Abstract Algebra

1 Preliminaries

1.1 Set Theory

1.1.1 Function & Map

Concepts:

$$A \xrightarrow{f} B$$

It denotes a function f from A to B and the value of f at a is denoted at $f(a)$. The set A is called the domain of f and B is called the codomain of f.

$$f(A) = \{b \in B \,|\, b = f(a), for\ a \in A\}$$

The set $f(A)$ is a subset of B, called the range or image of f.

$$f^{-1}(C) = \{a \in A \,|\, f(a) \in C\}$$

For each subset C of B, the set $f^{-1}(C)$ consisting elements of A mapping into C under f is called the preimage or inverse image of C. For each $b \in B$, the preimage of $\{b\}$ under f is called the fiber of b over f.

$$(g \circ f)(a) = g(f(a))$$

if $f : A \to B$ and $g : B \to C$, then the composite map $g \circ f : A \to C$ is defined by above equation.

$$f : A \to B$$

- f is injective or is an injection if whenever $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$
- f is surjective or is a surjection if for all $b \in B$ there is some $a \in A$ such that $f(a) = b$, i.e., the image of f is all of B.
- f is bijective or is a bijection if it is both injective and surjective. If such a bijection f exists from A to B, then we say A and B are in bijective correspondence.
- f has a left inverse if there is a function $g : B \to A$ such that $g \circ f : A \to A$ is the identity map on A, i.e., $(g \circ f)(a) = a$, for all $a \in A$.
- f has a right inverse if there is a function $g : B \to A$ such that $f \circ g : B \to B$ is the identity map on B, i.e., $(f \circ g)(b) = b$, for all $b \in B$.

Proposition: