Self-similar decomposition of digital signals *

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Abstract— **The behaviour of a physical quantity in time is described by signals. Traditionally, signals are analysed in the time domain either as time - amplitude relationship or in the frequency domain as frequency - signal power dependency. Both traditional representations have substantial limitations. New algorithms for signal representation and processing are required in order to give some additional useful information about observed processes. The present paper proposes a self-similar decomposition of digital signals, which gives rise to a multiscale description, preserving all features of the signals. The proposed description does not depend on predefined basis functions like sine waves, basic wavelets, etc. Instead, the newly proposed approach looks for self-similar associations of signal segments. The proposed signal description can be considered as an attempt to combine signal representation in time domain with signal representation in frequency domain**

Keywords—self-similarity, digital signal decomposition

I. INTRODUCTION

A signal can be defined as a detectable physical (i.e. finite) quantity that carries information. In practice, most of the signals are measured as functions of time. The acquisition of the finite physical signals is naturally related to the properties of the measurement apparatus. As an idealization, these properties are described by a transfer function, some delay of processing and integration or averaging. The resulting output signal can be regarded as a convolution of the physical signal and the measurement process function. On the other hand, the measurement is always contaminated by an unwanted signal, which is denoted broadly as "noise". The noise process is usually generated by different sources through the signal acquisition (measurement) process or corresponds to the influence of diverse external or internal factors on the studied physical process. In many occasions because of its perceived irregularity in time, it can be treated as a purely random process.

Signal analysis aims to collect, extract and understand the information carried by signals. Signal analysis processes signals in time, frequency and time-frequency domains. In the time domain, signal analysis aims to detect different components, shapes or features by filtering input signals in order to amplify and isolate useful signal characteristics or to remove some unwanted artifacts within the signals (i.e. noise). However, this representation is not the best one for some applications. Sometimes, the most interesting information is hidden in the frequency content of the signal. Thanks to the Fourier's representation theorem [1], applied spectrum analysis was developed for all types of signals. Fourier representation completely discards temporal coincidence of events, related to the signal of interest. This means that if a part of the signal has a specific form (and the corresponding to it frequency band) it does not matter where this part of the signal is - at the beginning of the signal, in the middle or somewhere at the end of the signal. From the theorem of decomposition of each signal to a sum of sine-wave periodic functions, the essential features in each signal can be represented by a band-limited spectrum. The Nyquist - Kotelnikov theorem states that for the full representation of a particular signal, the sampling rate must be no less than twice the frequency of the composite in the highest frequency band. Spectrum analysis today is realized by Fast Fourier Transform (FFT) algorithm [2]. FFT is computationally efficient and provides acceptable results for all discrete signals of finite length, but it has some fundamental limitations. The first limitation consists of narrow frequency resolution (the ability to distinguish the spectral lines of two or more signals), restricted by the sampling time interval (approximately reciprocal of it). Therefore, the frequency resolution does not depend on the type of the signal being analyzed at all. The second limitation is due to the calculation of the FFT and it appears in the frequency domain as a "leakage" of the energy of the main lobe of the spectral band to the side lobes. It results in distortion of the other spectral bands. Specific choices for the window function may reduce the leakage at the expense of lower frequency resolution [3]. These limitations are especially manifested in the case of short signals. Spectrum analysis cannot cope at all with discontinuous signals (i.e. with infinite slope at certain times). It needs an infinitely large number of terms in order to present these signals correctly. The spectrum of a signal depicts frequency properties over the entire interval due to integration. It does not give any information about instant (local) frequency characteristics of the signal.

Besides periodic sinusoidal functions, functions with local support have also been used to describe signals. In this case, the signal is decomposed on a set of wavelets. The key problem of wavelet analysis is the choice of a basis function. Only the knowledge of the properties of the observed signal in advance can help us with a proper choice of wavelets. However, wavelets give some frequency information in time by rescaling of the mother wavelet and representation of signals in time-frequency domain. For this space of signal representation, the Heisenberg uncertainty principle is in force - more selective wavelets have narrower compact support (less selective in time).

