Lesson 1

Steps In Performing A Significance Test

Lesson Contents

- Steps in significance testing
- Interpreting the p-value

Significance Testing

- ↳ Hypothesis
  - ↳ Null (original)
  - ↳ Alternative (change)

Significance Testing

- Also called "Hypothesis Testing".
- Uses probability to provide a way to quantify how plausible a parameter is while controlling the chance of an incorrect inference.
- Method for using data to summarize the evidence about a hypothesis.
- Determine whether there is enough evidence in favor of certain belief, or **hypothesis**, about a parameter.

Significance Testing

- Questions about the parameter.
  1. Is there statistical evidence, from a random sample of potential customers, to support the hypothesis that more than 10% of the potential customers will purchase a new product when seen in Facebook?
  2. Is a new drug effective in curing a certain disease? A sample of patients is randomly selected. Half of them are given the drug while the other half are given a placebo. The conditions of the patients are then measured and compared.

Steps in Conducting a Significance Test

1. Assumptions
2. State the hypotheses
3. Compute for the Test Statistic
4. Interpret the P-value
5. Make a conclusion

Step 1

Assumptions

Assumptions

- Each significance test makes certain assumptions or has certain conditions under which it applies.
- Assumptions may be about the sample size or about the shape of the population distribution.

getting a sample from the population

$(P \to S)$

$n \geq 30 \to$ z-test

$n < 30 \to$ t-test

Assumptions

Assumptions

Each test makes certain assumptions or has certain conditions for the test to be valid. These pertain to the following:

- ***Type of data***: Like other statistical methods, each test applies for either quantitative data or categorical data.
- ***Randomization***: Like the confidence interval method of statistical inference, a test assumes that the data were obtained using randomization, such as a random sample.
- ***Population distribution***: For some tests, the variable is assumed to have a particular distribution, such as the normal distribution.
- ***Sample size***: The validity of many tests improves as the sample size increases.

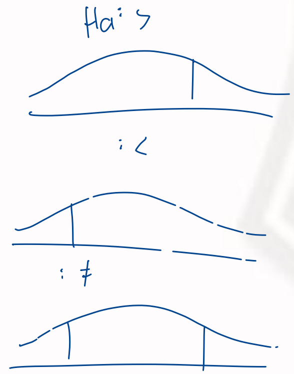
Step 2

Hypotheses

Null : =

Alternative: >, <, ≠

↳ right-tailed ↳ left-tailed ↳ two-tailed



Formulation of Hypotheses

- A **hypothesis** is a statement about a <u>population</u>, usually claiming that a parameter takes a particular numerical value or falls in a certain range of values.
  - For a categorical variable the parameter is a proportion, **ρ**
  - For a quantitative variable the parameter is a mean, **μ**
- The two types of hypothesis:
  - Null
  - Alternative

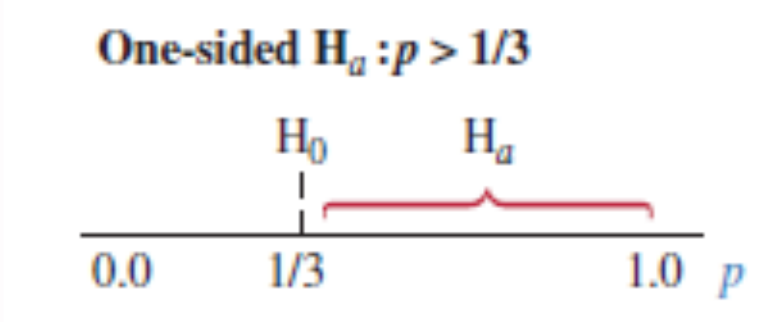
Null Hypothesis

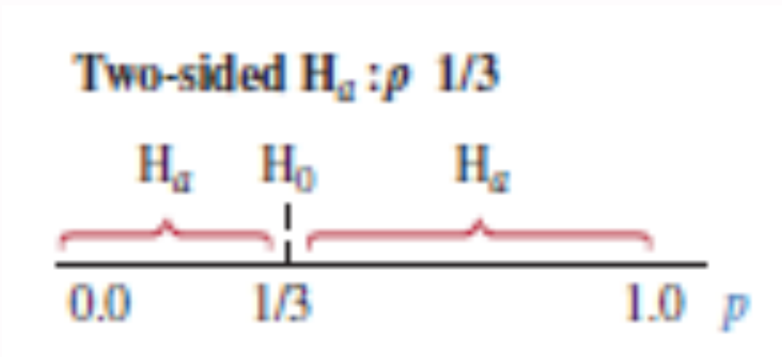
- $\boldsymbol{H_0}$ is the hypothesis to be tested.
- It is a statement that either a parameter takes on a particular value, or that there is no difference between parameters (i.e., there is no effect).
  - Examples
    - H_0: The probability of guessing the correct hand is 50% (ρ = 50%).
    - H_0: The mean IQ of UC students is 110. (μ = 110)
  - Symbols: ≥, ≤, or just =

