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LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

cambouropoulos˙lbdm˙2011

MIS

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

lucas˙ising˙2014

$A/B >= 2 \max(W)$ to weight the constraint term more heavily than any objective term

Phrase entropy

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

li˙automatic˙2019

References i