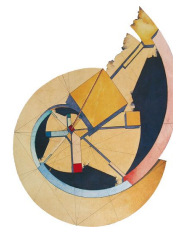


UNIVERSITÀ
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Università degli Studi di Padova
Scuola Galileiana di Studi Superiori

Classe di Scienze Naturali

Tesi di Diploma Galileiano

Towards an implementation in LambdaProlog of the two level Minimalist Foundation

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Contents

A	General Predicates	2
B	Dependent Products	8
C	Propositional Equalities	13

A General Predicates

```
1 %%— depends products: setPi
  type setPi mttType →
3     (mttTerm → mttType) →
      mttType.
5 type lambda mttType →
      (mttTerm → mttTerm) →
7       mttTerm.
  type app mttTerm →
9       mttTerm →
      mttTerm.
11
12 %%— propositional conjunction: and
13 type and mttType →
      mttType →
15       mttType.
  type pair_and mttType →
17       mttType →
      mttTerm →
19       mttTerm →
      mttTerm.
21 type p1_and, p2_and mttTerm → mttTerm.
23
24 %%— dependent sums: setSigma
25
26 type setSigma mttType →
27     (mttTerm → mttType) →
      mttType.
29 type pair mttType →
      (mttTerm → mttType) →
31       mttTerm → mttTerm →
      mttTerm.
33 type elim_setSigma mttTerm →
      (mttTerm → mttType) →
35       (mttTerm → mttTerm → mttTerm) →
      mttTerm.
37
38 %%— existential quantifier: exist
39 type exist mttType →
      (mttTerm → mttType) →
41       mttType.
  type pair_exist mttType →
43       (mttTerm → mttType) →
      mttTerm →
45       mttTerm →
      mttTerm.
```

```

47 type elim_exist mttTerm →
49   mttType →
51   (mttTerm → mttTerm → mttTerm) →
   mttTerm.

53 %%— universal quantifier: forall
   type forall mttType →
55   (mttTerm → mttType) →
   mttType.
57 type forall_lam mttType →
   (mttTerm → mttTerm) →
59   mttTerm.
   type forall_app mttTerm →
61   mttTerm →
   mttTerm.

63 %%— intensional propositional equality: propId
65 type propId mttType →
   mttTerm →
67   mttTerm →
   mttType.
69 type id mttType →
   mttTerm →
71   mttTerm.
   type elim_id mttTerm →
73   (mttTerm → mttTerm → mttType) →
   (mttTerm → mttTerm) →
75   mttTerm.

77 %%— implication: implies
   type implies mttType →
79   mttType →
   mttType.
81 type impl_lam mttType →
   (mttTerm → mttTerm) →
83   mttTerm.
   type impl_app mttTerm →
85   mttTerm →
   mttTerm.

87 %%— local definitions: letIn
89 type letIn mttType
   → mttTerm
91   → (mttTerm → mttTerm)
   → mttTerm
93   .

95 %%— propositional disjunction: or

```

```

type or mttType → mttType → mttType.
97 type inl_or , inr_or mttType
    → mttType
99    → mttTerm
    → mttTerm
101 .
type elim_or mttType
103   → mttTerm
    → (mttTerm → mttTerm)
105   → (mttTerm → mttTerm)
    → mttTerm
107 .

%%— disjoint sum: setSum
type setSum mttType → mttType → mttType.
111 type inl , inr mttType →
    mttType →
113    mttTerm →
    mttTerm.
115 type elim_setSum (mttTerm → mttType) →
    mttTerm →
117    (mttTerm → mttTerm) →
    (mttTerm → mttTerm) →
119    mttTerm.

%%— singleton set/unit type: singleton
121 type singleton mttType.
123 type star mttTerm.
type elim_singleton mttTerm →
125   (mttTerm → mttType) →
    mttTerm → mttTerm.
127

/* UNITYPED COMPUTATIONAL PREDICATES */
129 type hstep , dconv , hnf , conv , interp A → A → prop.

131 /*MIT PREDICATES*/
kind mttTerm , mttType , mttKind , mttLevel type.
133 type ext , int mttLevel.
type col , set , propc , props mttKind.
135

137 type locDecl mttTerm → mttType → prop.
type locDeclType mttType → mttKind → prop.
139 type ofType mttType → mttKind → mttLevel → prop.
type of , isa mttTerm → mttType → mttLevel → prop.
141
type locDef mttTerm → mttType → mttTerm → prop.
143
type forall mttType →

