Combinatorics

1 Permutation and Combination

1.1 Multinomial Theorem

$$(x_1 + x_2 + ... + x_m)^n = \sum_{k_1+k_2+...+k_m=n;\ k_1,k_2,...,k_m \geq 0} \binom{n}{k_1, k_2, ..., k_m} \prod_{t=1}^{m} x_t^{k_t}$$

where

$$\binom{n}{k_1, k_2, ..., k_m} = \frac{n!}{k_1!k_2!...k_m!}$$

When $m = 2$, we have binomial theorem:

$$(x_1 + x_2)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

$$= \sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!} x^k$$

$$= \sum_{k=0}^{n} \binom{n}{k,\ n-k} x^k$$

In python, we can use the following code to calculate the multinomial coefficient:

```python
from math import factorial, prod
def multinomial(arr):
    return (
        factorial(sum(arr)) //
        prod([factorial(num) for num in arr])
    )
```

1.2 Lucas Theorem