Alternative Hypothesis

- $\boldsymbol{H_a}$ **or** $\boldsymbol{H_1}$ states that either the parameter falls within an alternative range of values, or that there is a difference between parameters (i.e., there is an effect).
- It is a statement of what we believe is true if our sample data cause us to reject the null hypothesis.
  - Examples
    - H_a: The probability of guessing the correct hand is larger than 50% (ρ > 50%).
    - H_a: The probability of guessing the correct hand is less than 50% (ρ < 50%).
    - H_a: The mean IQ of UC students differs from 110. (μ ≠ 110).
  - Symbols: >, <, or ≠

Type of Test

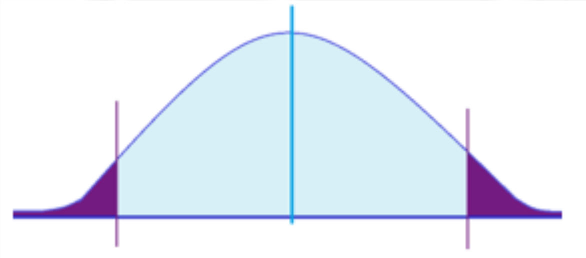




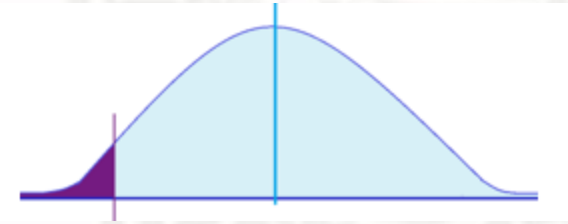
- The alternative hypothesis determines the type of test to be made
  - **One-sided alternative hypotheses**
    - Ha: $\rho > \rho_0$.
  - **Two-sided alternative hypotheses**
    - Ha: $p \neq \rho_0$.

Type of Test

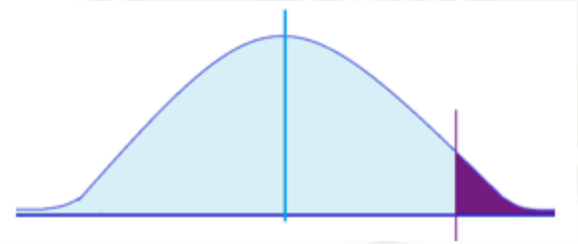
- Two – tailed test (H_a: $\neq$)



- One – tailed test
  - Left – tailed test (H_a: $<$)



  - Right – tailed test (H_a: $>$)



Example

- Provide the null and alternative hypotheses for the following statement:
  - Based on any person's horoscope, the probability p that an astrologer can correctly predict which of three personality charts applies to that person equals 1/3.
  - H_0: ρ = $\frac{1}{3}$
  - H_a: ρ ≠ $\frac{1}{3}$

Example

- Provide the null and alternative hypotheses for the following statement:
  - We want to test if college students take less than five years to graduate from college, on the average.
  - H_0: μ ≥ 5
  - H_a: μ < 5

Example

- Identify whether the following statements are null or alternative hypotheses:
  - In Canada, the proportion of adults who favor legalized gambling is 0.50. → null
  - The proportion of all Canadian college students who are regular smokers is less than 0.24, the value it was 10 years ago. → Alternative
  - The mean IQ of all students at UC is larger than 100. ↳ Alternative
  - The mean working hours for the working population differs from 40 hours. ↳ Alternative

Step 3

Compute the test statistic

Test Statistic

- A **test statistic** describes how far that point estimate falls from the parameter value given in the null hypothesis.
- It shows how closely the observed data match the distribution expected under the null hypothesis of that statistical test.

