Significance Testing About Proportions

Using the Rejection Region

Using the p - value

Population Proportion

- A ***population proportion*** is a parameter that describes a percentage value associated with a population.

ST → Proportion

Ratio

Basket 1
2 apples
3 mangoes
2:3

Basket 2
4 apples
6 mangoes
4:6 → 2:3

Example:

- Five years ago, 3.9% of children in a certain region lived with someone other than a parent. A sociologist wishes to test whether the current proportion is different. Perform the relevant test at the 5% level of significance using the following data: in a random sample of 2,759 children, 119 lived with someone other than a parent.

① $p_o = 3.9\%$ or 0.039

$a = 0.05$

$n = 2759$ ($n > 30$) → z-test

$p = 119$

② $H_o: p = 3.9\%$

$H_a: p \neq 3.9\%$

③ $se_o = \sqrt{\frac{p_o(1-p_o)}{n}}$

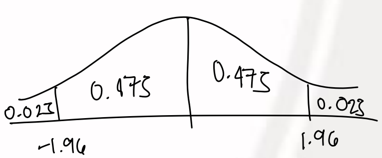
$= \sqrt{\frac{0.039(1-0.039)}{2759}}$

$= 0.003685684$

$z = \frac{\hat{p} - p_o}{se_o}$

$= \frac{\frac{119}{2759} - 0.039}{0.003685684}$

$= 1.12$



$a(M \rightarrow z_c) = 0.5 - 0.025$

$= 0.475$

Step 1: Assumptions

- Check assumptions
  - Population proportion p is defined
  - n is large enough ($n \geq 30$)

Step 1: Assumptions

- In a random sample of 2,759 children, 119 lived with someone other than a parent.
- Population Proportion:

$$\hat{p} = \frac{119}{2759}$$

- Sample size, $n = 2759$

Step 2: State the Hypotheses

- State the Hypotheses
  - let p_0 be the hypothesized value

Null Hypothesis

$H_0: p = p_0$ (or $p \geq p_0$ or $p \leq p_0$)

Alternative Hypothesis

$H_a: p \neq p_0$ (or $p < p_0$ or $p > p_0$)

