GCSE Mathematics Higher

AQA Specification 8300

Part 4

Chapter Seven: Sequences

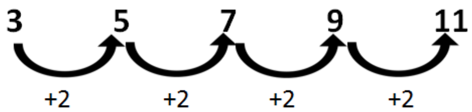
7.1 Linear Sequences

A sequence is a list of numbers, in order, connected by some rule. The individual numbers are called terms which are identified by their position: 1st, 2nd, 3rd etc

Term to Term Rules

Term to term rules are a way of generating sequences by beginning with one term and using a rule to move from this term on to the next one.

Example:
The first term is 3, the rule is “add 2”. Find the first five terms of the sequence.



You can also use term to term rules to find missing numbers in sequences by looking at how the numbers in the sequence relate to one another.

Example:
Fill the missing number in this sequence
2, 4, 8, ___, 32, 64

Looking at the numbers given we can see that each term is doubling the one before therefore the missing number is 16.

Position to Term Rules

Position to term rules are another way of generating sequences. You will be given a formula $T(n)$ for finding the nth term – for example, if you wanted to find the 3rd term then n would be equal to 3. We will cover how to find this formula for yourself in the next section.

Example:
A sequence has the position to term rule $T(n) = 3n + 1$
a. Find the first 4 terms of the sequence.
b. Find the 100th term of the sequence.

a. We need the 1st, 2nd, 3rd and 4th terms so we need to set $n = 1, n = 2, n = 3$ and $n = 4$.
$T(1) = 3 \times 1 + 1 = 4$
$T(2) = 3 \times 2 + 1 = 7$
$T(3) = 3 \times 3 + 1 = 10$
$T(4) = 3 \times 4 + 1 = 13$
So the sequence is 4, 7, 10, 13..

b. To find the 100th term we need $n = 100$
$T(100) = 3 \times 100 + 1 = 301$
So the 100th term is 301

Activity 7.1 A

1. Find the first five terms of the sequence when given the first term and the rule

a. First term: 4	Rule: +5	b. First term: 10	Rule: x2
c. First term: 128	Rule: $\div$ 2	d. First term: 68	Rule: -3
e. First term: $\frac{1}{2}$	Rule: $+\frac{1}{2}$	f. First term: 3.5	Rule: x2
g. First term: 4	Rule: $\div\frac{1}{2}$	h. First term: 3	Rule: -5

2. Find the missing number in each of these sequences

a. 2, 4, 6, 8, __, 12	b. 5, 10, 15, __, 25, 30	c. 90, 80, __, 60, 50, 40
d. 2, 1, 0, __, -2	e. 80, 40, 20, __, 5	f. __, 6, 9, 12, 15
g. 1, -2, 3, __, 5	h. $\frac{1}{2}, \frac{3}{2}$, __, $\frac{7}{2}, \frac{9}{2}$	i. 5, 10, 20, __, 80, 160
j. 4, 2, 1, __, __	k. 1, 3, 9, __, 81	l. __, 100, 1000, ____, 100 000

3. For each of the rules below i) generate the first five terms
ii) find the 20th term *You may use a calculator for g. ii.*

a. $T(n) = 5n$	b. $T(n) = 7n - 1$	c. $T(n) = 8 - n$	d. $T(n) = n^2$
e. $T(n) = \frac{n}{2}$	f. $T(n) = \frac{20}{n}$	g. $T(n) = 2^n$	h. $T(n) = 2(n^2 - n)$

<u>Finding the nth term</u>

Finding the nth term refers to finding the rule, like those used in the last section, that enables us to find the value of the term in any position. The nth term takes the form $T(n) = an + b$ for some values of a and b.

In a linear sequence the coefficient of n is the constant difference between the terms in the sequence. Once this has been determined you compare your value of an with the values in the sequence to find the value of b.

Example:
Find the nth term for the sequence
5, 8, 11, 14, 17, 20...

Here we can see that the terms are increasing by 3 each time so the value of a is 3.
We now compare $3n$ with the terms in the sequence.

$3n$: $3, \ 6, \ 9, \ 12, \ 15, \ 18, \ldots.$

+2

$Sequence$: $5, \ 8, \ 11, \ 14, \ 17, \ 20, \ldots$

The terms in the sequence are all 2 larger than the values of $3n$ so we have
$T(n) = 3n + 2$

Example:
Find the nth term for the sequence
15, 10, 5, 0, -5...

The terms in the sequence are decreasing by 5 so we must compare the value of $-5n$ with the terms in the sequence.

$-5n$: $\quad -5, \ -10, \ -15, \ -20, \ -25, \ldots$
$Sequence$: $\quad 15, \ 10, \ 5, \ 0, \ -5, \ldots$

The terms of the sequence are 20 larger than the values of $-5n$ so we have
$T(n) = -5n + 20$

Activity 7.1 B

1. Find the nth term for each of these sequences
   - a. 3, 5, 7, 9, 11...
   - b. 40, 50, 60, 70, 80...
   - c. 5, 9, 13, 17, 21...
   - d. 19, 18, 17, 16, 15...
   - e. 35, 30, 25, 20, 15...
   - f. -2, -4, -6, -8, -10...
   - g. $1, \ 1\frac{1}{2}, \ 2, \ 2\frac{1}{2}, \ 3 \ldots$
   - h. 2.1, 2.2, 2.3, 2.4, 2.5...
   - i. -12, -10, -8, -6, -4...
   - j. 100, 200, 300, 400, 500...
   - k. 1, 2, 3, 4, 5...
   - l. 3.5, 3, 2.5, 2, 1.5...

2. Find the 100^{th} term for each of these sequences
   - a. 7, 9, 11, 13, 15...
   - b. 33, 30, 27, 24, 21...
   - c. 3, 8, 13, 18, 23...
   - d. 3, 4.5, 6, 7.5, 9...
   - e. 30, 45, 60, 75, 90...
   - f. 5, 3, 1, -1, -3...

3. Does a sequence with the nth term $T(n) = 5n - 2$ contain the term 65?

4. Does a sequence with the nth term $T(n) = 2n + 1$ contain the term 84?

5. Does a sequence with the nth term $T(n) = 3n + 4$ contain the term 37?

6. Write down the nth term of a sequence containing only even numbers.

7. Find the nth term for each of these linear sequences
   - a. 5^{th} term is 34, increases by 3 each time
   - b. 2^{nd} term is 12, increases by 10 each time
   - c. 4^{th} term is 15, decreases by 4 each time

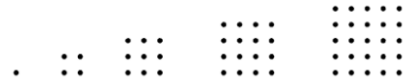
7.2 Special Sequences

In section 7.1 you met **arithmetic sequences** which have a constant difference between each term and **Geometric sequences** which have a constant ratio between terms. This means that you multiply by the same number each time.

For example, the sequence 4, 6, 8, 10... is arithmetic because it is generated by adding 2 each time. The sequence 4, 8, 16, 32.., on the other hand, is geometric because it is generated by multiplying by 2 each time.

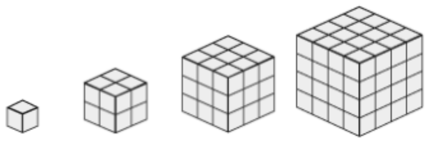
There are also a few special sequences that you should be aware of. You have met the sequence of square numbers previously.

1,   4,   9,   16,   25...



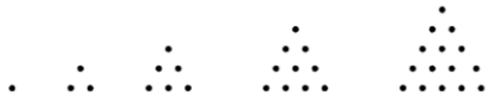
Two other sequences you should be familiar with are
cube numbers and

1,   8,   27,   64,   125 ...



triangular numbers

1,   3,   6,   10,   15 ...



Activity 7.2 A

Find the difference between the first 5 triangular numbers. Using your findings, or otherwise, calculate the first 10 triangular numbers.

The **Fibonacci sequence** is generated by adding the previous two terms together

1, 1, 2, 3, 5, 8, 13...

1+1=2   1+2=3   2+3=5   3+5=8....

A Fibonacci-type sequence is one which uses this rule but does not necessarily begin with 1.

This sequence takes its name from 13th century Italian mathematician Leonardo of Pisa who is thought to be responsible for introducing the Arabic number system, which took the place of Roman Numerals, to Europe. This number system is the one we still use today. In Europe at the time it was thought that he discovered the Fibonacci sequence but it was actually known in India as far back at the 6th century.

Activity 7.2 B

1. Calculate the first ten terms of the Fibonacci sequence.

2. Are these sequences arithmetic or geometric? Find the constant difference or ratio as appropriate
    a. 4, 6, 8, 10, 12...   b. 1, 2, 4, 8, 16...   c. The multiples of 7
    d. 1, -3, 9, -27..   e. 2, $2\sqrt{2}$, $4, 4\sqrt{2}$ ...   f. 10 000, 1000, 100, 10...

3. Generate the first five terms of the geometric sequence with the following first term and multiplier
    a. First term: 3   Multiplier: 2   b. First term: -4 Multiplier: 10

4. Research how the sequence of Fibonacci numbers relates to Pascal's Triangle.

5. Carl Gauss became one of the most influential mathematicians in the world. When he was in primary school his teacher asked the class to find the sum of the numbers from 1 to 100, he quickly found the answer to be 5050. Have a go at working out how he did this.

6. There are a few numbers that occur in both the square number and the cube number sequences. Find one of them.

7. Using a similar diagram to those shown for the square and triangular number sequences find the first five terms in the sequence of pentagonal numbers.

8. Write the number 32 of the sum of no more than four square numbers.

9. Write down the first five terms of each of these sequences

a. $T(n) = \frac{n^2}{n+1}$ b. $T(n) = (0.2)^n$ c. $T(n) = \frac{1}{n-2}$

10. Write down the first five terms of the geometric sequence with the following first term and multiplier

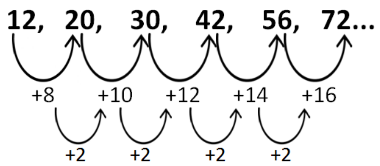
a. First term: 2 Multiplier: x b. First term: $x - 1$ Multiplier: x

c. First term: $x + y$ Multiplier: xy d. First term: x^2 Multiplier: $x + 1$

7.3 Quadratic Sequences

In a **quadratic sequence** the differences between the terms form an arithmetic sequence.

For example 12, 20, 30, 42, 56, 72... forms a quadratic sequence. The differences between the terms are 8, 10, 12, 14, 16.. this forms an arithmetic sequence with a constant difference of 2.



In order to find the nth term of a quadratic sequence you must first find these differences. Half of the second difference becomes the coefficient of n^2. As before this is then compared with the sequence to find if anything needs to be added or subtracted.

If there is a constant difference between the sequence of an^2 (where a is half of the second difference) and the original sequence then this is added on as with linear sequences.
If the difference between an^2 and the original sequence forms an arithmetic sequence you should find the nth term for this and add the rule to your value of an^2.

Example:
Find the nth term of the sequence 12, 20, 30, 42, 56, 72..

We can see from the diagram above that the second difference is +2 so the coefficient of n^2 becomes +1 to give n^2
Now we must compare n^2 with the original sequence

n^2:	1, 4, 9, 16, 25, 36..
Original:	12, 20, 30, 42, 56, 72..
Difference:	+11, +16, +21, +26, +31, +36

So the differences form a sequence of 11, 16, 21, 26, 31, 36...
Finding the nth term of this sequence gives 5n+6

So the nth term of the sequence is $n^2 + 5n + 6$

Activity 7.3

1. Calculate the 20th term in each of these sequences

   a. $T(n) = n^2 + 1$ b. $T(n) = 2n^2 - 3$ c. $T(n) = n^2 - 2n + 1$

2. Find the nth term for each of these sequences

   a. 2, 6, 12, 20... b. 9, 6, 1, -6... c. 4, 7, 12, 19...
   d. -4, -1, 4, 11... e. 2, 8, 18, 32, 50... f. 7, 16, 31, 52, 79...
   g. 4, 12, 24, 40, 60... h. 2, 7, 16, 29, 46... i. 8, 13, 20, 29, 40...

ASSIGNMENT SIX

Answers to these questions are not provided. You should send your work to your tutor for marking. Show all of your working.

