Quantum Annealing for Music Arrangement

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Overview

Motivations

Motivations

- Small lit review[1]
- Quantum computer music
- My own novel adaption of the method
- THIS IS MY OWN IDEA

[1] Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.

Theory

Adiabatic principle

A system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough.[2]

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Universal and guaranteed
- A system that starts in a ground state, ends in a ground state
- Not possible in practice

[2] Born and Fock, 'Beweis des Adiabatensatzes'.

Quantum annealing

- Relaxes the adiabaticity
- Rate of change determined heuristically
- Final state is probabilistic, not deterministic

Model

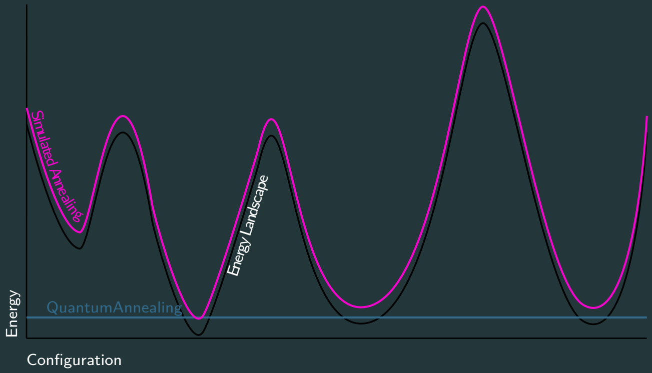
Initial state

$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

Ising model

$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^{N} h_i \sigma_i^z$$

Uses


Simulated Annealing
Energy Landscape
Energy
QuantumAnnealing
Configuration

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Sent to the QPU for optimisation

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

Beethoven's String Quartet No. 10

How to combine them?

Methods

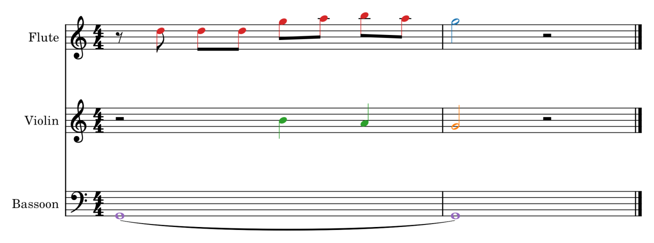
Problem formulation

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

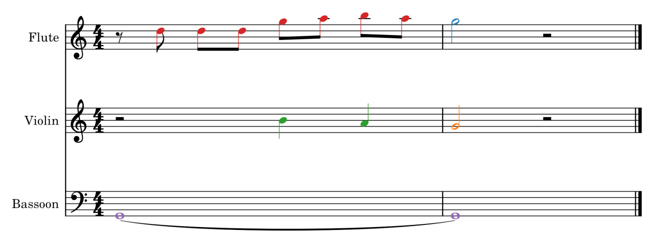
Local boundary detection model (LBDM)

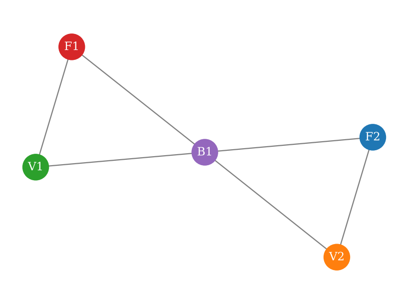
$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'



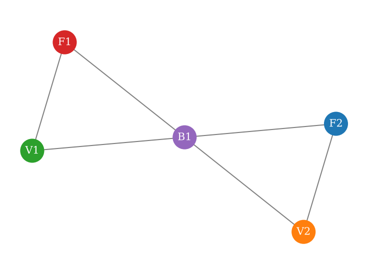
2. Create graph

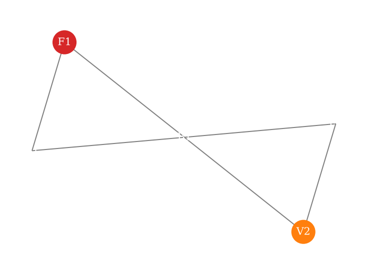




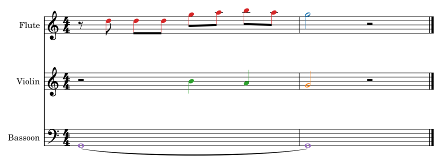
Maximal independent set (MIS)

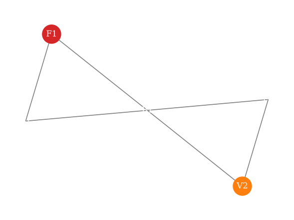
Largest subset of nodes such that no nodes within the subset are connected by an edge