Step 2: State the Hypotheses

- let p_0 be the hypothesized value

$$p_0 = 3.9\% = 0.039$$

$$H_0: p = 0.039$$
$$H_a: p \neq 0.039$$

Step 3: Compute the Test Statistic

- Compute for the Test Statistic

$$z = \frac{\hat{p} - p_0}{se_0}$$

where

$$se_0 = \sqrt{\frac{p_0(1 - p_0)}{n}}$$

Step 3: Compute the Test Statistic

$$se_0 = \sqrt{\frac{p_0(1-p_0)}{n}}$$

$$se_0 = \sqrt{\frac{0.039\,(1-0.039)}{2759}} = 0.0037$$

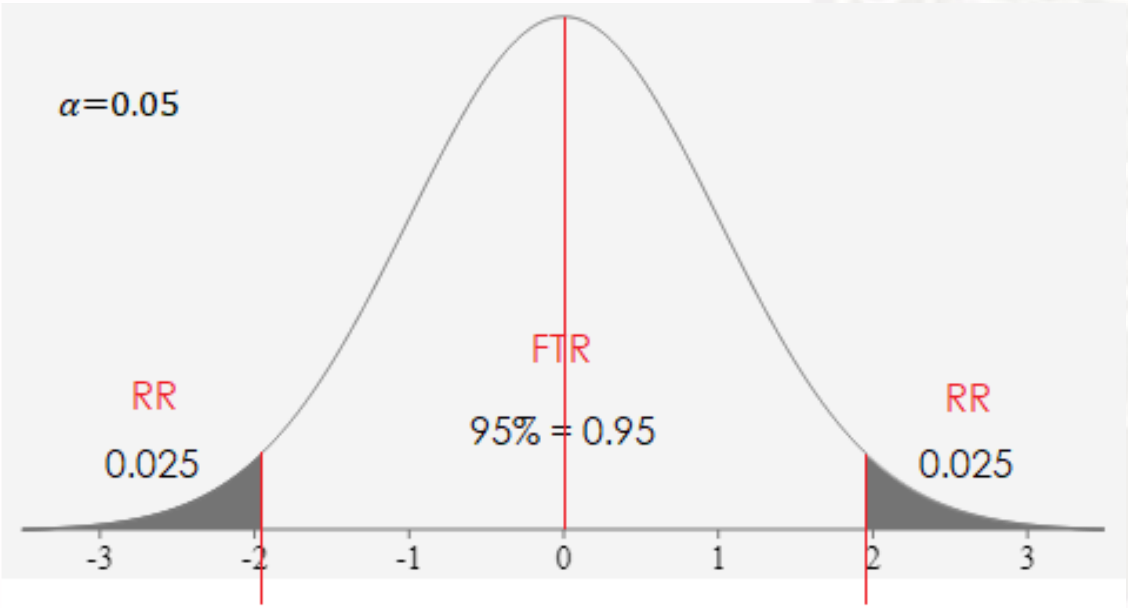
$$z = \frac{\hat{p}-p_0}{se_0} = \frac{\frac{119}{2759} - 0.039}{0.003685684} = 1.12$$

Step 4: Interpret the Test Statistic

- Using the Rejection Region
- Determine if one-tailed or two-tailed test based on H_a
- Determine the FTR (Fail To Reject) region and the RR (Rejection Region) based on the confidence level or the significance level

Step 4: Interpret the Test Statistic (Using Rejection Region)

- $H_a: p \neq 0.039$ (two-tailed test)
- 5% level of significance, $\alpha = 0.05$



