TWO-STREAM EFFECTS IN COHERENT BEAM-BEAM OSCILLATIONS IN VEPP-2000 COLLIDER NEAR THE LINEAR COUPLING RESONANCE

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Abstract

Synchro-betatron motion of colliding bunches may cause limitations of the high-luminosity performance. For a round beam collider operated near the linear coupling resonance, we present theoretical predictions of the beam-beam coherent synchro-betatron oscillation behavior under the influence of x-y coupling.

INTRODUCTION

Collective instability is an important problem in colliders where it can put a limit on its desired high-luminosity performance. Curing an instability is based on understanding its physical mechanism. A quantitative theory predicting the instability threshold and increment dependency on the machine parameters is very helpful in practice.

Available theories of the beam-beam coherent motion involve synchro-betatron motion and two-stream interaction of colliding bunches, however without the betatron coupling. In round-beam colliders such as VEPP-2000 [1] the betatron coupling resonances are inherent in the collider optics, that is why treatment of *coupled* coherent beam-beam oscillations is necessary.

In earlier studies [2-5] the two-stream interaction of colliding bunches was successfully included in their synchro-betatron motion, the theory similar to the TMCI was constructed including the effect of the lattice chromaticity and the transverse impedance of the machine.

In the present study we introduce the betatron coupling in the theoretical formalism and apply the circulant matrix approach. Such a technique exploits an essentially linear (macroparticle) construction in the time domain where the beam-beam kick is linearized and transport of synchro-betatron modes of the bunch motion is efficiently done with the circulant matrices. The full set of coherent mode tunes and waveforms are found from the one-turn map spectrum. Neither x-y coupling nor proximity of the tunes cause any problem in this formalism.

Similar coherent beam-beam problems were studied elsewhere, with application of circulants as one of the methods [6].

CIRCULANT EQUATIONS

The model employed here is not based on real macroparticles which move longitudinally while executing synchrotron oscillations. Instead, it considers the hollow beam model for the bunch and the synchrotron phase circle is divided into N fixed equal boxes with *fixed* longitudinal position in the bunch, see Fig. 1. A variable dipole moment d_i is ascribed to each ith box, $i = 1, \ldots, N$. The synchrotron oscillation just transports the dipole moment around the circle, across the fixed boxes, and we follow variations of d_i in each box: no need to interchange the boxes when evaluating their interaction.

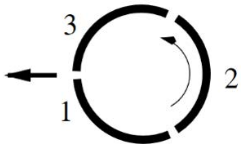

Figure 1: 3-boxes example of the synchrotron phase division.

The dipole moments obey the betatron oscillation equation in the Courant-Snyder normalization, with the interaction in the RHS:

$$\ddot{d}_i + \omega_b^2 d_i = -2\,\omega_b D_{ik}\, d_k\,. \tag{1}$$

Here ω_b is the betatron frequency, the machine azimuth is used as quasi-time, and the dynamic matrix D_{ik} approximates the linear integral operator of collective interaction between the boxes.

First of all we use the ansatz $d_i = a_i e^{-i\omega_b t}$ + c.c. (to introduce phasors a_i) and standard averaging to get rid of a_i^* in the shortened equations:

$$i\,\dot{a}_i = D_{ik}\, a_k\,. \tag{2}$$

N complex amplitudes a_i of oscillating dipole moments sitting in each box, form a complete set of dynamic variables of the averaged problem.

In fact, a_i are sampled values of the continuous function of the synchrotron phase φ: $a_i(t) = a(\varphi_i, t)$, its total time derivative reads:

$$\dot{a}(\varphi, t) = \frac{\partial a}{\partial t} + \dot{\varphi}\frac{\partial a}{\partial \varphi} = \frac{\partial a}{\partial t} + \omega_s\frac{\partial a}{\partial \varphi}, \tag{3}$$

here the synchrotron frequency $\omega_s = \dot{\varphi}$ is introduced.

The LHS of (2) involves *two terms*: $\partial a(\varphi, t)/\partial t \to \dot{a}_i(t)$, and a difference analog instead of $\partial a(\varphi, t)/\partial\varphi$, in (3).

The N-point approximation of differentiation operators for sampled periodic functions is given by the special circulant matrices γ_{ik}, see [3] for details.

Using this finite difference approximation $\partial a(\varphi, t)/\partial\varphi \to \gamma_{ik} a_k$, we rewrite the LHS of (2):

$$i\,(\dot{a}_i + \omega_s \gamma_{ik} a_k) = D_{ik}\, a_k\,. \tag{4}$$

We define another circulant $C = -i\omega_s\gamma$, and finally come to new equations:

$$i\,\dot{a}_i = (C_{ik} + D_{ik})\, a_k\,. \tag{5}$$

With division into 3 boxes as an example, the circulant reads:

$$C_3 = \frac{i\,\omega_s}{\sqrt{3}} \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}. \tag{6}$$

The synchrobetatron mode spectrum emerging from circulant equations should first be examined for **free oscillation**, $D_{ik} = 0$. We start with $N = 3$, we have the boxes numbered with $i, k = 1, 2, 3$ in Fig. 1a, and accordingly define the proper mode numbers m as -1, 0, $+1$.