```

```

145     (mttTerm → mttType) →
      mttType.
147
148 hnf A B ⊢ hstep A C, !, hnf C B.
149 hnf A A.
150
151 conv A A ⊢ !.
152 conv A B ⊢ (locDecl _ (propEq _ A B) ).
153 conv A B
      ⊢ spy(hnf A A')
154     , spy(hnf B B')
155     , spy(dconv A' B')
156     .
157
158 dconv A A ⊢ !.
159
160 pts_leq A A.
161 pts_leq props set.
162 pts_leq props col.
163 pts_leq props propc.
164 pts_leq set col.
165 pts_leq propc col.
166
167
168 pts_prop props props props ⊢ !.
169 pts_prop _ _ propc.
170
171 pts_fun A B set
172     ⊢ spy(pts_leq A set)
173     , spy(pts_leq B set)
174     , !
175     .
176 pts_fun _ _ col.
177
178 pts_for A props props ⊢ pts_leq A set, !.
179 pts_for _ _ propc.
180
181 %ofType A KIND IE ⊢ locDeclType A KIND.
182
183 isaType Type Kind IE
184     ⊢ spy(ofType Type Kind' IE)
185     , spy(pts_leq Kind' Kind)
186     .
187
188 of (fixMe2 M T ) T int
189     ⊢ !
190     , print "|<| Found a FixMe! |>|"
191     , print M
192     , term_to_string T S, print S
193

```

```

195      .
196
197 isa (fixMe M) T int
198   ⊢ !
199   , print "|<| Found a FixMe! |>|"
200   , print M
201   , term_to_string T S, print S
202   .
203
204 isa Term TY IE
205   ⊢ spy(of Term TY' IE)
206   , spy(conv TY' TY)
207   .
208
209 of X Y _ ⊢ locDecl X Y .
210
211
212 tau_proof_eq A A T H'
213   ⊢ interp_isa A T Ai
214   , setoid_refl T H
215   , H' = H Ai
216   .
217
218 tau_proof_eq A B T Hi
219   ⊢ spy(locDecl H (propEq T' A B)), !
220   , spy(interp_isa H (propEq T A B) Hi)
221   .
222
223 tau_proof_eq A B T Hi
224   ⊢ (locDecl H (propEq T' B A))
225   , spy(interp_isa H (propEq T B A) Hi')
226   , spy (setoid_symm T Q)
227   , Hi = Q Hi'
228   .
229
230
231
232
233 tau A A (x \ x) ⊢ !.
234
235 tau_trasp A A (x\y\h\ h) ⊢ !.
236
237
238 %interpret X:_ext T in un Xi di tipi Ti
239 interp_isa X T Xi
240   ⊢ spy(of X T_inf ext)
241   , spy(interp X Xi')
242   , spy(tau T_inf T F)

```

```

243     ,   spy(Xi = F Xi')
244     .
245
246 locDecl (k_propId Te)
247   (forall T t1\
248     forall T t1'\
249     implies (E t1 t1')
250     (forall T t2\ forall T t2'\
251       implies (E t2 t2')
252       (implies (E t1 t2)
253         (E t1' t2')))))
254
255   ⊢ interp Te T
256   ,   setoid_eq Te E
257   .
258
259 setoid_symm T (x\ fixMe "prova di simmetria").
260
261 macro_tau B B' Q
262   ⊢ spy(setoid_eq B EquB)
263   ,   spy(interp B Bi)
264   ,   spy(interp B' Bi')
265   ,   spy(pi x1 \ pi x2 \ pi h\
266     pi x1i\ pi x2i\ pi hi\
267     locDecl x1 B
268     ⇒ locDecl x2 B'
269     ⇒ locDecl x1i Bi
270     ⇒ locDecl x2i Bi'
271     ⇒ interp x1 x1i
272     ⇒ interp x2 x2i
273     ⇒ (locDecl h (propEq B x1 x2))
274     ⇒ (locDecl hi (EquB x1i x2i))
275     ⇒ interp h hi
276     ⇒ spy(Q x1 x2 h x1i x2i hi)
277   )
278   .
279 macro_interp B Q ⊢ macro_tau B B Q.