Z or t (test stat)

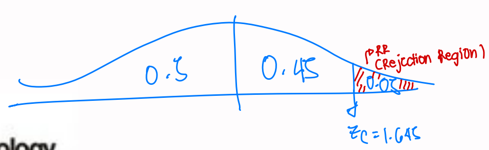
↳ used as a baris of rejecting the H_0 or not

$a = 0.05$ $H_a = >$

$a(\mu \to z_c) = 0.5 - 0.05$

$= 0.45$

↳ 1.645

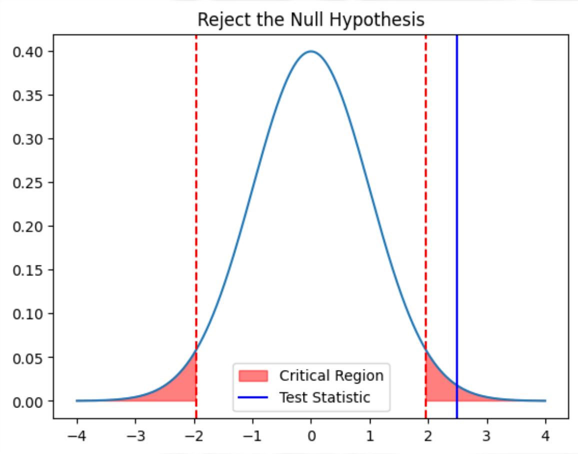


Interpreting the test statistic

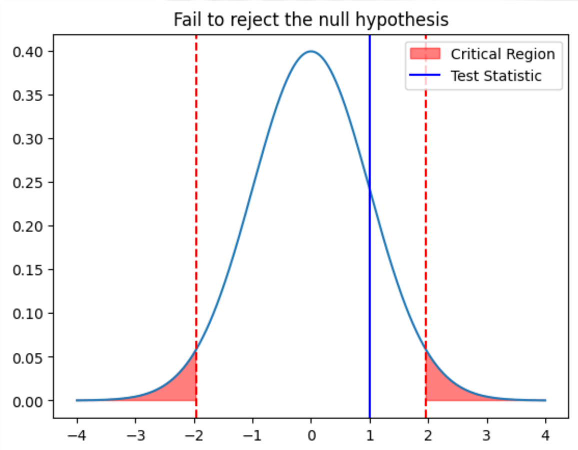
- A **test statistic** describes how far that point estimate falls from the parameter value given in the null hypothesis.
- It shows how closely the observed data match the distribution expected under the null hypothesis of that statistical test.

Interpreting the test statistic

- **Reject H_0:** When the test statistic falls within the critical region.



- **Fail to reject H_0:** When the test statistic does not fall within the critical region.



Step 4

Interpret the p-value

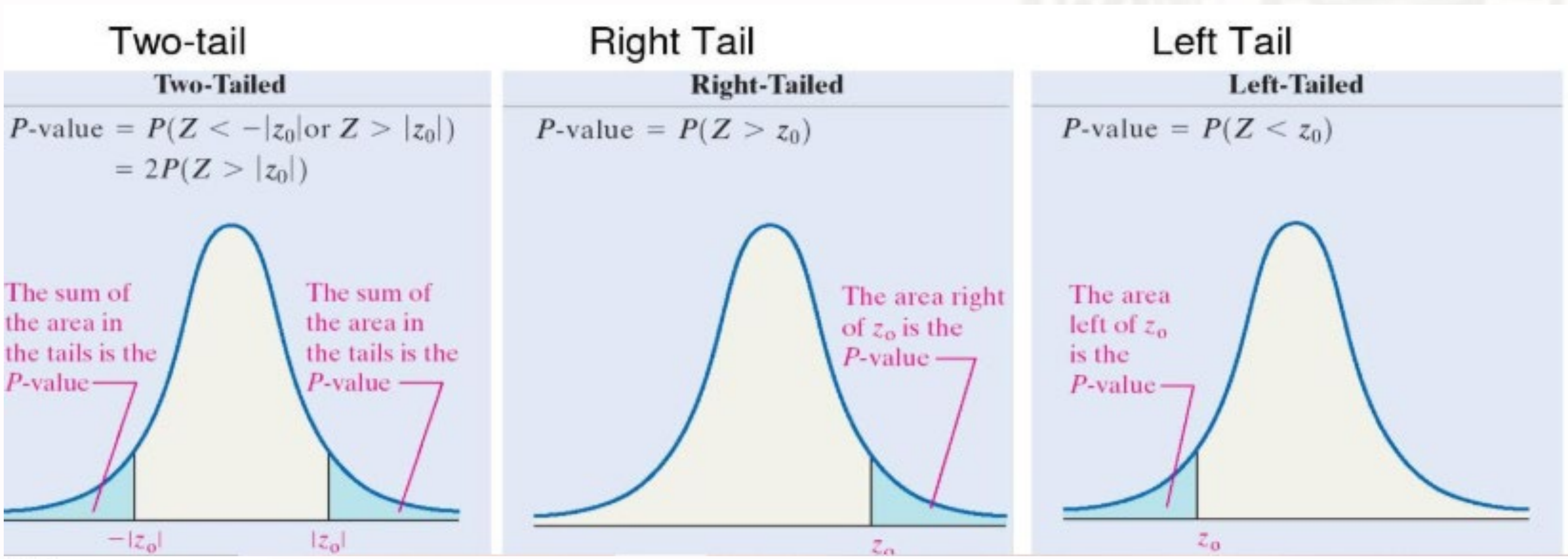
P-value

- The **p-value** is the probability of obtaining the observed value of the test statistic, or a more extreme value, assuming that the null hypothesis is true.
- It helps determine the strength of evidence against the null hypothesis
  - Smaller p – values represent stronger evidence against H_0.