Substituting for the proper modes $a_i(t) = v_i e^{-i\Omega t}$ in $i\,\dot{a}_i = (C_{ik} + W_{ik} + D_{ik})\,a_k$, with $D_{ik} = 0$, where we added $W = \mathrm{diag}\{\omega_\beta, \omega_\beta, \omega_\beta\}$, we have: $C_{ik} v_k = \Omega v_i$, and find the mode frequencies Ω and eigenvectors v from the eigensystem of C_3:

$$\begin{aligned} \Omega_0 &= \omega_\beta, & v_0^T &= (\ 1,\quad 1,\quad 1\ ); \\ \Omega_{+1,-1} &= \omega_\beta \pm \omega_s, & v_{+1,-1}^T &= (1,\ e^{\pm 2\pi i/3},\ e^{\pm 4\pi i/3}) \end{aligned} \tag{7}$$

The mode eigenvectors v should give sampling of the Fourier harmonics $e^{im\varphi}$, with $m = +1, 0, -1$, at 3 equidistant values of φ_k according to division into 3 boxes.

Betatron Coupling

For VEPP-2000 betatron coupling caused by skew quadrupoles is harmful because it breaks the round-beam condition, normally it is eliminated. However, non-compensated rotation from the final-focus solenoids is compatible with the round-beam collision concept and is present in the machine optics for tuning the main coupling resonance strength. Its phasor representation is given by the x-y coupling matrix for the phasors

$$\begin{pmatrix} 0 & iL \\ -iL & 0 \end{pmatrix}$$

Beam-Beam Kick

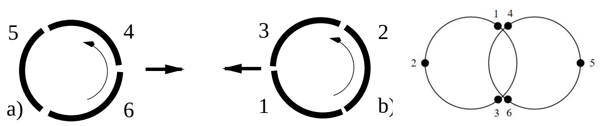

Figure 2: Schematic of the colliding bunches.

The transport matrices are doubled in dimension to map the two colliding bunches. Their interaction is represented by the linearized beam-beam kick matrix for phasors, according to Fig. 2,

$$B = \begin{pmatrix} 3 & 0 & 0 & -1 & -1 & -1 \\ 0 & 3 & 0 & -1 & -1 & -1 \\ 0 & 0 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & 0 & 0 \\ -1 & -1 & -1 & 0 & 3 & 0 \\ -1 & -1 & -1 & 0 & 0 & 3 \end{pmatrix} \frac{\xi}{3} k,$$

where ξ is the beam-beam parameter and k is the appropriate constant.

Synchrobetatron Beam-Beam Modes

The synchrobetatron mode spectrum is available from eigenvalues of $C + W + B$ as functions of the beam-beam parameter. It looks trivial for zero-length colliding bunches. However, even in the phasor technique we can introduce the bunch-length effect $\sim l_{\mathrm{bunch}}/\beta^*$ in the form of the betatron phase lag proportional to the particle's longitudinal position. We modify the phases of phasors accordingly to their numbers before the beam-beam kick and recover the phases after the kick. This is our model of the two-stream mechanism of the beam-beam interaction. In Figure 3 for symmetric round

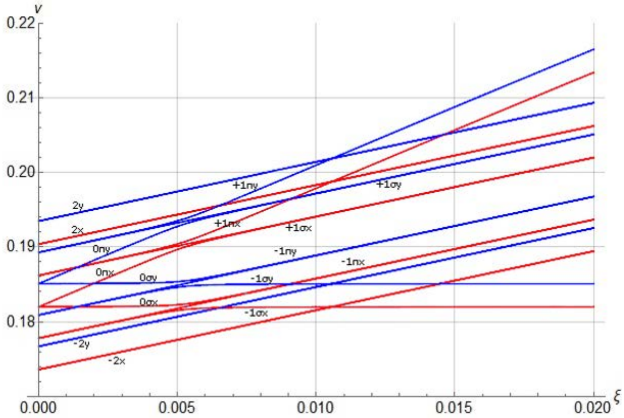

Figure 3: Synchrobetatron mode spectrum vs the beam-beam parameter for round beams.

bunch intensities, one can see repulsion of the mode tunes, no instability occurs in the two-stream beam-beam dynamic system unless transverse impedance elements are added.

The new feature of betatron coupling which we introduced here does not modify the mode-interaction scenario: the beam-beam modes only couple with own betatron family, the lines originating from another betatron mode are simply crossed.

This situation changes for non-round collision, see Fig. 4 where different betatron families do couple.

