

# Quantum Annealing for Music Arrangement

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4 March 2025

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# Overview

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# Motivations

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# Motivations

- Small lit review<sup>1</sup>
- Quantum computer music
- My own novel adaption of the method
- THIS IS MY OWN IDEA

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<sup>1</sup>Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

# Theory

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# Adiabatic quantum computing

## Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.<sup>2</sup>

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

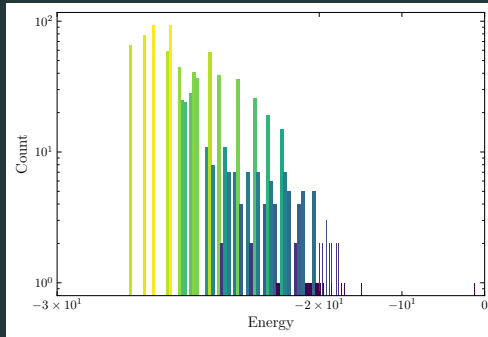
- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

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<sup>2</sup>Born and Fock, 'Beweis des Adiabatenatzes'.

# Quantum annealing

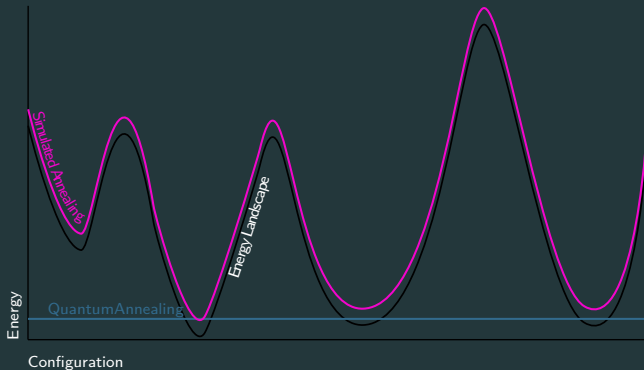
- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic



Own work

# Advantages

- Find the ground state of a complicated Hamiltonian
- Quantum tunneling overcomes local minima



By Brianlechthaler - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=112382195>



# Ising model

## Initial Hamiltonian

$$H_0 = h_0 \sum_{i=1}^N \sigma_i^x$$

## Problem Hamiltonian

$$H_p(\sigma^z) = \sum_{i < j}^N J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^N h_i \sigma_i^z$$

## Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i < j}^N Q_{i,j} x_i x_j + \sum_i^N Q_{i,i} x_i$$

- Difficult to solve analytically
- Mapped to  $H_p$  using simple change of variable
- Encodes problem solution into Hamiltonian's ground state

**What problems can we solve?**

# Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- *Reduction* can be shown to be computationally complex<sup>3</sup>

The image displays a page from a musical score, likely for a symphony or concerto. The top section shows the staves for Violin I, Violin II, Viola, and Violoncello. Each staff is labeled with its instrument name and the tempo 'Poco Adagio'. The music is written in a key signature of three flats (B-flat, E-flat, A-flat) and a 4/4 time signature. The score includes various musical notations such as notes, rests, and dynamic markings like 'cresc.' and 'p'. The bottom section of the page shows a continuation of the music, with a measure number '10' indicated. The overall layout is clean and professional, typical of a printed musical score.

REPLACE WITH HAYDN

<sup>3</sup>Moses and Demaine, 'Computational Complexity of Arranging Music'

# Methods

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## Problem formulation

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Formulate graph optimisation problem
4. Solve problem using QPU
5. Construct arrangement from solution

# 1. Split score

- Musical phrases chosen as smallest unit of music
- Preserve melody and structure when arranged

## Local boundary detection model (LBDM)<sup>4</sup>

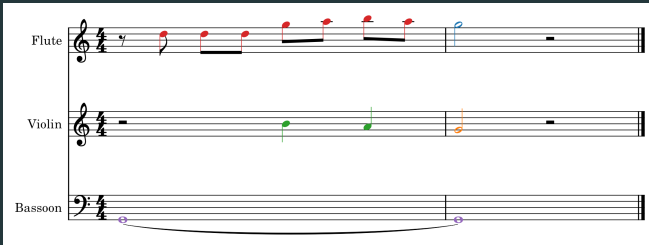
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

The image shows a musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a melody starting with a quarter rest, followed by eighth notes G4, A4, B4, C5, D5, E5, and F5, ending with a quarter rest. The Violin part (treble clef) has a melody starting with a quarter rest, followed by quarter notes G3, A3, and B3, ending with a quarter rest. The Bassoon part (bass clef) has a melody starting with a quarter rest, followed by a half note G2, ending with a quarter rest. A purple oval highlights the first measure of the Bassoon part, and a red oval highlights the last measure of the Bassoon part, indicating the boundaries of a musical phrase.

