Music Arrangement via Quantum Annealing

Lucas Kirby

30 January 2025

Department of Physics, Durham University

Music Arrangement via Quantum Annealing

Music Arrangement via Quantum Annealing

Lucas Kirby
30 January 2025
Department of Physics, Durham University

Overview

Overview

Theory
Music arrangement
Quantum annealing
Methods
Results
Conclusions

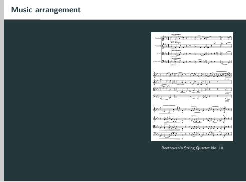
- What is music arrangement? What is quantum annealing?
- Methods used to solve the music arrangement problem
- Preliminary results from application of the method
- Concluding thoughts about this process

Theory

Theory

Music arrangement

Beethoven's String Quartet No. 10

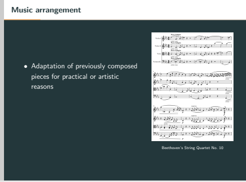


- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet)

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons

Beethoven's String Quartet No. 10

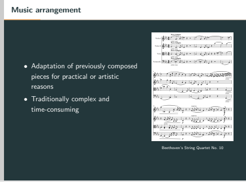


- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet)

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming

Beethoven's String Quartet No. 10

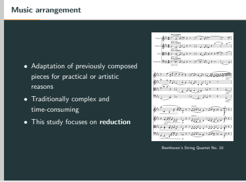


- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet)

Music arrangement

- Adaptation of previously composed pieces for practical or artistic reasons
- Traditionally complex and time-consuming
- This study focuses on **reduction**

Beethoven's String Quartet No. 10



- Adaptation of music in terms of instrumentation, medium, or style
- Traditionally a complex process that requires a deep understanding of musical theory and structure
- Musically interesting whilst still remaining faithful to the source material
- Interest in automating this process
- Reduction is the rewriting of music for a smaller number of instruments (for example string quartet)

Adiabatic quantum computing (AQC)

Adiabatic quantum computing (AQC)

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties

Adiabatic quantum computing (AQC)
- *Materials* — heating and cooling a material to alter its physical properties

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

Adiabatic quantum computing (AQC)

- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

[Lucas, 2014]

Adiabatic quantum computing (AQC)
- *Materials* — heating and cooling a material to alter its physical properties
- *Quantum* — changing a quantum system from one Hamiltonian to another
- Done slowly and adiabatically to remain in the ground state
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$
[Lucas, 2014]

- Materials science, annealing is a slow heating/cooling process to make a material softer and less brittle
- Quantum computing, slow evolution of a system between Hamiltonians
- Done adiabatically (closed system), system remains in ground state

Quantum annealing

Quantum annealing

- How is this used to solve problems?
- Ising model, create a lattice of variables with two discrete values (spin up/down)
- Problem Hamiltonian, qubits σ^z, coupling strengths J_{ij} and field strengths h_i
- Initial state is a superposition of all possible states
- If problem solution is encoded within the ground state, system will give solution after evolution

Ising model

$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$

Quantum annealing
Ising model
$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$

- How is this used to solve problems?
- Ising model, create a lattice of variables with two discrete values (spin up/down)
- Problem Hamiltonian, qubits σ^z, coupling strengths J_{ij} and field strengths h_i
- Initial state is a superposition of all possible states
- If problem solution is encoded within the ground state, system will give solution after evolution

Quantum annealing

Ising model

$$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$$

Initial state

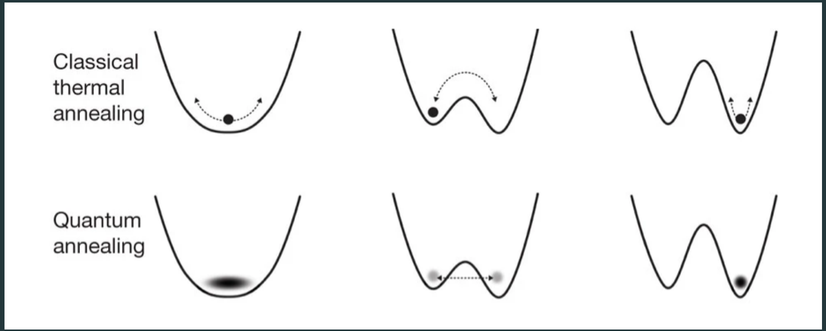
$$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$$

