GCSE Mathematics Higher

AQA Specification 8300

Part 1

Glossary

Symbols & Annotations

+	Addition
-	Subtraction
×	Multiplication
÷	Division
$\pm$	Positive or negative
=	Equal to
≠	Not equal to
≅	Approximately equal to
<	Less than
>	Greater than
$\leq$	Less than or equal to
≥	Greater than or equal to
∝	Proportional
( )	Brackets
[ ]	Brackets used when ( ) are already in place
{ }	Set of elements
HCF	Highest Common Factor
LCM	Lowest Common Multiple
LHS	Left hand side
RHS	Right hand side
π	Pi (3.141592....)
$\sqrt{}$	Square root
$\sqrt[3]{}$	Cube root
dp	Decimal place
sf	Significant figure
∴	Therefore
≡	Identical to
≢	Not identical to
%	Percent
$f(x)$	Function of x
θ	Theta - A Greek letter often used to represent unknowns such as angles
α	Alpha – A Greek letter often used to represent unknowns such as angles
β	Beta - A Greek letter often used to represent unknowns such as angles
∞	Infinity
°	Degrees – used to measure angles
∈	Is an element of
ε	Universal set
∅	Empty set
∩	Intersection/And
∪	Union/Or

Congruent Triangles

SSS	Three equal sides
SAS	Two sides and the angle between them is equal
ASA	Two angles and a corresponding side are equal
RHS	A right angle, the hypotenuse and another side equal

Terminology

Acute Angle	An angle less than 90°
Arc	Part of the circumference of a circle
Arithmetic Sequence	A sequence of numbers that increases or decreases by a constant difference
BIDMAS	Brackets, Indices, Divide, Multiply, Add, Subtract
Chord	A line inside a circle touching the circumference at either end but not passing through the centre
Circumference	The perimeter of a circle
Coefficient	The number before a variable
Congruent	Shapes that are identical sizes
Cube Number	The result of multiplying a number by itself 3 times
Cube Root	The number multiplied by three times itself to give the cube number
Cubic	The highest power in an expression or equation is 3
Decimal Place	The number of digits after the decimal point
Degree (of expression)	The highest power
Denominator	The number at the bottom of a fraction
Diameter	A line that passes from one side of a circle to the other through the centre
Discrete Data	Data whereby only a finite number of values is possible.
Equation	A collection of terms with an equal sign
Equilateral	An equilateral triangle has all the sides the same and all the angles the same
Expression	A collection of terms with no equal sign
Factorise	Put into brackets
Factors	Numbers that divide into a given number exactly
Fibonacci Sequence	A sequence in which each term is the sum of the previous two terms
Formula	An equation with one or more variable

Geometric Sequence	A sequence of numbers with a constant ratio between the terms
Gradient	The steepness of a line
Highest Common Factor	The HCF of two numbers is the largest number that is a factor of them both
Hypotenuse	The longest side of a triangle
Identity	An equation that is true for all values of the variables
Index / Indices	See "power"
Integer	Whole number
Interquartile Range	The difference between the upper and lower quartiles
Improper Fraction	Also "top heavy fraction". A fraction where the numerator is bigger than the denominator.
Irrational Number	Infinite number of decimals places in no apparent pattern
Isosceles	A triangle with two equal sides and two equal angles.
Linear	The highest power is 1
Lowest Common Multiple	The LCM of two numbers is the smallest number that is a multiple of them both
Mean	An average calculated by adding all the data values and dividing by the number of data values
Median	An average calculated by finding the middle value of a data set that is arranged in ascending order
Mixed Number	A number consisting of an integer and a fraction
Mode	An average calculated by finding the most common data value within a set
Multiples	Result of multiplying the given number by an integer
Negative Number	Less than zero
Numerator	The number at the top of a fraction
Obtuse Angle	An angle bigger than 90° but less than 180°
Origin	The point (0,0) where the axes cross