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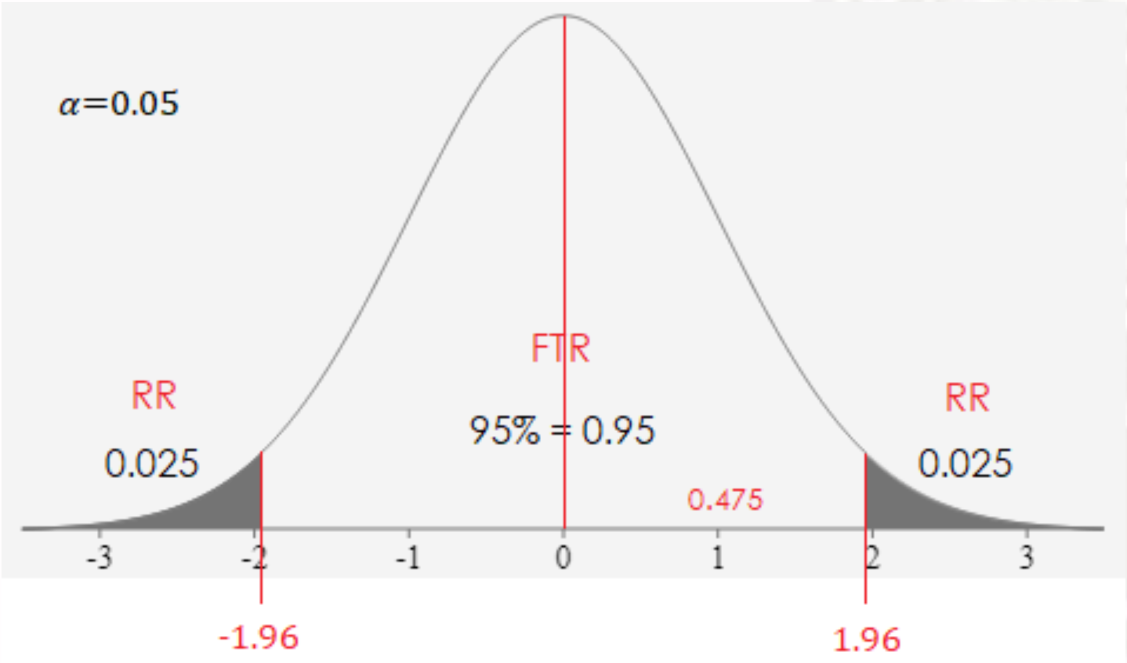
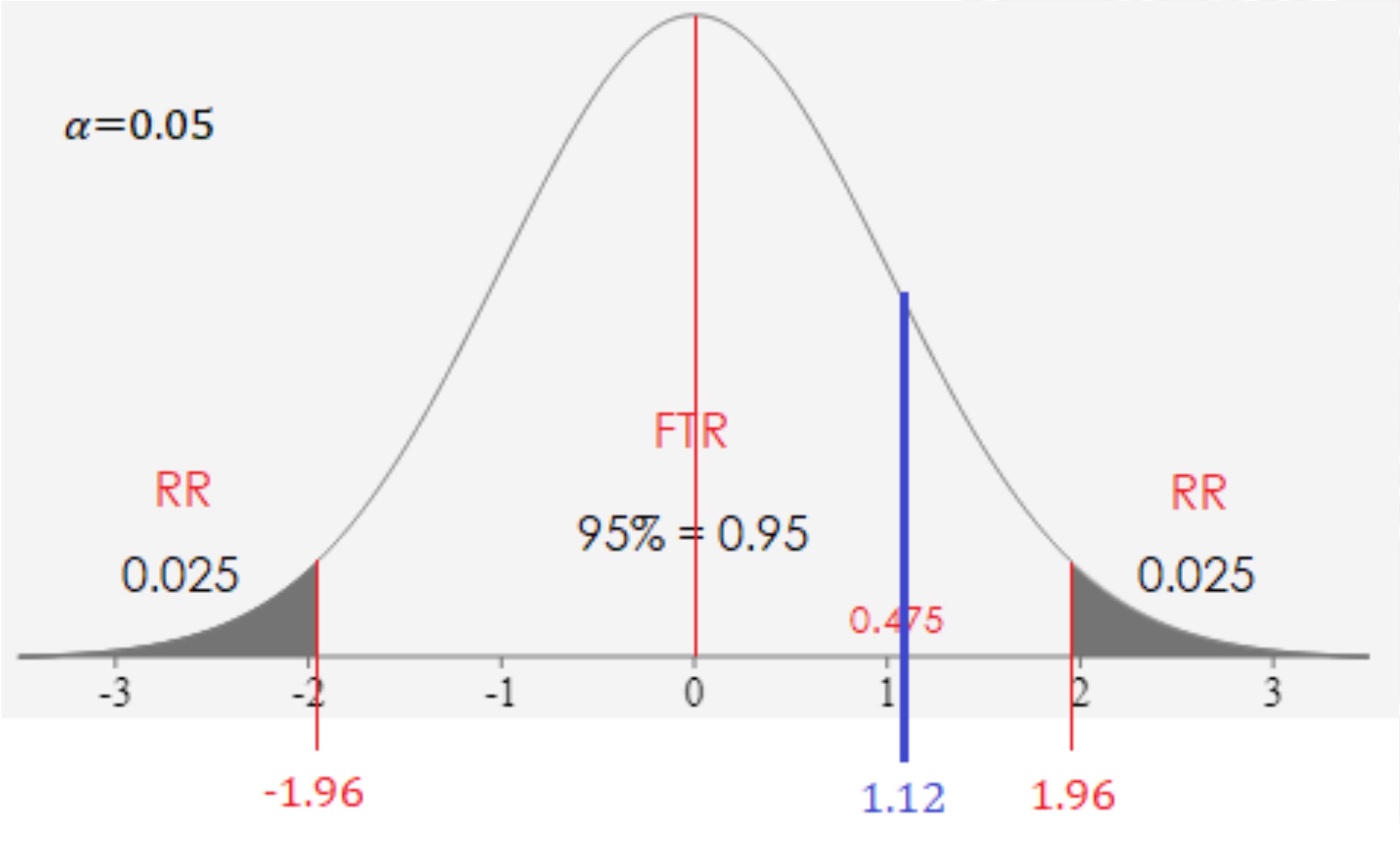


Step 4: Interpret the Test Statistic (Using Rejection Region)



Step 5: Make a Conclusion

- Since the test statistic lies in the FTR, then we fail to reject the null hypothesis.
- Therefore, there is no significant difference between the current and the past proportion.