1. Solve $\frac{2(x+2)}{5} = 8x$ (4)

2. Ian, John and Fred go on holiday. Ian takes £x, John takes half as much as Ian and Fred takes £150 more than John. They take £2000 between them. Find out how much Fred took. (5)

3. Solve each of these equations
a. $x^2 + 60x + 500 = 0$ b. $6x^3 + 7x^2 - 3x = 0$ c. $x^2 - 2x - 5 = 0$ (8)

4. Use a graph to solve $x^2 + 4x = 0$ (3)

5. Set up and solve a pair of simultaneous equations to find the required values.
Two numbers have a difference of 12. The smaller number plus double the larger number also equals 12. What numbers are they? (4)

6. Find the value of x and y when $4x - y = 2$ and $y = x^2$ (4)

7. Derek has a choice between buying 4 sheep and 10 pigs for £590 or 6 sheep and 5 pigs for £435. Find the cost of one pig. (4)

8. Solve each of these inequalities a. $2(4 - 3x) < 6$ b. $2 < \frac{5x-1}{2} \leq 7$ (5)

9. Use the iterative formula $x_{n+1} = \sqrt[3]{2x_n - 1}$ to find a solution to $x^3 - 2x + 1 = 0$ to 3dp. Start with $x_1 = 0.5$ (3)

10. For the sequences below find the following information:
i. Is it arithmetic or geometric?
ii. The rule explaining how the sequence works
iii. The next two terms
For the arithmetic sequences only:
iv. The nth term
v. The 150th term

a. 34, 37, 40, 43, 46...
b. 896, 448, 224, 112, 56...
c. 1.2, 0.9, 0.6, 0.3, 0... (15)

11. Find the next three terms of the following sequences
a. Arithmetic 2, 4.....
b. Geometric 2, 4.....
c. Fibonacci-type 2, 4..... (3)

12. Find the nth term of the sequence: 2, 3, 6, 11, 18... (3)

Total marks 61

End of Section Revision Quiz

These questions are based upon everything you have covered in the algebra section of the course. It is not compulsory to complete this quiz but it is recommended that you use it in order to make sure you have a firm understanding of all of the topics covered. Answers are provided to these questions – if you're finding a question or group of questions particularly difficult you should go back to that section in your notes and go over it to make sure you understand it fully.
The majority of these questions are exam style questions so you should familiarise yourself with them now.

1. Solve $3x - 11 = 2(5 - x)$

2. Solve $x^2 - 3x - 40 = 0$

3. Expand the brackets $(3x + 2)(5 - 4x)$

4. Solve $\frac{1}{2}(a - 3) = 2a + 1$

5. Find the next four terms in the sequence $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}$ ...

6. The nth term of a sequence is given by $T(n) = 2^n + 2^{n+1}$. Find the first 4 terms and the 20th term.

7. Use the iterative formula $x_{n+1} = \sqrt{3x_n + 2}$ to find a solution to $x^2 - 3x - 2 = 0$ to 2dp. Start with $x_1 = 3.1$

8. Expand $-5d(3d - 4)$

9. Given $f(x) = 2x^2 - 1$ and $g(x) = \frac{x+2}{3}$, find $gf(x)$ and $fg(x)$

10. Mary says that $89^2 - 11^2 = 7800$. Without using a calculator show that she is correct.

11. Sabrina says that "an odd number multiplied by an even number is always odd". Either show that this is always true or find an example to show that it's false.

12. Make x the subject of $2y = \frac{3x}{4} + \frac{x}{2}$

13. Solve $6 - 4x \geq 2x - 3$

14. Use an algebraic method to find the point of intersection of the lines
$4x + 6y = 14$ and $3x - 2y = 23\frac{1}{2}$

15. Solve $3x^2 + 14x + 8 = 0$ by factorising

16. Simplify $4a - (2a + 5) + 3a(2 + b)$

17. Draw the graphs of $y = 5 - x$ and $y = x + 1$.
Use your diagram to solve this pair of simultaneous equations.

18. Factorise $3x^2 - 9x$

19. Find the nth term of the sequence 1, 11, 21, 31 ...

20. Complete the square to solve $2x^2 + 4x = 8x - 1$

21. Given that $x_{n+1} = \frac{x_n^4 - 2}{5}$ and $x_1 = 1$, find x_4

22. Tanya says that "the sum of two even numbers is always even". Either show that this is always true or find an example to show that it's false.

23. A book costs £b, a DVD costs £d and a CD costs £c. What do each of the following statements mean in words?

a. $c = 9$ b. $b + d + c = 24$ c. $b = \frac{1}{2}d$ d. $c + d = 19$

e. Find the value of c, b and d

24. The area of a trapezium is given by $A = \frac{(a+b)\times h}{2}$. Find the value of A when $a = 2x^2, b = 3x^2$ and $h = 4x$

25. Expand and simplify $(2x + 5)(3 - x)$

26. x and y are integers. If $x > 30$ and $y < 20$ work out the smallest possible value of $x - y$

27. Factorise $6x^2 - 5x - 4$

28. Make v the subject of $s = \frac{v^2}{123}$

29. Sarah is trying to work out the values of x for which $3x - x^3 - 2 = 0$. She says $x = 1$ and -1. Is she correct?

30. The nth term of a sequence is $2n + 1$, the nth term of a different sequence is $3n - 1$. Work out two numbers that are in both sequences and are between 20 and 40.

31. Harvey says that "the sum of three consecutive numbers is always a multiple of 6". Either show that this is always true or find an example to show that it's false.

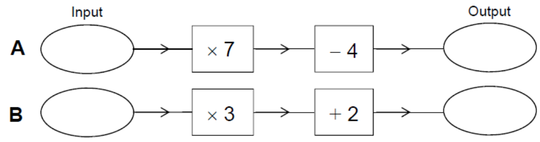
32. Simplify $\frac{3x}{x+1} + \frac{2}{x}$ as far as possible

33. Simplify $a + a + b \times b - c + c$

34. The cost in £ to hire a car for n days is given by $C = 14 + \frac{27}{4}(n + 1)$. Find the cost of hiring a car for nine days.

35. The product of x and y is 34. Write a formula for y in the form $y =$...

36. Here are two function machines

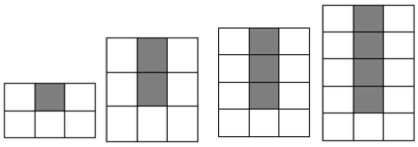


Both machines have the same input. Work out the value of the input when the output of A is three times the output of B

37. Which sequence is geometric?

1, 2, 3, 4   1, 2, 4, 7   1, 2, 4, 8   1, 2, 3, 5

38. Here is a sequence of white and coloured squares



a. Find the total number of squares in the 100th term.
b. Find the number of coloured squares in the 50th term.

39. Lucy says "when $y^2 = 25$ the only possible value of y is 5". Is she correct? Explain your answer.

40. An arrow is fired and its height above ground is modelled by $h = (14 - t)(t + 1)$ where h =height in cm and t = length of shot in metres. Find the maximum height of the arrow.

41. Prove that $\frac{6x^3+30x}{3x^2+15}$ is even for all positive integers x

42. Graham says that (-2, -1), (0,2) and (3,9) are all points on the line $y = 2x + 3$. Is he correct? Explain your answer.

43. Find the gradient of $14x + 5y = 37$

44. Holly goes for a walk to a park which is 1km away, it takes her half an hour to get there. Once she reaches the park Holly stays there for an hour before walking to the shop a further 500m away, this takes her 15 minutes. She spends quarter of an hour in the shop before taking half an hour to walk home. Show her journey on a distance-time graph.

45. Solve $8x^2 + 3x - 4 = 0$ using the quadratic formula

46. Solve $\frac{2(m+4)}{5} = 8m$

47. Write down an equation with roots 4 and -8.

48. Solve the equations

a. $16 = 2x^2 + 4x$ b. $4x^2 - 25 = 0$

49. Solve $2x + 9 \geq 12 - x$. Show your answer on a number line.

50. Find an estimate for the area between the x axis, $y = x$ and $y = 2 - x^2$

51. Given the sequence 1, 10, 20, 40, 61... Find the 50th term.

Section Three: Geometry

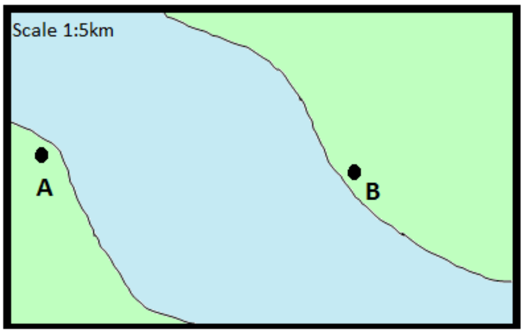
Chapter Eight: Properties and Construction

8.1 Measuring

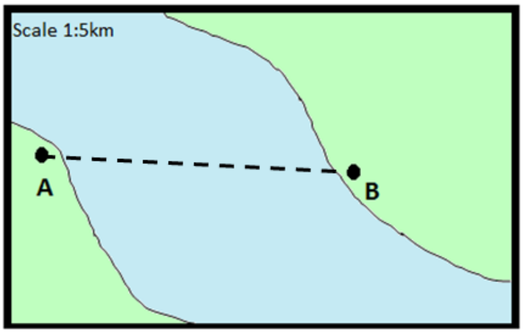
Scales

Recall that distances are always measured in metric units, we dealt with the correct units of measurement in section 3.4. On maps and other plans, it is impractical to use the exact distances and for this reason they use scales. The ratio used is 1:a for some value of a. This means that the actual length is a x the length on the map.

Example:
A ferry travels from point A to point B. How far does it travel?



First measure the actual distance on the map



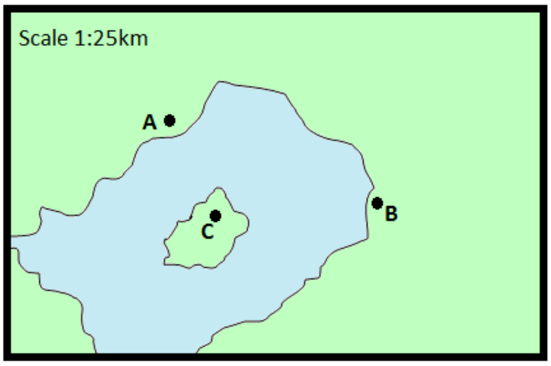
Since the distance on the map is 4cm and the scale tells us that 1cm represents 5km we find the real distance travelled by the ferry 4 x 5 = 20km.

Activity 8.1 A

1. Fill in the missing values in this table.

Scale	Distance on map	Real distance
1:10km	4.5cm	
1:20km	2cm	
1:5km	5.2cm	
1:2km	2.5cm	
1:100km		589km
1:50km	1.2cm	
1:10km		23km
1:20km		240km

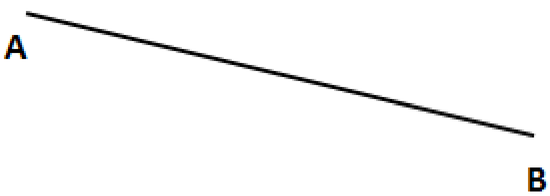
2. Using the map below find the real distance between each of the points.



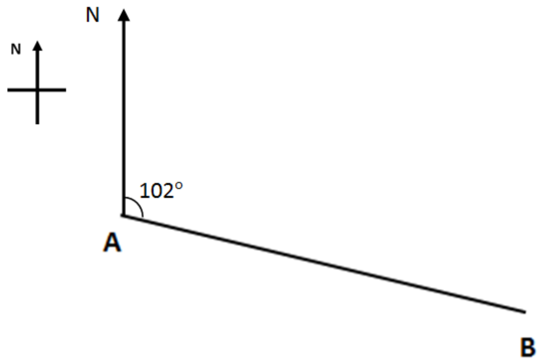
<u>Angles and Bearings</u>

A bearing is an angle that is always measured clockwise from the north line. It should always be given as three figures so the number should be rounded to the nearest integer. If the size of the angle is a two digit number you simply place a zero in front of it.

Example:
Find the bearing of B from A



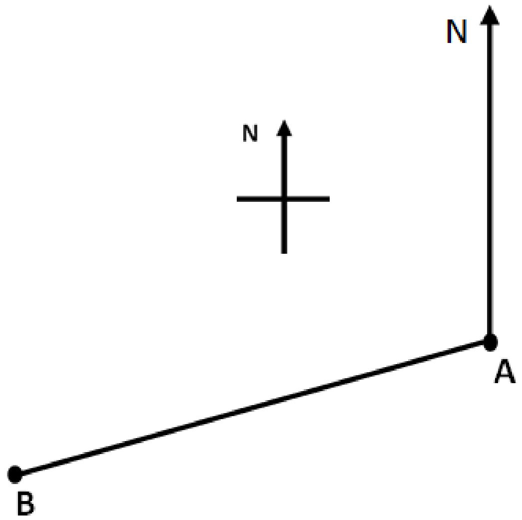
The first thing we need to do is draw the north line then we measure the angle between north and the line between A and B, working in a clockwise direction.
A bearing question will always ask you to find a bearing of a point from another point. The north line should be drawn at the point you're measuring *from*. Since the question asked for a bearing from A the north line is drawn at A.
It is good practice to draw a small compass beside the diagram.



So we can say that the bearing of B from A is 102°

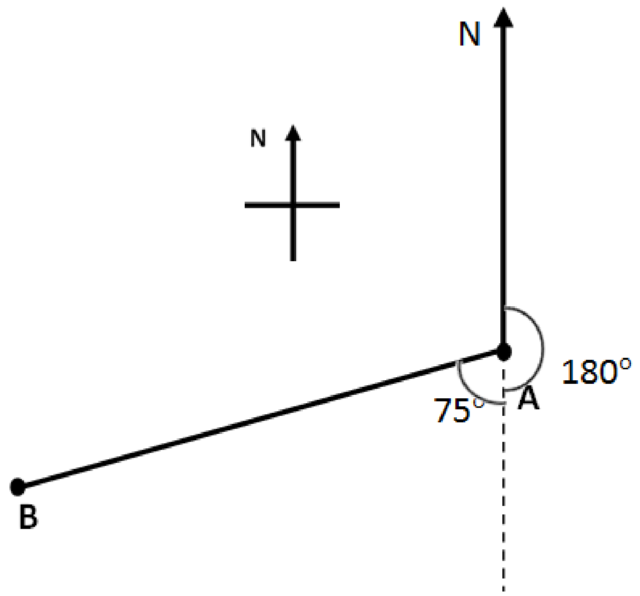
Example:
Find the bearing of B from A

First draw the line to join the points and then draw the north line as before.