The different presentations of signals listed above are not sufficiently descriptive in many cases. Finding "useful" information in a large amount of raw data is a serious problem,

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which can be often alleviated by partitioning of the signal. The segmentation process requires a rule for border points determination. The signal presentation in segments should be postulated in advance or in a more general form - to have a model of the signal behavior [4]. The signal information can be detected also in signal statistics, signal shapes, signal autocorrelation, signal coherence or something else [5].

The texture of a signal is another important signal characteristic. Signal texture describes regions with certain regular pattern or repetitive structure. Patterns may be defined in advance or adaptively chosen from analyzed signal.

The approach proposed in the present paper is closer to the structural texture analysis, performing detection and localization of texture elements (primitives) with repeating patterns. Furthermore, an algorithm for special segmentation of the observed signal is formulated. The input signal is initially partitioned in segments, which are compared for similarity. If distinct sets of consecutive segments are similar, the segments from the sets are merged together in longer ones. The segmentation procedure does not need any a priori defined patterns. What is more, non-continuous signals do not disrupt the algorithm. Due to this fact, the newly proposed self-similar decomposition of signals is especially suitable for digitally encoded signals, which are widely spread at present. They can be presented in a very compressed form.

The rest of the paper is organized as follows: The problem formulation, similarity estimation and segmentation are given in Section 2. The suggested algorithm is presented in Section 3. Section 4 contains some experimental results. The final Section 5 provides conclusions and discussion.

II. Self-similar decomposition (SSD) of signals

The proposed signal decomposition is an attempt to combine signal representation in the time domain with a representation in the frequency domain. The main idea is to use some basic signal shapes for signal representation. It is expected that they are repetitive within the observed signal. This idea is not new. There are two different approaches for signal description using the properties of shapes. The first one is simpler. It uses a set of a priori defined signal shapes or dictionaries [6, 7, 8]. Each fragment of signal is compared with this set and classified according to its similarity to the corresponding pattern. The second approach comes from fractal analysis where a signal is described using its self similarity. Usually fractal analysis is applied in order to estimate the degree of repetition of a given shape at different scales. The application in the context of the second approach is limited due to the small number of self similar (fractal) signals in practice. The approach proposed here may be regarded as an extension of this approach.

The place of SSD algorithm in general scheme of signal processing is shown on Fig.1.

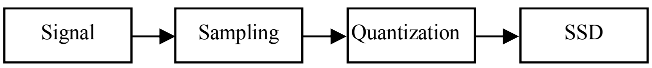


Figure 1. General scheme of signal processing with SSD algorithm

Many segmentation techniques are described in literature [5, 7, 8, 9]. The most popular between them are similarity-based thresholding, histogram-based thresholding and template matching. The last two presume signal model assumptions that strongly restrict flexibility of the chosen approach. The proposed algorithm attempts to group measurements into connected regions, based on similarity measures.

A. SSD algorithm description

The structure of SSD algorithm is shown on Fig.2.

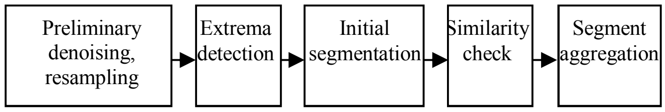


Figure 2. Structure of SSD algorithm

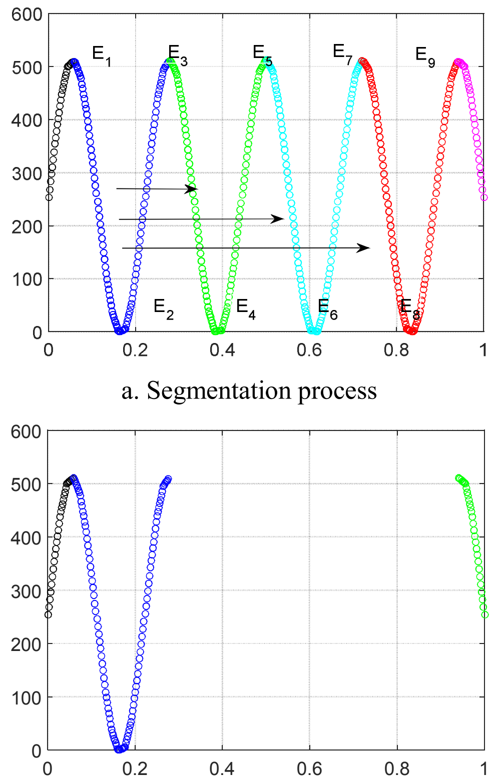
The first step of the algorithm is preliminary denoising and resampling, if necessary. On the next step of SSD algorithm the extrema are detected. The accuracy of signal extrema localization plays an important role for quality of segmentation. The extrema detection algorithm identifies all local peaks and troughs of the observed signal. The found signal extrema define the borders of segments. The segmentation, third step of the SSD algorithm, consists of breaking down input signal into non-overlapping parts. Every segment is enclosed between one maximum and one minimum or vice versa - between one minimum and one maximum. From this definition of the segmentation procedure it is clear that every segment of the signal is a monotone function. The remaining two partially unfinished edge parts of the signal - the first one at the signal onset, before the first found peak and the last one - after the last peak are also monotone onset functions. These two parts of signal are defined as original segments and will not be explored further for similarity. All segments preserve their position in the signal. The arrangement of all segments onto their positions will reconstruct the signal.

