Music Arrangement via Quantum Annealing

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30 January 2025

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Overview

Overview
Theory
Music arrangement
Quantum annealing
Methods
Results
Conclusions

- What is music arrangement? What is quantum annealing?
- Methods used to solve the music arrangement problem
- Preliminary results from application of the method
- Concluding thoughts about this process

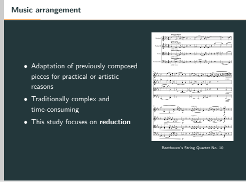
Theory

Theory

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

Beethoven's String Quartet No. 10



- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet to solo)

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

Adiabatic quantum computing (AQC)
- *Materials* — heating and cooling a material to alter its physical properties
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- Done slowly and adiabatically to remain in the ground state
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

Quantum annealing

Ising model

$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$

Initial state

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

Quantum annealing

Ising model

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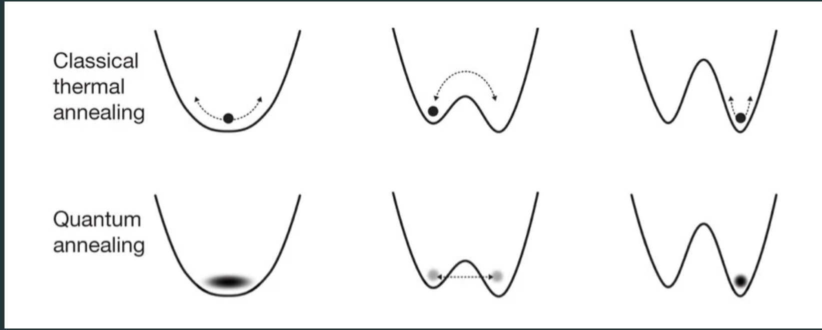
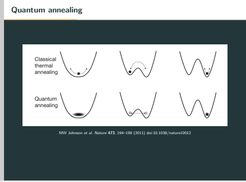
- How is this used to solve problems?
- Ising model, create a lattice of variables with two discrete values (spin up/down)
- Problem Hamiltonian, qubits σ^z, coupling strengths J_{ij} and field strengths h_i
- Initial state is a superposition of all possible states
- If problem solution is encoded within the ground state, system will give solution after evolution

Quantum annealing



MW Johnson *et al.* *Nature* **473**, 194–198 (2011) doi:10.1038/nature10012



- What does this look like?
- Evolution of superposition to a particular state
- More efficiently escape from local minima via quantum tunneling
- Can solve harder problems with a more turbulent energy landscape

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

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- How to encode a problem into a Hamiltonian?
- QUBO is a function to be minimised
- Set of binary variables x, matrix Q of real weights that describes interactions between variables

How to combine them?

How to combine them?

- How to apply quantum annealing to the problem of music arrangement?

Methods

Methods

Problem formulation

1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

Problem formulation

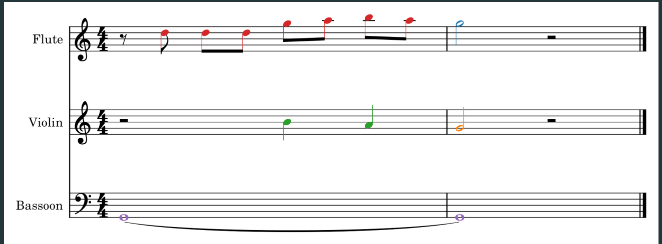
1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

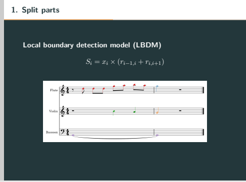
- Formulating arrangement as a problem to be solved via annealing
- Four-step process
- Split parts into musical phrases
- Arrange phrases into a graph (nodes and edges)
- Solve corresponding graph problem using quantum computing
- Construct final arrangement from the solution returned

1. Split parts

Local boundary detection model (LBDM)

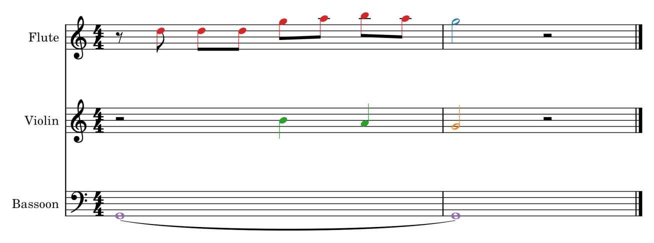
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

