

Music Arrangement via Quantum Annealing

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Overview

Theory

Music arrangement

Quantum annealing

Methods

Results

Conclusions

Theory

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

Poco Adagio

Violin I
sotto voce
Poco Adagio

Violin II
sotto voce
Poco Adagio

Viola
sotto voce
Poco Adagio

Violoncello
sotto voce

cresc.

cresc.

cresc.

cresc.

p

p

cresc.

p

p

f

f

f

p

Beethoven's String Quartet No. 10

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another (AQC)
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

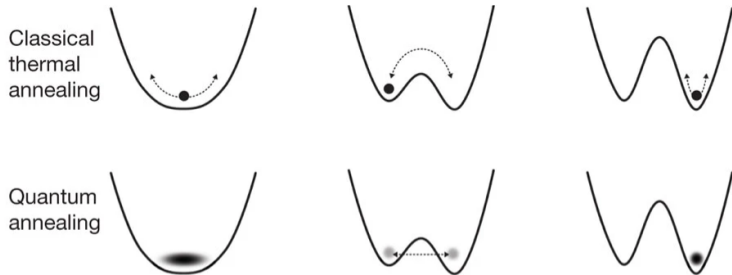
Ising model

$$H_p(\sigma^z) = \sum_{i < j} J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

Initial state

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

Quantum annealing



MW Johnson *et al.* *Nature* **473**, 194–198 (2011) doi:10.1038/nature10012

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j} Q_{i,j} x_i x_j + \sum_i Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

How to combine them?

Methods

Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

1. Split parts

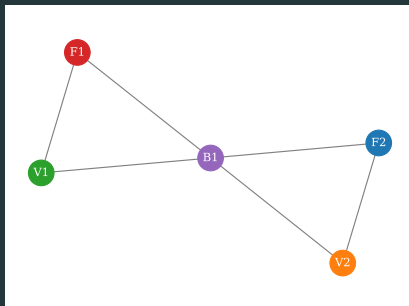
Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a sequence of notes: a quarter rest, a quarter note (red), an eighth note (red), a quarter note (red), an eighth note (red), a quarter note (red), and a quarter note (blue) followed by a whole rest. The Violin part (treble clef) has a whole rest, followed by two quarter notes (green), and a quarter note (orange) followed by a whole rest. The Bassoon part (bass clef) has a whole rest, followed by a whole note (purple) with a slur extending to the end of the measure. The instruments are labeled on the left: Flute, Violin, and Bassoon.

2. Create graph

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (top staff) begins with a quarter rest, followed by a sequence of eighth notes: G4, A4, B4, C5, D5, E5, and F5. The Violin part (middle staff) has a whole rest for the first two measures, then plays a half note G4 in the third measure and a half note A4 in the fourth measure. The Bassoon part (bottom staff) plays a whole note G2 in the first measure and a whole note A2 in the second measure, with a slur connecting the two notes. The score ends with a double bar line.

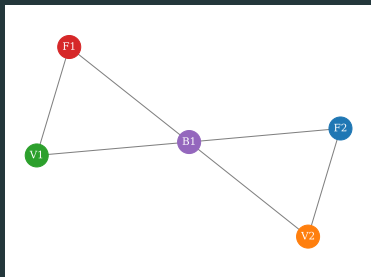


3. Solve graph

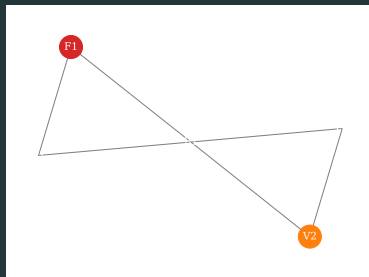
Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge.

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$



Problem graph



Solution graph

4. Construct arrangement

Flute

Violin

Bassoon

The image shows a musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a quarter rest in the first measure, followed by eighth notes G4, A4, B4, C5, D5, and E5 in the second measure, and a whole note F5 in the third measure. The Violin part (treble clef) has a whole rest in the first measure, eighth notes G4 and A4 in the second measure, and a whole note B4 in the third measure. The Bassoon part (bass clef) has a whole note G2 in the first measure, a whole rest in the second measure, and a whole note G2 in the third measure. A slur connects the G2 notes in the first and third measures.



The image shows a simplified musical score for two instruments: Flute and Violin, in 4/4 time. The Flute part (treble clef) has a quarter rest in the first measure, followed by eighth notes G4, A4, B4, C5, D5, and E5 in the second measure, and a whole note F5 in the third measure. The Violin part (treble clef) has a whole rest in the first measure, eighth notes G4 and A4 in the second measure, and a whole note B4 in the third measure.