[Lucas, 2014]

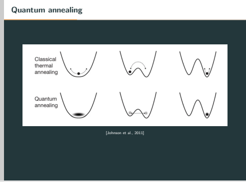
Quantum annealing
Ising model
$H_p(\sigma^z) = \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i=1}^{N} h_i\sigma_i^z$
Initial state
$H_0 = h_0 \sum_{i=1}^{N} \sigma_i^x$
[Lucas, 2014]

- How is this used to solve problems?
- Ising model, create a lattice of variables with two discrete values (spin up/down)
- Problem Hamiltonian, qubits σ^z, coupling strengths J_{ij} and field strengths h_i
- Initial state is a superposition of all possible states
- If problem solution is encoded within the ground state, system will give solution after evolution

Quantum annealing



[Johnson et al., 2011]



- What does this look like?
- Evolution of superposition to a particular state
- More efficiently escape from local minima via quantum tunneling
- Can solve harder problems with a more turbulent energy landscape

QUBO

QUBO

- How to encode a problem into a Hamiltonian?
- Similar form to the Ising model, but with binary variables (0 or 1)
- QUBO is a function to be minimised
- Set of binary variables x, matrix Q of real weights that describes interactions between variables
- Can read out variable values after evolution to give solution

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

QUBO
Quadratic Unconstrained Binary Optimisation
$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$

- How to encode a problem into a Hamiltonian?
- Similar form to the Ising model, but with binary variables (0 or 1)
- QUBO is a function to be minimised
- Set of binary variables x, matrix Q of real weights that describes interactions between variables
- Can read out variable values after evolution to give solution

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

QUBO

Quadratic Unconstrained Binary Optimisation

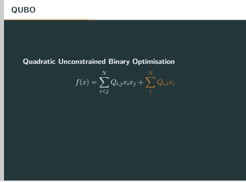
$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- How to encode a problem into a Hamiltonian?
- Similar form to the Ising model, but with binary variables (0 or 1)
- QUBO is a function to be minimised
- Set of binary variables x, matrix Q of real weights that describes interactions between variables
- Can read out variable values after evolution to give solution

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$



- How to encode a problem into a Hamiltonian?
- Similar form to the Ising model, but with binary variables (0 or 1)
- QUBO is a function to be minimised
- Set of binary variables x, matrix Q of real weights that describes interactions between variables
- Can read out variable values after evolution to give solution

QUBO

Quadratic Unconstrained Binary Optimisation

$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$

- Encodes problem solution into Hamiltonian's ground state
- Sent to the QPU for optimisation

QUBO
Quadratic Unconstrained Binary Optimisation
$$f(x) = \sum_{i<j}^{N} Q_{i,j} x_i x_j + \sum_{i}^{N} Q_{i,i} x_i$$
- Encodes problem solution into Hamiltonian's ground state
- Sent to the QPU for optimisation

- How to encode a problem into a Hamiltonian?
- Similar form to the Ising model, but with binary variables (0 or 1)
- QUBO is a function to be minimised
- Set of binary variables x, matrix Q of real weights that describes interactions between variables
- Can read out variable values after evolution to give solution

How to combine them?

- How to apply quantum annealing to the problem of music arrangement?

Methods

Methods

Problem formulation

Problem formulation

- Formulating arrangement as a problem to be solved via annealing, four-step process
- Split parts into musical phrases
- Arrange phrases into a graph (nodes and edges)
- Solve corresponding graph problem using quantum computing
- Construct final arrangement from the solution returned

Problem formulation

1. Split score into musical phrases

Problem formulation
1. Split score into musical phrases

- Formulating arrangement as a problem to be solved via annealing, four-step process
- Split parts into musical phrases
- Arrange phrases into a graph (nodes and edges)
- Solve corresponding graph problem using quantum computing
- Construct final arrangement from the solution returned

Problem formulation

1. Split score into musical phrases
2. Arrange phrases into a graph

Problem formulation
1. Split score into musical phrases
2. Arrange phrases into a graph

- Formulating arrangement as a problem to be solved via annealing, four-step process
- Split parts into musical phrases
- Arrange phrases into a graph (nodes and edges)
- Solve corresponding graph problem using quantum computing
- Construct final arrangement from the solution returned

