



Logical Equivalence laws

1. Reflexivity

$$p \equiv p$$

2. Double Negation

$$\neg(\neg p) \equiv p$$

3. De Morgan's Law

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

4. Identity

$$p \vee F \equiv p \quad p \wedge T \equiv p$$

5. Inverse

$$p \wedge \sim p \equiv F \quad p \vee \sim p \equiv T$$

6. Dominance

$$p \wedge F \equiv F \quad p \vee T \equiv T$$

7. Distributivity

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

8. Associativity

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

9. Commutativity

$$p \vee q \equiv q \vee p$$

10. Material Implication

$$p \rightarrow q \equiv \sim p \vee q$$

11. Material Equivalence

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

12. Idempotency

$$p \wedge p \equiv p \quad p \vee p \equiv p$$

13. Absorption

$$p \wedge (p \vee q) \equiv p \quad p \vee (p \wedge q) \equiv p$$

14. Contrapositive law

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Logical Inference Laws

1. Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array} \quad /$$

7. Simplification

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$$

2. Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array} \quad /$$

3. Hypothetical Syllogism

$$\begin{array}{ll} p \wedge q & p \rightarrow q \\ q & q \rightarrow r \\ \hline \therefore p & p \rightarrow r \end{array}$$

4. Modus Ponens

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array} \quad /$$

5. Modus Tollens

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array} \quad /$$

6. Addition

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

Logical Equivalence laws

1 Reflexivity

$$p \equiv p$$

2 Double Negation

$$\sim(\sim p) \equiv p$$

3 Associativity

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

4 Material Interference

$$p \rightarrow q \equiv \sim p \vee q$$

5 Material Equivalence

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

6 Distributivity

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

7 Idempotency

$$p \vee p \equiv p$$

8 Commutativity

$$p \vee q \equiv q \vee p$$

9 De Morgan's Law

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

10 Dominance

11 Inverse

12 Identity

13 Absorption

14

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

1. $[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ MI
2. $[(\sim p \vee q) \wedge (\sim q \vee r)] \rightarrow (p \rightarrow r)$ MI
3. $[(\sim p \vee q) \wedge (\sim q \vee r)] \rightarrow (\sim p \vee r)$ MI
4. $\sim [(\sim p \vee q) \wedge (\sim q \vee r)] \vee (\sim p \vee r)$ MI
5. $[\sim(\sim p \vee q) \vee \sim(\sim q \vee r)] \vee (\sim p \vee r)$ DM
6. $[(\sim(\sim p) \wedge \sim q) \vee \sim(\sim q \vee r)] \vee (\sim p \vee r)$ DM
7. $[(p \wedge q) \vee \sim(\sim q \vee r)] \vee (\sim p \vee r)$ DN
8. $[(p \wedge q) \vee (\sim(\sim q) \wedge r)] \vee (\sim p \vee r)$ DM
9. $[(p \wedge q) \vee (q \wedge \sim r)] \vee (\sim p \vee r)$ DN
10. $(\sim p \vee r) \vee [(p \wedge q) \vee (q \wedge \sim r)]$ Com
11. $(\sim p \vee r) \vee [(p \wedge q) \vee (q \wedge \sim r)]$ Associa
12. $\sim p \vee r \vee (p \wedge q) \vee (q \wedge \sim r)$ Associa
13. $\sim p \vee (p \wedge q) \vee r \vee (q \wedge \sim r)$ Com
14. $[\sim p \vee (p \wedge q)] \vee r \vee (q \wedge \sim r)$ Associa
15. $[\sim p \vee (p \wedge q)] \vee [r \vee (q \wedge \sim r)]$ Associa
16. $[(\sim p \vee p) \wedge (\sim p \wedge q)] \vee [r \vee (q \wedge \sim r)]$ Distri
17. $[(p \vee \sim p) \wedge (\sim p \wedge q)] \vee [r \vee (q \wedge \sim r)]$ Com
18. $[T \wedge (\sim p \wedge q)] \vee [r \vee (q \wedge \sim r)]$ Inverse
19. $[(\sim p \wedge q)] \vee [r \vee (q \wedge \sim r)]$ Identity
20. $(\sim p \wedge q) \vee [r \vee (q \wedge \sim r)]$ Associativity
21. $(\sim p \wedge q) \vee [(r \vee q) \wedge (r \vee \sim r)]$ Distri
22. $(\sim p \wedge q) \vee [(r \vee q) \wedge T]$ Inverse
23. $(\sim p \wedge q) \vee [(r \vee q)]$ Identity
24. $(\sim p \wedge q) \vee (r \vee q)$ Associa
25. $\sim p \vee q \vee (r \vee q)$ Associa
26. $\sim p \vee q \vee r \vee q$ Associa
27. $\sim p \vee r \vee q \vee \sim q$ Com

- 28 $\sim p \vee r \vee (q \vee \sim q)$ Associa
29 $\sim p \vee r \vee T$ Inverse
30 $\sim p \vee (r \vee T)$ Associa
31 $\sim p \vee T$ Dominance
32 TRUE Dominance

QED