The initial set of segments, determined on the third step of the algorithm, is the input information for the next step of the SSD algorithm - similarity check. It appears to be the most sensitive element of the proposed signal decomposition algorithm. In the SSD algorithm the segments have to be compared quantitatively, allowing them to differ by scale factor (in both dimensions - amplitude and time). There are many classical signal processing operations that may be regarded as measures of similarity, such as subtracting signals, matched filtering, auto correlation, cross correlation, beam formation, etc. [10].

The last step of SSD algorithm consists of segments aggregation. The found similar segments are processed further and if there are found doubles, triples, quadruples,..., n-tuples of similar segments they are aggregated.

B. An example of SSD algorithm on sinusoidal signal

In the example below the most popular signal, i.e. sinusoidal, will be used in order to demonstrate how the SSD algorithm is performed. The input signal is shown on Fig.3. The first step concerning denoising and resampling is omitted due to the usage of synthetic signal. The algorithm begins with localization of all tops and bottoms. The peaks determine the segments. Initially, a set $\mathbf{M_1}$ is defined, consisting of all possible segments, restricted by any two neighboring extrema: $\mathbf{M_1} = \{(E_1,E_2), (E_2,E_3), ..., (E_7,E_8), (E_8,E_9)\}$. Here n denotes the number of found signal extrema and E_i is the i-th found extremum. The index of $\mathbf{M}$ denotes the depth level of recursion procedure, i.e. $\mathbf{M_1}$ denotes the considered set on the first level of recursion procedure. In our example the segmentation process starts with the first segment, defined by the first two extrema points E_1 and E_2. They constitute the segment (E_1,E_2). The algorithm finds all extrema points - E_1,...,E_9, then it defines the first segment (E_1,E_2) and looks for similar to (E_1,E_2) segments. The algorithm finds that there are four similar segments - the segments (E_1,E_2), (E_3,E_4), (E_5,E_6) and (E_7,E_8) (Fig.3a). If the set consists of only one element, the search for similar segments continues with the next (the second) segment (E_2,E_3). If the number of similar segments is greater than 1, all segments in the set $\mathbf{M_1}$, found as similar, are marked as used (for our example - (E_1,E_2), (E_3,E_4), (E_5,E_6) and (E_7,E_8)). Then the next set $\mathbf{M_2}$ is formed, containing the segments, following the corresponding segments from those in the set $\mathbf{M_1}$. In the considered example, these elements are (E_2,E_3), (E_4,E_5), (E_6,E_7) and (E_8,E_9). The set $\mathbf{M_2}$ is the input set for the next step of the recursive procedure. The formation of $\mathbf{M_2}$ = {(E_2,E_3), (E_4,E_5), (E_6,E_7), (E_8,E_9)} finishes the first pass of recursive procedure. The output result from the first pass is the found similarity between (E_1,E_2), (E_3,E_4), (E_5,E_6) and (E_7,E_8).



a. Segmentation process

b. Three primitive shapes describe the signal

Figure 3. An example for SSD algorithm for sinusoidal function

The second pass of the recursive procedure in depth repeats the same procedure for the new set $\mathbf{M_2}$. Let us consider the second pass in detail. The first segment of the updated set $\mathbf{M_2}$ is segment (E_2,E_3). The algorithm looks for segments, similar to the chosen segment (E_2,E_3), in the set $\mathbf{M_2}$. If there are no similar segments, the segment (E_2,E_3) is marked as a new original shape**.** If some similar segments are found (in our case there are three similar segments (E_4,E_5), (E_6,E_7) and (E_8,E_9)), a merging of disjoint segments is committed. The result is four new shapes - (E_1,E_3) and similar to it (E_3,E_5), (E_5,E_7) and (E_7,E_9). The next pass follows the same logic. The signal decomposition produces three primitive shapes, displayed on Fig. 3b.