Problem formulation

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU

Problem formulation

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU

- Formulating arrangement as a problem to be solved via annealing, four-step process
- Split parts into musical phrases
- Arrange phrases into a graph (nodes and edges)
- Solve corresponding graph problem using quantum computing
- Construct final arrangement from the solution returned

Problem formulation

1. Split score into musical phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

Problem formulation
1. Split score into musical phrases
2. Arrange phrases into a graph
3. Solve graph problem using QPU
4. Construct arrangement from solution

- Formulating arrangement as a problem to be solved via annealing, four-step process
- Split parts into musical phrases
- Arrange phrases into a graph (nodes and edges)
- Solve corresponding graph problem using quantum computing
- Construct final arrangement from the solution returned

1. Split score

1. Split score

- First stage to separate each part of original score into phrases
- Phrases — smallest unit of music that preserves melody and structure
- Boundaries between phrases found using LBDM
- Measures the degree of change of a certain parameter between notes (explain equation)
- Check pitch and IOI
- Strengths above a threshold value are considered phrases

1. Split score

Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

[Cambouropoulos, 2011]

1. Split score
Local boundary detection model (LBDM)
$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$
[Cambouropoulos, 2011]

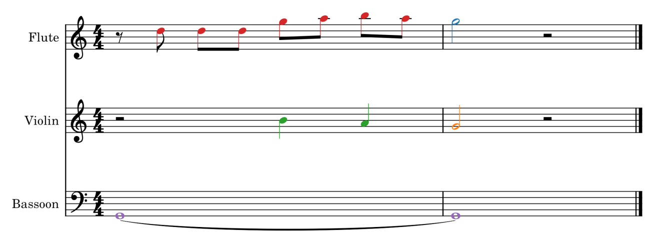
- First stage to separate each part of original score into phrases
- Phrases — smallest unit of music that preserves melody and structure
- Boundaries between phrases found using LBDM
- Measures the degree of change of a certain parameter between notes (explain equation)
- Check pitch and IOI
- Strengths above a threshold value are considered phrases

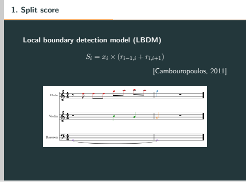
1. Split score

Local boundary detection model (LBDM)

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$

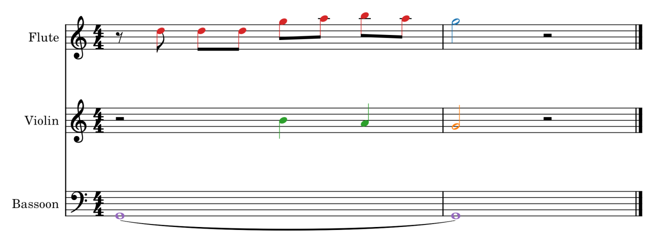
[Cambouropoulos, 2011]





- First stage to separate each part of original score into phrases
- Phrases — smallest unit of music that preserves melody and structure
- Boundaries between phrases found using LBDM
- Measures the degree of change of a certain parameter between notes (explain equation)
- Check pitch and IOI
- Strengths above a threshold value are considered phrases

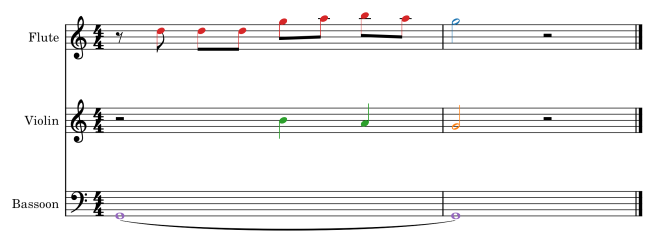
2. Create graph

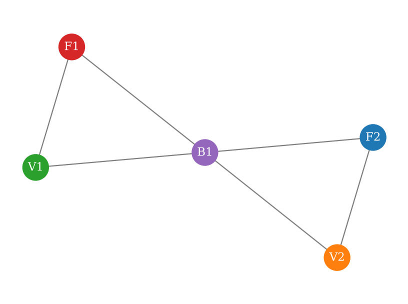


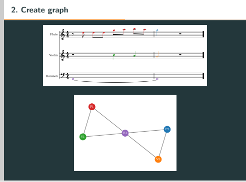
2. Create graph

- Construct problem graph
- Each phrase becomes a node, edges between nodes if phrases overlap

