Music Arrangement via Quantum Annealing

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Overview

Theory

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

Beethoven's String Quartet No. 10

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

[Lucas, 2014]

Quantum annealing

Ising model

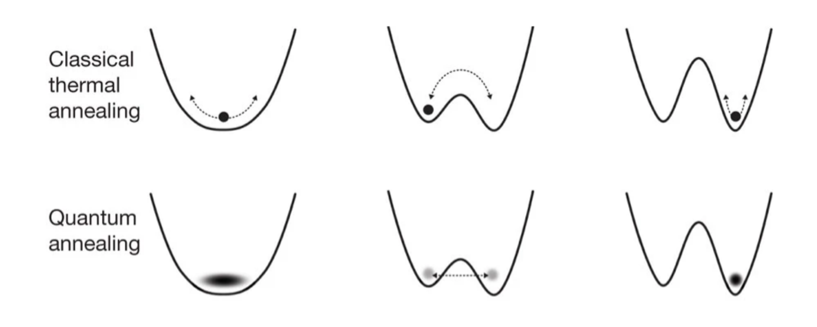
$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$

Initial state

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

[Lucas, 2014]

Quantum annealing



[Johnson et al., 2011]

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Remains in low-energy state via quantum tunneling

How to combine them?

Methods

Problem formulation

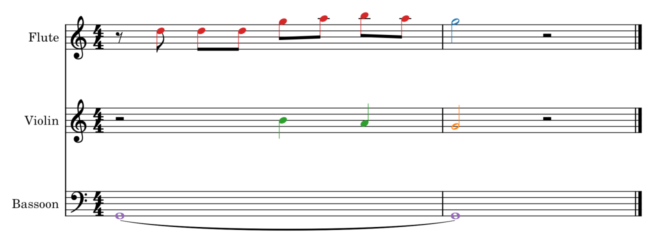
1. Split parts into phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

1. Split parts

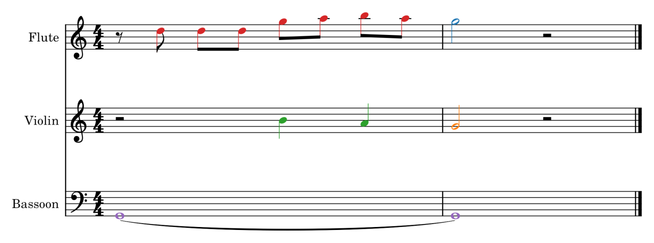
Local boundary detection model (LBDM)

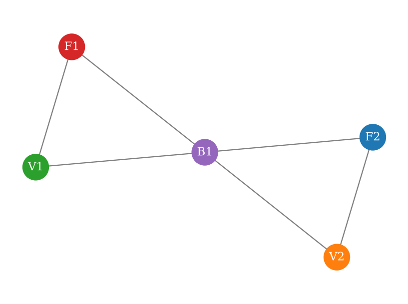
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

[Cambouropoulos, 2011]



2. Create graph



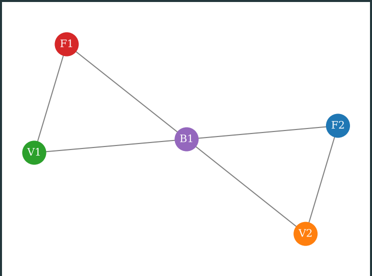


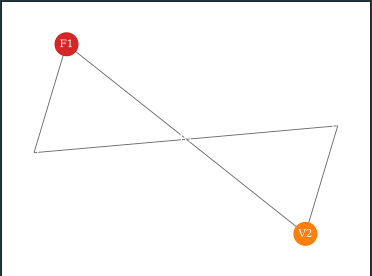
Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge

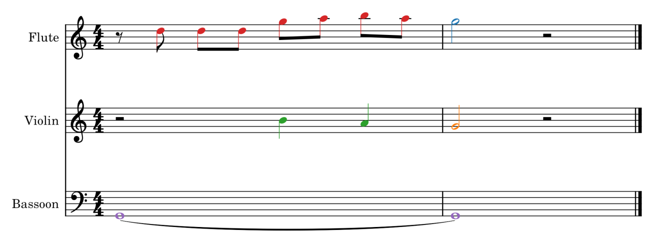
$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i x_i$$

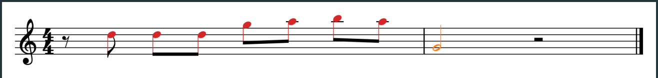
[Lucas, 2014]