The recursive procedure is organized as follows. The recursion goes in depth for the every set of similar shapes until no more connected similar segments are detected. Horizontally, the processing is executed until the considered set $\mathbf{M_i}$ becomes empty.

C. Similarity measures

Similarity estimation should guarantee correct results even for segments on different scales. The signal scalability may be considered in time and in amplitude. There are several significant differences between the definition of the problem given above and the well explored problem of dependence estimation between sequences. In both cases, correlation analysis will be applied. But the correlation analysis, in principle, can detect linear functional dependency. The mutual information approach [11] and its generalizations like dual total correlation [12], excess entropy [13], binding information [14], deal with linear and nonlinear dependencies. These methods are referred to as multi-moment correlation measures. Another way to find a more complex dependence is to consider a copula [15] between them. The requirement for similarity estimation up to scale coefficient limits our investigation only to correlation coefficient and relationships into the class of simple linear regression. The slope of the linear regression line will determine the scale coefficient. It is obvious that the scale coefficient is sensitive to the sampling procedure and the measurement noise.

Let us denote the compared signal segments with $s_1(t)$, $t \in [t_{b1}, t_{e1}]$ and $s_2(t)$, $t \in [t_{b2}, t_{e2}]$. The samplings of s_1 and s_2 may differ due to phase shift or different time scale. To alleviate this discrepancy the proposed algorithm applies new resampling. Two types of resampling could be applied.

If the time scalability is not considered, the resampling should use uniform equal time intervals for the considered segments. In this case the similarity analysis can be vastly accelerated by using the number of sampling points. Different numbers simply demonstrate the dissimilarity of these signals. However, if the number of sampling points of both segments is equal, a criterion for similarity calculation has to be applied.

When the time scale is considered, it can be calculated through the ratio $m_{t_{1,2}} = \frac{t_{e1} - t_{b1}}{t_{e2} - t_{b2}}$ of the compared segments. The resampling procedure generates equal number of sampling points for the observed segments. After resampling, the segments are examined for similarity.

As a consequence from the statements above, it can be seen that all compared segments have the same number of points. Let us denote the points, describing the signal in segment $s_1(t)$ as $s_{1_1}, s_{1_2}, \dots, s_{1_N}$, and the points of the signal in segment $s_2(t)$ as $s_{2_1}, s_{2_2}, \dots, s_{2_N}$.

Correlation analysis is a very popular tool for similarity detection. It estimates the closeness of normalized functions. It means, that equal signals in different amplitude scale will be found as similar. A value of correlation coefficient equal to 1 indicates a perfect coincidence between the compared segments. If the correlation coefficient is 0, no correlation at all could be found. What is more, correlation analysis will discover symmetric signals (negative correlation). In this case, the correlation coefficient is equal to -1. Four types of correlations may be used - Pearson, Spearman, point-biserial and Kendall rank correlation but the Pearson correlation criterion is preferred to be used for normally distributed $s_{1_1}, s_{1_2}, \dots, s_{1_N}$ and $s_{2_1}, s_{2_2}, \dots, s_{2_N}$.