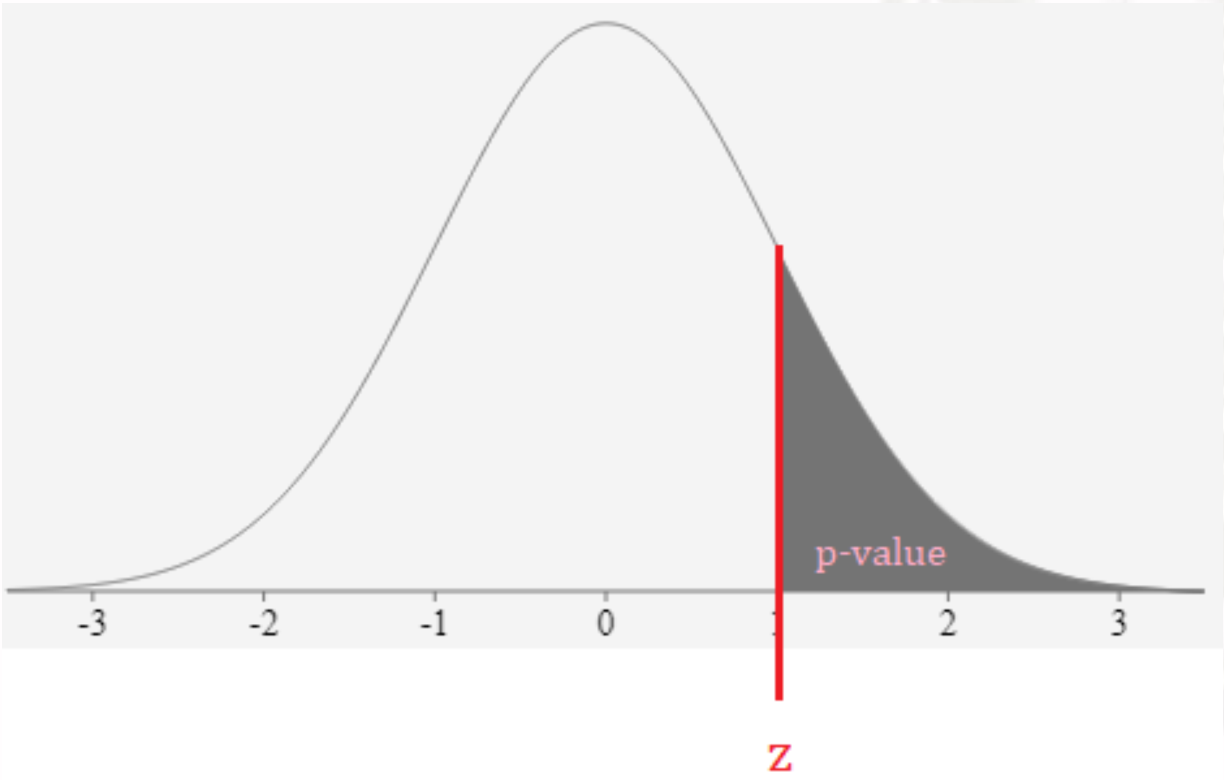
Step 4: Interpret the Test Statistic (Using p-values)

- Compute for the p – value. The ***p – value*** is the probability that the test statistic takes the observed value or a value more extreme if we presume H_0 is true.