2. Create graph

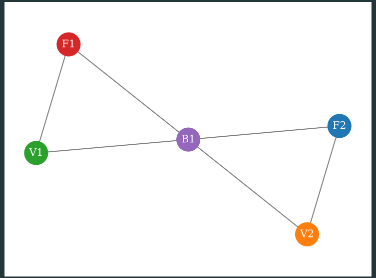


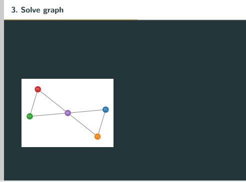




- Construct problem graph
- Each phrase becomes a node, edges between nodes if phrases overlap

3. Solve graph



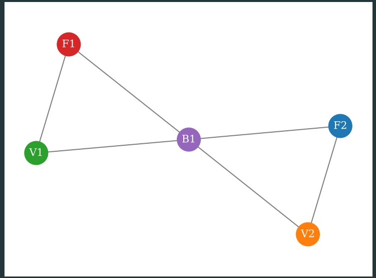


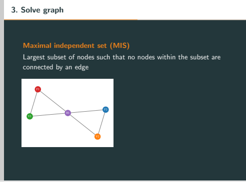
- Special case for reducing to a single monophonic part (one phrase played at a time)
- Solve problem graph using a graph theory problem called MIS
- Enforces that only one simultaneous phrase can be played at once
- Nodes can also be weighted to introduce preference for certain nodes e.g. musicality

3. Solve graph

Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge



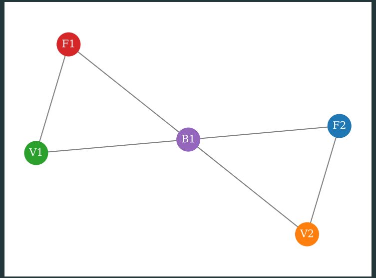


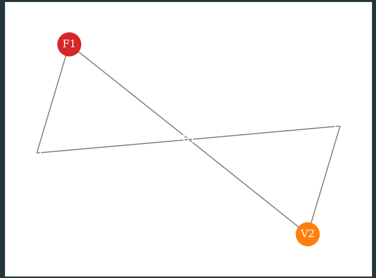
- Special case for reducing to a single monophonic part (one phrase played at a time)
- Solve problem graph using a graph theory problem called MIS
- Enforces that only one simultaneous phrase can be played at once
- Nodes can also be weighted to introduce preference for certain nodes e.g. musicality

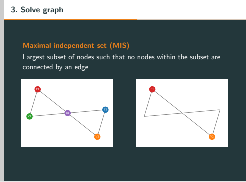
3. Solve graph

Maximal independent set (MIS)

Largest subset of nodes such that no nodes within the subset are connected by an edge

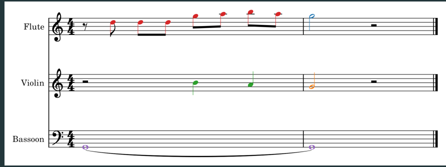


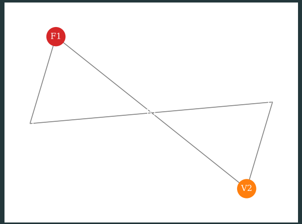


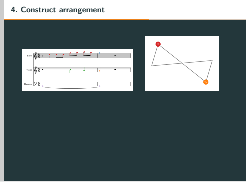


- Special case for reducing to a single monophonic part (one phrase played at a time)
- Solve problem graph using a graph theory problem called MIS
- Enforces that only one simultaneous phrase can be played at once
- Nodes can also be weighted to introduce preference for certain nodes e.g. musicality