Visualizing the p-value

SUMMARY: P-values for Different Alternative Hypotheses

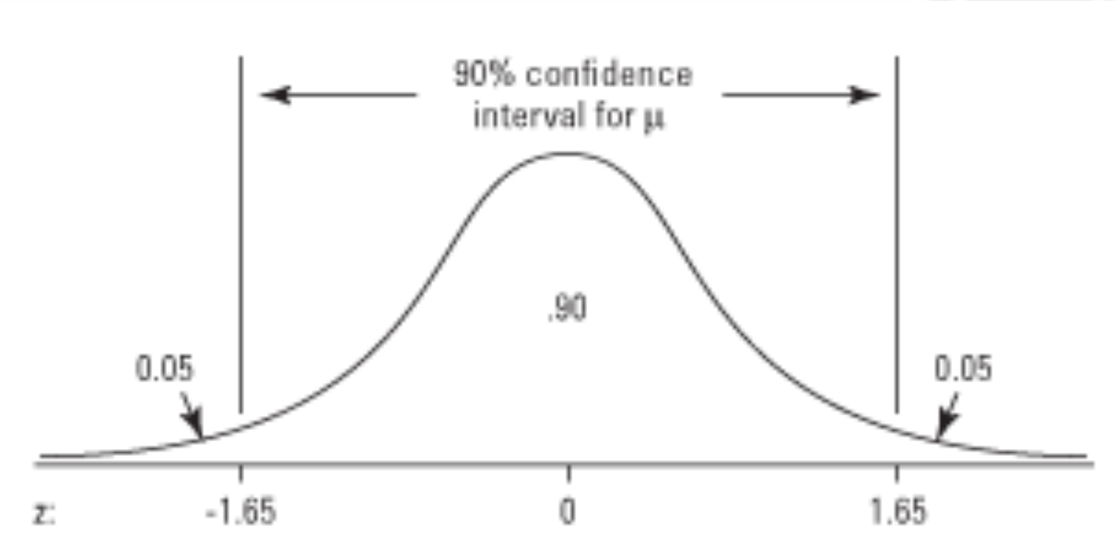
Alternative Hypothesis	P-value
$H_a: p > p_0$	Right-tail probability
$H_a: p < p_0$	Left-tail probability
$H_a: p \neq p_0$	Two-tail probability



Confidence Level and Significance Level (α)

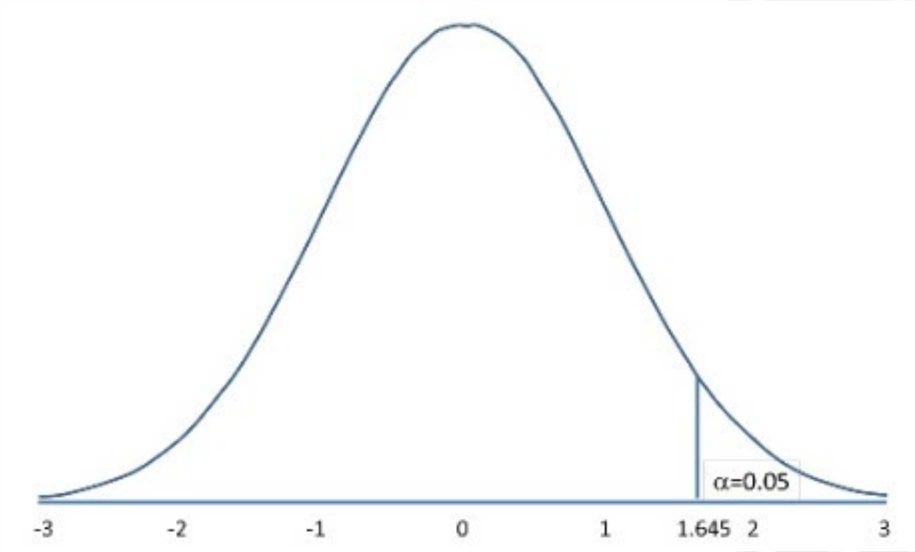
- A **confidence level** refers to the percentage of all possible samples that can be expected to include the true population parameter.
- This is also $1 - \alpha$
- Indicates where the ***fail to reject region*** lie
- Usually has the value 90%, 95% and 99%