- The computation of the ***p – value*** from the test statistic depends on H_a

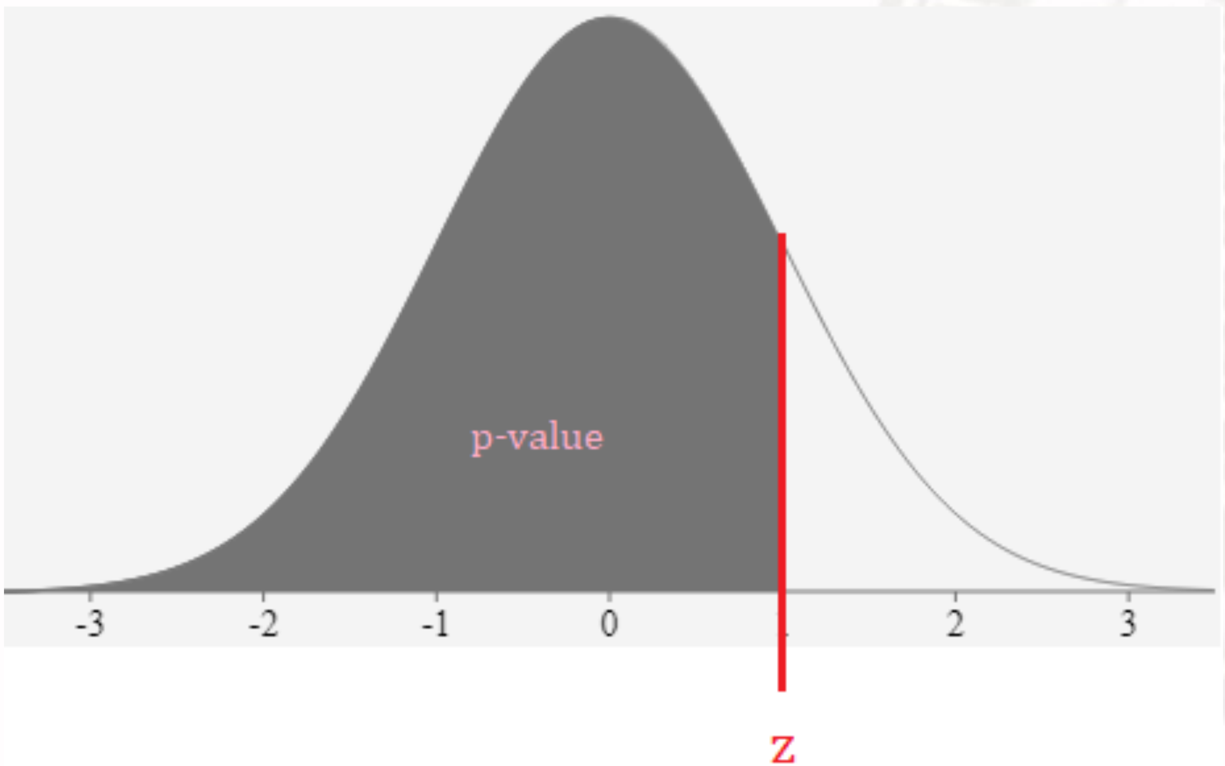
Step 4: Interpret the Test Statistic (Using p-values)

- $H_a: p > p_0$ (right-tailed test)