4. Construct arrangement

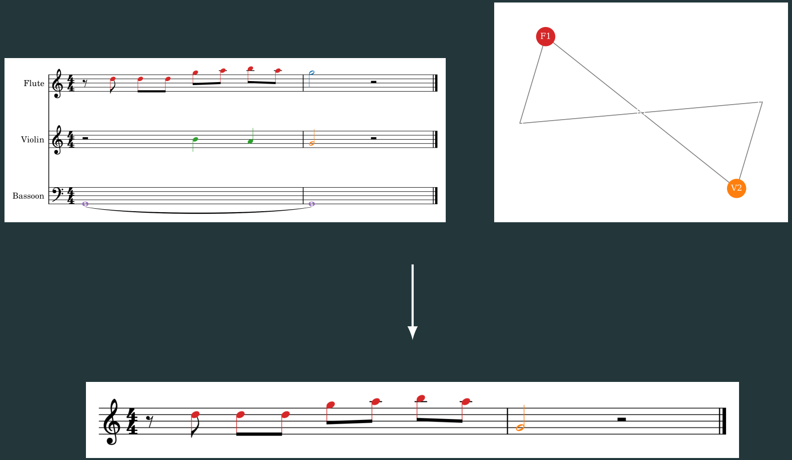


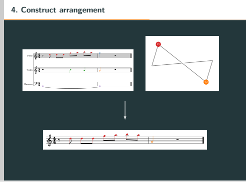




- Take solution graph and combine selected nodes to create final arrangement

4. Construct arrangement



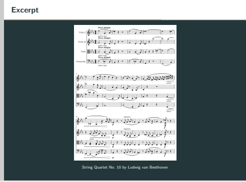


- Take solution graph and combine selected nodes to create final arrangement

Results

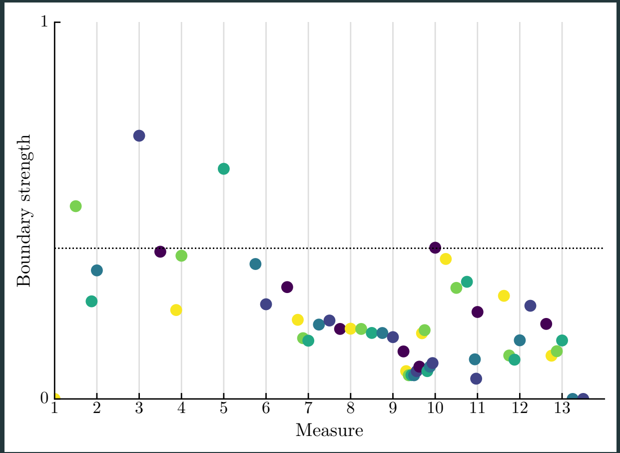
Excerpt

String Quartet No. 10 by Ludwig van Beethoven

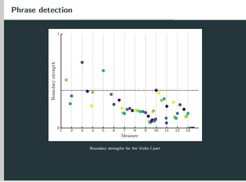


- Excerpt of String Quartet No. 10 in E-flat major, Op. 74, by Ludwig van Beethoven
- Chosen due to its relatively simple structure and smaller instrumentation, keeping the problem graph small

Phrase detection

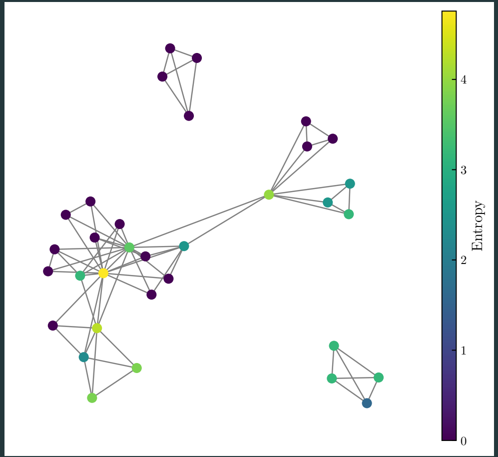


Boundary strengths for the Violin I part

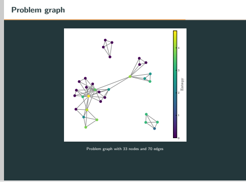


- Example of the LBDM finding suitable boundaries for phrases
- Threshold value of 0.4 chosen, finds five phrases