Outlier	A data value within a set that is distinctly different from the rest of the values
Parallel	Parallel lines have the same gradient and are the always the same distance apart
Perpendicular	Perpendicular lines meet at right angles
Polygon	A 2D shape with straight sides
Prime Factor	A factor of a number that is also prime
Prime Number	A number that only has two factors; 1 and itself
Proper Fraction	A fraction where the numerator is smaller than the denominator
Quadratic	The highest power in the expression or equation is 2
Quadratic Sequence	A sequence in which the differences between the terms form an arithmetic sequence.
Radius	The distance from the centre to the circumference of the circle.
Range	An average calculated by subtracting the lowest data value from the highest data value
Reciprocal	Interchange the numerator and the denominator
Reflex Angle	An angle bigger than 180°
Right Angle	An angle of 90°
Roots	The solution to $f(x)=0$ for some function $f(x)$
Sector	The area formed between the circumference and two radii
Segment	The area enclosed between the chord and the circumference
Significant Figure	The number of digits after the first non zero number
Scalene	A triangle whereby all of the sides and angles are different.
Solve	Find the value(s) of the unknown(s)
Square Number	The result of multiplying a number by itself
Square Root	The number multiplied by itself to give the square number
Subject	The subject of an equation is the term that is alone of one side
Surd	A square root with an irrational answer

Tangent	A line outside of a circle that touches the circumference
Term	Group of numbers/symbols/letters separated by +, - or =
Top Heavy Fraction	Also “improper fraction”. A fraction where the numerator is bigger than the denominator.
Triangular Numbers	A sequence of numbers with differences 2, 3, 4, 5...
Turning Point	The point at which a graph changes from rising to falling or falling to rising.
Variables	Unknown values represented by letters or symbols
x intercept	The value of x when a line crosses the x axis
y intercept	The value of y when a line crosses the y axis

Section One: Number

Chapter One: Structure and Calculations

1.1 Positive and Negative Numbers

Ordering

Positive numbers are those that are larger than zero, negative numbers are smaller than zero.

-5 -4 -3 -2 -1 0 1 2 3 4 5

Negative Numbers Positive Numbers

Signs and symbols you need to know:

< means less than	≤ means less than or equal to
> means greater than	≥ means greater than or equal to

The easy way to remember how to use these is to place the smaller end of the symbol pointing towards the smaller number. For example, 3 < 5 reads "three is less than five" and 6 > 4 reads "six is greater than four".

Other signs that are important to remember:

= means equal to	≠ means not equal to

Activity 1.1A

1. Put the correct symbol = or ≠ between each pair of numbers:

3	4
390	three hundred and ninety
9.6	nine point 5
3	-3

2. Put the correct symbol < or > between each pair of numbers:

6	9
3,900	three thousand and nine
- 5	5
42	40 + 3
- 6	- 7

- 10 - 100

- 9 2

3. Put each list of numbers in order, starting with the smallest:
   a. 47, 78, 32, 14, 11
   b. 7, 77, 17, 117, 777
   c. -7, 0, 2, -5, 3
   d. -9, -4, -11, -5, 8
   e. 10, 19, -10, -19, -18, -17
   f. 29, -38, 56, -90, -100, 100

Adding and Subtracting

Addition and subtraction are inverses, they undo each other.
For example, 4 – 3 = 1. 1 + 3 = 4

There are two important rules to remember when adding and subtracting negative numbers:
- Adding a negative number is the same as subtracting a positive number
- Subtracting a negative number is the same as adding a positive number

When you have two signs written together you can think of it as:
- Two of the same sign make a positive
- Different signs make a negative

Example:
- 5 + - 4 = - 5 - 4 = -9

6 - - 3 = 6 + 3 = 9

+ 3 + - 4 = 3 – 4 = - 1
(Sometimes positive numbers have + in front of them to show that they are positive but this sign is usually omitted, if a number doesn't have a sign in front of it assume it is positive.)