Confidence Level and Significance Level (α)



- Confidence Level: 90%
- Significance Level: $\alpha = 0.1$
- Critical z – scores: - 1.645 and 1.645

Confidence Level and Significance Level (α)



- Confidence Level: 95%
- Significance Level: $\alpha = 0.05$
- Critical z – score: 1.645

Calculating the p-value

- Calculating p-value depends on the alternative hypothesis, H_A.
- Suppose, we compute z-statistic, z_C, we compute the p-value using the following:

Alternative Hypothesis:	$H_A: \mu \neq \mu_o$	$H_A: \mu > \mu_o$	$H_A: \mu < \mu_o$
P-Value =	$2 \times P(Z > \lvert z_c \rvert)$	$P(Z > z_c)$	$P(Z < z_c)$
Significance Level =	α		
Decision:	Reject H_o if P-value $< \alpha$.		

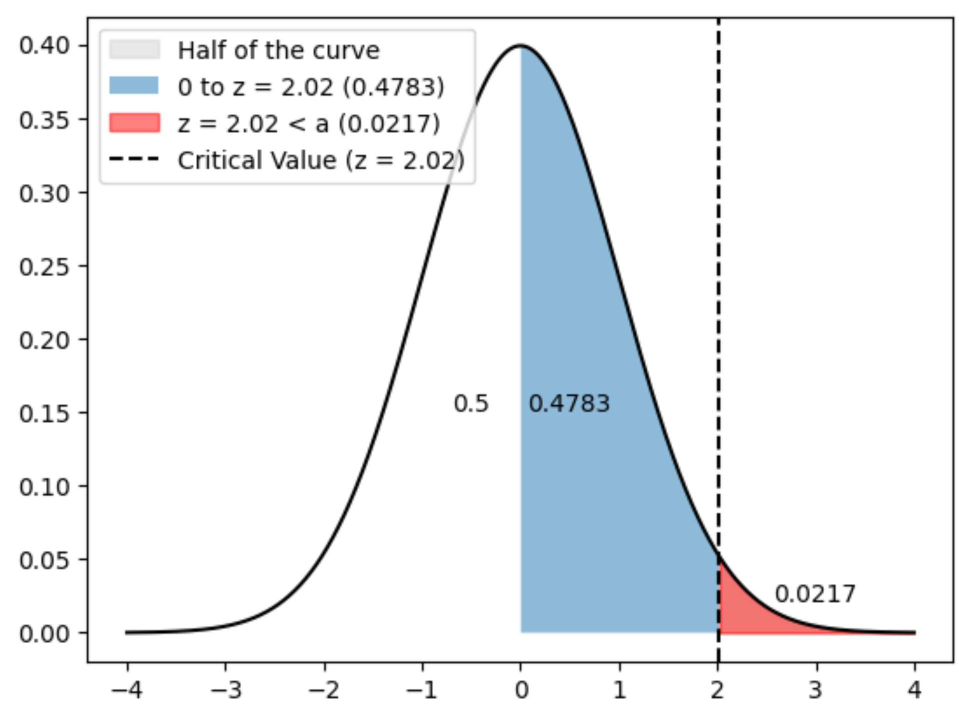
Interpreting the p-value

- The **significance level (α)** is the probability of rejecting a null hypothesis.
  - In practice, the most common significance level is 0.05.
- It is a number such that we reject H_0 if the P-value is less than or equal to that number.

