Exercise: The Vertical Fault – Part 1

Linear inversion through Tikhonov regularization

A vertical fault separates two areas of the subsurface. The first area is a homogeneous quarter space (to the left of the fault in Figure 1) with everywhere the same density $\rho = 2600\ kg/m^3$. On the other side of the fault the density varies with depth z (positive downwards). Points where the gravity is measured are indicated by triangles in Figure 1.

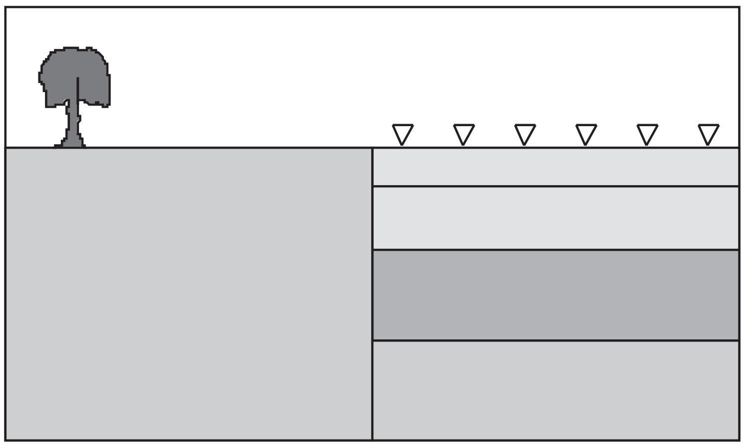

Figure 1: Outline of a vertical fault model.

The right column in the files "gravdata.txt" $(n = 1, 2, 3, \dots)$ contains the observed data, $\mathbf{d}_{obs}$, which is the horizontal gravity gradient (in units of s^{-2}) measured at 18 points along a linear, horizontal profile, starting at the fault, on which it is perpendicular. The left column contains the corresponding x-coordinates in units of km for the 18 observation points. It is assumed that the vertical fault is located at $x = 0\ km$.

If $\Delta\rho(z)$ is the vertical density variation in the quarter space on the right, minus the constant density in the quarter space on the left, the horizontal gravity gradient, as a function of x, is theoretically given by

$$d_j = \frac{\partial g}{\partial x}(x_j) = \int_0^\infty \frac{2Gz}{x_j^2 + z^2} \Delta\rho(z) dz$$

where G is the Gravity constant and x_j is the horizontal coordinate of the j'th observation, measured from the fault plane.

The inverse problem

We wish to estimate $\Delta\rho(z)$ from observations of the horizontal gravity gradient along the x-axis. The first step of the analysis is a discretization of the problem.

- Perform such a discretization by representing the subsurface to the right of the fault by 100 horizontal layers of thickness 1 km each. The 100 layers are resting on an infinite quarter space of density 2600 kg/m^3 everywhere. It can be shown that the contribution to data from a homogeneous "half layer" (one of the layers in the layer series to the right of the fault, after the discretization) is

$$G\Delta\rho \log\left(\frac{z_{base}^2 + x^2}{z_{top}^2 + x^2}\right),$$

where z_{top} is the depth to the top of the homogeneous half layer, z_{base} is the depth to the base of the layer, $\Delta\rho$ is the density contrast (to the material to the left of the half layer), and x is the horizontal distance to the edge of the half layer.

- Show that the problem is linear (and hence can be written $\mathbf{d} = \mathbf{G}\mathbf{m}$).
- Is the solution to this problem unique? Why/why not?
- Data is measured with an uncertainty of $\pm 1.0 \cdot 10^{-9} s^{-2}$. Find a solution $\mathbf{m}_\epsilon$ to the problem using *Tikhonov Regularization*:

$$\mathbf{m}_\epsilon = [\mathbf{G}^T\mathbf{G} + \epsilon^2\mathbf{I}]^{-1}\mathbf{G}^T\mathbf{d}_{obs}. \tag{1}$$

The regularization parameter ϵ should be chosen such that the solution becomes physically acceptable, gives the best possible resolution of the model, and at the same time fits the data "within its uncertainties".