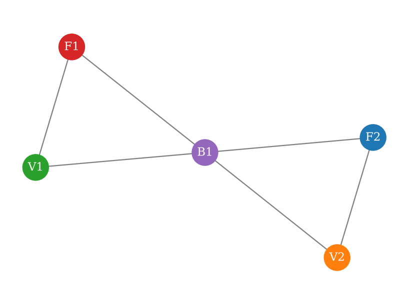


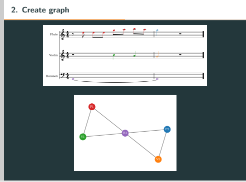


- First stage to separate each part of original score into phrases
- Phrases — smallest unit of music that preserves melody and structure
- Boundaries between phrases found using LBDM
- Measures the degree of change of a certain parameter between notes (explain equation)
- Check pitch and IOI
- Strengths above a threshold value are considered phrases

2. Create graph





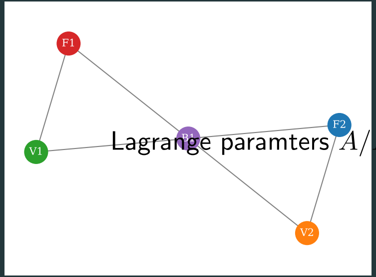


- Construct problem graph
- Each phrase becomes a node, edges between nodes if phrases overlap

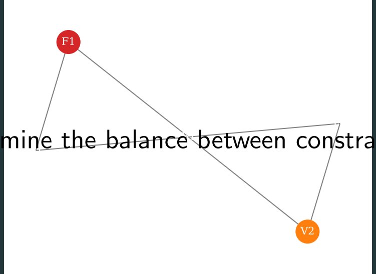
Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge.

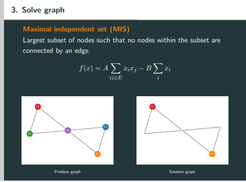
$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$



Problem graph



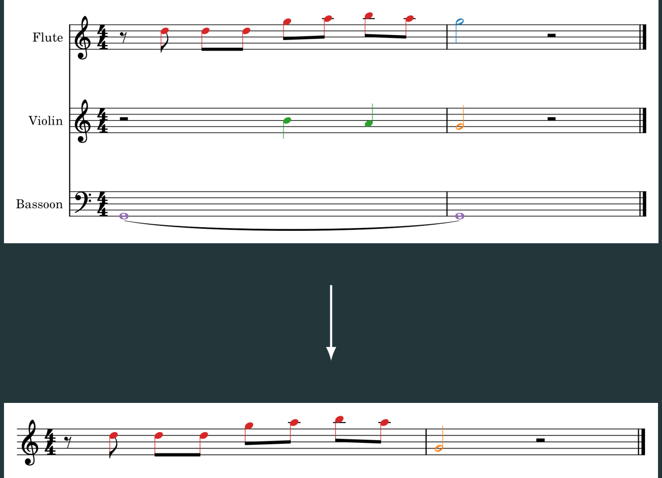
Solution graph

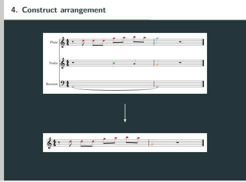


- Solve problem graph using a graph theory problem called MIS
- Special case for reducing to a single monophonic part (one phrase played at a time)
- Find largest subset of nodes such that no nodes within the subset are connected by an edge
- Enforces that only one simultaneous phrase can be played at once
- Constraint term enforces no edges
- Objective term is quantity to be minimised

Lagrange paramters A/B determine the balance between constraint and objective

4. Construct arrangement





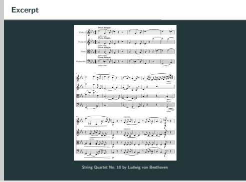
- Take solution graph and combine selected nodes to create final arrangement

Results

Results

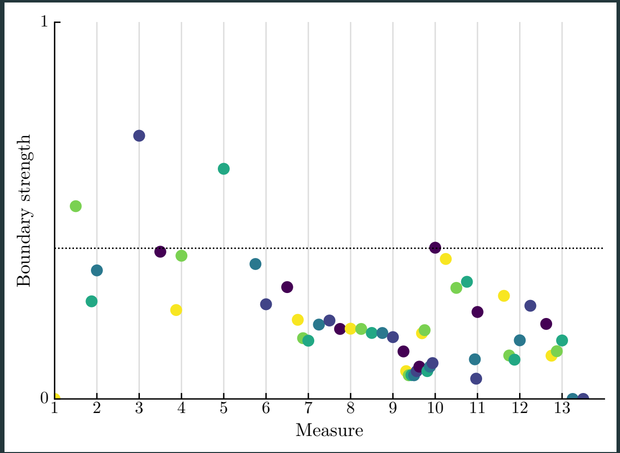
Excerpt

String Quartet No. 10 by Ludwig van Beethoven

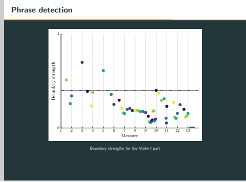


- Excerpt of String Quartet No. 10 in E-flat major, Op. 74, by Ludwig van Beethoven
- Chosen due to its relatively simple structure and smaller instrumentation, keeping the problem graph small