```

elpi/main.elpi

B Dependent Products

```

2 ofType (setPi B C) KIND3 IE
4   ⊢ (ofType B KIND1 IE)
   , (pi x\ locDecl x B
6     ⇒ (ofType (C x) KIND2 IE))
   , spy(pts_fun KIND1 KIND2 KIND3)
8   .

10 of (lambda B F) (setPi B C) IE
    ⊢ spy (ofType B _ IE)
    , spy (pi x\ locDecl x B ⇒ isa (F x) (C x) IE)
12   .

14 of (app Lam X) (CX) IE
    ⊢ spy (of Lam (setPi B C) IE)
    , spy (isa X B IE)
16   , CX = C X
18   .

20

22 hstep (app LAM Bb) (F Bb)
    ⊢ of LAM (setPi B C) IE
    , (ofType B _ IE)
24   , (isa Bb B IE)
    , hnf LAM (lambda B' F)
26   , conv B B'
    , (pi x\ locDecl x B ⇒ isa (F x) (C x) IE)
28   , (pi x\ locDecl x B ⇒ ofType (C x) _ IE)
30   .

32 dconv (setPi B C) (setPi B' C')
    ⊢ (conv B B')
34   , (pi x\ locDecl x B ⇒ conv (C x) (C' x))
    .

36 dconv (app F X) (app F' X')
    ⊢ (conv F F')
38   , (conv X X')
    .

40

42 dconv (lambda B F) (lambda B' F')
    ⊢ (conv B B')
    , pi x\ locDecl x B ⇒ (conv (F x) (F' x))
44   .

46 interp (setPi B C) T

```

```

48   ⊢ spy(interp B Bi)
    , spy(pi x\ pi xi\ locDecl x B
      ⇒ locDecl xi Bi
50      ⇒ interp x xi
      ⇒ interp (C x) (Ci xi))
52   , spy(setoid_eq B EquB)
    , spy(pi x\ pi xi\ locDecl x B
54      ⇒ locDecl xi Bi
      ⇒ interp x xi
56      ⇒ setoid_eq (C x) (EquC xi))
    , spy(pi x1 \ pi x2 \ pi h\
58      pi x1i\ pi x2i\ pi hi\ locDecl x1 B
      ⇒ locDecl x2 B
60      ⇒ locDecl x1i Bi
      ⇒ locDecl x2i Bi
62      ⇒ interp x1 x1i
      ⇒ interp x2 x2i
64      ⇒ (locDecl h (propEq B x1 x2))
      ⇒ (locDecl hi (EquB x1i x2i))
66      ⇒ interp h hi
      ⇒ spy(tau (C x1) (C x2)
68      (TauC x1i x2i hi)))
    , T = setSigma (setPi Bi Ci) f\
70      (forall (Bi) x1\
      (forall Bi x2\
72      (forall (EquB x1 x2) h\
      (EquC x2
74      (TauC x1 x2 h (app f x1))
      (app f x2))))))
76   .

78
interp (app F X) R
80   ⊢ spy(of F (setPi B C) ext)
    , spy(interp_isa X B Xi)
82   , spy(interp F Fi)
    , spy(of Fi T int)
84   , spy(T = (setSigma PI _))
    , R = (app
86      (elim_setSigma Fi (_\PI) (x\y\y) )
      Xi)
88   .

90
interp (lambda B F) R
92   ⊢ spy(of (lambda B F) (setPi B C) ext)
    , spy(interp (setPi B C)
94      (setSigma (setPi Bi Ci) H ))
    , macro_interp B