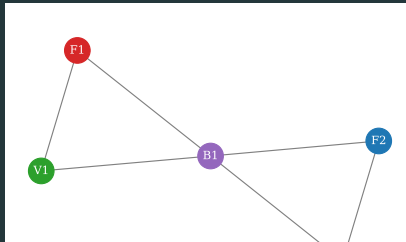
<sup>4</sup>Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

## 2. Create graph

- Vertices (nodes) connected by edges
- Models pairwise relations between objects



A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (treble clef) has a sequence of notes: a quarter rest, a quarter note G4, a quarter note A4, a quarter note B4, a quarter note C5, a quarter note D5, and a half note E5. The Violin part (treble clef) has a sequence of notes: a half rest, a quarter note G4, a quarter note A4, and a half note B4. The Bassoon part (bass clef) has a sequence of notes: a half rest, a quarter note G3, a quarter note A3, and a half note B3. A curved line connects the first and last notes of the Bassoon part, indicating a sustained note.





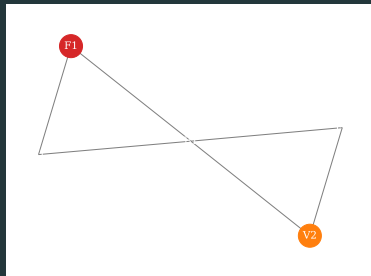
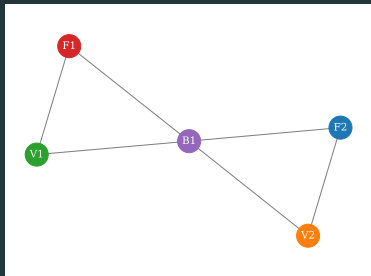
### 3. Create optimisation problem

#### Proper vertex colouring

Colour each vertex such that no edge connects two vertices of the same colour

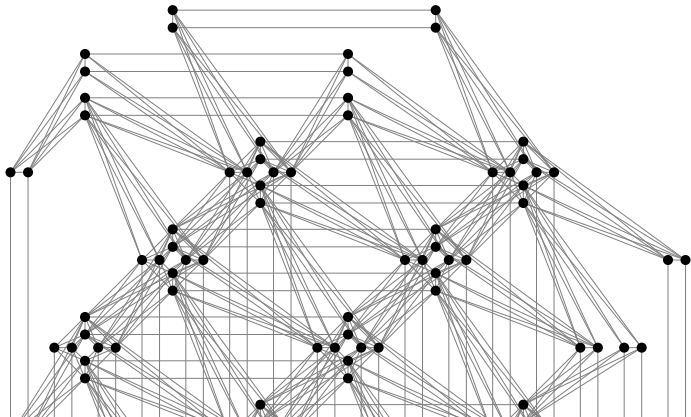
$$\begin{aligned} f(x) = & A \sum_{v \in V} \left( 1 - \sum_{i=1}^n x_{v,i} \right)^2 + B \sum_{(u,v) \in E} \sum_{i=1}^n x_{u,i} x_{v,i} \\ & - C \sum_{v \in V} \sum_{i=1}^n W_v x_{v,i} - D \sum_{(u,v) \in E} W_{uv} \sum_{i=1}^n \sum_{j=1}^n x_{u,i} x_{v,j} \end{aligned}$$

### 3. Create optimisation problem



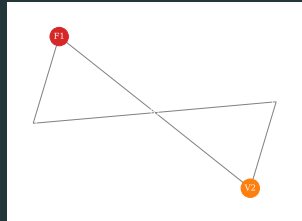
## 4. Solve problem

- Problem embedded onto D-Wave quantum hardware
- True quantum annealer optimises QUBO formulation
- Returns a sample set of results