Problem graph

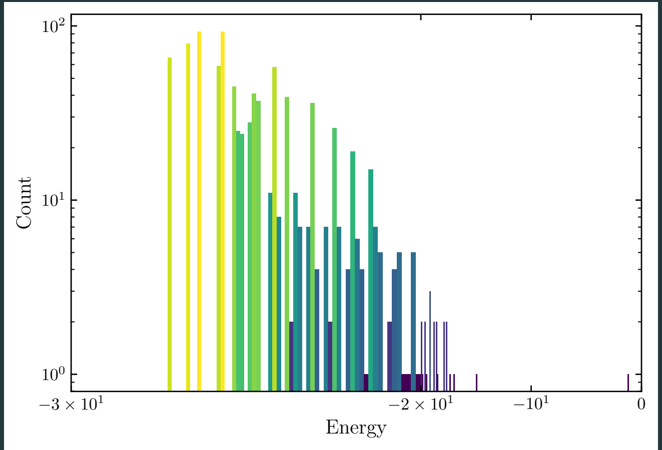


Problem graph with 33 nodes and 70 edges

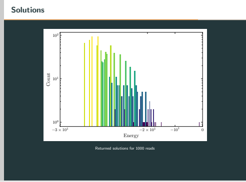


- 33 identified phrases (nodes) with 70 overlaps (edges)
- Nodes are weighted by the phrase entropy, how musically interesting the distribution of notes is

Solutions

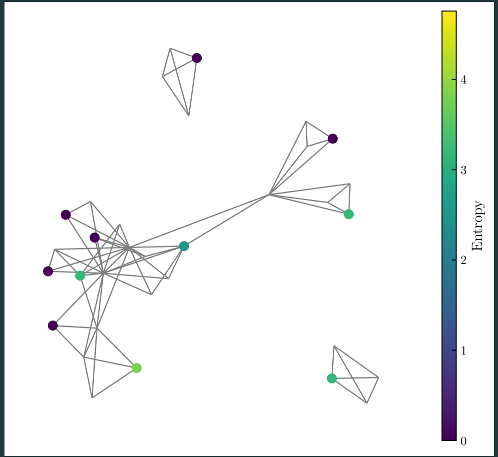


Returned solutions for 1000 reads

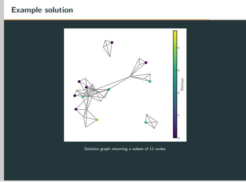


- Histogram of the returned solutions, only energies below zero shown
- Distribution of solutions due to the stochastic nature of annealing
- Lowest energy solution most musically interesting due to construction of weighted MIS QUBO
- (Lowest energy solution -26.8 with a degeneracy of 34)

Example solution



Solution graph returning a subset of 11 nodes

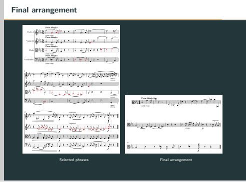


- Selected nodes from one of the lowest energy solutions
- Note in the cliques only one node could be selected

Final arrangement

Selected phrases

Final arrangement

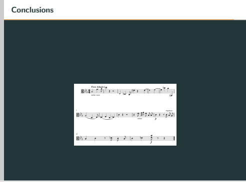


- Selected phrases from solution graph highlighted
- Phrases concatenated to create the final arrangement

Conclusions

Conclusions

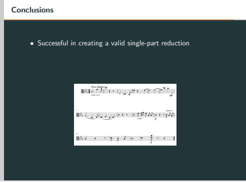
Conclusions



- Valid reduction gained from implemented method
- Monophonic lowest-energy solution
- Phrase identification and selection result in musically interesting final arrangement
- Displays some advantages over classical algorithms
- Does not require training data, costly in both time and resources
- Faster solve time compared to similarly-sized problems
- Method and more advanced versions of it removes the skill barrier for music arrangement

Conclusions

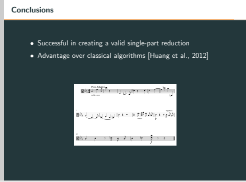
- Successful in creating a valid single-part reduction



- Valid reduction gained from implemented method
- Monophonic lowest-energy solution
- Phrase identification and selection result in musically interesting final arrangement
- Displays some advantages over classical algorithms
- Does not require training data, costly in both time and resources
- Faster solve time compared to similarly-sized problems
- Method and more advanced versions of it removes the skill barrier for music arrangement

Conclusions

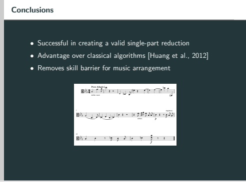
- Successful in creating a valid single-part reduction
- Advantage over classical algorithms [Huang et al., 2012]