Activity 1.1B

1. Calculate:

a. 150 + 180	b. 130 + 280	c. 120 + 180	d. 190 + 310
e. 370 – 190	f. 270 – 120	g. 800 – 210	h. 170 – 160

2. Calculate:

a. 23 + 26	b. 19 + 39	c. 389 + 193	d. 189 – 24	e. 178 – 173
f. 268 + 183	g. 178 – 37	h. 289 – 182	i. 839 + 182	j. 289 – 134

3. Calculate:

a. 3 – 4	b. 6 – 8	c. – 5 – 6	d. – 7 – 3	e. – 6 + 3

f. – 7 + 8	g. 5 – + 6	h. 3 – + 4	i. 6 – – 6	j. – 2 – – 4
k. 7 – 10	l. 3 – 5	m. – 3 – 5	n. – 8 – 10	o. – 3 + 5
p. – 2 + 6	q. 8 – + 6	r. – 5 + – 4	s. 9 – – 10	t. – 3 – – 2

Multiplication and Division

Multiplication and division are inverses. For example 12 ÷ 4 = 3. 3 x 4 = 12
It is important that you know the times tables up to 12 x 12.

The important rules to remember when multiplying or dividing with negative numbers are:

Positive x Positive = Positive	Positive ÷ Positive = Positive
Negative x Negative = Positive	Negative ÷ Negative = Positive
Positive x Negative = Negative	Positive ÷ Negative = Negative
Negative x Positive = Negative	Negative ÷ Positive = Negative

The easiest way to remember all of these is:
Same signs = Positive   Difference signs = Negative

Order of Operations

If there is more than one operation the order you complete them in is brackets, indices (also known as powers), division, multiplication, addition and subtraction. The last two have equal importance and are completed from left to right. This is remembered by the acronym : BIDMAS.

Brackets
Indices
Division
Multiplication
Addition
Subtraction

Example:
50 ÷ (15 – 10)
= 50 ÷ 5   *Brackets first*
= 10

Example:
$\frac{45-5}{2\times 2}$
= (45 – 5) ÷ (2 x 2)   *Remember that a fraction is the same as dividing the top by the bottom*
= 40 ÷ 4
= 10

Example:
4 + 5 x 2 – 3
= 4 + 10 – 3   *Multiplication first*
= 11

Activity 1.1C

1. Fill in the blank spaces:

a. 8 x 5 = 40
5 x __ = 40
40 ÷ 5 = __
__ ÷ 8 = 5

b. 9 x 2 = 18
__ x 9 = 18
18 ÷ __ = 2
18 ÷__ = 9

2. Calculate:

a. – 2 x – 4	b. – 3 x 6	c. – 4 ÷ – 2	d. – 10 ÷ 5	e. – 5 x 6	f. 10 x – 5
g. 50 ÷ – 5	h. – 40 ÷ 8	i. 6 x – 2	j. – 8 x – 2	k. – 30 ÷ – 5	l. 10 ÷ – 2
m. 5 x – 3	n. – 2 x – 6	o. – 40 ÷ 4	p. – 5 ÷ 5	q. – 15 ÷ 3	r. 11 x – 3

3. Calculate:

a. 3 x (15 – 7)	b. 12 + 7 – 8	c. 4 + (10 – (3 – 2))
d. 100 ÷ (5 x (3 + 2))	e. 4 + 1 – 6 + 5	f. 1 x 2 x (4 – 1)
g. 10 + 8 ÷ 2 – 1	h. 35 ÷ 7 + 11 x 2 – 11	i. 3 x 4 + 2 – 5 + 10

4. Calculate:

a. $5^2 + 18 \div 2 - 3$ b. $18 - 10 \div 5 + 3^2$

5. Calculate:

a. $\frac{9-1}{1 \times 2}$ b. $\frac{4\times 4}{8}$ c. $4 + \frac{5\times 3}{1+2}$

1.2 Place Value

The value of each digit in a number is determined by its place value.
For example, in the case of the number 444 the first digit represents 400, the second 40 and the third 4. That is 4 hundreds, 4 tens and 4 units.

Thousands	Hundreds	Tens	Units	•	Tenths	Hundredths	Thousandths
3	5	7	1	.	3	2	4

In the example above the number 3571.324 can be seen to have 3 thousands, 5 hundreds, 7 tens, 1 unit, 3 tenths, 2 hundredths and 4 thousandths.

To multiply a number by 10, 100, 1000... move all of the digits to the left. The number of spaces the number is moved is determined by the number of zeros in the number you're multiplying by. So, if you're multiplying by 100, for example, you would move the number two places to the left because 100 has two zeros. When dividing the digits are moved to the right, again with the number of places determined by the number of zeros.