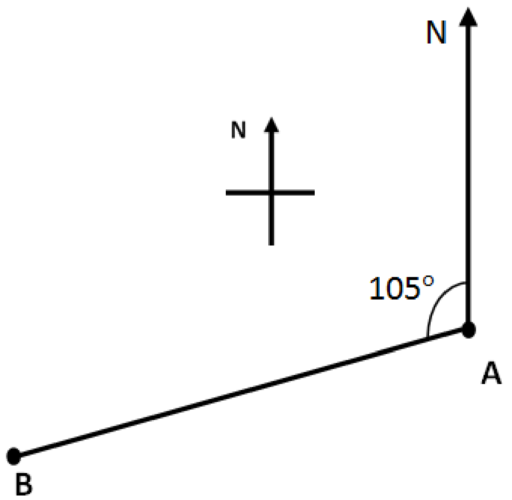
Remembering that bearings always have to be measured clockwise there are two options finding larger bearings such as these. Both options are illustrated below.

Option One:
One option is to extend the north line beyond the point, essentially creating a south line. You can then use the fact that **angles on a straight line add up to 180°** to find the total bearing.



Therefore the total bearing of B from A is 180°+75°=255°

Option Two:
You can measure the angle anti clockwise then use the fact that **angles about a point add up to 360°** to find the required bearing.

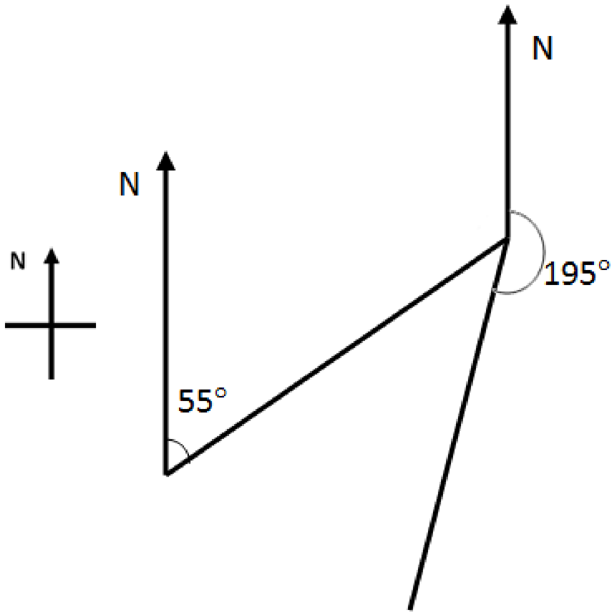


Therefore the required bearing is 360°-105°=255°

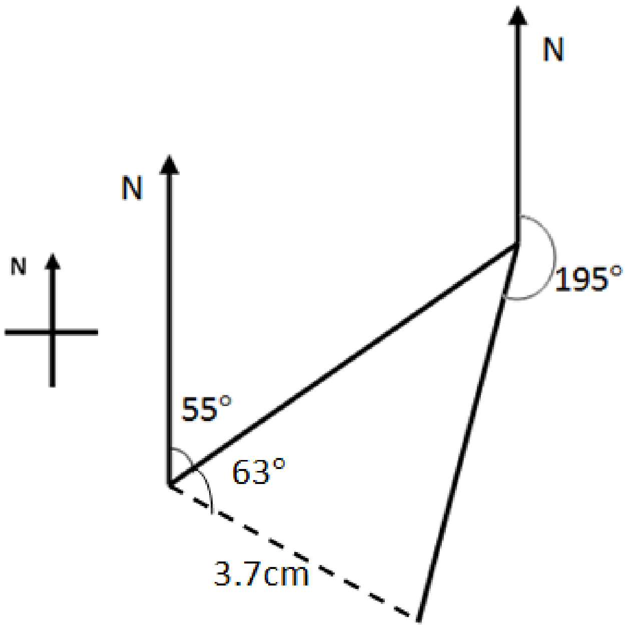
Example:
A car travels for 27.5 miles on a bearing of 055° then turns and travels a further 25 miles on a bearing of 195°. Find the direction and the length of the return journey.

In order to find the required values we need to draw a diagram. Clearly it will be necessary to use a scale as it is not possible to draw lengths of 25 miles! It is also not practical to draw these as 25cm so we will decide to use a 1cm:5miles scale. Unless it is specified in the question you can choose your own scale for a diagram such as this as long as you choose one which is practical for your drawing.

The distances drawn on the diagram are worked out as 27.5 ÷ 5 = 5.5cm and 25 ÷ 5 = 5cm



Now all the remains to be done is to draw on the line of the return journey and to find the bearing.



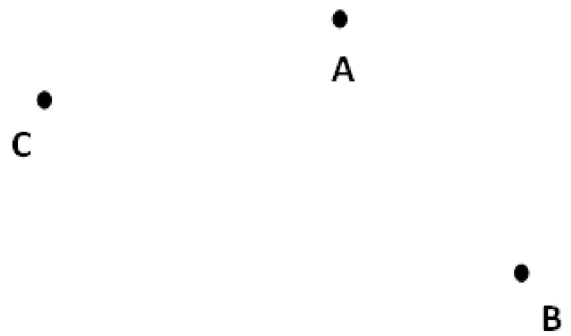
So the direction of the return journey is a bearing of 55° + 63°=118°
The distance of the return journey is 3.7 x 5 = 18.5 miles

It is important to be as accurate as possible when measuring angles but it is likely that your answers may differ one or two degrees from those published. This is due to the fact that it is almost impossible to be completely accurate when using a protractor. This is taken into account in mark schemes so, as long as you take your time and are careful to measure as accurately as you can don't worry if you find your answers differ slightly.
It's also important to remember that we are usually working with scale diagrams. The larger the diagram the more accurate it will be as errors in measuring are less important.

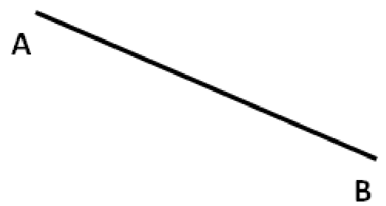
Activity 8.1 B
For questions 1-3 you may find it helpful to use tracing paper and copy the diagrams onto another sheet of paper so that you have more room.

1. Find the bearing of B from A

2. Find the bearing of B and C from A



3. Find the bearing of A from B



4. A boat leaves a harbour and sails 30 miles on a bearing of 127°, it then sails a further 26 miles on a bearing of 211°. Find the direction and the length of the return journey.

8.2 Area and Perimeter

The area of a shape is the amount of space inside. The perimeter is the total length of all of the sides.
Perimeter is measured in the same units as the sides, area is measured in this unit squared. For example if the sides are measured in cm then the area is measured in cm^2.

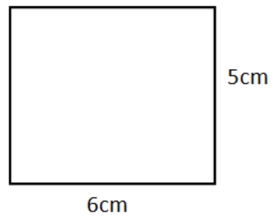
Questions of this sort are rarely drawn to scale so you should not measure the sides.

To find the perimeter of any shape you simply find the total of the sides. There are a few different rules for finding the area which will be looked at in turn.

Rectangle

To find the area of a rectangle or a square you multiply the length by the width.

Example:
Find the area and perimeter of this rectangle

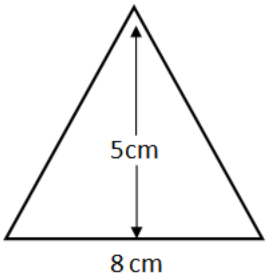


Perimeter: 5 + 6 + 5 + 6 = 22cm
Area: 5 x 6 = 30 cm^2

Triangle

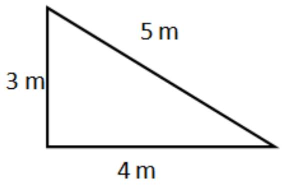
The area of a triangle can be found with the formula area = ½ x base x height

Example:
Find the area of this triangle



Area = ½ x 5 x 8 = 20 cm^2

Example:
Find the area and perimeter of this triangle

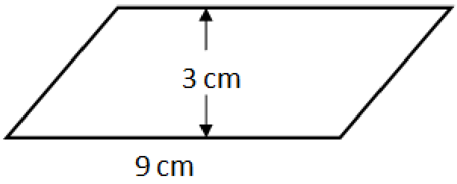


Perimeter: 3+4+5=12m
Area: ½ x 3 x 4 = 6m^2

Parallelogram

The area of parallelogram is found by multiplying the base by the perpendicular height

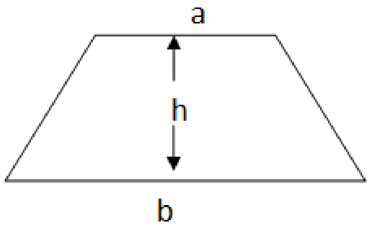
Example:
Find the area of this shape



Area = 3 x 9 = 27cm^2

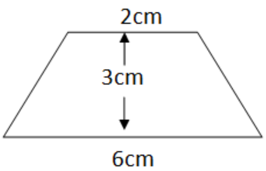
Trapezium

The area of a trapezium is found using the formula area = $\frac{a+b}{2} \times h$
where *h* is the height and *a* and *b* are the parallel sides as follows



You can also think of this as area = (sum of parallel sides ÷ 2) x height

Example:
Find the area of this shape

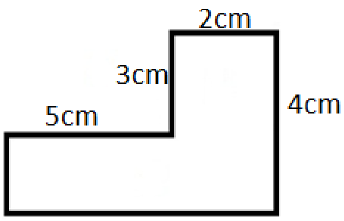


$\text{Area} = \frac{2+6}{2} \times 3 = 4 \times 3 = 12\text{cm}^2$

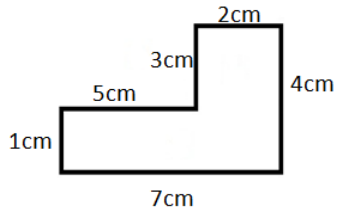
Compound Shapes

You will often find that you're given a shape that you can't find the area of with one formula. When this is the case you should split it into shapes you can find the area of and then add your answers together.

Example:
Find the area and perimeter of this shape

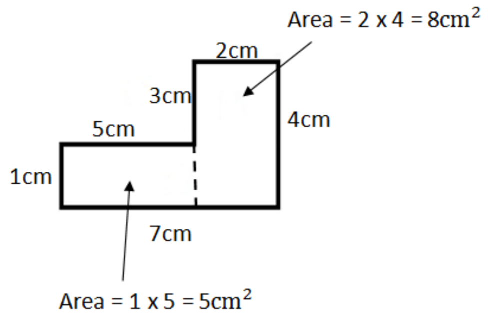


Before we can find the perimeter we need to find the missing sides.
The missing vertical side is 4 – 3 = 1cm and the missing horizontal side is 2 + 5 = 7cm
So now we have



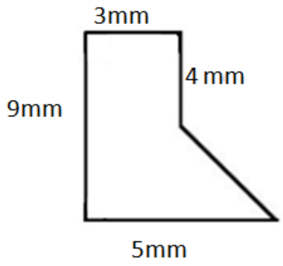
And we can find the perimeter: 2+4+7+1+5+3=22cm

To find the area we must split the shape into two rectangles and find the area of each of them

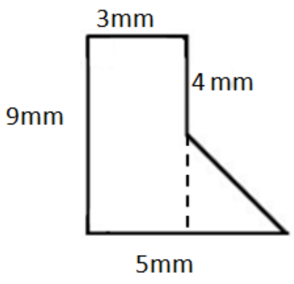


So the total area is 8 + 5 = 13cm^2

Example:
Find the area of this shape



Splitting this into two shapes we can find the area of gives

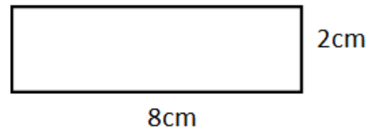


The area of the rectangle is 3 x 9 = 27mm^2
The height of the triangle is 9 – 4 = 5mm and the base is 5 – 3 = 2mm so the area is ½ x 5 x 2 = 5mm^2
So the total area is 27 + 5 = 32mm^2

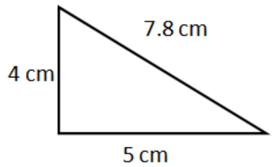
Activity 8.2

1. Find the area and perimeter of each of these shapes

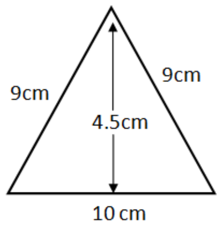
a.



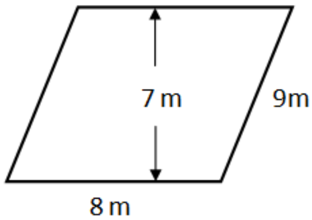
b.



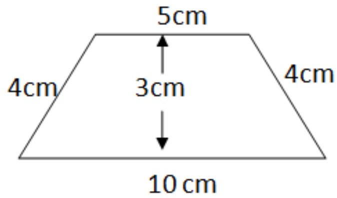
c.



d.

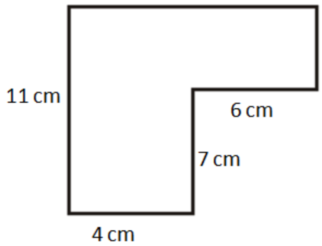


e.

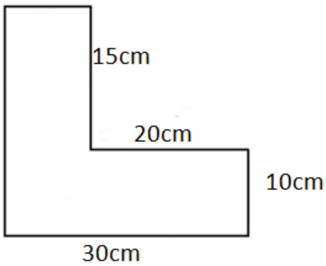


2. Find the area and perimeter of each of these shapes

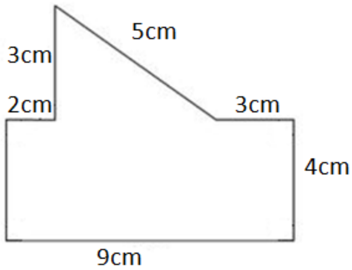
a.



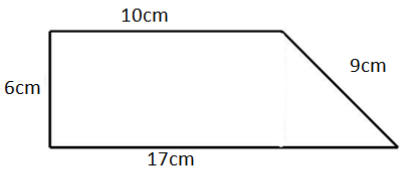
b.



c.



d.



3. A square has an area of 49cm^2. Find the perimeter.

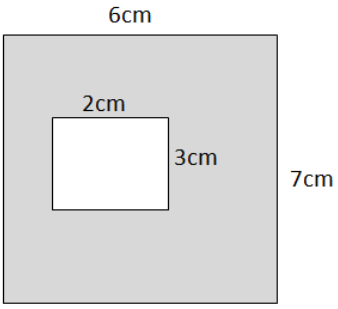
4. A triangle with base 8cm has an area of 20cm^2, what is the height?

5. An equilateral triangle has a perimeter of 90mm, find the length of each side.

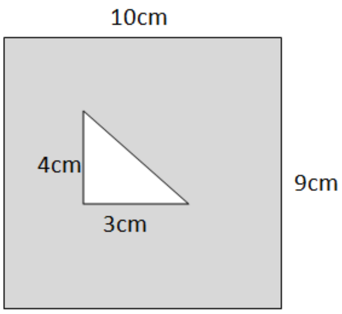
6. A parallelogram has height 4cm and area 24cm^2, find the length of the base.

7. Find the area of the shaded part for each of these shapes.

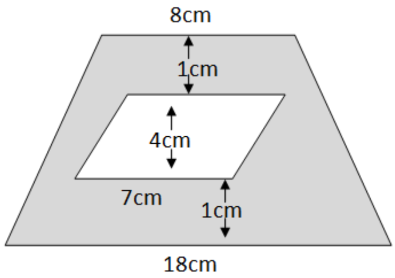
a.



b.



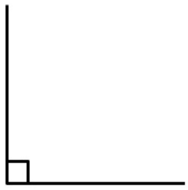
c.



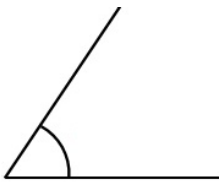
8.3 Angle Rules

Types of Angles

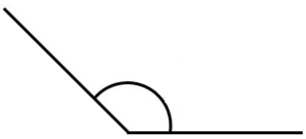
A **right angle** is 90° and is denoted by a small square.



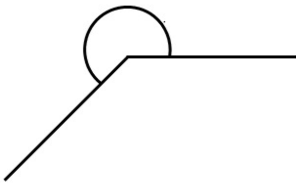
An **acute** angle is less than 90°.



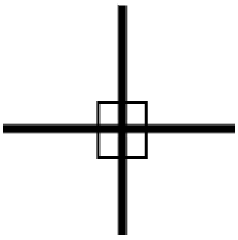
An **obtuse** angle is more 90° but less than 180°



A **reflex** angle is more than 180°



Perpendicular lines meet at right angles



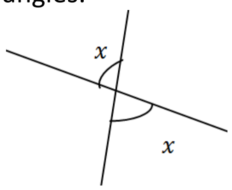
Angles and Lines

Two important rules were used in the last section, they will be repeated again here:

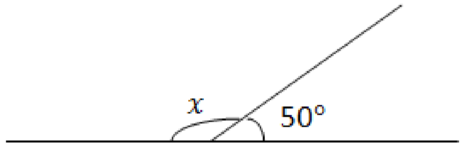
Angles on a straight line add up to 180°

Angles about a point add up to 360°

You also need to know that vertically opposite angles are equal. This is unofficially referred to as X angles.

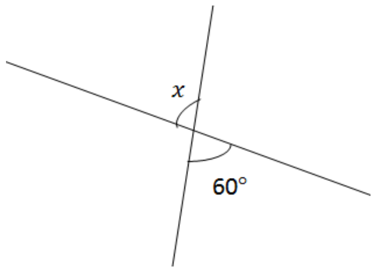


Example:
Find the value of x



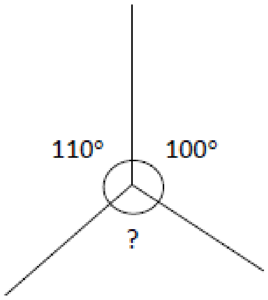
We know that angles on a straight line add up to 180° so x = 180° – 50° = 30°

Example:
Find the value of x



Since vertically opposite angles are equal x = 60°

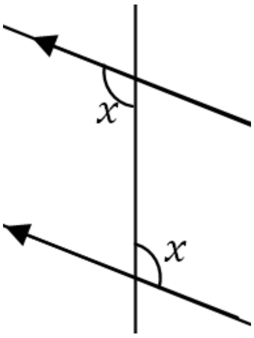
Example:
Find the value of the missing angle



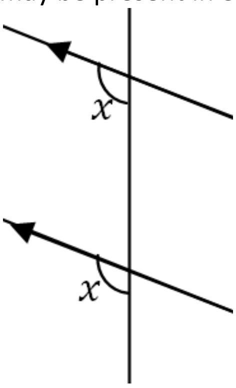
Angles about a point add up to 360° so we have 110 + 100 + x = 360 which gives x = 150°

There are two more rules you should be aware of. These arise when a pair of parallel lines cross another line – note that parallel lines are usually denoted by arrows.

Alternate angles are equal, this is unofficially referred to as Z angles – the Z shape may be present in either direction.

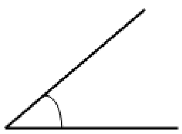


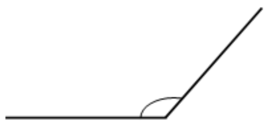
Corresponding angles are equal, this is unofficially referred to as F angles – as above the F shape may be present in either direction.



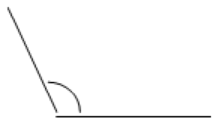
Activity 8.3 A

1. Write down the type of each of these angles

a.


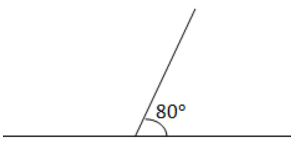
b.


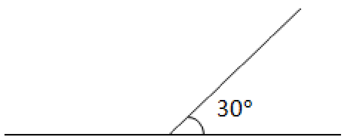
c.

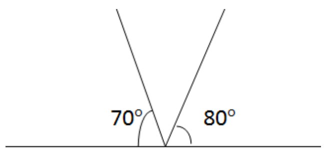

d.


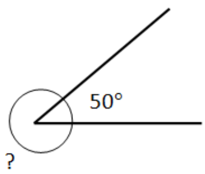
e. 154° f. 35° g. 283° h. 78° i. 90° j. 91° k.89°

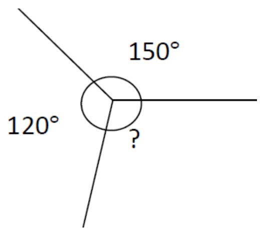
2. In each of the diagrams below find the missing angle(s)

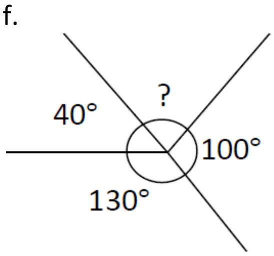
a.


b.


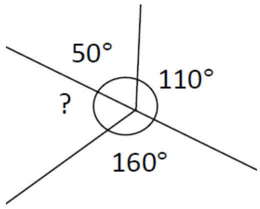
c.


d.


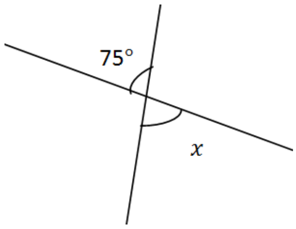
e.


f.


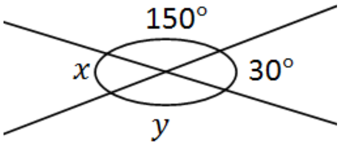
g.



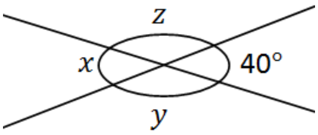
h.



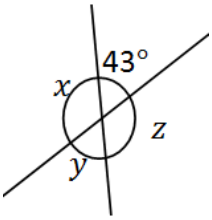
i.



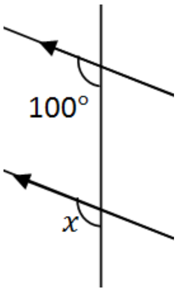
j.



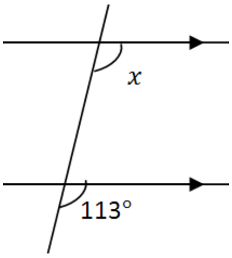
k.



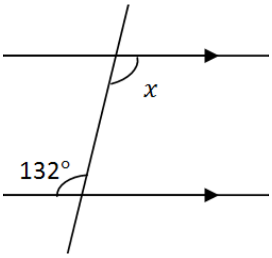
l.



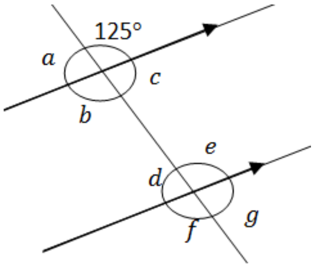
m.



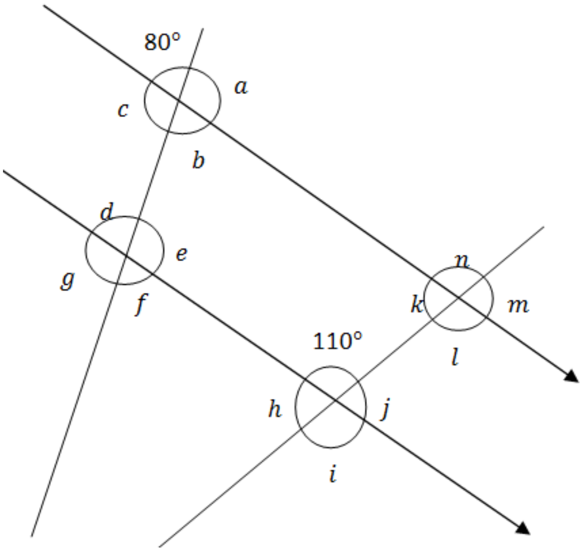
n.



o.



3. Find the value of all of the missing angles in the diagram below



Angles in a Triangle and Quadrilateral

There is one important rule to remember when working with triangles:

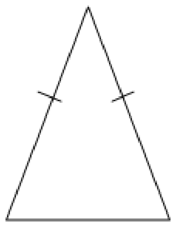
Angles in a triangle add up to 180°

In a **scalene** triangle all of the sides and angles are different. An **isosceles** triangle has two equal sides and two equal angles and an **equilateral** triangle has 3 equal sides and 3 equal angles.

Activity 8.3 B

Find the size of each angle in an equilateral triangle

If sides of any shape are of the same length they may be marked with lines as shown below



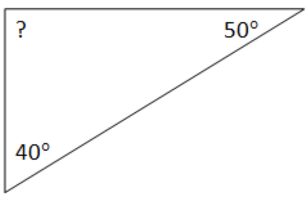
The two angles that are the same are the ones that connect the equal length sides to the base, the angle where the two equal sides meet is the one that is different.

A quadrilateral is a shape with four sides. Angles in a quadrilateral add up to 360°.

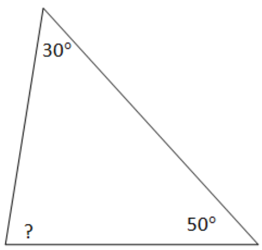
Activity 8.3 C

1. In each of the diagrams below find the missing angles

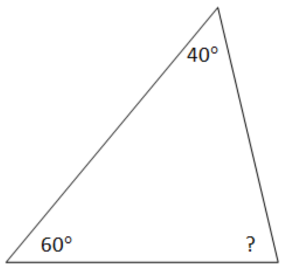
a.



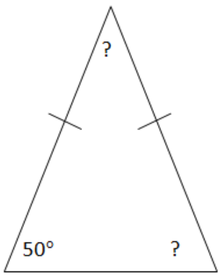
b.



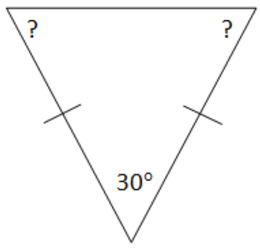
c.