- Valid reduction gained from implemented method
- Monophonic lowest-energy solution
- Phrase identification and selection result in musically interesting final arrangement
- Displays some advantages over classical algorithms
- Does not require training data, costly in both time and resources
- Faster solve time compared to similarly-sized problems
- Method and more advanced versions of it removes the skill barrier for music arrangement

Conclusions

- Successful in creating a valid single-part reduction
- Advantage over classical algorithms [Huang et al., 2012]
- Removes skill barrier for music arrangement



- Valid reduction gained from implemented method
- Monophonic lowest-energy solution
- Phrase identification and selection result in musically interesting final arrangement
- Displays some advantages over classical algorithms
- Does not require training data, costly in both time and resources
- Faster solve time compared to similarly-sized problems
- Method and more advanced versions of it removes the skill barrier for music arrangement

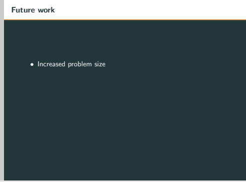
Future work

Future work

- Problem size — full piece has upwards of 1000 phrases, compare solve times with classical algorithms
- LBDM parameters — phrases should be short and similar in length, prevents important notes being hidden in long phrases with low entropy
- Physical limitations — note ranges, note change speed
- Multiple parts — need different problem formulation, colouring problem with each colour representing a reduced part
- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

Future work

- Increased problem size



- Problem size — full piece has upwards of 1000 phrases, compare solve times with classical algorithms
- LBDM parameters — phrases should be short and similar in length, prevents important notes being hidden in long phrases with low entropy
- Physical limitations — note ranges, note change speed
- Multiple parts — need different problem formulation, colouring problem with each colour representing a reduced part
- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

Future work

- Increased problem size
- Parametric variation of LBDM

Future work

- Increased problem size
- Parametric variation of LBDM

- Problem size — full piece has upwards of 1000 phrases, compare solve times with classical algorithms
- LBDM parameters — phrases should be short and similar in length, prevents important notes being hidden in long phrases with low entropy
- Physical limitations — note ranges, note change speed
- Multiple parts — need different problem formulation, colouring problem with each colour representing a reduced part
- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments

- Problem size — full piece has upwards of 1000 phrases, compare solve times with classical algorithms
- LBDM parameters — phrases should be short and similar in length, prevents important notes being hidden in long phrases with low entropy
- Physical limitations — note ranges, note change speed
- Multiple parts — need different problem formulation, colouring problem with each colour representing a reduced part
- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part

- Problem size — full piece has upwards of 1000 phrases, compare solve times with classical algorithms
- LBDM parameters — phrases should be short and similar in length, prevents important notes being hidden in long phrases with low entropy
- Physical limitations — note ranges, note change speed
- Multiple parts — need different problem formulation, colouring problem with each colour representing a reduced part
- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements [Pearce and Wiggins, 2001]

Future work

- Increased problem size
- Parametric variation of LBDM
- Physical limitations of instruments
- Reduction to more than one part
- Quality comparison of computer arrangements [Pearce and Wiggins, 2001]

- Problem size — full piece has upwards of 1000 phrases, compare solve times with classical algorithms
- LBDM parameters — phrases should be short and similar in length, prevents important notes being hidden in long phrases with low entropy
- Physical limitations — note ranges, note change speed
- Multiple parts — need different problem formulation, colouring problem with each colour representing a reduced part
- Quality judgement — Turing-like test, present subjects with human-/computer-generated scores

Thank you!

Thank you!

Music Arrangement via Quantum Annealing

Lucas Kirby

30 January 2025

Department of Physics, Durham University

2025-01-29

Music Arrangement via Quantum Annealing

Music Arrangement via Quantum Annealing

Lucas Kirby
30 January 2025
Department of Physics, Durham University

References i

- Cambouropoulos, E. (2011).
  The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing.
  International Computer Music Association.
- Huang, J.-L., Chiu, S.-C., and Shan, M.-K. (2012).
  Towards an automatic music arrangement framework using score reduction.
  ACM Trans. Multimedia Comput. Commun. Appl., 8(1):8:1–8:23.

References i

Cambouropoulos, E. (2011).
The Local Boundary Detection Model (LBDM) and its Application in the Study of Expressive Timing.
International Computer Music Association.
Huang, J.-L., Chiu, S.-C., and Shan, M.-K. (2012).
Towards an automatic music arrangement framework using score reduction.
ACM Trans. Multimedia Comput. Commun. Appl., 8(1):8:1–8:23.