The Pearson correlation coefficient is calculated as:

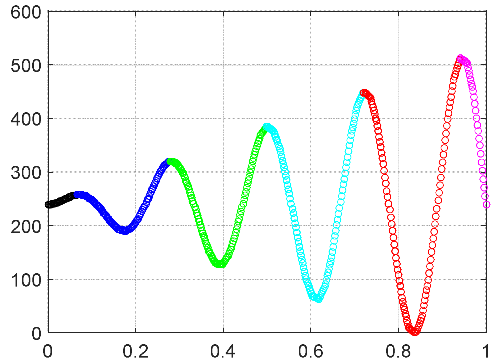
$$R = \frac{N\sum_{i=1}^{N} s_{1_i}s_{2_i} - \left(\sum_{i=1}^{N} s_{1_i}\right)\left(\sum_{i=1}^{N} s_{2_i}\right)}{\sqrt{\left[N\sum_{i=1}^{N} s_{1_i}^2 - \left(\sum_{i=1}^{N} s_{1_i}\right)^2\right]\left[N\sum_{i=1}^{N} s_{2_i}^2 - \left(\sum_{i=1}^{N} s_{2_i}\right)^2\right]}}$$

The biggest Pearson correlation criterion downside is that $R = 0$ does not imply independence. The implementation of Pearson approach for similarity estimation avoids this drawback because similarity estimation does not concern with the independence of data, but in similarity only.

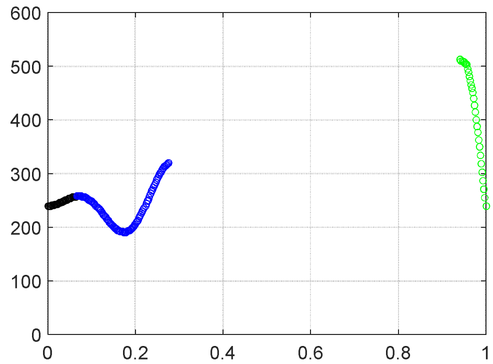
III. Experimental Results

The SSD Algorithm has been applied to several typical signals to evaluate its performance and to demonstrate the main advantages of the proposed method. The noise is not considered (or it is supposed that optimal low-pass filter is applied, assuring high signal to noise ratio).

In the first example SSD decomposes a sinusoidal function with linearly increasing amplitude (Fig. 4). The input signal is displayed on Fig. 4a, where the compared shapes are depicted with different colours. There are four similar segments. Despite the linearly increasing signal amplitude, the algorithm successfully detects their similarity and stores only the shapes, depicted on Fig. 4b. The applied SSD algorithm determines 3 primitive shapes to fully describe this signal. The algorithm demonstrates good multiscale properties.

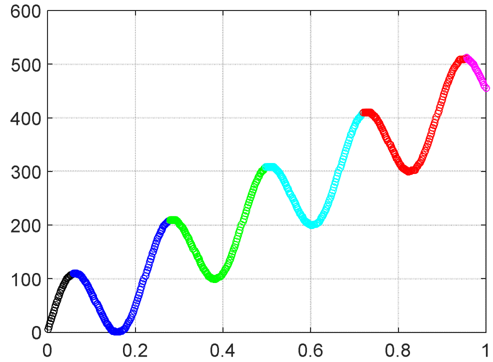


a. Segmentation

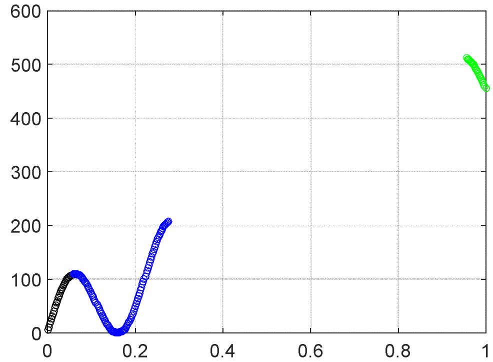


b. Four primitive shapes describe the signal

Figure 4. SSD algorithm for sinusoidal function with linearly increasing amplitude