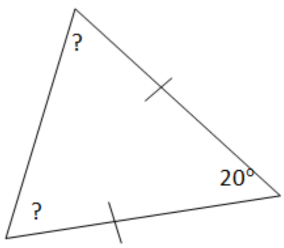
d.



e.



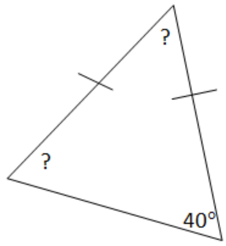
f.

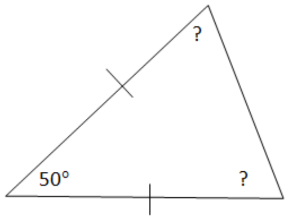


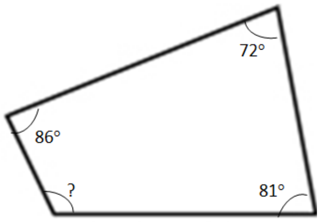
g.

h.

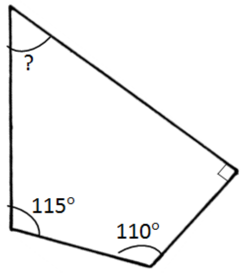
i.





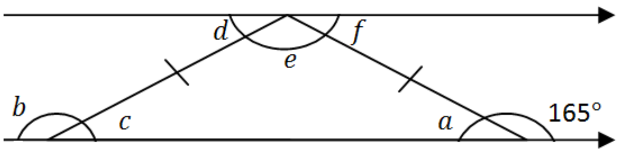


j.

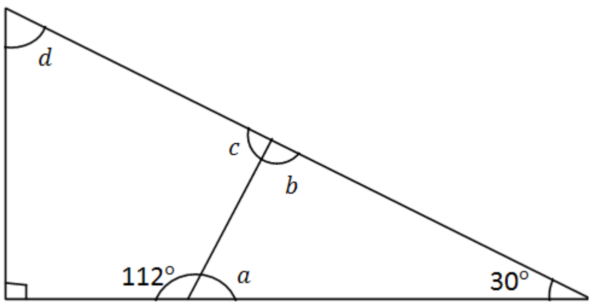


2. Find all of the missing angles in the diagrams below

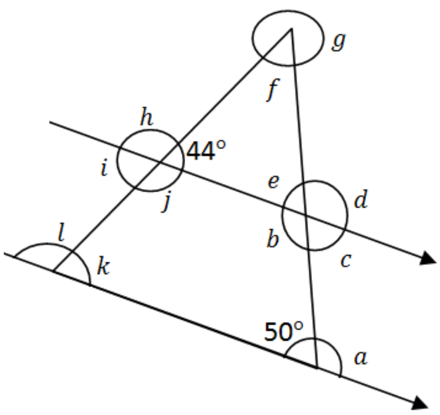
a.



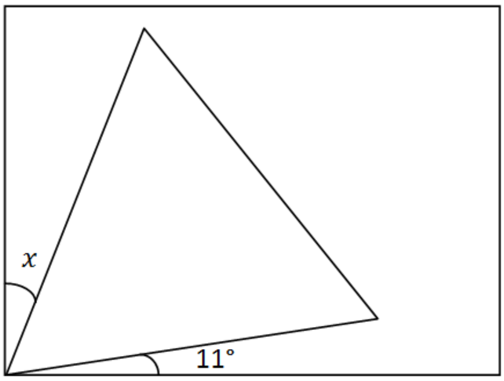
b.



c.



3. An equilateral triangle is inside a rectangle as shown. Find the size of the missing angle.



Angles in Polygons

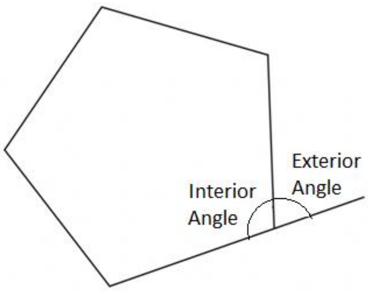
A polygon is any 2D shape with straight sides. It is said to be regular if all of the sides are equal and all of the angles are equal.

Activity 8.3 D

Write down the number of sides/angles each of these polygons has.

a. Hexagon	b. Octagon	c. Triangle	d. Quadrilateral	e. Decagon
f. Pentagon	g. Heptagon	h. Nonagon	i. Dodecagon	

All polygons have interior and exterior angles as shown in the diagram below.



There are three important rules relating to interior and exterior angles that you should remember.

At each vertex the interior angle and the exterior angle add up to 180°. You should be able to see this from the diagram as they lie on a straight line.

The sum of the exterior angles equals 360°.

The sum of interior angles is (number of sides – 2) x 180°

Example:
A regular polygon has 16 sides. Find the size of the interior and the exterior angles.

Since the polygon is regular we know that all of the angles are the same size.

Sum of interior angles = (16 – 2) x 180° = 2520°

So the size of one interior angle is 2520° ÷ 16 = 157.5°.

There are then two ways to find the size of the exterior angles. You can use the fact that the interior angle + the exterior angle = 180° to give 180° - 157.5° = 22.5°.
Or you can use the fact that the sum of the exterior angles is 360° to give 360° ÷ 16 = 22.5°
(Note: you could also use this method to find the exterior angle first and then use this value to find the interior angle)

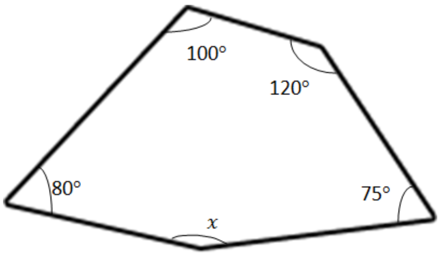
Activity 8.3 E

1. Find the size of the interior and exterior angles for all of the polygons in activity 8.3 D. Assume they are all regular polygons.

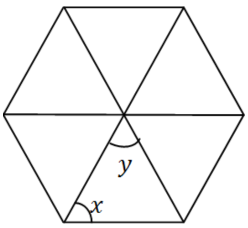
2. A regular polygon has 20 sides. Find the size of the interior angles.

3. A hexagon has 5 interior angles of 130°. Find the size of the remaining interior angle.

4. Find the value of x



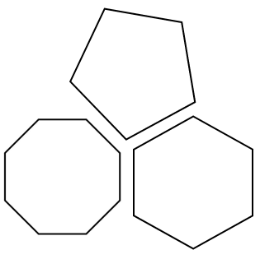
5. A regular hexagon is split into equal sized triangles as shown. Find the value of x and y.



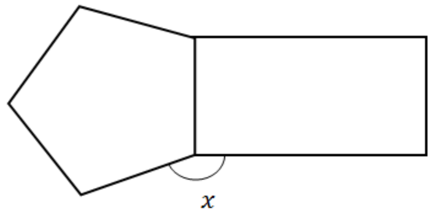
6. A regular polygon has interior angles of size 156°, find the number of sides the polygon has.

7. A regular polygon has exterior angles of size 18°, find the number of sides the polygon has.

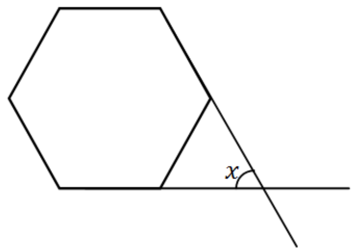
8. Lisa has some tiles. She picks up one regular hexagon, one regular octagon and one regular pentagon. Can they fit together? Explain your answer.



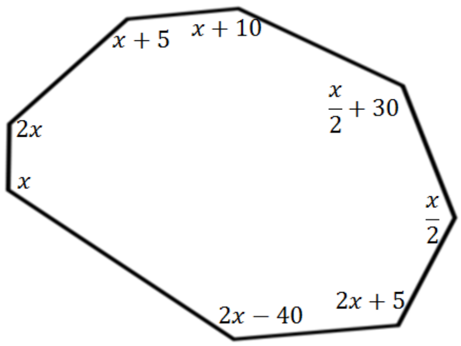
9. A regular pentagon is attached to a rectangle as shown. Find the size of x



10. Two sides of a regular hexagon are extended to form a triangle as shown. Find the size of x



11. This shape is not to scale. Find the value of x



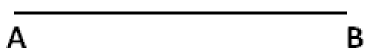
<u>8.4 Loci and Construction</u>

<u>Perpendicular Bisector</u>

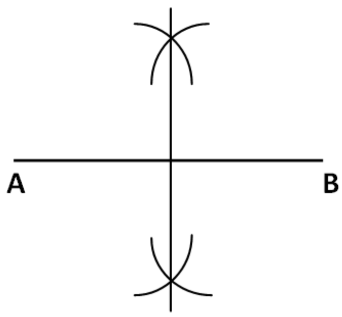
You already know that perpendicular lines cross at right angles. A bisector of a line cuts it in the middle so that it is the same distance from either end.

A perpendicular bisector of line AB is constructed as follows:

- Measure the length of the line AB



- Set your compasses to over half the length of this line. There is no set distance you should use as long as it is greater than half but it is generally advised to stick to less than the full length of the line.
- Put the point of the compass at A and draw arcs above and below the line. Repeat with the point at B.
- Draw a straight line through the points where these arcs cross.

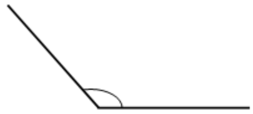


<u>Angle Bisector</u>

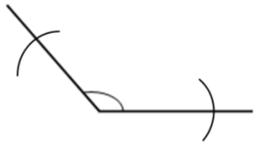
As before a bisector cuts exactly in half, in this case it is an angle that you are cutting in half.

An angle bisector is constructed as follows:

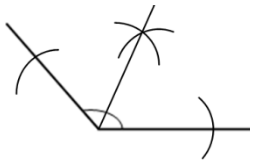
- Set the compasses so they are less than the length of the lines extending from the angle. If this is not practical simply extend the lines.



- Place the point of the compass on the angle and draw arcs that cross each of the lines extending from the angle.



- Place the point of the compass at the point where the arcs cross the lines (doing each one in turn) and draw an arc inside the angle.
- Draw a line connecting the angle to the point where these arcs cross.



When drawing constructions you should always leave your arcs and constructions lines in. Do not rub them out. Correct answers without the construction lines showing will not receive full marks.

Activity 8.4 A

1. Draw a horizontal line of 8cm. Construct the perpendicular bisector of this line.

2. Draw a horizontal line of 10cm. Construct the perpendicular bisector of this line.

3. Draw a vertical line of 13cm. Construct the perpendicular bisector of this line.

4. Draw a horizontal line of 15 cm. Label this AB. Mark the point on the line that is 4cm away from A, label this C. Draw a line that is perpendicular to AB that cuts through C.

5. Draw any obtuse angle and construct the bisector.

6. Draw any acute angle and construct the bisector.

<u>Loci</u>

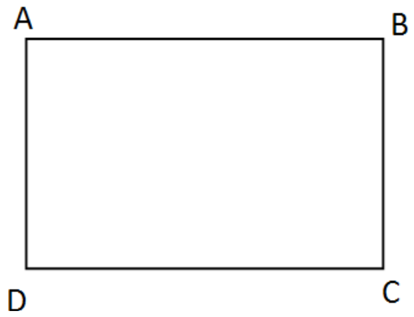
A locus (plural: loci) is a set of points that follow particular rules or it is the path that a moving point follows.

If you are given a point A and you want to find the set of points that are a given distance away from point A you would use a set of compasses to draw the locus, which takes the form of a circle, with centre A.

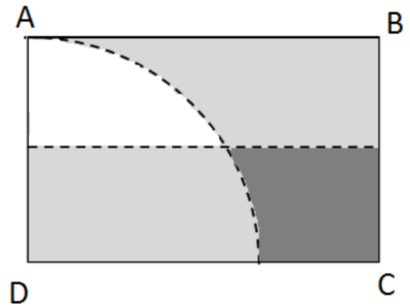
The points that are equidistant from two points lie on the perpendicular bisector of the line joining them.

The points that are equidistant from two lines that meet at point A lie on the angle bisector of the angle A.

Example:
Given the rectangle below sketch the locus of the points that are more than 3cm away from D and are nearer to C than B.



First we need to draw the perpendicular bisector of BC and shade the side of this line that it nearer to C. Next we need to set the compasses to 3cm and draw an arc with centre D, since we need the points more than 3cm away we shade in the points outside the circle. The darker section shows the points at which the loci overlap. This is the required region.



Activity 8.4 B

1. Draw a point and label it A. Draw the locus of points that are 3cm away from this point.

2. Using the same point as above shade the locus of points that are more than 3cm away from A, but less than 5 cm away from A.

3. Draw a line that is 8cm long. Label one end A and the other end B. Draw the locus of points that are equidistant from each end.

4. Draw a rectangle with sides of 5cm and 8cm. Draw the locus of the points that are no more than 1cm away from this rectangle.

5. A business owner wants to open a new shop. They want it to be closer to town A than town B but they want it to be no more than 5km away from town B. Sketch the locus of points such that this is possible.
Scale 1km:1cm

6. Using the rectangle from the example – you may find it easier to use tracing paper and make another copy of it – sketch the locus of points such that the points are closer to A than B and are less than 3cm away from C.