P-value	Decision About H_0
≤ 0.05	Reject H_0
> 0.05	Do not reject H_0

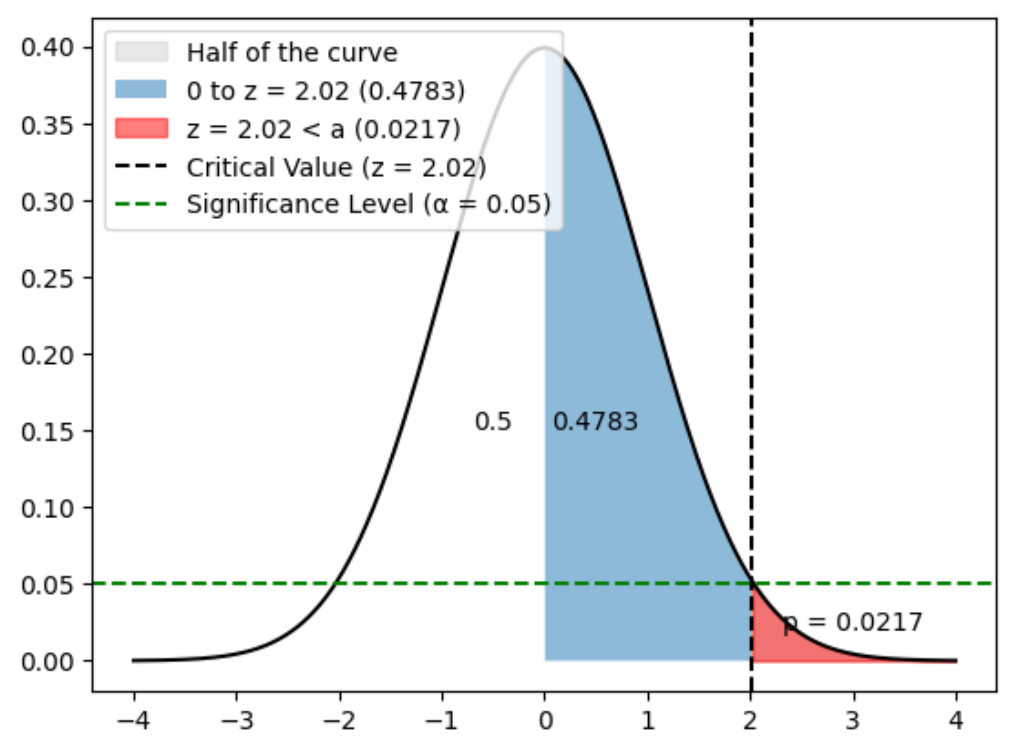
Calculating the p-value: Example 1

- Computed z-score, z_c = 2.02
- Alternative hypothesis, $H_{A:}$ ρ > 70.
- Area between 0 and 2.02 is 0.4873
- P-value = P$(p_0 > z_c)$ = 0.0217
- $\alpha = 0.05$



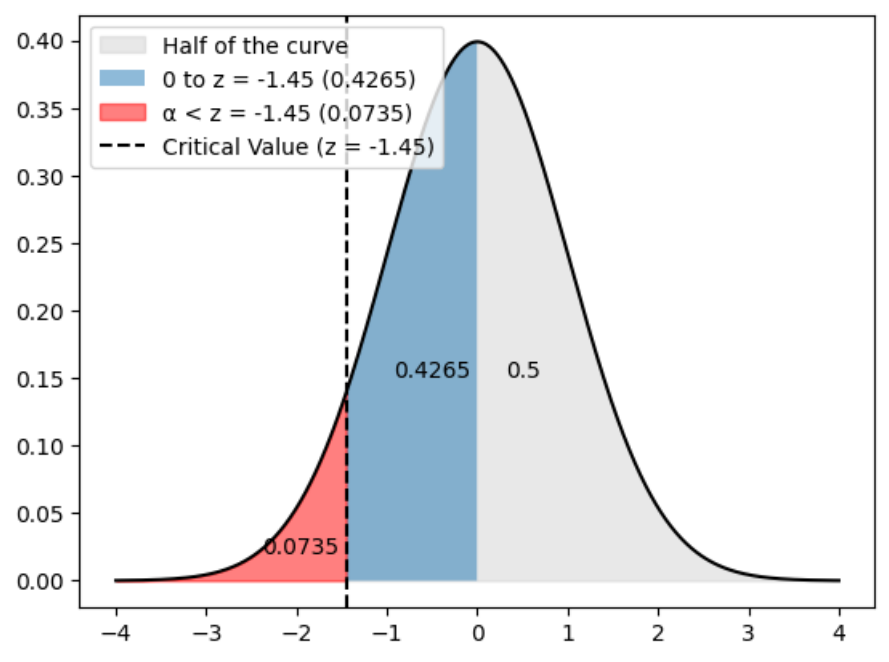
Interpreting the p-value: Example 1

- Decision:
  - Since p-value < α, that is, $0.0217 < 0.05$, we **reject the null hypothesis** and thus, accept the alternative hypothesis.



