

Music Arrangement via Quantum Annealing

Lucas Kirby

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Department of Physics, Durham University

Overview

Theory

Music arrangement

Quantum annealing

Methods

Results

Conclusions

Theory

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

The image displays the first system of a musical score for Beethoven's String Quartet No. 10. The score is written for four instruments: Violin I, Violin II, Viola, and Violoncello. The key signature is three flats (B-flat, E-flat, A-flat) and the time signature is 4/4. The tempo marking is 'Poco Adagio'. The first system shows the initial measures of the piece, with each instrument part clearly delineated. The second system continues the music, featuring dynamic markings such as 'cresc.' (crescendo) and 'p' (piano). The third system includes the marking 'espress.' (espressivo) and continues the musical development. The fourth system concludes the excerpt with a 'p' marking. The notation includes various musical symbols such as notes, rests, and dynamic markings.

Beethoven's String Quartet No. 10

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

[Lucas, 2014]

Ising model

$$H_p(\sigma^z) = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

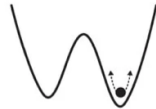
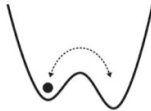
Initial state

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

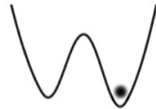
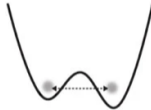
[Lucas, 2014]

Quantum annealing

Classical
thermal
annealing



Quantum
annealing



[Johnson et al., 2011]

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

How to combine them?

Methods

Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

1. Split parts

Local boundary detection model (LBDM)

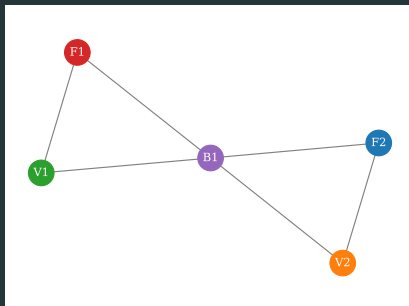
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

[Cambouropoulos, 2011]

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a sequence of red notes: a quarter rest, an eighth note, a quarter note, a half note, a quarter note, and a half note, followed by a blue quarter note and a whole rest. The Violin part (treble clef) has a whole rest, followed by two green quarter notes, and an orange quarter note with a whole rest. The Bassoon part (bass clef) has a purple whole note with a slur extending to the end of the measure. The notes are color-coded to represent different segments or boundaries in the LBDM model.

2. Create graph

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (top staff) begins with a quarter rest, followed by a sequence of eighth notes: G4, A4, B4, C5, D5, E5, and F5. The Violin part (middle staff) has a whole rest in the first measure, followed by two eighth notes: G4 and A4, and then a half note: B4. The Bassoon part (bottom staff) has a whole rest in the first measure, followed by a half note: G2, and then a whole note: A2. A purple line connects the Bassoon part to the Violin part, indicating a relationship or connection between the two parts.



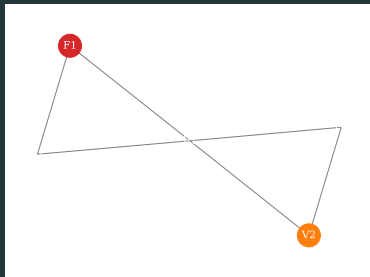
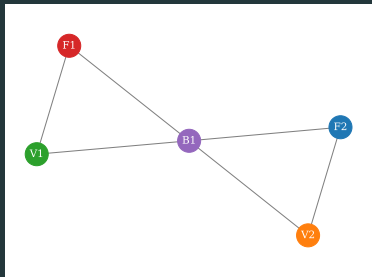
3. Solve graph

Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$

[Lucas, 2014]



4. Construct arrangement

Flute

Violin

Bassoon

The image shows a musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a quarter rest followed by six eighth notes (G4, A4, B4, C5, B4, A4) and a quarter rest. The Violin part (treble clef) has a whole rest followed by two eighth notes (G4, A4) and a quarter rest. The Bassoon part (bass clef) has a whole note (G3) and a whole rest. A blue bracket connects the two whole notes in the Bassoon part.



The image shows a reduced musical score for two instruments: Flute and Violin, in 4/4 time. The Flute part (treble clef) is identical to the original score. The Violin part (treble clef) has a quarter rest followed by two eighth notes (G4, A4) and a quarter rest. The Bassoon part has been removed.

Results

Excerpt

Poco Adagio

Violin I
sotto voce
Poco Adagio

Violin II
sotto voce
Poco Adagio

Viola
sotto voce
Poco Adagio

Violoncello
sotto voce
Poco Adagio

6

cresc.

cresc.

cresc.

10

espress.

p

f

espress.

p

f

cresc.