## 5. Construct arrangement

A musical score for three instruments: Flute, Violin, and Bassoon, in 4/4 time. The Flute part (top staff) begins with a quarter rest, followed by a sequence of eighth notes: G4 (red), A4 (red), B4 (red), C5 (red), D5 (red), E5 (blue), and a final quarter rest. The Violin part (middle staff) has a quarter rest, followed by a half note G4 (green), a half note A4 (green), and a final quarter rest. The Bassoon part (bottom staff) has a half note G2 (purple) and a half note A2 (purple) tied across the first two measures.



A single-staff musical score in 4/4 time, showing the combined melody from the Flute part of the previous score. It begins with a quarter rest, followed by a sequence of eighth notes: G4 (red), A4 (red), B4 (red), C5 (red), D5 (red), E5 (red), and a final quarter rest.

# Results

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# Excerpt

**Poco Adagio**

Violin I  
sotto voce  
**Poco Adagio**

Violin II  
sotto voce  
**Poco Adagio**

Viola  
sotto voce  
**Poco Adagio**

Violoncello  
sotto voce  
**Poco Adagio**

6

cresc.

cresc.

cresc.

10

espress.

*p*

*f*

espress.

*p*

*f*

cresc.

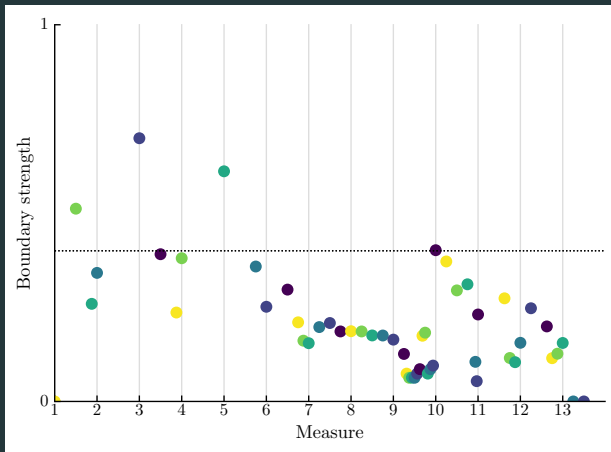
*p*

*f*

*p*

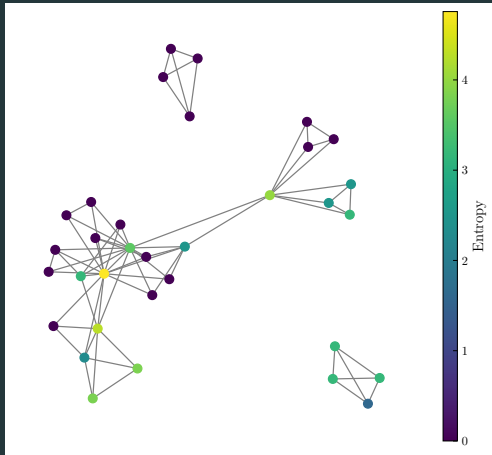
String Quartet No. 10 by Ludwig van Beethoven

