

Music Arrangement via Quantum Annealing

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19th January 2025

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Overview

Theory

Music arrangement

Quantum annealing

Methods

Results

Conclusions

Theory

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

The image displays a musical score for Beethoven's String Quartet No. 10, specifically the first movement, 'Poco Adagio'. The score is written for four instruments: Violin I, Violin II, Viola, and Violoncello. The key signature is three flats (B-flat, E-flat, A-flat), and the time signature is 4/4. The tempo is marked 'Poco Adagio'. The score includes various dynamic markings such as *p* (piano), *f* (forte), *cresc.* (crescendo), and *decresc.* (decrescendo). The notation includes slurs, ties, and other musical symbols typical of a string quartet score. The score is presented in a clean, professional layout with clear instrument labels and musical notation.

Beethoven's String Quartet No. 10

Quantum annealing

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

Encodes problem solution into Hamiltonian's ground state

Ising model

$$H(s) = - \sum_{i < j} J_{ij} s_i s_j - \sum_{i=1}^N h_i s_i$$

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j} Q_{i,j} x_i x_j + \sum_i Q_{i,i} x_i$$

How to combine them?

Methods

Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

1. Split parts

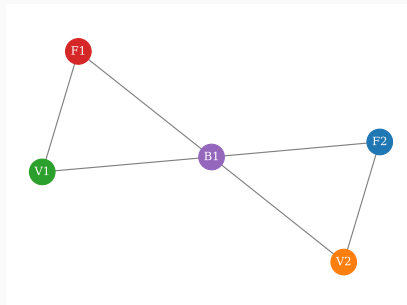
Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a sequence of notes: a quarter rest, an eighth note G4, an eighth note A4, a quarter note B4, a quarter note C5, a quarter note D5, a quarter note E5, and a quarter note F5. The Violin part (treble clef) has a quarter rest, a quarter note G4, a quarter note A4, a quarter note B4, and a quarter note C5. The Bassoon part (bass clef) has a quarter rest, a quarter note G3, a quarter note A3, a quarter note B3, and a quarter note C4. A curved line with purple dots at the ends connects the first and last notes of the Bassoon part, indicating a boundary or split. The notes are color-coded: red for Flute, green for Violin, and blue for Bassoon.

2. Create graph

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (top staff) begins with a quarter rest, followed by a sequence of eighth notes: G4, A4, B4, C5, D5, E5, and F5. The Violin part (middle staff) has a whole rest for the first two measures, then plays a half note G4 in the third measure and a half note A4 in the fourth measure. The Bassoon part (bottom staff) has a whole rest for the first two measures, then plays a half note G2 in the third measure and a half note A2 in the fourth measure. A purple line connects the G2 note in the Bassoon part to the G4 note in the Violin part, indicating a pitch relationship.

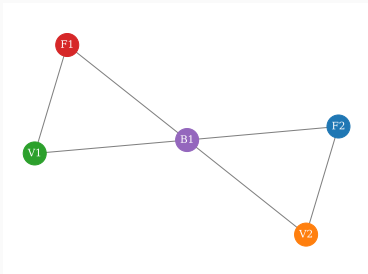


3. Solve graph

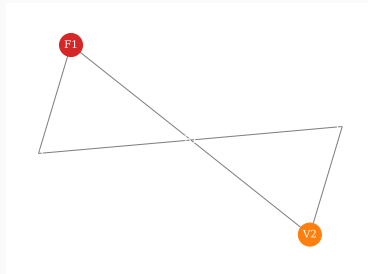
Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge.

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$



Problem graph



Solution graph

4. Construct arrangement

Flute

Violin

Bassoon

The image shows a musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a melody of eighth notes: G4 (quarter rest), A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a melody of eighth notes: G4 (quarter rest), A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Bassoon part (bass clef) has a melody of eighth notes: G3 (quarter rest), A3, B3, C4, D4, E4, F4, G4, followed by a whole rest. A blue slur connects the G3 and G4 notes in the Bassoon part.



Flute

Violin

The image shows a musical score for two instruments: Flute and Violin, in 4/4 time. The Flute part (treble clef) has a melody of eighth notes: G4 (quarter rest), A4, B4, C5, D5, E5, F5, G5, followed by a whole rest. The Violin part (treble clef) has a melody of eighth notes: G4 (quarter rest), A4, B4, C5, D5, E5, F5, G5, followed by a whole rest.

Results

Excerpt

Poco Adagio

Violin I
sotto voce
Poco Adagio

Violin II
sotto voce
Poco Adagio

Viola
sotto voce
Poco Adagio

Violoncello
sotto voce
Poco Adagio

6

cresc.

cresc.

cresc.