Example:

63 x 10 = 630

Hundred	Tens	Units
	6	3

Hundred	Tens	Units
6	3	0

This can also be thought of as moving the decimal place

Example:
2831 ÷ 100 = 28.31

Thousands	Hundred	Tens	Units	.	Tenths	Hundredths	Thousanths
2	8	3	1				

Thousands	Hundred	Tens	Units	.	Tenths	Hundredths
		2	8	.	3	1

Activity 1.2

1. Write each of these numbers in figures
   - a. Ninety two
   - b. Six hundred and forty three
   - c. Four hundred and two
   - d. One thousand, one hundred and one
   - e. Twenty two thousand, six hundred and forty two
   - f. One hundred and twenty six thousand, three hundred and ninety one
   - g. Two hundred thousand, three hundred and fifteen
   - h. Nine hundred and two thousand, four hundred and two
   - i. Six million and fourteen

2. Write each of these numbers in words

a. 823
b. 124
c. 5729
d. 2048
e. 1070
f. 6003
g. 80 820
h.273 948
i. 30 021
j. 900 008
k. 8 902010

3. What value does the 3 represent in each of these numbers?

a. 4300 b. 130 c. 13000 d. 19 903 e. 7 867 839

4. Calculate:

a. 12 x 10	b. 827 x 100	c. 298 ÷ 10
d. 378 x 1000	e. 8290 ÷ 10	f. 18200 x 10
g. 9.2 x 10	h. 0.039 x 100	i. 39.2 ÷ 10
j. 0.0002 x 1000	k. 28 ÷ 1000	l. 394 ÷ 100

1.3 Powers and Roots

An index is a power. A base is a number that is raised to a power. The index tells you how many times to multiply the base by itself.
For example, with 2^4 the base is 2 and the index is 4.
$2^4 = 2 \times 2 \ \times 2 \times 2 = 16$

A **square** number is the result of multiplying an integer (whole number) by itself; the index is two.

3^2 is read as "three squared"
The square numbers are found as follows
$1^2 = 1 \ \times 1 = 1$
$2^2 = 2 \ \times 2 = 4$

You need to know all of the square numbers up to 15 x 15.

Activity 1.3A

Find all of the square numbers from 1^2 to 15^2

The **square root** of a given number is a number that, when multiplied by itself, results in the given number.
For example, the square roots of 144 are 12 and – 12 because 12 x 12 = 144 and – 12 x – 12 = 144
In most cases we only need the positive answer but it is important to remember that every square number has both positive and negative square roots.

"The square root of 144" is written as $\sqrt{144}$
Very rarely you will see $\sqrt[2]{144}$ which means the same thing but the 2 is usually omitted.

A **cube** number is the result of multiplying an integer by itself three times; the index is three.

9^3 is read as "nine cubed"

The **cube root** of a given number is a number that, when multiplied by itself three times, results in the given number.
For example the cube root of 8 is 2 because 2 x 2 x 2 = 8. In this case there is only one root, no positive cube number can have a negative cube root. For example – 2 x – 2 x – 2 = – 8.

"The cube root of 8" is written as $\sqrt[3]{8}$

Other facts you need to know:
$10^3 = 1000$
$10^6 = 1\ 000\ 000 = 1 \text{ million}$

Activity 1.3B

1. Work out the first five cube numbers.

2. Find a cube number that is the sum of two square numbers.

3. Write down the value of:
   a. 3^2 b. 4^2 c. 8^2 d. 7^2 e. 9^2 f. 10^2 g. 6^2 h. 5^2 i. 13^2

4. Write down the value of:
   a. $\sqrt{36}$ b. $\sqrt{196}$ c. $\sqrt{4}$ d. $\sqrt{64}$ e. $\sqrt{225}$ f. $\sqrt{121}$

5. Using trial and error work out the value of:
   a. $\sqrt[3]{125}$ b. $\sqrt[3]{27}$ c. $\sqrt[3]{64}$

6. Work out the value of:
   a. 2^5 b. 3^4 c. 10^5 d. 1^{15} e. 2^6

1.4 Factors and Multiples

The factors of a number divide into it leaving no remainder. For example, the factors of 6 are 1, 2, 3 and 6.
Multiples of a given number are the numbers we get after multiplying said number by another whole number. For example the first three multiples of 10 are 10, 20 and 30. The first three multiples of 5 are 5, 10 and 15.