Phrase detection

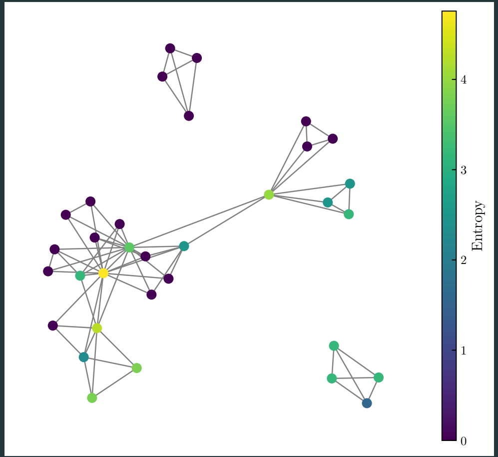


Boundary strengths for the Violin I part

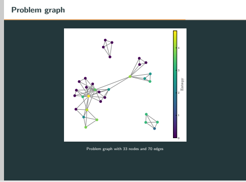


- Example of the LBDM finding suitable boundaries for phrases
- Threshold value of 0.4 chosen, finds five phrases

Problem graph

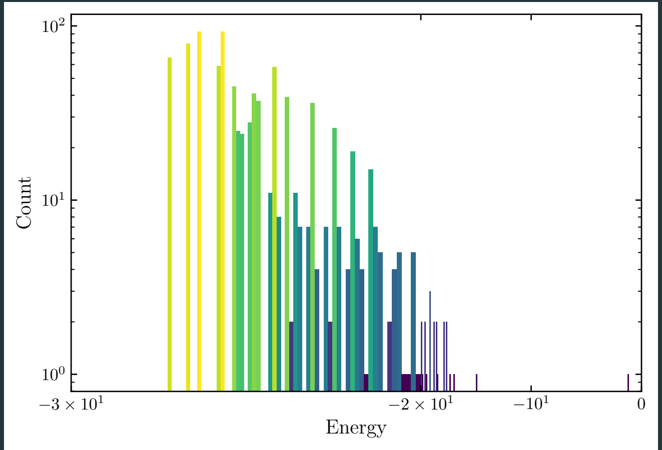


Problem graph with 33 nodes and 70 edges

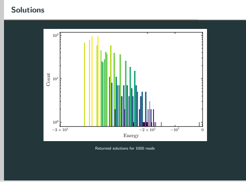


- 33 identified phrases (nodes) with 70 overlaps (edges)
- Nodes are weighted by the phrase entropy, how musically interesting the distribution of notes is

Solutions

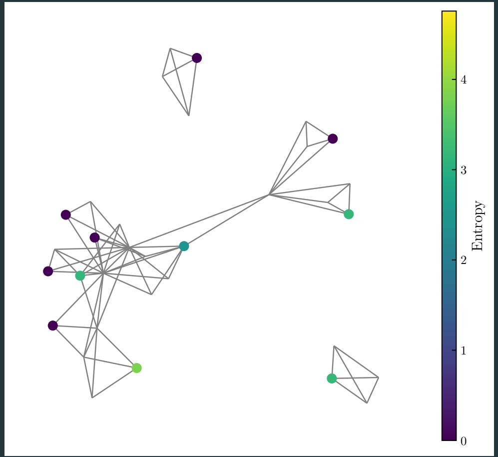


Returned solutions for 1000 reads

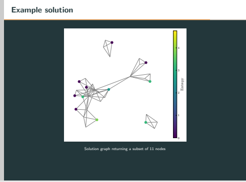


- Histogram of the returned solutions, only energies below zero shown
- Distribution of solutions due to the stochastic nature of annealing
- Not always guaranteed the ground state during evolution
- Lowest energy solution -26.8 with a degeneracy of 34

Example solution



Solution graph returning a subset of 11 nodes

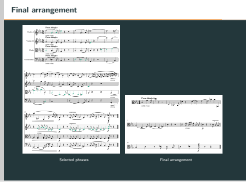


- Selected nodes from one of the lowest energy solutions
- Note in the cliques only one node could be selected

Final arrangement

Selected phrases

Final arrangement



- Selected phrases from solution graph highlighted
- Phrases concatenated to create the final arrangement

Conclusions

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- Successful in creating a valid single-part reduction
- Advantage over classical algorithms (deep learning, genetic)
  - No training data needed
  - Faster solve time

└─Conclusions

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-

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements

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LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

Degree of change

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S_i' = \frac{S_i - \min(S)}{\max(S) - \min(S)}$$

Weighting

$$S = \frac{1}{3}\left(S_{\text{pitch}} + 2S_{\text{IOI}}\right)$$

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Phrase entropy

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

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