

LOGICAL EQUIVALENCE LAWS

1	Reflexivity	$p \equiv p$
2	Double Negation	$\neg(\neg p) \equiv p$
3	Commutativity	$p \wedge q \equiv q \wedge p$
4	Associativity	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
5	Distributivity	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
6	Idempotency	$p \wedge p \equiv p$
7	Identity	$p \wedge T \equiv p$
8	Inverse	$p \wedge \neg p \equiv F$
9	Dominance	$p \wedge F \equiv F$
10	Absorption	$p \wedge (p \vee q) \equiv p$
11	De Morgan's Laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
12	Contrapositive Law	$p \rightarrow q \equiv \neg q \rightarrow \neg p$
13	Material Implication	$p \rightarrow q \equiv \neg p \vee q$
14	Material Equivalence	$p \leftrightarrow \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

IDEMPOTENCY and IDENTITY - proving what 'p' is

INVERSE and DOMINANCE - proving the value of the different manipulations of 'p'

<u>IDENTITY</u>			<u>INVERSE</u>			<u>DOMINANCE</u>	
P	$P \wedge T$	$P \vee F$	$\neg P$	$P \wedge \neg P$	$P \vee \neg P$	$P \wedge F$	$P \vee T$
T	T	T	F	F	T	F	T
F	F	F	T	F	T	F	T

$$P \wedge (P \vee Q) = (P \wedge P) \vee (P \wedge Q)$$

$$P \vee (P \wedge Q) = (P \vee P) \wedge (P \vee Q)$$

$$P \wedge (P \vee Q) = (P \wedge P) \vee (P \wedge Q) =$$

P	Q	$P \vee Q$	$P \vee (P \wedge Q)$	$P \wedge (P \vee Q)$			
T	T	T	T	T			
T	F	T	T	T			
F	T	T	F	F			
F	F	F	F	F			

$$\{(P \vee \neg P) \vee [(P \vee \neg P) \wedge (M \text{ xor } Q)]\} \vee (p \vee T) = T$$

$$\{\neg T \vee [\neg T \wedge (M \text{ xor } Q)]\} \vee (p \vee T) \text{ Inverse}$$

$$\neg T \vee (p \vee T) \text{ Absorption}$$

$$\neg T \vee T \text{ Dominance}$$

$$T$$

Prove that $\{(p \vee T) \wedge [(p \vee T) \vee I]\} \wedge (p \wedge \neg p)$ is a contradiction.

$$\{(p \vee T) \wedge [(p \vee T) \vee I]\} \wedge (p \wedge \neg p)$$

$$(p \vee T) \wedge (p \wedge \neg p)$$

$$T \wedge (p \wedge \neg p)$$

$$T \wedge F$$

Abs

Dom

Inv

FALSE

DML:

$$\sim(P \vee Q) = \sim P \wedge \sim Q$$

$$\sim(P \wedge Q) = \sim P \vee \sim Q$$

$$\sim[(P \vee Q) \vee (\sim R \wedge W)] =$$

$$\sim(P \vee Q) \wedge \sim(\sim R \wedge W)$$

$$(\sim P \wedge \sim Q) \wedge (R \vee \sim W)$$

$$(P \vee Q) \rightarrow (R \wedge S)$$

$$\sim(R \wedge S) \rightarrow \sim(P \vee Q) \quad CL$$

$$(\sim R \vee \sim S) \rightarrow (\sim P \wedge \sim Q) \quad DMLS$$

$$\sim(\sim R \vee \sim S) \vee (\sim P \wedge \sim Q) \quad MI$$

$$(\sim \sim R \wedge \sim \sim S) \vee (\sim P \wedge \sim Q)$$

$$(R \wedge S) \vee (\sim P \wedge \sim Q)$$

Argument Form:

$$p \rightarrow (\neg r \rightarrow q)$$

$$\neg(s \wedge \neg p)$$

$$\neg r$$

$$\therefore s \rightarrow q$$

Premises	CNF		
$p \rightarrow (\neg r \rightarrow q)$	$\neg p \vee (\neg r \rightarrow q)$	MI	
	$\neg p \vee (r \vee q)$	MI	
	$\neg p \vee r \vee q$	Ass	(1)
$\neg(s \wedge \neg p)$	$(\neg s \vee \neg \neg p)$	DML	
	$\neg s \vee p$	DN	(2)
$\neg r$	$\neg r$		(3)
Negated Conclusion	CNF		
$\neg(s \rightarrow q)$	$\neg(\neg s \vee q)$	MI	
	$(s \wedge \neg q)$	DML	
	s		(4)
	$\neg q$		(5)

Resolution Table

1	$\neg p \vee r \vee q$	Clause 1
2	$\neg s \vee p$	Clause 2
3	$\neg r$	Clause 3
4	s	Clause 4
5	$\neg q$	Clause 5
6	p	2, 4
7	$r \vee q$	1, 6
8	r	7, 5
9	FALSE	8, 3
		QED

Argument Form:

$$p \vee (q \rightarrow r)$$

$$p \rightarrow s$$

$$(q \rightarrow t) \rightarrow (r \rightarrow u)$$

$$(\neg p \wedge \neg s) \rightarrow (r \rightarrow t)$$

$$\neg s$$

$$\neg u \vee \neg t$$

$$\therefore \neg r \vee \neg q$$

Premises	CNF		
$p \vee (q \rightarrow r)$	$p \vee (\neg q \vee r)$	MI	
	$p \vee \neg q \vee r$	Ass	(1)
$p \rightarrow s$	$\neg p \vee s$	MI	(2)
$(q \rightarrow t) \rightarrow (r \rightarrow u)$	$\neg(\neg q \vee t) \vee (\neg r \vee u)$	MI	
	$(q \wedge \neg t) \vee (\neg r \vee u)$	DML	
	$(\neg r \vee u) \vee (q \wedge \neg t)$	Comm	
	$[(\neg r \vee u) \vee q] \wedge ((\neg r \vee u) \vee \neg t)$	Dist	
	$(\neg r \vee u \vee q) \wedge (\neg r \vee u \vee \neg t)$	Ass	
	$\neg r \vee u \vee q$		(3)
	$\neg r \vee u \vee \neg t$		(4)
$(\neg p \wedge \neg s) \rightarrow (r \rightarrow t)$	$\neg(\neg p \wedge \neg s) \vee (\neg r \vee t)$	MI	
	$(p \vee s) \vee (\neg r \vee t)$	DML	
	$p \vee s \vee \neg r \vee t$	Ass	(5)
$\neg s$	$\neg s$		(6)
$\neg u \vee \neg t$	$\neg u \vee \neg t$		(7)

Negated Conclusion	CNF		
$\neg(\neg r \vee \neg q)$	$\neg \neg r \wedge \neg \neg q$	DML	
	$r \wedge q$	DN	
	r		(8)

	q		(9)
--	-----	--	-----

Resolution Table:

1	$p \vee \neg q \vee r$	Clause 1
2	$\neg p \vee s$	Clause 2
3	$\neg r \vee u \vee q$	Clause 3
4	$\neg r \vee u \vee \neg t$	Clause 4
5	$p \vee s \vee \neg r \vee t$	Clause 5
6	$\neg s$	Clause 6
7	$\neg u \vee \neg t$	Clause 7
8	r	Clause 8
9	q	Clause 9
10	$\neg p$	2, 6
11	$p \vee \neg r \vee t$	5, 6
12	$p \vee t$	8, 11
13	$p \vee r$	1, 9
14	t	10, 12
15	$\neg r \vee u$	4, 14
16	u	8, 15
17	$\neg u$	7, 14
18	FALSE	16, 17
		QED