10

espress.

p

espress.

p

cresc.

p

f

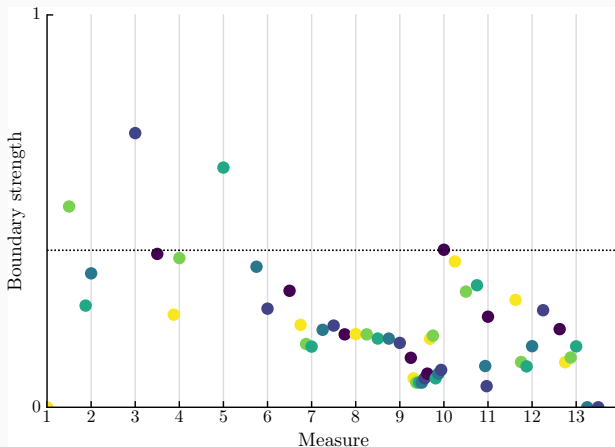
f

p

String Quartet No. 10 by Ludwig van Beethoven

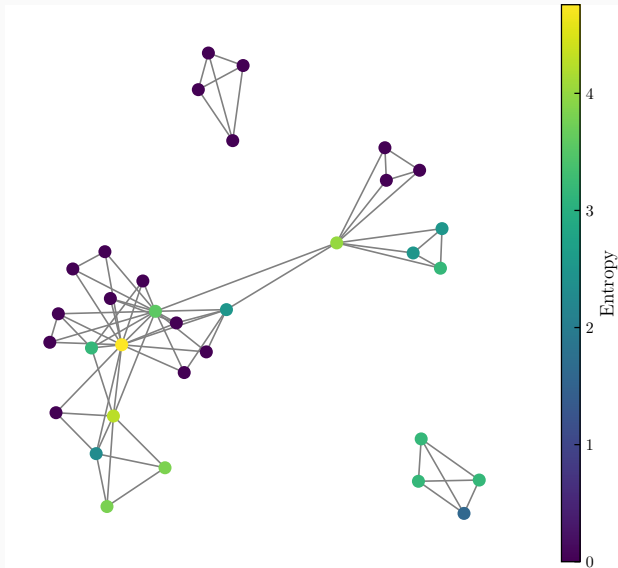
Phrase detection

Local boundary detection model (LBDM)

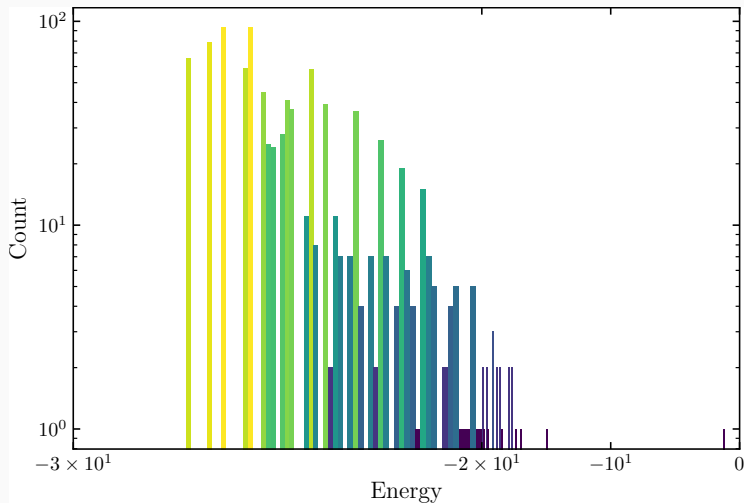


Boundary strengths for the Violin I part

Problem graph



Solutions



Solutions returned by the QPU

Example solution

$$\oiint_A \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

The *net electric flux* through any **closed** surface is proportional to the **enclosed charge**.

Alert

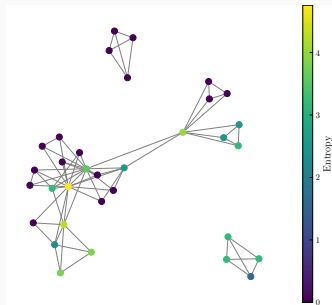
This is an alert.

Example

This is an example.

Apperance sync

- Volume rate of flow equal to divergence
- Summed over entire volume
- Equal to net flow across the boundary



Source: Wikimedia Commons

$$\iiint_V \nabla \cdot \mathbf{F} dV = \oiint_A \mathbf{F} \cdot d\mathbf{A}$$

Conclusions

Equation gather

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{I}$$