# Phrase detection



Boundary strengths for the Violin I part

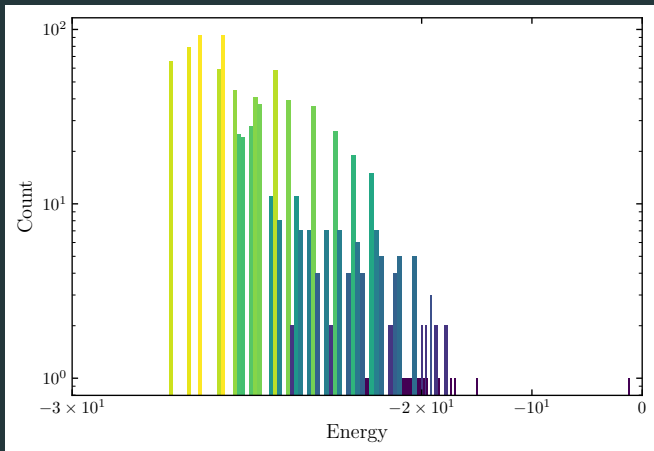
# Problem graph



Problem graph with 33 nodes and 70 edges

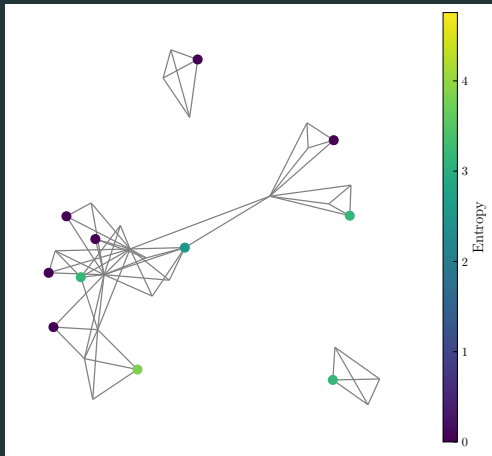


# Solutions



Returned solutions for 1000 reads

## Example solution



Solution graph returning a subset of 11 nodes

# Final arrangement

**Poco Adagio**

Violin I  
sotto voce  
**Poco Adagio**

Violin II  
sotto voce  
**Poco Adagio**

Viola  
sotto voce  
**Poco Adagio**

Violoncello  
sotto voce

6

cresc.

cresc.

cresc.

10

espress.

*p*

*f*

espress.

*p*

*f*

cresc.

*p*

*f*

*p*

*f*

Selected phrases

**Poco Adagio**

sotto voce

7

espress.

cresc.

*p*

12

*f*

Final arrangement

# Conclusions

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# Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms Huang, Chiu and Shan, 'Towards an automatic music arrangement framework using score reduction'
- Removes skill barrier for music arrangement



## Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'

**Thank you!**

# Quantum Annealing for Music Arrangement

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## Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

## Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

## Weighting

$$S = \frac{1}{3} (S'_{\text{pitch}} + 2S'_{\text{IOI}})$$

Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

Lucas, 'Ising formulations of many NP problems'

$A/B \geq 2 \max(W)$  to weight the constraint term more heavily than any objective term

# Phrase entropy


## Shannon entropy

$$H(X) := - \sum_i P(x_i) \log_2 P(x_i)$$

## Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'

-  Born, M. and V. Fock. **‘Beweis des Adiabatenatzes’**. de. In: *Zeitschrift für Physik* 51.3 (Mar. 1928), pp. 165–180. ISSN: 0044-3328. DOI: 10.1007/BF01343193. URL: <https://doi.org/10.1007/BF01343193> (visited on 01/03/2025).
-  Cambouropoulos, Emiliós. **‘The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing’**. In: *International Computer Music Association* (2011). ISSN: 2223-3881.
-  Huang, Jiun-Long, Shih-Chuan Chiu and Man-Kwan Shan. **‘Towards an automatic music arrangement framework using score reduction’**. In: *ACM Trans. Multimedia Comput. Commun. Appl.* 8.1 (Feb. 2012), 8:1–8:23. ISSN: 1551-6857. DOI: 10.1145/2071396.2071404. (Visited on 05/12/2024).



Li, You et al. **‘Automatic Piano Reduction of Orchestral Music Based on Musical Entropy’**. In: *2019 53rd Annual Conference on Information Sciences and Systems (CISS)*. Mar. 2019, pp. 1–5. DOI: 10.1109/CISS.2019.8693036. URL: <https://ieeexplore.ieee.org/document/8693036> (visited on 27/12/2024).



Lucas, Andrew. **‘Ising formulations of many NP problems’**. English. In: *Frontiers in Physics* 2 (Feb. 2014). Publisher: Frontiers. ISSN: 2296-424X. DOI: 10.3389/fphy.2014.00005. (Visited on 14/10/2024).



Moses, William S. and Erik D. Demaine. '**Computational Complexity of Arranging Music**'. In: *arXiv* (July 2016).

*arXiv*:1607.04220. DOI: 10.48550/*arXiv*.1607.04220. (Visited on 09/11/2024).



Pearce, M. and Geraint A. Wiggins. '**Towards A Framework for the Evaluation of Machine Compositions**'. In:

*Proceedings of the AISB'01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences*. 2001.