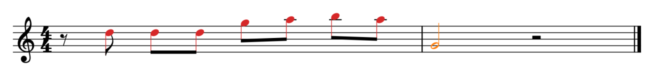




4. Construct arrangement





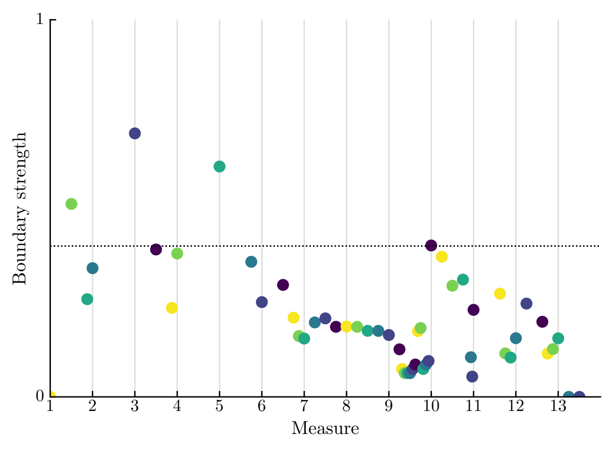


Results

Excerpt

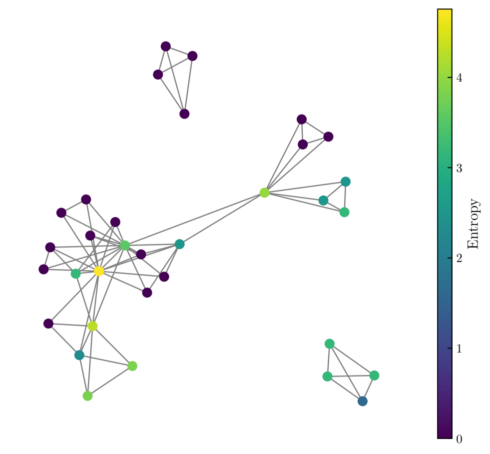
String Quartet No. 10 by Ludwig van Beethoven

Phrase detection



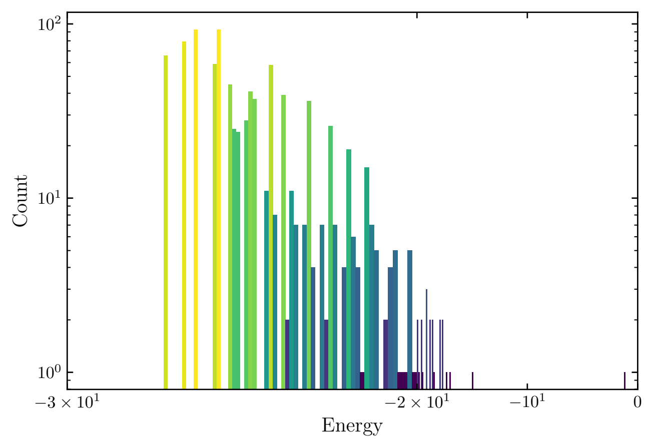
Boundary strengths for the Violin I part

Problem graph



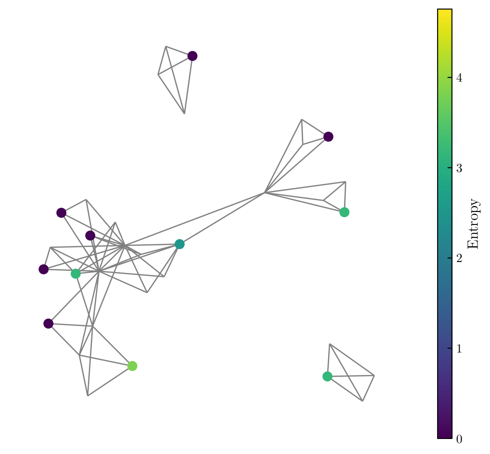
Problem graph with 33 nodes and 70 edges

Solutions



Returned solutions for 1000 reads

Example solution



Solution graph returning a subset of 11 nodes

Final arrangement

Selected phrases

Final arrangement

Conclusions

Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms Huang, Chiu and Shan, ‘Towards an automatic music arrangement framework using score reduction’
- Removes skill barrier for music arrangement

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements Pearce and Wiggins, 'Towards A Framework for the Evaluation of Machine Compositions'

Thank you!

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LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$
$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S'_i = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

Cambouropoulos, 'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'

MIS

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

Lucas, 'Ising formulations of many NP problems'

$A/B >= 2\max(W)$ to weight the constraint term more heavily than any objective term

Phrase entropy

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

Li et al., 'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'

- Born, M. and V. Fock. **'Beweis des Adiabatensatzes'.** de. In: *Zeitschrift für Physik* 51.3 (Mar. 1928), pp. 165–180. ISSN: 0044-3328. DOI: 10.1007/BF01343193. URL: https://doi.org/10.1007/BF01343193 (visited on 01/03/2025).
- Cambouropoulos, Emilios. **'The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing'.** In: *International Computer Music Association* (2011). ISSN: 2223-3881.
- Huang, Jiun-Long, Shih-Chuan Chiu and Man-Kwan Shan. **'Towards an automatic music arrangement framework using score reduction'.** In: *ACM Trans. Multimedia Comput. Commun. Appl.* 8.1 (Feb. 2012), 8:1–8:23. ISSN: 1551-6857. DOI: 10.1145/2071396.2071404. (Visited on 05/12/2024).

- Li, You et al. **'Automatic Piano Reduction of Orchestral Music Based on Musical Entropy'.** In: *2019 53rd Annual Conference on Information Sciences and Systems (CISS)*. Mar. 2019, pp. 1–5. DOI: 10.1109/CISS.2019.8693036. URL: https://ieeexplore.ieee.org/document/8693036 (visited on 27/12/2024).
- Lucas, Andrew. **'Ising formulations of many NP problems'.** English. In: *Frontiers in Physics* 2 (Feb. 2014). Publisher: Frontiers. ISSN: 2296-424X. DOI: 10.3389/fphy.2014.00005. (Visited on 14/10/2024).

Pearce, M. and Geraint A. Wiggins. **'Towards A Framework for the Evaluation of Machine Compositions'.** In: *Proceedings of the AISB'01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences*. 2001.