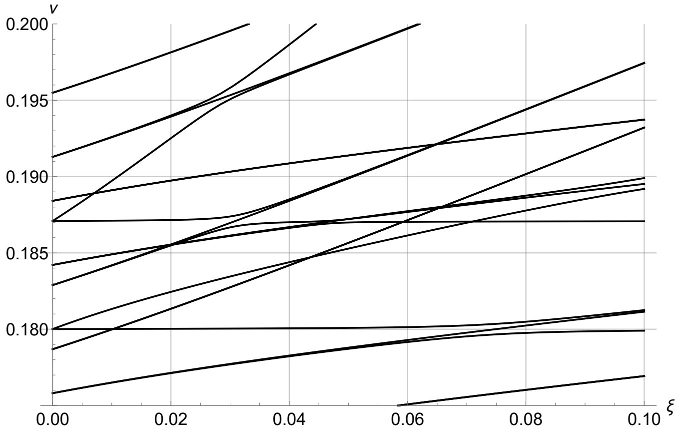

Figure 4: Synchrobetatron mode spectrum vs the beam-beam parameter for non-round beams.

For detailed analysis of localized elements we need to return from phasors to complete equations for Courant-Snyder normalized betatron variables (x, p_x, y, p_y). To this end matrix exponents of the circulants will provide for the transport matrices, and the eigensystem of the full-turn mapping will give the beam-beam modes' spectrum and waveform.

TRACKING

To confirm the predictions of the circulant equation theory, a simple code was written for tracking simulation of the coherent beam-beam oscillations with the finite bunch length.

A hollow-beam distribution in the synchrotron oscillation phase plane was populated with a large number of particles equidistant in the synchrotron phase, with an initial betatron amplitude. After one revolution these were rotated by the synchrotron oscillation phase advance. And the betatron variables (including x-y coupling) were transformed by their one-turn matrix.

The two-stream interaction of colliding bunches is a more complicated story. Prior the beam-beam transformation, the particles of the both colliding bunches were deployed into the finite-lengths bunches, with re-ordering at each turn w.r.t. their new longitudinal coordinates. Next the bunches were pulled against each other and the transverse beam-beam kick was applied to each pair of the particles from opposite bunches at their pre-computed encounter point. Between the kicks the particles perform a drift with the new momentum each time. After completion, the longitudinal positions are transformed back into the IP's transverse positions, the bunches' lengths are contracted at the IP.

The synchrotron mode spectrum is found from the Fourier transform of the bunch dipole moment history. A special algorithm is developed for automated search and identification of small spectral peaks, see Fig. 5. An agreement with the spectra from the circulant equations is very good, see Fig. 6.

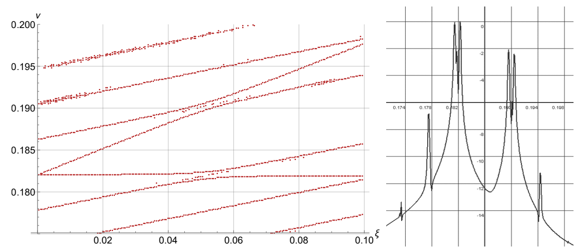

Figure 5: Left: Part "x" of the mode spectra from tracking with VEPP-2000 parameters:
$\nu_x = 0.18, \nu_y = 0.1851, \nu_s = 0.0042, l/\beta^* = 0.35$.
Right: Example of the Fourier transform at $\xi = 0.05$.

CONCLUSIONS

In the round-beam collider VEPP-2000, the x-y betatron tunes lie in the band of the main coupling resonance, and their separation is small. This is a difficulty for conventional perturbative approach.

On the other hand, a time-domain approach using circulanat matrices can be applied for constructing a full set of

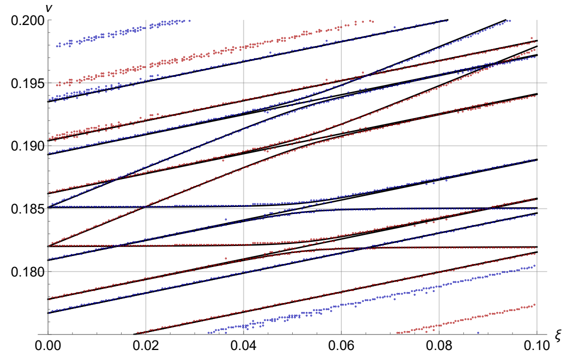

Figure 6: Spectra from circulant equation and from tracking compared.

the synchrobetatron modes of colliding bunches which interact in a two-stream manner due to finite lengths of the colliding bunches. The beam-beam kick is linearized, and the mode-set spectrum is obtained from the one-turn map, to conclude on the stability of the beam-beam coherent oscillations. Neither x-y coupling nor closeness of the tunes cause any problem in this formalism.

In this paper we studied the coherent beam-beam oscillations using the circulant-matrix approach. We have made theoretical predictions on synchrobetatron mode spectrum vs. the beam-beam parameter for round-beam colliders. We assumed the operating point located near the main coupling resonance (e.g. for e+e- collider VEPP-2000).

The results have been compared with a simple macroparticle simulation and have shown a good agreement.

The wakefields if taken into account have been shown to cause instability. However, the wake effects are small compared with beam-beam effects and do not have to be worried about.

The obtained results can help one to better understand the role of coherent oscillations in the beam-beam limit problem in colliders.

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