Significance Testing About Means

Using the Rejection Region

Population Mean

- A ***population mean*** is an average of a group characteristic.

Step 1: Assumptions

- Check assumptions
  - Population mean μ is defined
  - Sample mean, $\bar{x}$, and standard deviation is defined or can be determined
  - If n is large enough, $(n > 30)$, use the z distribution table
  - If n is small, $(n \leq 30)$, use the t distribution table

Example 1:

- Suppose a test has been given to all high school students in a certain state. The mean test score for the entire state is 70, with standard deviation equal to 10. Members of the school board suspect that female students have a higher mean score on the test than male students, because the mean score $\bar{x}$ from a random sample of 64 female students is equal to 73. Does this provide strong evidence that the overall mean for female students is higher considering a 95% confidence level?

Step 1: Assumptions

- Example 1: 64 female students
  - Use the z-table

Step 2: State the Hypotheses

- State the Hypotheses
  - let μ_0 be the hypothesized value

Null Hypothesis

$H_0: \mu = \mu_0 (\text{or} \mu \geq \mu_0 \text{or } \mu \leq \mu_0)$

Alternative Hypothesis

$H_a: \mu \neq \mu_0 (\text{or } \mu < \mu_0 \text{or } \mu > \mu_0)$

Step 2: State the Hypotheses

- Example 1:

$$H_0: \mu \leq 70$$
$$H_a: \mu > 70$$

Step 3: Compute the Test Statistic

- Compute for the Test Statistic

$$t \text{ or } z = \frac{\bar{x} - \mu_0}{se_0}$$

where

$$se_0 = \frac{s}{\sqrt{n}}$$

$\bar{x}$ is the sample mean

s is the sample standard deviation

n is the sample size

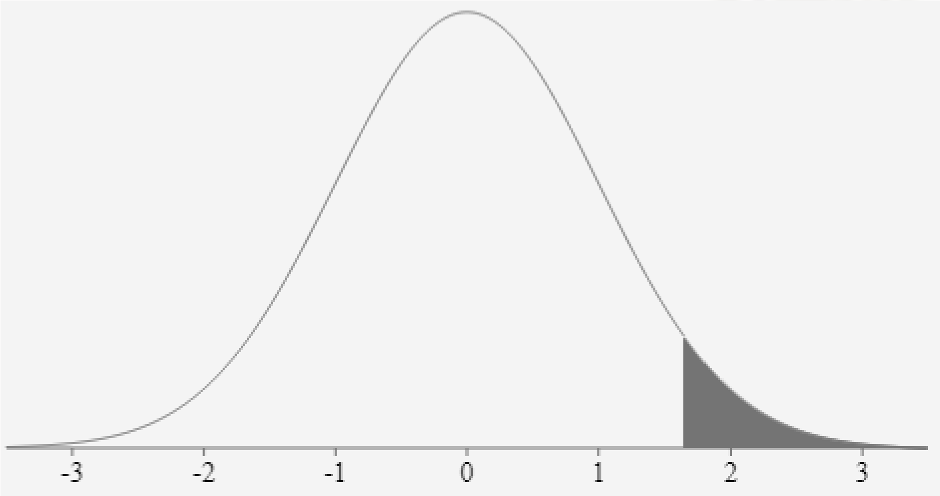
Step 3: Compute the Test Statistic

$$se_0 = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{64}} = 1.25$$

$$z = \frac{73 - 70}{1.25} = 2.4$$

Step 4: Interpret the Test Statistic (Using Rejection Region)

- $H_a: \mu > 70$ (right-tailed test)
- 95% confidence level, $\alpha = 0.05$
- $z_c = 1.645$



Step 5: Make a Conclusion

- Since the test statistic lies in the RR, then we reject the null hypothesis.
- Therefore, the overall mean for female students is higher.

Example 2:

Consider the NCHS-reported mean total cholesterol level in 2002 for all adults of 203. Suppose a new drug is proposed to lower total cholesterol. A study is designed to evaluate the efficacy of the drug in lowering cholesterol. Fifteen patients are enrolled in the study and asked to take the new drug for 6 weeks. At the end of 6 weeks, each patient's total cholesterol level is measured and the sample statistics are as follows: $n=15$, $\bar{x}=195.9$ and $s=28.7$. Is there statistical evidence of a reduction in mean total cholesterol in patients after using the new drug for 6 weeks? Consider $\alpha = 0.05$

Step 1: Assumptions

- Example 2: $n = 15$
  - Use the t-table

Step 2: State the Hypotheses

- Example 2:

$$H_0: \mu \geq 203$$
$$H_a: \mu < 203$$

Step 3: Compute the Test Statistic

$$se_0 = \frac{s}{\sqrt{n}} = \frac{28.7}{\sqrt{15}} = 7.41$$

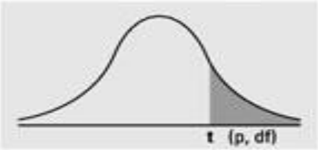
$$t = \frac{195.9 - 203}{7.41} = -0.96$$

Step 4: Interpret the Test Statistic (Using Rejection Region)

- $H_a: \mu < 203$ (left-tailed test)
- $\alpha = 0.05$
- $n = 15$
- $df = n - 1$
- $df = 14$

$$t_c = -1.761310$$

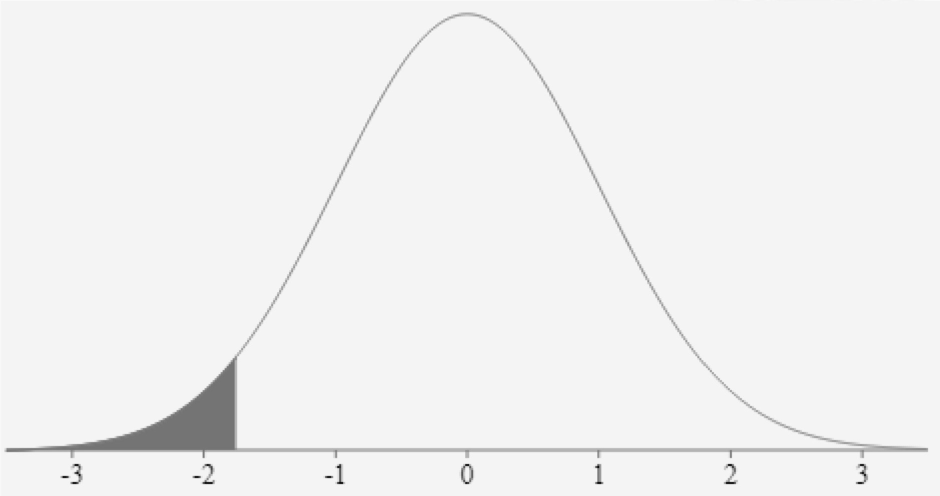
Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	—	—	80%	90%	95%	98%	99%	99.9%

Step 4: Interpret the Test Statistic (Using Rejection Region)

$$t_c = -1.761310$$
$$t = -0.96$$



Step 5: Make a Conclusion

- Since the test statistic lies in the FTR, then we fail to reject the null hypothesis.

- Therefore, there no statistical evidence of a reduction in mean total cholesterol in patients after using the new drug for 6 weeks