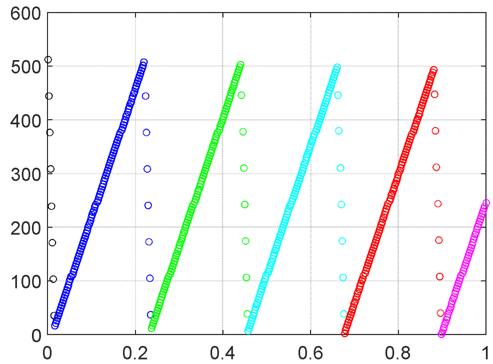


a. Segmentation

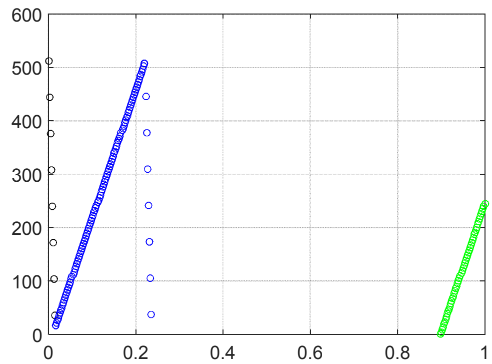


b. Three primitive shapes describe the signal

Figure 5. SSD algorithm for sinusoidal with linear trend

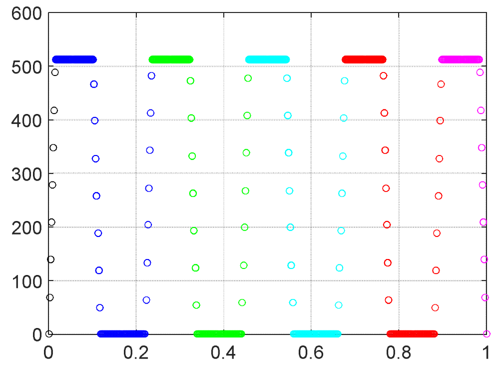


a. Segmentation

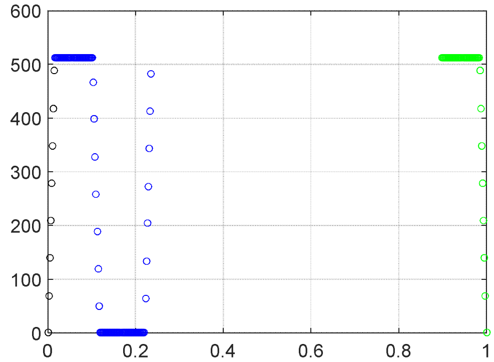


b. Three primitive shapes describe the signal

Figure 6. SSD algorithm for sawtooth signal

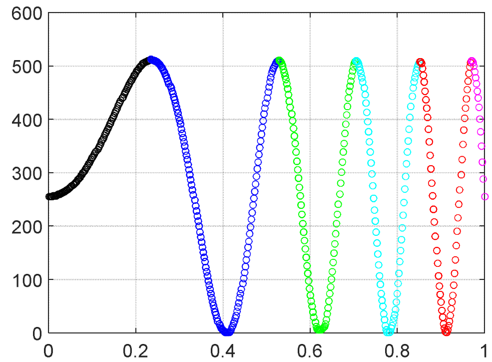


a. Segmentation

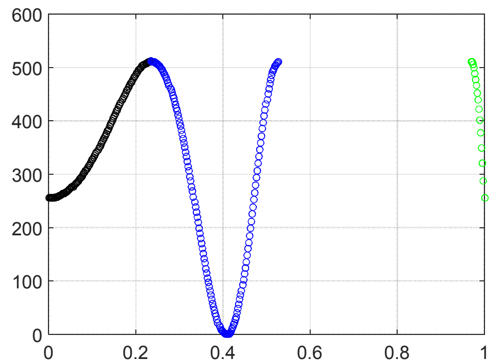


b. Three primitive shapes, describing function

Figure 7. SSD algorithm for rectangular pulse train

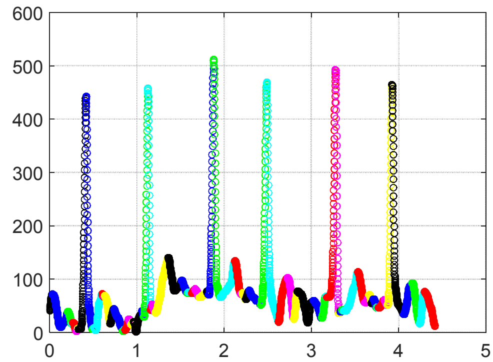


a. Segmentation

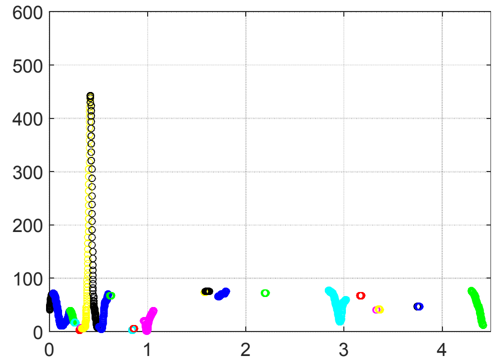


b. Four primitive shapes, describing function

Figure 8. SSD algorithm for sinusoidal function with time scale



a. Segmentation



b. Primitive shapes, describing ECG signal

Figure 9. SSD algorithm for ECG signal

On Fig. 5 a sinusoidal signal with linear trend is depicted. The SSD algorithm is not influenced by the trend and decomposes the signal into three primitives (Fig. 5b).