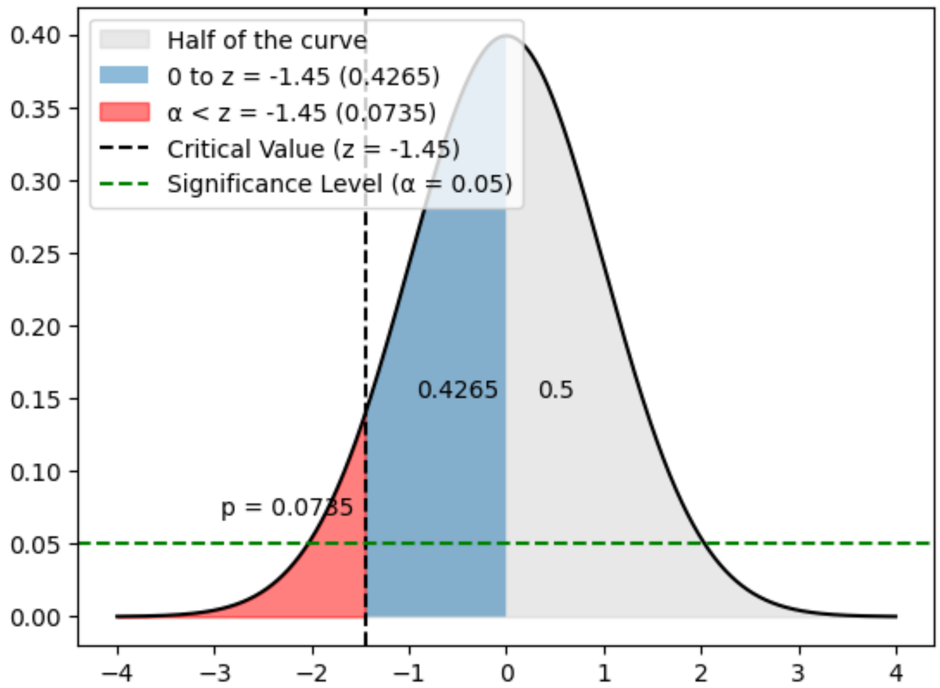
Calculating the p-value: Example 2

- Computed z-score, z_C = -1.45
- Alternative hypothesis, $H_{A:}$ μ < 70.
- Area between 0 and -1.45 is 0.4265
- P-value = P$(p_0 < z_c)$ = 0.0735
- $\alpha = 0.05$



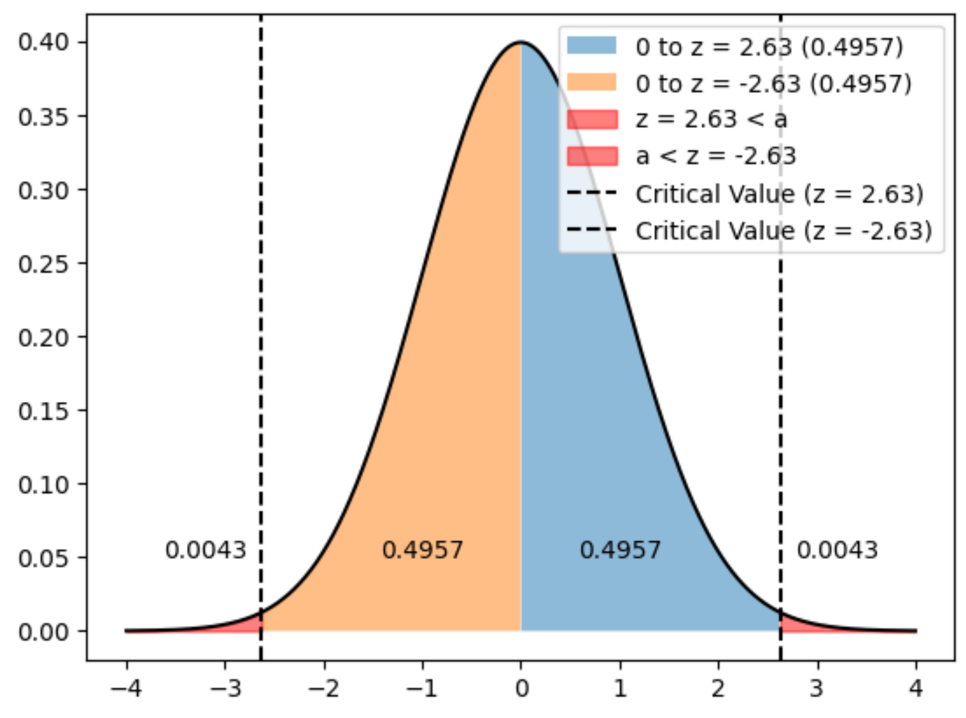
Interpreting the p-value: Example 2

- Decision:
  - Since p-value > α, that is, 0.0735 > 0.05, we **fail to reject the null hypothesis**, in other words, accept the null hypothesis.