```

```

96      ( x\_\_xi\_\_ \interp (F x) (Fi xi))
,   setoid_eq B EquB
98 ,   macro_interp B
      (x1\x2\h\x1i\x2i\hi\
100      tau_proof_eq (F x1) (F x2) (C x2)
      (K_EQU x1i x2i hi))
102 ,   R = pair (setPi Bi Ci) (H) (lambda Bi Fi)
      (forall_lam Bi x1\
104      forall_lam Bi x2\
      forall_lam (EquB x1 x2) h\
106      K_EQU x1 x2 h)
.

108

110 setoid_eq (setPi B C) P
  ⊢ spy(interp B Bi)
112 ,   spy(pi x\ pi xi\ locDecl x B
      ⇒ locDecl xi Bi
114      ⇒ interp x xi
      ⇒ interp (C x) (Ci xi))
116 ,   spy(pi x\ pi xi\locDecl x B
      ⇒ locDecl xi Bi
118      ⇒ interp x xi
      ⇒ (interp (C x) (Ci xi)
120      , setoid_eq (C x) (EquC xi)))
,   P = (f\ g\
122      forall Bi x\
      EquC x
124      (app (elim_setSigma f
      (\setPi Bi Ci) (x\y\ x) ) x)
126      (app (elim_setSigma g
      (\setPi Bi Ci) (x\y\ x) ) x))
128 .

130 tau_proof_eq (app F X1) (app F X2) T H
  ⊢ of F (setPi B T') ext
132 ,   spy(tau_proof_eq X1 X2 B G)
,   spy(interp F Fi)
134 ,   spy(of Fi (setSigma TyF MorF) int)
,   P11 = (c\ elim_setSigma c
136      (\ TyF) (x \ y\ x))
,   P2Fi = elim_setSigma Fi
138      (c \ MorF (P11 c))
      (x\ y\ y)
140 ,   spy(interp_isa X1 B X1i)
,   spy(interp_isa X2 B X2i)
142 ,   spy(tau
      (propEq (T' X2) (app F X1) (app F X2))
144      (propEq (T) (app F X1) (app F X2)))

```

```

146     TAU)
147   , H = TAU
148     (forall_app
149     (forall_app
150     (forall_app P2Fi X1i) X2i) G)
151   .
152 tau (setPi B C) (setPi B' C') P
153   ⊢ spy(interp (setPi B C) (setSigma T1 T2))
154   , spy(T1 = setPi Bi Ci)
155   , spy(interp (setPi B' C') (setSigma T1' T2'))
156   , spy(T1' = setPi Bi' Ci')
157   , spy(setoid_eq B' EquB')
158   , spy(macro_interp B
159         (_\x2\_ \_ \x2i\_ \_
160         setoid_eq (C x2) (EquC x2i)))
161   , spy(tau B' B FB)
162   , spy(macro_tau B B'
163         (x\x' \_ \xi \xi' \_ \_
164         tau (C x) (C' x') (FC' xi xi'))))
165   , spy(macro_interp B (x1\x2\_ \_ \xi \xi' \_ \_
166         tau (C x1) (C x2) (FCC x1i x2i)))
167   , spy(tau_trasp B' B KB)
168   , spy(macro_tau B B' x\x' \_ \xi \xi' \_ \_
169         tau_trasp (C x) (C' x') (KC' xi xi' hi))
170   , P = (w\ elim_setSigma w
171         (_\setSigma T1' T2') f\p\
172         pair T1' T2'
173         (lambda Bi' x\ FC' (FB x) x (app f (FB x)))
174         (forall_lam Bi' y1' \
175         forall_lam Bi' y2' \
176         forall_lam (EquB' y1' y2') d' \
177         KC' (FB y2')
178         y2'
179         d'
180         (FCC (FB y1') (FB y2') (app f (FB y1'))))
181         (app f (FB y2'))
182         (forall_app
183         (forall_app
184         (forall_app p (FB y1'))
185         (FB y2'))
186         (KB y1' y2' d')))))
187   .
188 tau_trasp (setPi B C) (setPi B' C') P
189   ⊢ spy(macro_tau B B' x\x' \_ \xi \xi' \_ \_
190         tau_trasp (C x) (C' x') (KC' xi xi' hi))
191   , spy(tau B' B FB)
192   , P = f\g\d\ forall_lam B' y' \