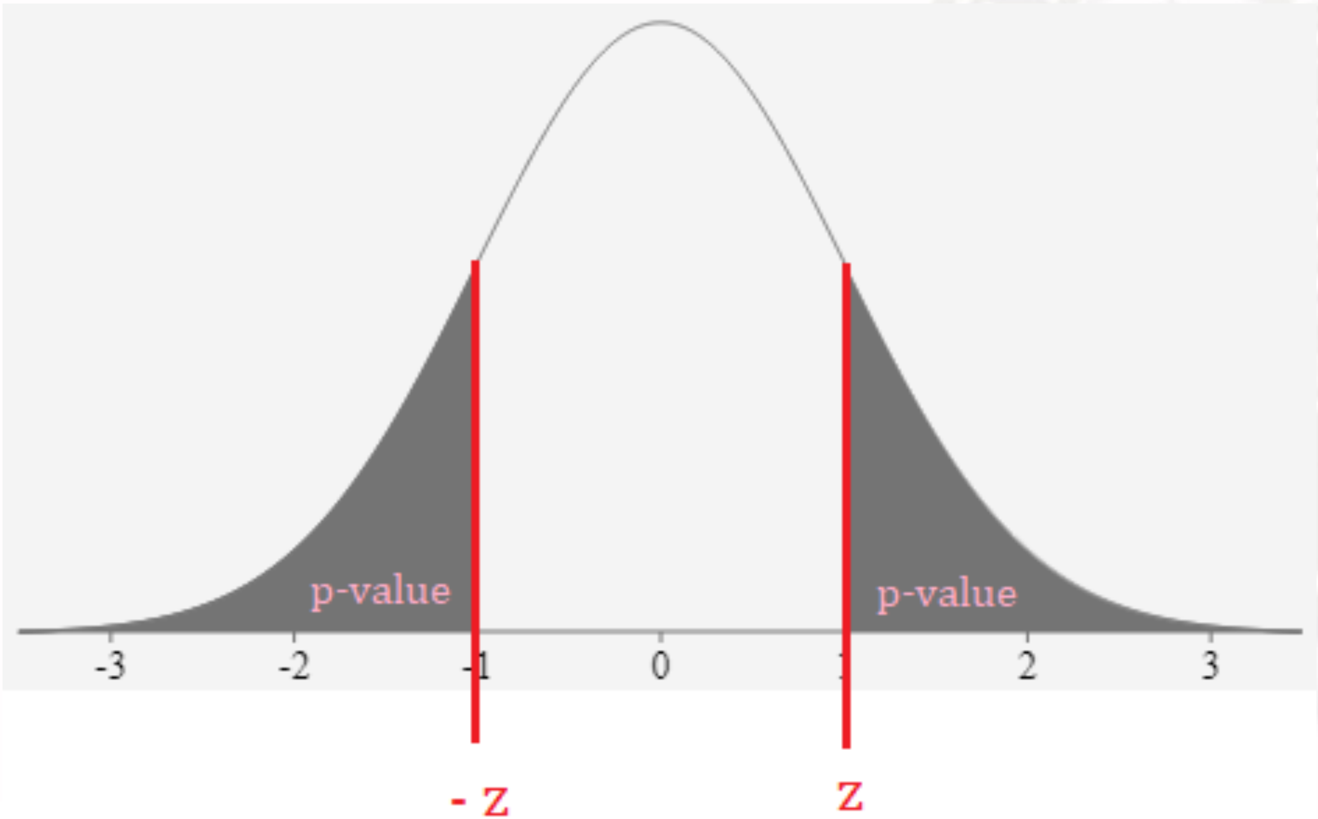
Step 4: Interpret the Test Statistic (Using p-values)

- $H_a: p < p_0$ (left-tailed test)



Step 4: Interpret the Test Statistic (Using p-values)

- $H_a: p \neq p_0$ (two-tailed test)



Step 4: Interpret the Test Statistic (Using p-values)

- $H_a: p \neq 0.039$ (two-tailed test)
- $z = 1.12$
- p – value $= 2(0.5 - 0.3686)$

$$= 2(0.1314)$$

$$= 0.2628$$

Step 5: Make a Conclusion

- If p – value < significance level, then reject H_0
- If p – value ≥ significance level, then do not reject H_0

Step 5: Make a Conclusion

- 5% level of significance, $\alpha = 0.05$
- p – value = 0.2628

- p-value > α, so, do not reject the null hypothesis.

- Therefore, there is no difference between the current and the past proportion.

You Try

- Professional astrologers prepared horoscopes for 83 adults. Each adult was shown three horoscopes, one of which was the one an astrologer prepared for him or her and the other two were randomly chosen from ones prepared for other subjects in the study. Each adult had to guess which of the three was his or hers. Of the 83 subjects, 28 guessed correctly. Would you conclude that people are more likely to select their horoscope?
- Set up the hypotheses to test that the probability of a correct prediction is 1/3 against the astrologers' claim that it is greater than 1/3. Consider a 95% confidence level