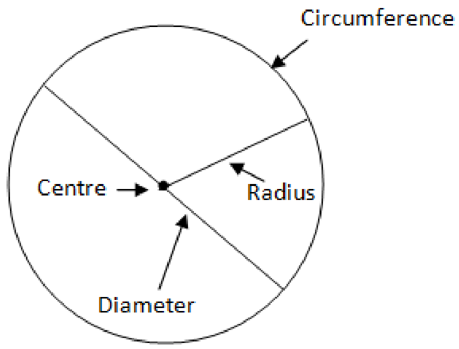
8.5 Circles

Terminology

You already know that the perimeter is the total length of the sides of a shape. When we have a circle the perimeter is called the **circumference**.

The **radius** is the distance from the centre to the circumference of the circle.

The **diameter** is twice the length of the radius, it passes from one side to the other through the centre.



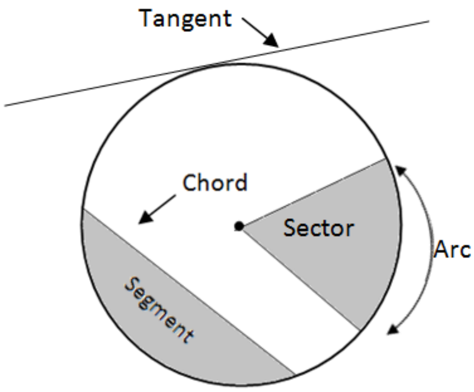
An **arc** is part of the circumference.

A **sector** is the area formed between the circumference and two radii (radii is the plural of radius).

A **chord** is a line inside the circle touching the circumference at either end but not passing through the centre.

A **segment** is the area enclosed between the chord and the circumference.

A **tangent** is a line outside of the circle that touches the circumference.



Area and Circumference

We have already seen that diameter = 2 x radius. When working with circles we write d = diameter and r = radius. The formulae you need to know are:

$\text{Circumference} = \pi\text{d} = 2\pi\text{r}$

$\text{Area} = \pi\text{r}^2$

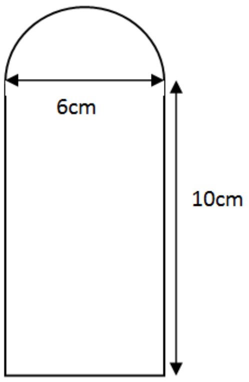
We met π in a previous chapter. To ensure the most accurate answers you should use the π button on the calculator when carrying out calculations. If you have problems with this you can use the approximation π = 3.14.
All answers should be rounded to a minimum of 3 significant figures or 2 decimals places, whichever is more accurate.
When the value is needed for further calculations you should leave it in its exact form, that is leaving π in the answer, until you reach the final step. This ensures accuracy.

Example:
A circle has a radius of 5cm. Find the area and the circumference.

Circumference = 2 x π x 5 = 10π = 31.41492654... ≅ 31.41cm

Area = $\pi \times 5^2 = 25\pi = 78.53981634 \ldots \cong 78.54\text{cm}^2$

Example:
A shape is made up of a rectangle and a semi circle as shown. Find the area and the perimeter of the shape.



Notice that you are given the diameter of the semi circle, the radius is 6 ÷ 2 = 3cm
Area:
Area of rectangle = 10 x 6 = 60cm^2
Area of semi circle = area of circle ÷ 2 = $(\pi \times 3^2) \div 2 = \frac{9}{2}\pi\ \text{cm}^2$
Total area = 60 + $\frac{9}{2}\pi$ = 74.1371669... $\cong 74.14\ \text{cm}^2$

Perimeter:
Length of curved side = circumference of circle ÷ 2 = (π x 6) ÷ 2 = 3π
Total perimeter = 10 + 6 + 10 + 3π = 35.42477796... $\cong 35.42\ \text{cm}^2$

Example:
Find the perimeter of this semi circle.

Length of curved side = (π x 7) ÷ 2 = $\frac{7}{2}\pi$
Total perimeter = $\frac{7}{2}\pi$ + 7 =17.99557429...≅ 18.00cm

This last example has been included because it is a common mistake when finding the perimeter of semi-circles to forget to include the straight side.

Activity 8.5 A

1. A circle has a radius of 8cm, find the area and the circumference.

2. A circle has a radius of 3.5cm, find the area and the circumference.

3. A circle has a diameter of 10cm, find the area and the circumference.

4. A circle has a diameter of 9cm, find the area and the circumference.

5. A circle has a circumference of approximately 56.55cm. Find the radius.

6. A circle has a circumference of approximately 69.12cm. Find the diameter.

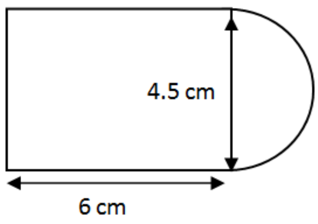
7. A circle has a circumference of approximately 43.98m. Find the area.

8. A circle has an area of approximately 153.94cm^2, find the circumference.

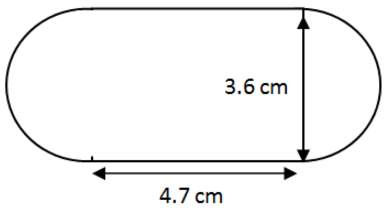
9. A circle has an area of approximately 38.48cm^2, find the circumference.

10. Find the area and the perimeter of each of these shapes.

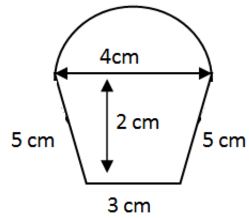
a.



b.

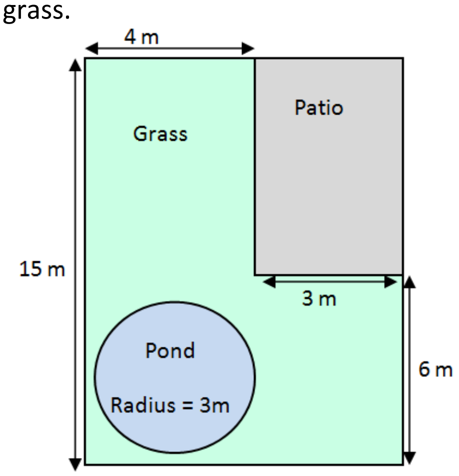


c.



11. A square garden with sides of length 30m contains a circular pond. The remainder of the garden is grass. The pond has a radius of 5m. Find the area of the garden that is covered in grass.

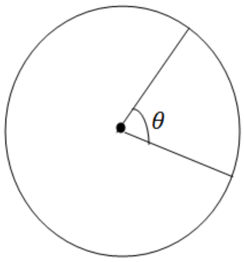
12. The diagram below shows the plan of a garden. Find the area of the garden that is covered in grass.



<u>Arcs and Sectors</u>

The area of a sector is found by multiplying the fraction of the circle that the sector takes up by the area of the circle.

Given a sector as shown below



The fraction of the circle taken by the sector is $\frac{\theta}{360}$

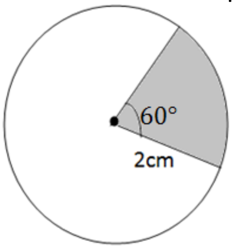
The area of the circle is πr^2

Therefore the area of the sector is $\frac{\theta}{360} \times \pi r^2$

The sector is enclosed by two radii and one arc so the perimeter is found by adding the lengths of the two radii to the length of the arc.

The length of the arc is found using a similar logic as above, you multiply the fraction of the circumference that is taken up by the arc by the circumference of the circle. Therefore the length of an arc is $\frac{\theta}{360} \times 2\pi r$

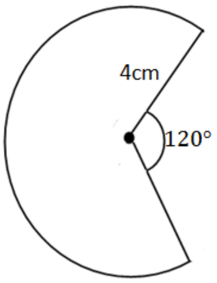
Example:
Find the area and the perimeter of the shaded area.



Area: $\frac{60}{360} \times \pi \ \times 2^2 = \frac{2}{3}\pi \cong 2.09\text{cm}^2$

Length of arc: $\frac{60}{360} \times 2 \times \pi \times 2 = \frac{2}{3}\pi$
Perimeter: $\frac{2}{3}\pi + 2 + 2 \cong 6.09\text{cm}$

Example:
Find the area and perimeter of this sector



The angle inside the sector is 360° - 120° = 240°

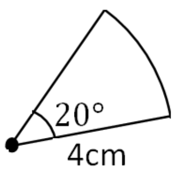
Area: $\frac{240}{360} \times \pi \times 4^2 \cong 33.51\text{cm}^2$

Length of arc: $\frac{240}{360} \times 2 \times \pi \times 4 = \frac{16}{3}\pi$
Perimeter: $\frac{16}{3}\pi + 4 + 4 \cong 24.76\text{cm}$

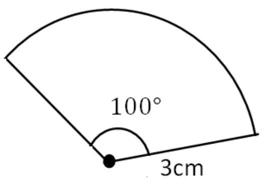
Activity 8.5 B

1. Find the perimeter and area of each of these shapes

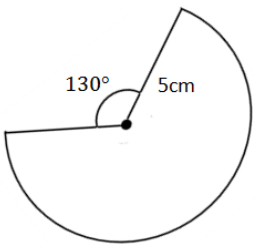
a.



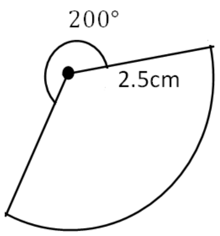
b.



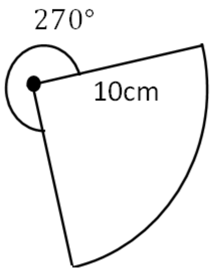
c.



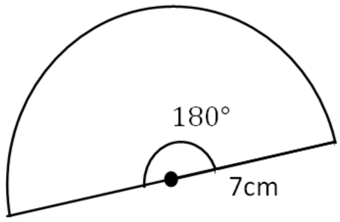
d.



e.



f.

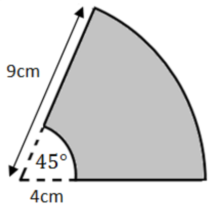


2. A circle has a radius of 5cm. A sector of this circle has an area of approximately 17.45cm^2. Find the angle of the sector.

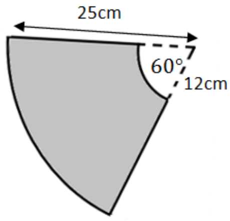
3. A sector has an area of approximately 235.62cm^2 and an angle of 120°. Find the radius.

4. Find the area and perimeter of the shaded area.

a.



b.



5. A sector with radius 3cm and angle 30° is cut out of a square with sides of 5cm. Find the area of the remaining shape.

Equation of a Circle

It is possible to plot circles on graphs. A circle with a centre at (0,0) and a radius r will have an equation of the form

$$x^2 + y^2 = r^2$$

For example, the circle $x^2 + y^2 = 25$ has radius $\sqrt{25} = 5$

Activity 8.5 C

1. Write down the centre and the radius of each of these circles

a. $x^2 + y^2 = 9$ b. $x^2 + y^2 = 36$ c. $x^2 + y^2 = 100$ d. $y^2 + x^2 = 4$

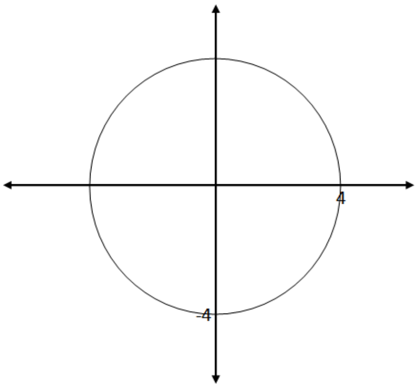
2. Sketch the graph of each of the circles in question 1. A sketch does not require all gridlines, it should show the main features of a graph (shape and points of intersection) on otherwise unlabelled axes. See question 4 for an example.

3. The equation of a circle is $x^2 + y^2 = 121$. Find the coordinates of the point where

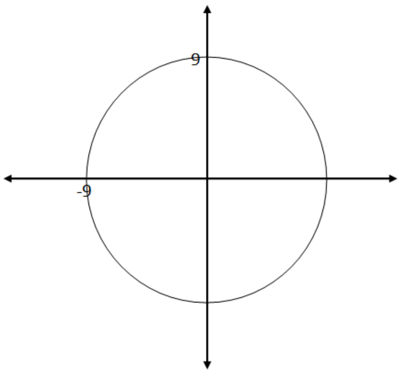
a. $x = 0$ b. $y = 1$ c. $x = -2$ d. $x = 5$ e. $y = 0$

4. Write down the equation of each circle

a.



b.



5. Find the points of intersection for each of these lines

a. $x^2 + y^2 = 100, y = x$

b. $x^2 + y^2 = 25, y = 2x + 1$

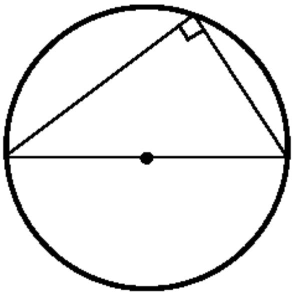
c. $x^2 + y^2 = 144, y = 2x$

d. $x^2 + y^2 = 49, y = \frac{x+1}{2}$

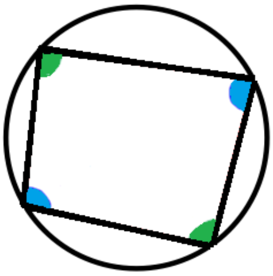
<u>Circle Theorems</u>

There are a number of theorems about angles, lines and triangles in circles. You need to ensure that you are familiar with and able to remember all of them.