```

```

194      KC' (FB y' )
195      y'
196      d
197      (app f (FB y' ))
198      (app g (FB y' ))
199      (forall_app d (FB y' ))
200
.
```

elpi/calc/setPi.elpi

C Propositional Equalities

Extensional

```
2 %% calc_Eq.elpi
4
6 type propEq mttType → mttTerm →
   mttTerm → mttType.
8 type eq mttType →
   mttTerm → mttTerm.
10 pts_eq K props ⊢ pts_leq K set, !.
   pts_eq _ propc.
12
14 ofType (propEq A AA1 AA2) KIND ext
   ⊢ ofType A KIND' ext
   , pts_eq KIND' KIND
   , isa AA1 A ext
   , isa AA2 A ext
18 .
20
22 of (eq C Cc) (propEq C Cc Cc) ext
   ⊢ spy(of Cc C ext)
   .
24
26
28 %dstep A B ⊢ of _ ()
30
32 dconv (propEq A AA1 AA2)
   (propEq A' AA1' AA2')
   ⊢ spy(conv A A')
   , spy(conv AA1 AA1')
   , spy (conv AA2 AA2')
34 .
36
38 dconv (eq A AA) (eq A' AA')
   ⊢ conv A A'
   , conv AA AA'
   .
40
42 interp (propEq A Aa1 Aa2) R
   ⊢ spy(setoid_eq A EquA)
   , spy(interp_isa Aa1 A Aa1')
   , spy(interp_isa Aa2 A Aa2')
44 , spy(R = (EquA Aa1' Aa2'))
```

```

46  .
48  interp (eq A Aa) T
    ⊢ spy (setoid_refl A ReflA)
50    , spy (interp Aa Aa')
    , T = (ReflA Aa')
52  .

54  setoid_refl (propEq _ _ _)
    (id singleton star).
56  setoid_eq (propEq A Aa1 Aa2)
    (_\ _\ (propId singleton star star)).
58
60  tau (propEq T_ T1 T2) (propEq T T1' T2') (F)
    ⊢ spy (tau_proof_eq T1 T1' T F1)
    , spy (tau_proof_eq T2 T2' T F2)
62    , spy (interp_isa T1 T T1i)
    , spy (interp_isa T2 T T2i)
64    , spy (interp_isa T1' T T1i')
    , spy (interp_isa T2' T T2i')
66    , spy (interp T Ti)
    , F = x\ impl_app (
68      impl_app (
69        forall_app (
70          forall_app (
71            impl_app (
72              forall_app (
73                forall_app (k_propId T)
74                  T1i)
75                T1i')
76              F1)
77            T2i)
78            T2i')
79          F2) x
80  .
82  tau_trasp (propEq _ _ _ )
    (propEq _ _ _ )
    (h\h'\k\ k).
84
86  tau_proof_eq _ _ (propEq T A B)
    (id singleton star).

```

elpi/calc/propEq.elpi

Intensional

```

2 ofType (propId A AA1 AA2) KIND IE

```

```

4      ⊢ isa AA1 A int
      ,   isa AA2 A int
      ,   ofType A KIND1 int
6      ,   (spy(pts_leq KIND1 set , KIND = props), !
          ;   KIND = propc)
8      .

10 of (id A AA) (propId A AA AA) int
    ⊢ ofType A _ int
12    ,   isa AA A int
    .

14 of (elim_id P C CC) (C AA1 AA2) int
16   ⊢ (of P (propId A AA1 AA2) int)
    ,   (pi x\ pi y\ locDecl x A
18         ⇒ locDecl y A
         ⇒ isaType (C x y) propc int)
20   ,   (pi x\ locDecl x A
         ⇒ of (CC x) (C x x) int)
22   .

24 hstep (elim_id (id A AA) C CC) (CC AA)
    ⊢ (isa AA A int)
26    ,   (pi x\ pi y\ locDecl x A
         ⇒ locDecl y A
28         ⇒ isaType (C x y) propc int)
    ,   (pi x\ locDecl x A
30         ⇒ of (CC x) (C x x) int)
    .

32 dconv (id A AA) (id A' AA')
34   ⊢ (conv A A')
    ,   (conv AA AA')
36   .

38 dconv (propId A AA1 AA2) (propId A' AA1' AA2')
    ⊢ spy (conv A A')
40    ,   spy (conv AA1 AA1')
    ,   spy (conv AA2 AA2')
42    .

```

elpi/calc/propId.elpi