Results

Excerpt

Poco Adagio

Violin I
sotto voce
Poco Adagio

Violin II
sotto voce
Poco Adagio

Viola
sotto voce
Poco Adagio

Violoncello
sotto voce
Poco Adagio

6

cresc.

cresc.

cresc.

10

espress.

p

f

espress.

p

f

cresc.

p

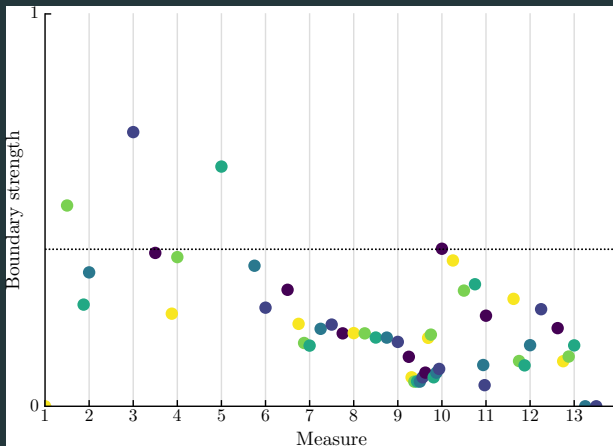
f

p

String Quartet No. 10 by Ludwig van Beethoven

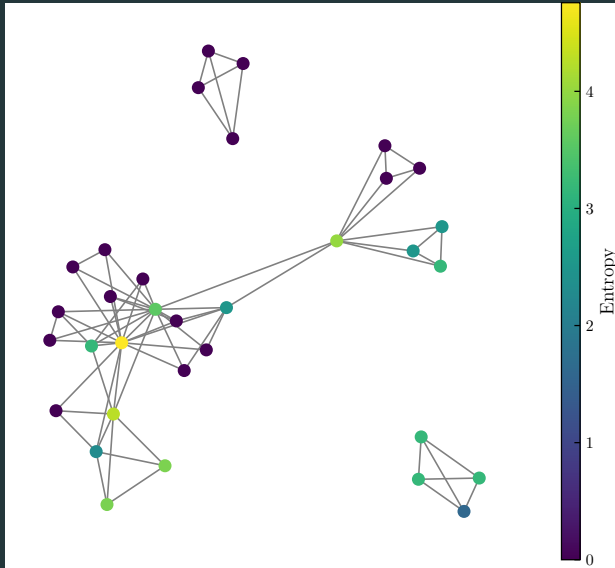
Phrase detection

Local boundary detection model (LBDM)

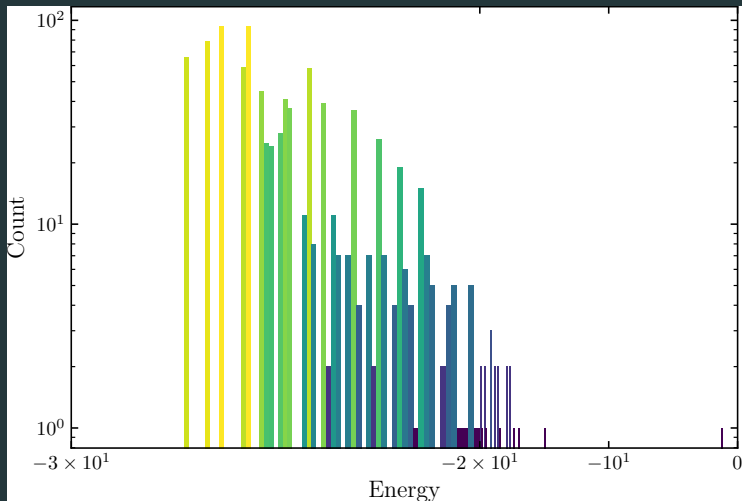


Boundary strengths for the Violin I part

Problem graph



Solutions



Solutions returned by the QPU

Example solution

$$\oiint_A \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

The *net electric flux* through any **closed** surface is proportional to the **enclosed charge**.

Alert

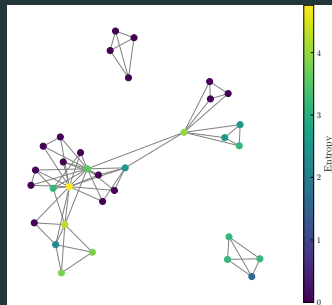
This is an alert.

Example

This is an example.

Apperance sync

- Volume rate of flow equal to divergence
- Summed over entire volume
- Equal to net flow across the boundary



Source: Wikimedia Commons

$$\iiint_V \nabla \cdot \mathbf{F} dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$$

Conclusions

Equation gather

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{I}$$