4. Construct arrangement



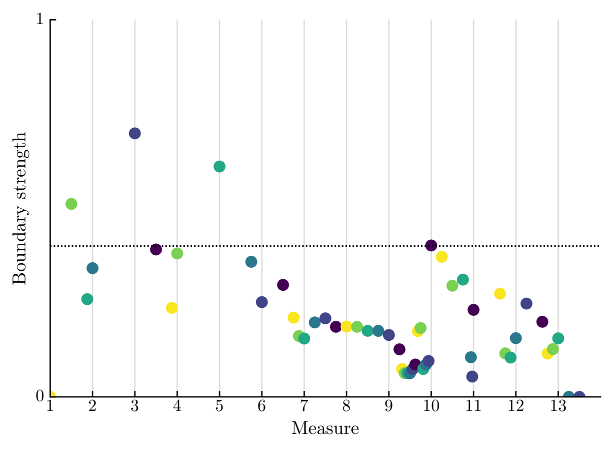


Results

Excerpt

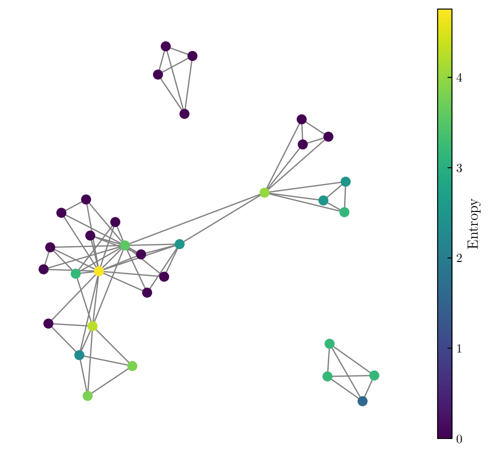
String Quartet No. 10 by Ludwig van Beethoven

Phrase detection



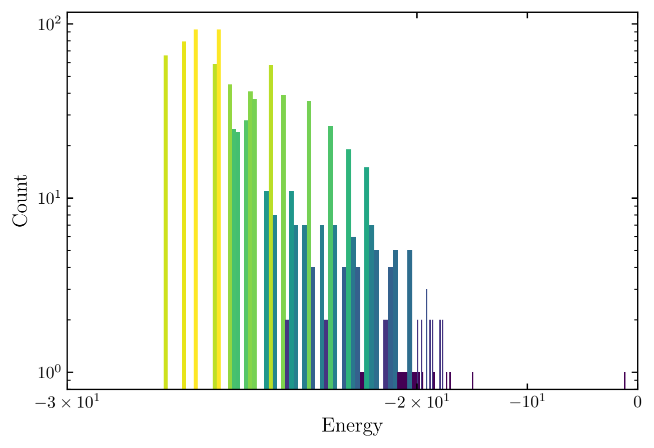
Boundary strengths for the Violin I part

Problem graph



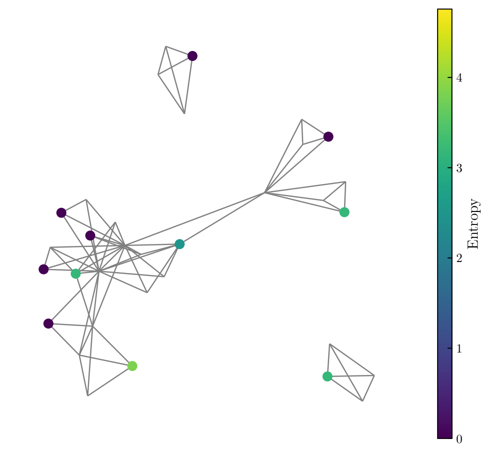
Problem graph with 33 nodes and 70 edges

Solutions



Returned solutions for 1000 reads

Example solution



Solution graph returning a subset of 11 nodes

Final arrangement

Selected phrases

Final arrangement

Conclusions

Conclusions

- Successful in creating a valid single-part reduction
    - Monophonic lowest-energy solution
    - Sufficient number of phrases selected
    - Musically interesting final arrangement
- Advantage over classical algorithms (deep learning, genetic)
    - No training data needed
    - Faster solve time [Huang et al., 2012]

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements [Pearce and Wiggins, 2001]

Thank you!

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References i

Cambouropoulos, E. (2011).
The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing.
International Computer Music Association.

Huang, J.-L., Chiu, S.-C., and Shan, M.-K. (2012).
Towards an automatic music arrangement framework using score reduction.
ACM Trans. Multimedia Comput. Commun. Appl., 8(1):8:1–8:23.

Johnson, M. W., Amin, M. H. S., Gildert, S., Lanting, T., Hamze, F., Dickson, N., Harris, R., Berkley, A. J., Johansson, J., Bunyk, P., Chapple, E. M., Enderud, C., Hilton, J. P., Karimi, K., Ladizinsky, E., Ladizinsky, N., Oh, T., Perminov, I., Rich, C., Thom, M. C., Tolkacheva, E., Truncik, C. J. S., Uchaikin, S., Wang, J., Wilson, B., and Rose, G. (2011).
Quantum annealing with manufactured spins.
Nature, 473(7346):194–198.
Publisher: Nature Publishing Group.

Li, Y., Wilk, C. M., Hori, T., and Sagayama, S. (2019).
Automatic Piano Reduction of Orchestral Music Based on Musical Entropy.
In *2019 53rd Annual Conference on Information Sciences and Systems (CISS)*, pages 1–5.

Lucas, A. (2014).
Ising formulations of many NP problems.
Frontiers in Physics, 2.
Publisher: Frontiers.

Pearce, M. and Wiggins, G. A. (2001).
Towards A Framework for the Evaluation of Machine Compositions.
In *Proceedings of the AISB'01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences*.

LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

Degree of change

$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S)}{\max(S) - \min(S)}$$

Weighting

$$S = \frac{1}{3}\,(S_{\text{pitch}} + 2S_{\text{IOI}})$$

[Cambouropoulos, 2011]

Phrase entropy

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

[Li et al., 2019]