Extra Practice Questions for Math 200 Final Exam

1. Give a function (a formula, not a graph) that is discontinuous at $x = 2$, but continuous everywhere else. Explain your reasoning.

2. Sketch the graph of a function with domain $(-\infty, 5) \cup (5, \infty)$,
   that is continuous on $(-\infty, 1) \cup (1, 5) \cup (5, \infty)$,
   and differentiable on $(-\infty, -3) \cup (-3, 1) \cup (1, 5) \cup (5, \infty)$.

3. For $f(x) = \dfrac{x+2}{x-1}$, find $f'(x)$ using the limit definition of the derivative.

4. Find both the derivative and integral of $f(x) = \dfrac{x^4 - 3x + \pi^2 x}{x}$

5. Find both the derivative and integral of $g(x) = x(\sqrt{x} - 2)$

6. Find both the derivative and integral of $h(x) = (x+2)^2$

7. Find both the derivative and integral of $P(t) = 3e^t - 5t^2$

8. Find an equation of the tangent line to $y = 3x^2 + 10x - 2$ that is parallel to $2y - 8x = 4$.

9. Evaluate the following limits analytically:

   (a) $\displaystyle\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)}$

   (b) $\displaystyle\lim_{x \to \infty} \frac{e^{2x}}{x^2}$

   (c) $\displaystyle\lim_{x \to -\infty} \frac{e^{3x}}{x^3}$

   (d) $\displaystyle\lim_{x \to 0^+} x \ln x$

   (e) $\displaystyle\lim_{x \to 0} \frac{\tan x - x}{x^3}$

   (f) $\displaystyle\lim_{\theta \to 0} \frac{\sin\theta - \sin\theta\cos\theta}{\theta^2}$

10. A 6 meter ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 1.5 m/sec, how fast will the bottom of the ladder be moving away from the wall when the top of the ladder is 4m above the ground?

11. A cistern used to catch rain water has the shape of a cone. The diameter at the top is three times the depth of the cistern. The cistern is 2 meters deep. On a rainy day water is collecting at a rate of 20 Liters per minute (0.02 m^3/min), but being drained out the bottom at the same time. If the depth of the water is increasing at a rate of 0.001 meters per minute when the water level is 1.5 meters deep, find the rate at which water is draining out the bottom.

12. A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 80ft, find the dimensions of the window so that the greatest possible amount of light is admitted. (Maximize the area of the window).

13. A metal can is to be constructed with a volume of 100 cubic inches. The can is a right circular cylinder. If the cost of the metal for the sides of the can is 4 cents per square inch, and the cost of the top and bottom is 5 cents per square inch, find the cost of the cheapest such can.

14. For $g(x) = \sec(\arctan(x))$, find $g'(x)$

15. For $h(x) = (\sin x)^{\ln x}$, find $h'(x)$

16. Find the absolute max and absolute min of $f(x) = \dfrac{\ln(2x)}{x}$ on the interval $[1, e]$.

17. Find $\frac{dy}{dx}$ by implicit differentiation for $e^{\left(\frac{x}{y}\right)} = \sqrt{x} + \sqrt{y}$

18. Find the equation of the tangent line to $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at $(3, 1)$.

19. Use a Riemann sum with right-hand endpoints to evaluate $\displaystyle\int_1^3 (3x - x^2)\, dx$.

20. Evaluate the limit by writing it as an integral: $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \sec\left(\frac{\pi i}{4n}\right) \tan\left(\frac{\pi i}{4n}\right)$ on the interval $\left[0, \frac{\pi}{4}\right]$.

21. For a particle moving along an s-axis, the velocity is given by $v(t) = t - \sqrt{t}$.

    (a) Find the displacement and distance traveled by the particle on the time interval $[0, 4]$.

    (b) Find when the particle is speeding up and slowing down.

22. A model rocket is launched vertically from the ground. For the first 3 seconds of its flight, the rocket motor propels it up with an acceleration of $50 m/s^2$. The acceleration due to gravity is $9.8 m/s^2$.

    (a) How long will it take the rocket to reach its maximum height?

    (b) What is the maximum height?

    (c) How long will it take for the rocket to hit the ground?

    (d) What will be its speed at impact?

23. Find the value of k so that the average value of $f(x) = \sqrt{2x}$ over the interval $[0, k]$ is 4.

24. Evaluate $\displaystyle\int_1^1 \frac{1}{\sqrt{x}\sqrt{4-x}}\, dx$

25. Evaluate $\displaystyle\int_{-1}^4 \frac{x}{\sqrt{5+x}}\, dx$

26. Evaluate $\displaystyle\int \frac{4}{x(\ln x + 1)^2}\, dx$

27. Evaluate the following derivatives using the FTC:

    (a) $\dfrac{d}{dx} \displaystyle\int_2^{x^2+1} \sin t\, dt$ (b) $\dfrac{d}{dx} \displaystyle\int_x^{\pi/2} e^t \cos t\, dt$ (c) $\dfrac{d}{dx} \displaystyle\int_{x^2}^{x^3} \ln(t^2+1)\, dt$