The following two examples demonstrate the properties of the SSD algorithm to decompose digital signals with infinite slopes. For these signals, a linear interpolation is applied. On Fig. 6 a sawtooth signal is processed. The signal is decomposed on only three primitives. The rectangular pulse train from Fig. 7 is also decomposed to three primitives only. The examples above demonstrate significantly better results in signal decomposition compared to the well-known FFT. Fourier decomposition of a simple digital pulse train characterises with unlimited spectrum of the fundamental frequency and odd harmonics.

The test signal on Fig. 8 is scaled in time. The SSD algorithm deals successfully with the time scaled signal and decomposes it in three primitive shapes. Fig. 9 shows a SSD decomposition of fluctuating and noisy repetitive signals like ECG.

IV. Discussion

The experimental results demonstrate the ability of the proposed SSD algorithm to deal with highly repetitive artificially generated signals. The SSD operates successfully with digital signals, which are problematic for FFT and wavelet transform. Its multiscale property allows to process amplitude scales signals, time scaled signals or even both simultaneously. This feature leads to a very compact description of the corresponding signals. The SSD algorithm does not need a priori preset patterns. It adapts to the particular signal patterns within the signal and looks for the most repetitive ones. The proposed algorithm, however, is noise sensitive. When the noise exceeds a certain level, the procedure of extrema detection/localization gives many false extrema. Something more, the extrema positioning fails too. Incorrect results from the segmentation process lead to poor signal decomposition. The denoising of the input signal is a prerequisite for the SSD implementation. It has not been investigated quantitatively in this paper to what extent noise filtering has to be applied. Nevertheless, it is clear that there should not to be false extrema. It is the most important requirement to the observed signal. Even when the false extrema are smoothed, the presence of noise will influence on the similarity estimate. An adaptive choice of similarity threshold as a function of SNR will resolve the problem.

The computation complexity of the proposed algorithm is determined mostly by the filtering procedure. There are two variants to realize the filtration. The filtering in time domain is usually expressed as $O(n^2)$ real multiplications and $O(n^2)$ real additions. The order of FFT computation is O(n*log n) [21]. To realize a convolution in frequency domain the FFT has to be applied twice on signal and convolution function (O(n*log n)), then the results are multiplied (O(n)) and transformed back by IFFT (O(n*log n)). The total complexity of FFT realization of filtering algorithm is (O(n*log n)). Wavelet transforms is calculated by passing the signal through a series of filters with a constant length. This diminishes the computational load of Wavelet transform to O(n). In summary, the computation complexity of the proposed SSD is the highest one of the three, if time-domain filtering is applied. Its complexity is comparable with those of FFT, when FFT filtering is used in the SSD algorithm, but both algorithms (SSD and FFT) defers to the Wavelet transform complexity.

V. Conclusion

The paper proposes an approach for local multiscale associative parameterization of a signals. The algorithm does not require prior choice of basis functions like sine waves, basic wavelets, etc. Instead, the newly proposed approach (Self-Similarity Decomposition of Signals) looks for self-similarity associations of signal segments. The proposed signal description can be considered as an attempt to combine signal representation in the time domain with presentation in frequency domain. The algorithm for Self-Similarity Decomposition of Signals was demonstrated on several examples and the results were discussed. The output results can be used for automatic detection, classification, localization, association of highly repetitive signals, signal texture description, compact digital signal presentation, etc.

References

[1] J. Fourier, The Analytical Theory of Heat, Cosimo Inc., New York, 2007, ISBN: 978-1-60206-107-1, originally published in 1822.