Johnson, M. W., Amin, M. H. S., Gildert, S., Lanting, T., Hamze, F., Dickson, N., Harris, R., Berkley, A. J., Johansson, J., Bunyk, P., Chapple, E. M., Enderud, C., Hilton, J. P., Karimi, K., Ladizinsky, E., Ladizinsky, N., Oh, T., Perminov, I., Rich, C., Thom, M. C., Tolkacheva, E., Truncik, C. J. S., Uchaikin, S., Wang, J., Wilson, B., and Rose, G. (2011). **Quantum annealing with manufactured spins.** *Nature*, 473(7346):194–198. Publisher: Nature Publishing Group.

References ii

Johnson, M. W., Amin, M. H. S., Gildert, S., Lanting, T., Hamze, F., Dickson, N., Harris, R., Berkley, A. J., Johansson, J., Bunyk, P., Chapple, E. M., Enderud, C., Hilton, J. P., Karimi, K., Ladizinsky, E., Ladizinsky, N., Oh, T., Perminov, I., Rich, C., Thom, M. C., Tolkacheva, E., Truncik, C. J. S., Uchaikin, S., Wang, J., Wilson, B., and Rose, G. (2011). **Quantum annealing with manufactured spins.** *Nature*, 473(7346):194–198. Publisher: Nature Publishing Group.

References iii

Li, Y., Wilk, C. M., Hori, T., and Sagayama, S. (2019).
Automatic Piano Reduction of Orchestral Music Based on Musical Entropy.
In *2019 53rd Annual Conference on Information Sciences and Systems (CISS)*, pages 1–5.

Lucas, A. (2014).
Ising formulations of many NP problems.
Frontiers in Physics, 2.
Publisher: Frontiers.

References iii

Li, Y., Wilk, C. M., Hori, T., and Sagayama, S. (2019).
Automatic Piano Reduction of Orchestral Music Based on Musical Entropy.
In *2019 53rd Annual Conference on Information Sciences and Systems (CISS)*, pages 1–5.

Lucas, A. (2014).
Ising formulations of many NP problems.
Frontiers in Physics, 2.
Publisher: Frontiers.

Pearce, M. and Wiggins, G. A. (2001).
Towards A Framework for the Evaluation of Machine Compositions.
In *Proceedings of the AISB'01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences.*

References iv

Pearce, M. and Wiggins, G. A. (2001).
Towards A Framework for the Evaluation of Machine Compositions.
In *Proceedings of the AISB'01 Symposium on Artificial Intelligence and Creativity in the Arts and Sciences.*

LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$
$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S_i' = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

[Cambouropoulos, 2011]

LBDM

Boundary strength

$$S_i = x_i \times (r_{i-1,i} + r_{i,i+1})$$
$$r_{i,i+1} = \frac{|x_i - x_{i+1}|}{x_i + x_{i+1}}$$

Normalisation

$$S_i' = \frac{S_i - \min(S_i)}{\max(S_i) - \min(S_i)}$$

Weighting

$$S = \frac{1}{3}\left(S'_{\text{pitch}} + 2S'_{\text{IOI}}\right)$$

[Cambouropoulos, 2011]

- Boundaries always taken at beginning/end of piece
- Weightings derived by trial and error

MIS

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

[Lucas, 2014]

$A/B >= 2\max(W)$ to weight the constraint term more heavily than any objective term

MIS

$$f(x) = A \sum_{ij \in E} x_i x_j - B \sum_i W_i x_i$$

[Lucas, 2014]

$A/B >= 2\max(W)$ to weight the constraint term more heavily than any objective term

- Weighted MIS QUBO
- Lagrange paramters A/B determine the balance between constraint and objective

Phrase entropy

Shannon entropy

$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$

Probability distribution

$$P(x_i) = \frac{n_i}{N}$$

[Li et al., 2019]

Phrase entropy
Shannon entropy
$$H(X) := -\sum_i P(x_i) \log_2 P(x_i)$$
Probability distribution
$$P(x_i) = \frac{n_i}{N}$$
[Li et al., 2019]

- Shannon entropy units in bits due to $\log_2$
- Distribution calculated for pitch and duration