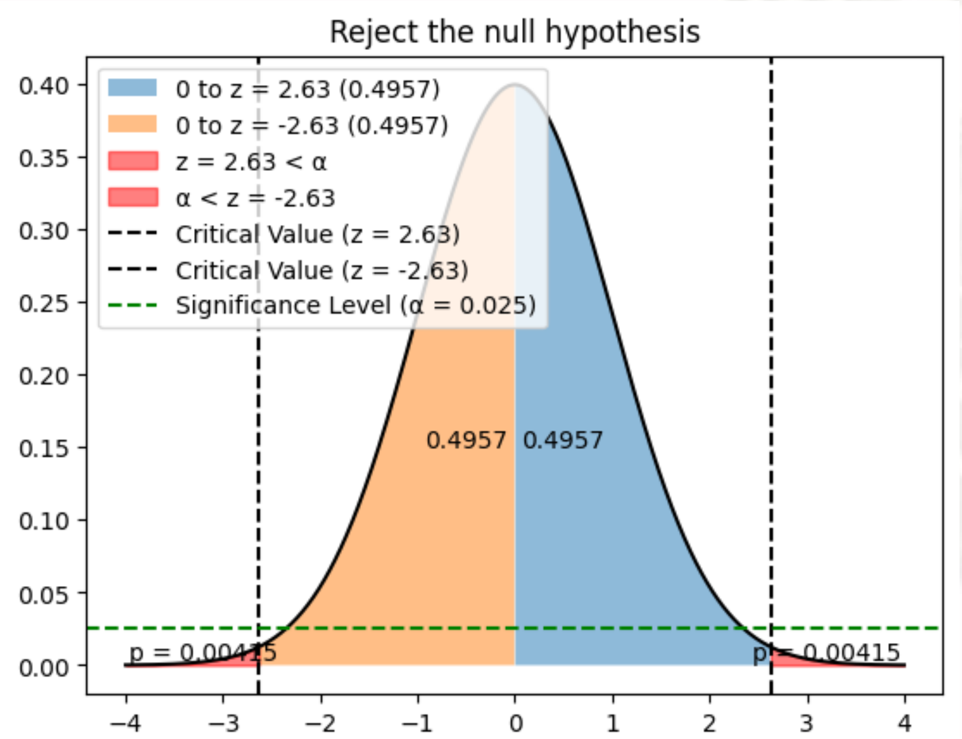
Calculating the p-value: Example 3

- Computed z-score, z_c = 2.63
- Alternative hypothesis, $H_{A:}$ $\mu \neq 70$
- Area between 0 and ± 2.63 is 0.4957
- P-value = P$(p_0 \neq \pm z_c)$ $= \frac{0.0086}{2}$ = 0.0043
- $\alpha = 0.05/2$ = 0.025



Interpreting the p-value: Example 3

- Decision:
  - Since p-value < α, that is, $0.0043 < 0.05$, we **reject the null hypothesis**, in other words, accept the null hypothesis.



Step 5

Make a conclusion

Conclusion

- The **conclusion** determines whether there is sufficient evidence to reject the null hypothesis and supports decision-making based on the statistical analysis.
- **Evidence** refers to the data and results from the experiment or study that support or contradict the null hypothesis. In significance testing, this data determines whether the data supports rejecting or failing to reject the null hypothesis.
  - **Reject H_0:** There is not enough evidence to support the null hypothesis, but there is enough evidence to support the alternative hypothesis.
  - **Fail to reject H_0:** There is not enough evidence to support the alternative hypothesis, but there is enough evidence to maintain the null hypothesis.

Null Hypothesis Cases

- **True Positive (Correct Rejection):** This occurs when we reject the null hypothesis when it is actually false. It's the correct decision.
- **False Positive (Type I Error):** This occurs when we reject the null hypothesis when it is actually true. It's an incorrect decision, also known as a Type I error.
- **False Negative (Type II Error):** This occurs when we fail to reject the null hypothesis when it is actually false. It's an incorrect decision, also known as a Type II error.
- **True Negative (Correct Non-rejection):** This occurs when we fail to reject the null hypothesis when it is actually true. It's the correct decision.

		Condition of Null Hypothesis	
		True	False
Possible Action	Fail to reject H_0	Correct action	Type II error
	Reject H_0	Type I error	Correct action

In summary…

1. **Assumptions**
   First, specify the variable and parameter. The assumptions commonly pertain to the method of data production (randomization), the sample size, and the shape of the population distribution.
2. **Hypotheses**
   State the null hypothesis, H_0 (a single parameter value, usually no effect), and the alternative hypothesis, H_a (a set of alternative parameter values)
3. **Test statistic**
   The test statistic measures distance between the point estimate of the parameter and its null hypothesis value, usually by the number of standard errors between them.
4. **P-value**
   The P-value is the probability that the test statistic takes the observed value or a value more extreme if we presume H_0 is true. Smaller P-values represent stronger evidence against H_0.
5. **Conclusion**
   Report and interpret the P-value in the context of the study. Based on the P-value, make a decision about H_0 (either reject or do not reject H_0) if a decision is needed.

END