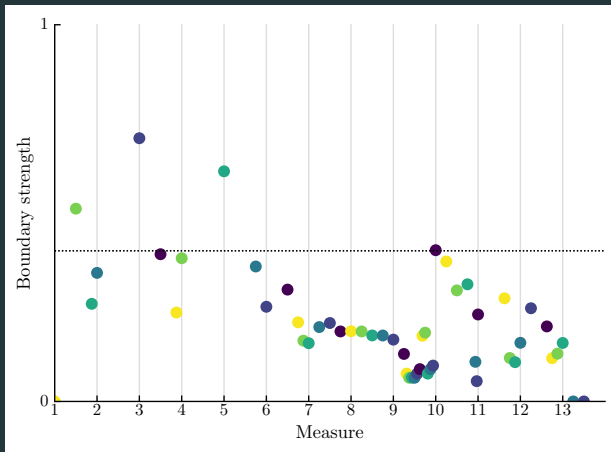
p

f

p

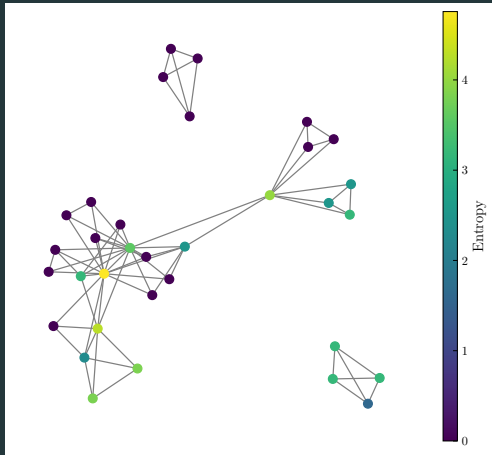
String Quartet No. 10 by Ludwig van Beethoven

Phrase detection



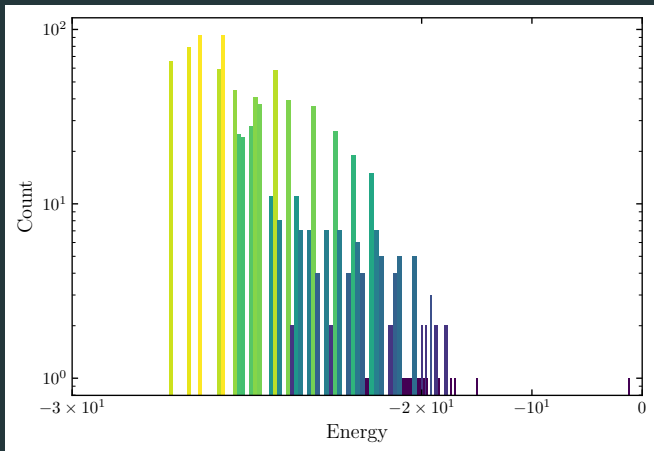
Boundary strengths for the Violin I part

Problem graph



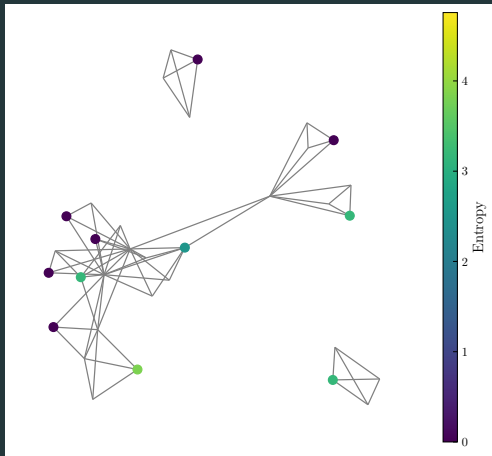
Problem graph with 33 nodes and 70 edges

Solutions



Returned solutions for 1000 reads

Example solution



Solution graph returning a subset of 11 nodes

Final arrangement

Poco Adagio

Violin I
sotto voce

Violin II
sotto voce

Viola
sotto voce

Violoncello
sotto voce

6

cresc.

cresc.

cresc.

10

espress.

espress.

cresc.

p

f

p

f

p

f

Selected phrases

Poco Adagio

sotto voce

7

espress.

cresc.

p

12

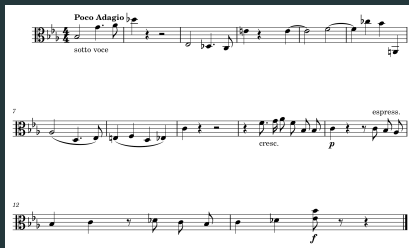
f

Final arrangement

Conclusions

Conclusions

- Successful in creating a valid single-part reduction
 - Monophonic lowest-energy solution
 - Sufficient number of phrases selected
 - Musically interesting final arrangement
- Advantage over classical algorithms (deep learning, genetic)
 - No training data needed
 - Faster solve time [Huang et al., 2012]



Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements
[Pearce and Wiggins, 2001]

Thank you!

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Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

Degree of change

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S)}{\max(S) - \min(S)}$$

Weighting

$$S = \frac{1}{3} (S_{\text{pitch}} + 2S_{\text{IOI}})$$

Phrase entropy

Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

[Li et al., 2019]