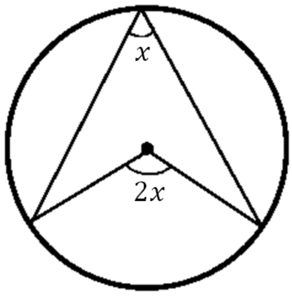
The angle in a semi circle is a right angle.
If a triangle is within a semi circle – that is where one side is the diameter – the angle at the circumference is 90°



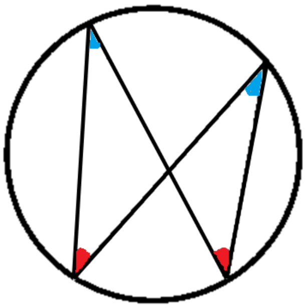
Opposite angles in a cyclic quadrilateral add up to 180°



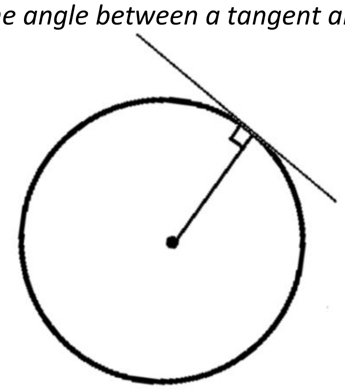
The angle at the circumference is half the angle at the centre
You can also think of this as "the angle at the centre is double the angle at the circumference"
This theorem is used when you see this distinctive arrowhead shape.



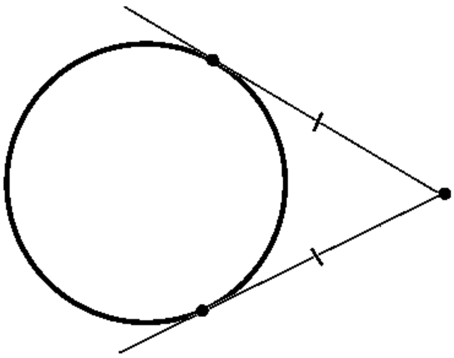
Angles in the same segment are equal



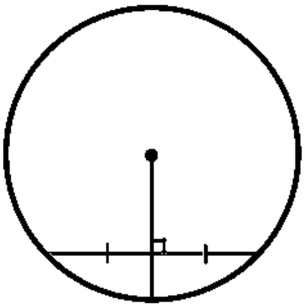
The angle between a tangent and radius is 90°



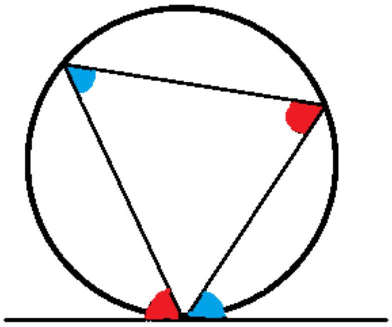
Tangents that meet at a point are equal in length
This allows you to use isosceles triangle rules when calculating angles.



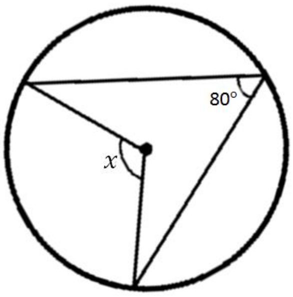
The radius that bisects the chord does so at 90°
Recall that bisects means it cuts through the midpoint. This can also be used the opposite way round – if the radius cuts the chord at 90°, it cuts through the midpoint of the chord.



The alternate segment theorem states that the angle between a tangent and a chord is equal to the angle in the alternate segment.
This is the most difficult one to get your head around. If you have an angle between a tangent and a chord the angle opposite it is equal to it.



Example:
Find angle x

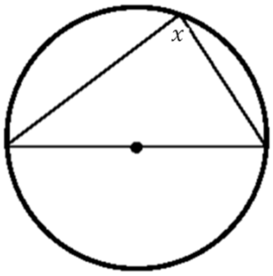


The angle in the centre is double the angle at the circumference so $x = 160°$

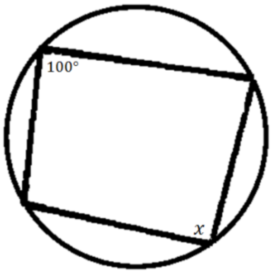
Activity 8.5 D

1. In each case find angle x

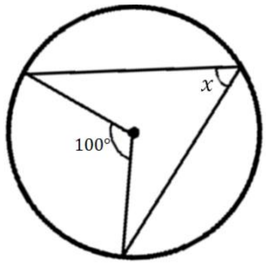
a.



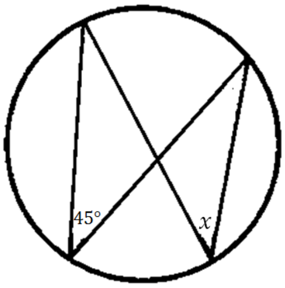
b.



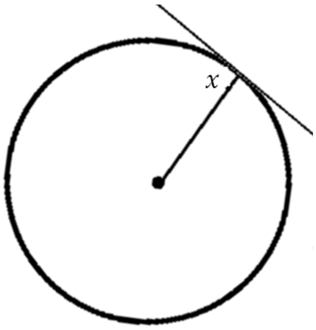
c.



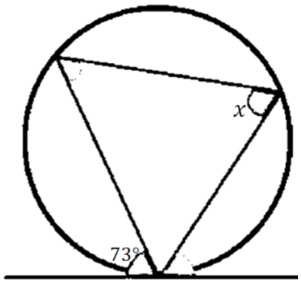
d.



e.

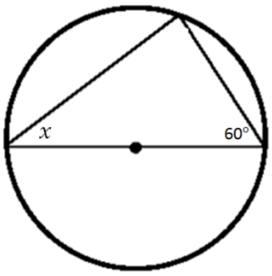


f.

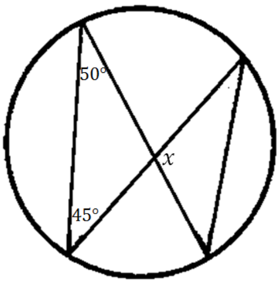


2. Find the value of x and, where appropriate, y

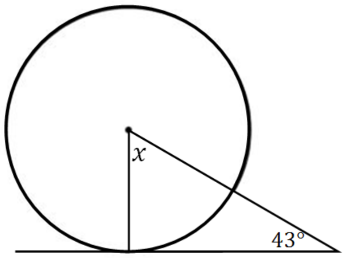
a.



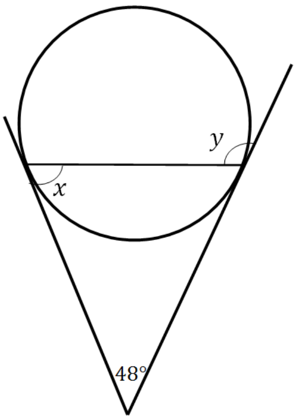
b.



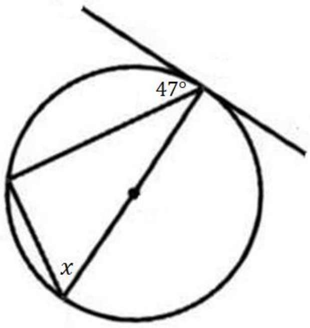
c.



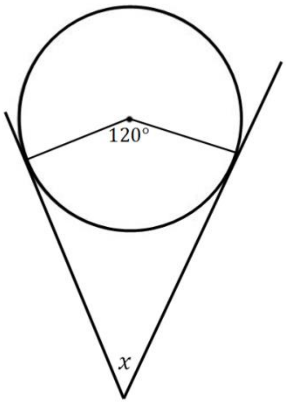
d.



e.



f.

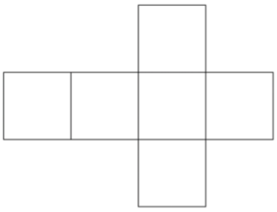


8.6 3D Shapes

Plans, Elevations and Nets

A net is a pattern or 2D shape that can be folded up to make a 3D shape.

For example the net of a cube is



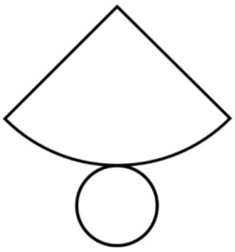
If you find this difficult to visualise you may find it helpful to copy the net out onto another piece of paper – or look online for one you can print – then cut it out and fold it up.

Activity 8.6 A

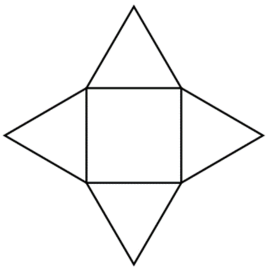
1. Draw nets for each of these shapes
   a. Triangular prism b. Cuboid c. Cylinder d. Triangular based pyramid

2. Write down the name of the 3D shape represented by each of these nets.

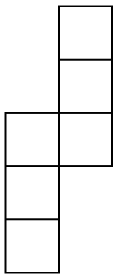
a.



b.

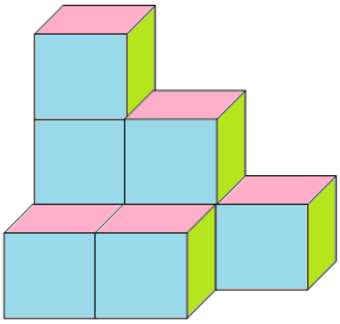


c.

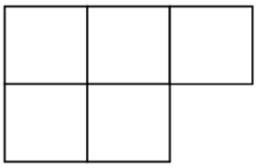


3D shapes can also be represented by plans and elevations.
The plan is the view from the top of the shape, it is what you would see if you were looking down at a shape. The elevation can be taken from any of the sides.

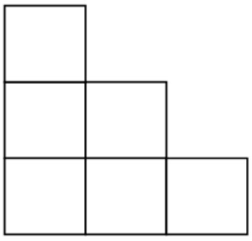
Example:
Given the 3D shape below, draw the plan, the front elevation and the side elevation.
The colours have been added for ease of reference.



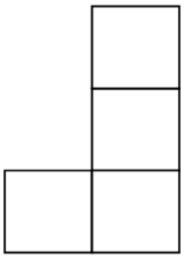
Plan: The plan is drawn as if you were looking down at the shape, in this case you would see the pink squares.



Front elevation: This is drawn as if you were looking at the shape from the front, in this case you would see the blue squares.



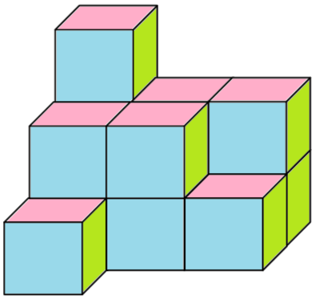
Side elevation: This is draw as if you were looking at the shape from the side, in this case you would see the green squares.



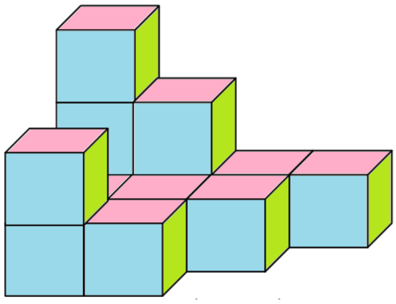
Activity 8.6 B

1. Draw the plan, the front elevation and the side elevation for each of these shapes.

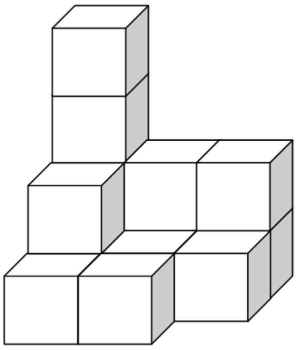
a.



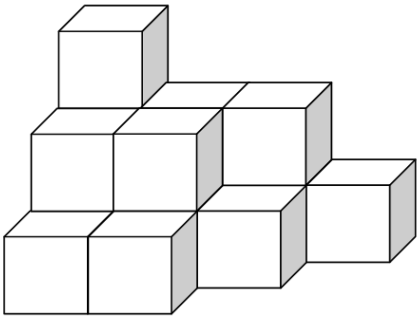
b.



c.

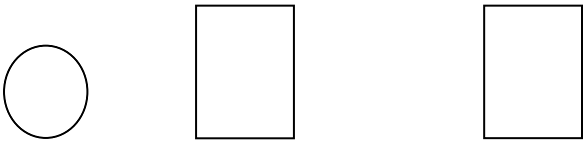


d.

