Matrix Calculus

1 Basics

1.1 Trace Concepts

$$\operatorname{tr}(A) = \sum_{i=1}^{n} a_{ii}$$

If A is n x m, B is m x n, then

$$AB = [\sum_{k=1}^{m} a_{ik} b_{kj}]$$
$$\operatorname{tr}(AB) = \sum_{i=1}^{n} \sum_{k=1}^{m} a_{ik} b_{ki}$$

If A is m x n, B is m x n, then

$$A^T B = [\sum_{k=1}^{m} a_{ki} b_{kj}]$$
$$\operatorname{tr}(A^T B) = \sum_{i=1}^{n} \sum_{k=1}^{m} a_{ki} b_{ki}$$

1.2 Trace Properties

If AB and BA are both defined, then A^T and B have the same shape, and $\operatorname{tr}(AB) = \operatorname{tr}(BA)$, which is equal to $\sum_{i,j} a_{ij} b_{ji}$.

Element-wise product (Hadamard product): $tr(A^T(B \odot C)) = tr((A \odot B)^T B)$