[2] J. W. Cooley, J. W. Tukey, "An algorithm for the machine calculation of complex Fourier series", Mathematics of Computation. 19 (90): 297–301, doi:10.1090/S0025-5718-1965-0178586-1, ISSN 0025-5718

[3] V. I. Krivosheew, Contemporary methods for digital signal processing (digital spectral analysis) - in Russian, В.И. Кривошеев, Современные методы цифровой обработки сигналов (цифровой спектральный анализ), Учебно-методический материал по программе повышения квалификации "Современные системы мобильной цифровой связи, проблемы помехозащищенности и защиты информации", Нижний Новгород, 2006.

[4] P. J. Besl, R. C. Jain, Segmentation Through Variable-Order Surface Fitting, IEEE Trans. on PAMI, vol. IO, № 2, March 1988, pp. 167-192.

[5] R.d L. Allen, D. W. Mills, Signal Analysis Time, Frequency, Scale, And Structure, Copyright © 2004 by The Institute of Electrical and Electronics Engineers, Inc., ISBN: 0-471-23441-9.

[6] B. Bhanu, Representation and shape matching of 3-D objects, IEEE Trans. On PAMI, vol. PAMI-6, №. 3, May 1984, pp. 340-350.

[7] M. Hebert and J. Ponce, A new method for segmenting 3-D scenes into primitives, in Proc. 6th Int. Conf. Pattern Recognition, Munich, West Germany, Oct. 19-22, 1982, pp. 836-838.

[8] P. J. Besl, R. C. Jain, Segmentation Through Variable-Order Surface Fitting, IEEE Trans. on PAMI, Vol. IO, № 2, March 1988.

[9] C. Panagiotakis, G. Tziritas, Signal Segmentation and Modelling Based On Equipartition Principle, 16th International Conference on Digital Signal Processing, Print ISBN: 978-1-4244-3297-4, CD-ROM ISBN: 978-1-4244-3298-1, DOI: 10.1109/ICDSP.2009.5201105.

[10] H. L. Kennedy, A New Statistical Measure of Signal Similarity, Conference: Information, Decision and Control, 2007, IDC '07, DOI: 10.1109/IDC.2007.374535.

[11] T. M. Cover, J. A. Thomas, Elements of Information Theory (Wiley ed.), 1991, ISBN:978-0-471-24195-9.

[12] T. S. Han, Nonnegative entropy measures of multivariate symmetric correlations, Information and Control 36, 1978, pp.133–156,.

[13] E. Olbrich, N. Bertschinger, N. Ay, J. Jost, How should complexity scale with system size?, The European Physical Journal B - Condensed Matter and Complex Systems, 2008, doi:10.1140/epjb/e2008-00134-9.

[14] S. A. Abdallah, M. D. Plumbley, A measure of statistical complexity based on predictive information, ArXiv e-prints. arXiv:1012.1890v1, 2010.

[15] R.K.Y. Low, J. Alcock, R. Faff, T. Brailsford, Canonical vine copulas in the context of modern portfolio management: Are they worth it?, Journal of Banking & Finance. 37 (8), 2013, doi:10.1016/j.jbankfin.2013.02.036.

[16] E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons, 1978, ISBN: 0-471-50731-8.

[17] L. P. Chew, K. Kedem, Getting around a lower bound for the minimum Hausdorff distance. Computational Geometry, Theory and Applications, v. 10(3), pp. 197-202.

[18] L. P. Chew, K. Kedem, S. Schirra, On characteristic points and approximate decision algorithms for the minimum Hausdorff distance. Technical report MPI-I-94-150, Max-Planck-Institut für Informatik, Saarbrücken, DE, 1994, 10 pages.

[19] M. P. Dubuisson, A. K. Jain (1994), Modified Hausdorff distance for object matching. Proc. of IAPR Int. Conf. on Pattern Recognition (ICPR'94, Jerusalem, IS), v. A, pp. 566-568.

[20] L. Rabiner, R. Schafer, Theory and Applications of Digital Speech Processing, 1st Edition, ISBN-13: 978-0136034285, ISBN-10: 0136034284.

[21] V. Y. Pan, The trade-off between the additive complexity and the asynchronicity of linear and bilinear algorithms, Information Processing Letters, 22(1), pp.11–14, DOI:10.1016/0020-0190(86)90035-9.