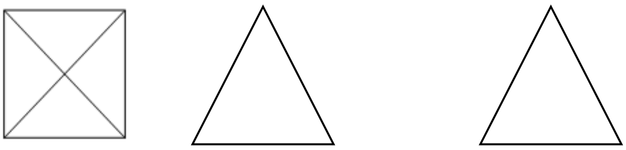


2. Write down the name of each of these shapes

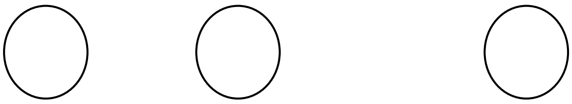
a. Plan Front elevation Side elevation



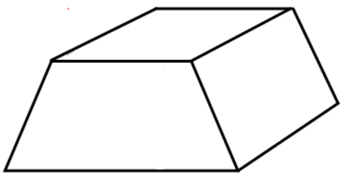
b. Plan Front elevation Side elevation

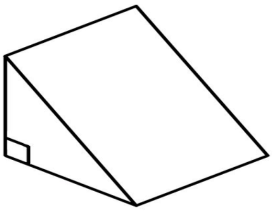


c. Plan Front elevation Side elevation



3. Draw a plan, front elevation, side elevation and net for these shapes.





Volume and Surface Area

The volume of a 3D shape is the amount of space inside it, when it contains liquid we talk of its capacity instead of volume though they are found in the same way.
Volume is measured in units cubed.

The volume of a prism – this includes cubes, cuboids and cylinders – is found by finding the area of a cross section and multiplying by the height. By "cross section" we usually mean one end.

The volumes of other 3D shapes are found as follows:

$\text{Volume of a pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$

$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$ (where r is the radius)

$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$ (where r is the radius and h is the perpendicular height)

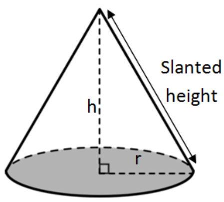
The surface area of a 3D shape is the total area of each of the faces. For example, a cube has six square faces so you would find the area of one of these squares and multiply it by 6 to find the total surface area. When the 3D shape has faces that are polygons it is relatively straight forward to find the surface area by finding the area of each face in turn.

The surface areas of other 3D shapes are found as follows:

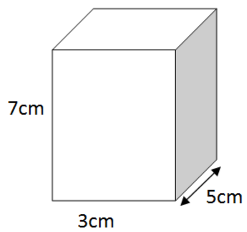
$\text{Surface area of a sphere} = 4\pi r^2$

$\text{Curved surface area of a cylinder} = 2\pi rh$ (The total surface area would include the circles at either end as well)

$\text{Curved surface area of a cone} = \pi r \times \text{slanted height}$ (As above, total surface area also includes the area of the circle at the base)



Example:
Find the volume and the surface area



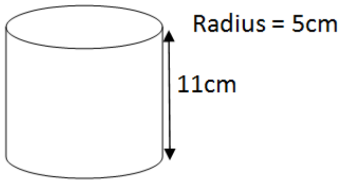
As this is a prism the volume is found by multiplying the area of a cross section by the height. In the case of cuboids this is as simple as doing height x length x width. So we have

Volume = 7 x 3 x 5 = 105cm^3

The area of the front and back faces are 3 x 7 = 21cm^2
The area of the side faces are 5 x 7 = 35cm^2
The area of the top and bottom faces are 3 x 5 = 15cm^2

Therefore the total area is 21 + 21 + 35 + 35 + 15 + 15 = 142cm^2

Example:
Find the volume and the surface area

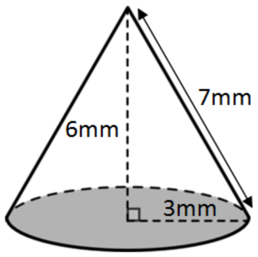


The volume is the area of a cross section multiplied by the height. The area of the circle is $\pi \times 5^2 = 25\pi$
Therefore the volume is 25π x 11 = 275π $\cong 863.94\text{cm}^3$

The area of the curved surface is $2\pi\text{rh} = 2 \times \pi \times 5 \times 11 = 110\pi\ \text{cm}^2$
As found earlier the area of each of the circles is 25π

Therefore the surface area of the cylinder is 25π + 25π + 110π = 160π $\cong 502.65\text{cm}^2$

Example:
Find the volume and the surface area



The volume of a cone is found with $\frac{1}{3}\pi\text{r}^2\text{h} = \frac{1}{3} \times \pi \times 3^2 \times 6 = 18\pi \cong 56.55\text{mm}^3$

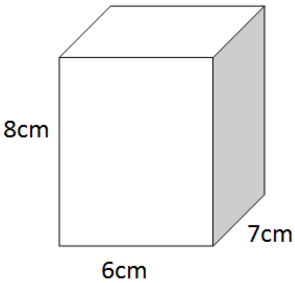
The area of the curved surface is $\pi\text{r} \times \text{slanted height} = \pi \times 3 \times 7 = 21\pi\ \text{mm}^2$
The area of the circular face is $\pi\text{r}^2 = \pi \times 3^2 = 9\pi\ \text{mm}^2$

Therefore the total surface area is $21\pi + 9\pi = 30\pi \cong 94.25\text{mm}^2$

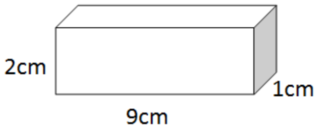
Activity 8.6 C

1. Find the volume and total surface area for each of these shapes

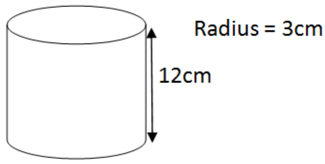
a.



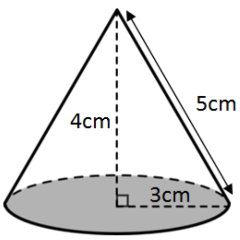
b.



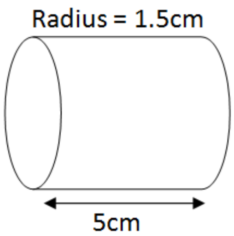
c.



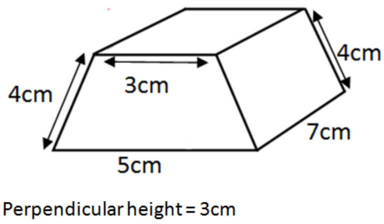
d.



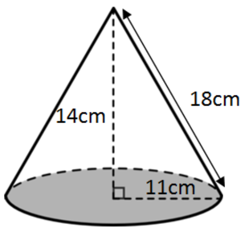
e.



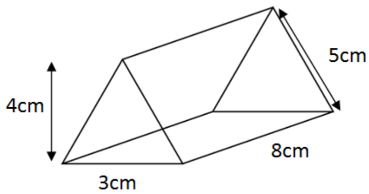
f.



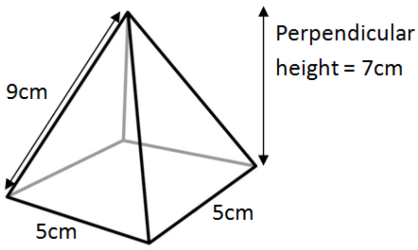
g.



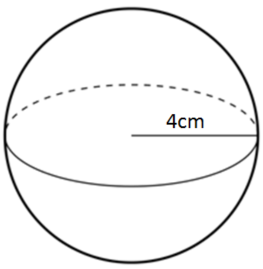
h.



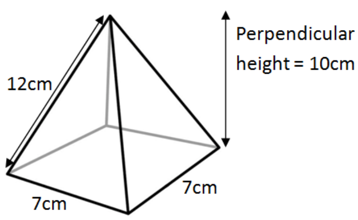
i.



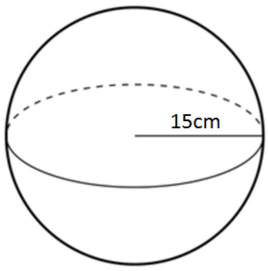
j.



k.

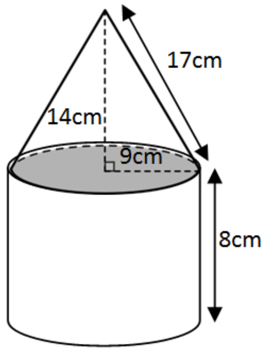


l.

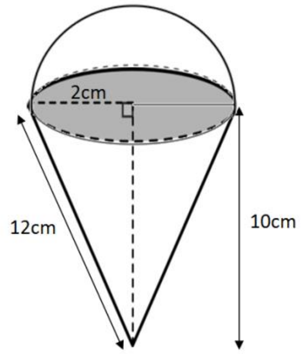


2. Find the volume and surface area of each of these shapes

a.



b.



3. Recall that $\text{Mass} = \text{Volume} \times \text{Density}$. Using the fact that the density of gold is 19.3g/cm^3, find the mass of a gold bar with height 5cm, width 4cm and length 11cm.

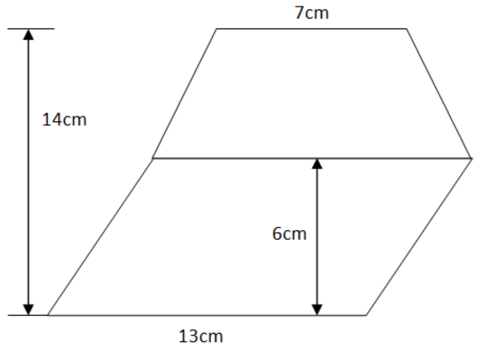
4. A cylinder with height 6cm has a volume of 320cm^3. Find the radius.

5. Unsharpened pencils have a diameter of 9mm and a length of 12cm. They are composed of a cylinder of graphite, with a diameter of 2mm, which is surrounded by wood. Assume that graphite has a density of 2.3 g/cm^3 and wood has a density of 0.5g/cm^3. Find the mass of 100 pencils.

ASSIGNMENT SEVEN

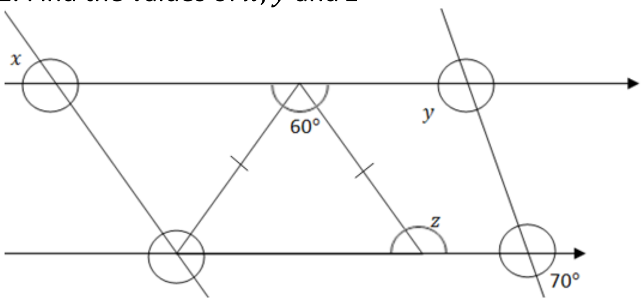
Answers to these questions are not provided. You should send your work to your tutor for marking.

1. Find the area of this shape



(4)

2. Find the values of x, y and z



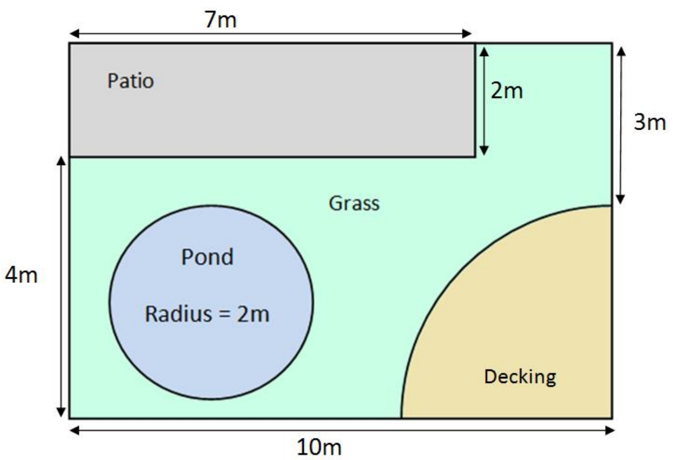
(6)

3. A regular pentagon has angles of 95°, 182°, 63° and 75°. What size is the remaining angle? (3)

4. A regular quadrilateral has angles $x°, 2x°, (x + 10)°$ and $(x + 20)°$. Find the value of x. (3)

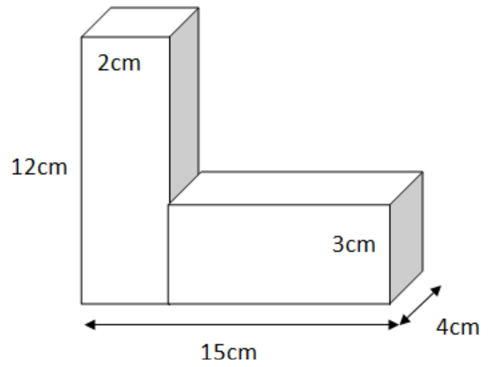
5. Draw a rectangle with sides of 4cm and 5cm. Draw the locus of the points that are no more than 2cm away from this rectangle. (3)

6. Given the plan of the garden below find the area of the grass (7)

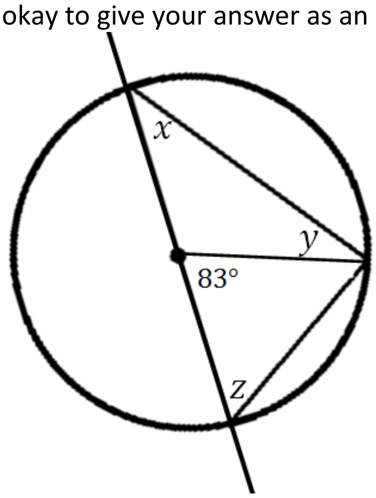


7. A hemisphere has a radius of 5.5cm. Find the volume. (3)

8. Find the volume and surface area of this shape (10)



9. Find the values of x, y and z. Give reasons for your answers. (3)
It is okay to give your answer as an annotated diagram provided that it is clear.



Total marks 42