Divisibility Tests

Two

Is the number even? (Ending with 0,2,4,6 or 8)

- Yes: Two is a factor
- No: Two is not a factor

Three

Add the digits in the number together
Is this number divisible by 3?

- Yes: Three is a factor
- No: Three is not a factor

Four

Is the number even? (Ending with 0,2,4,6 or 8)

- Yes
  - Halve the number
    Is this answer even?
    - Yes: Four is a factor
    - No: Four is not a factor
- No: Four is not a factor

Five

Does the number end in 0 or 5?

- Yes: Five is a factor
- No: Five is not a factor

Six

Is the number even? (Ending with 0,2,4,6 or 8)

- Yes
  - Is the number divisible by 3?
    - Yes: Six is a factor
    - No: Six is not a factor
- No: Six is not a factor

Eight

Is the number even? (Ending with 0,2,4,6 or 8)

Yes

No
Eight is not a factor

Halve the number
Is this answer even?

Yes

No
Eight is not a factor

Halve the number again
Is this answer even?

Yes
Eight is a factor

No
Eight is not a factor

Nine

Add the digits in the number together
Is this number divisible by 9?

Yes
Nine is a factor

No
Nine is not a factor

Ten

Does the number end in 0?

Yes
Ten is a factor

No
Ten is not a factor

Activity 1.4A

1. List all of the factors of the following numbers:
   a. 12 b. 10 c. 20 d. 24 e. 18 f. 9

2. Write down the first five multiples of the following numbers:
   a. 3 b. 2 c. 5 d. 11 e. 10 f. 9

3. Is 172 divisible by 3? How do you know?

4. Is 283 290 divisible by 10? How do you know?

5. Is 279 274 191 divisible by 2? How do you know?

6. Is 918 divisible by 9? How do you know?

Prime Numbers

A **prime number** is a number that has only two factors: 1 and itself.

Sieve of Eratosthenes – Optional
If you wish to find all of the prime numbers between 1 and 100 yourself draw a 1-100 grid then cross out all except the first multiples of 2, 3, 5, 7 and 11. (Using these also covers 4, 6, 8, 10 and 12 because any multiples of these numbers must be even and therefore must also be multiples of 2. Multiples of 9 are also multiples of 3). Cross out number 1. The numbers you are left with are the prime numbers up to 100.

The first ten prime numbers are:
2, 3, 5, 7, 11, 13, 17, 19, 23 and 29

One is not a prime number because it does not have two factors, it only has one.

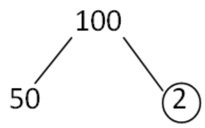
A **prime factor** is a factor of a number that is also prime. Every integer can be written as a product of its prime factors. In order to find the prime factors of a number we will use a factor tree.

First you split the number into a factor pair, this pair need not necessarily contain a prime number. With each of your new numbers repeat the process of splitting it into a factor pair, if you reach a prime number circle it and stop that branch.

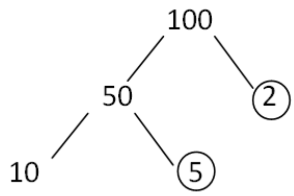
The first example below shows step by step what you would write; when you do your own factor trees simply continue the tree until you have finished as in the second example.

Example: Write 100 as a product of its prime factors

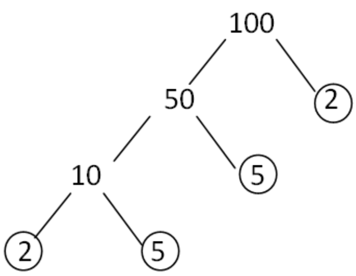
100 = 50 x 2 so we will split 100 into these factors. Since 2 is a prime number we will circle it.



Next we will split 50 into two factors, here we use 50 = 5 x 10. Five is a prime number so we circle it.



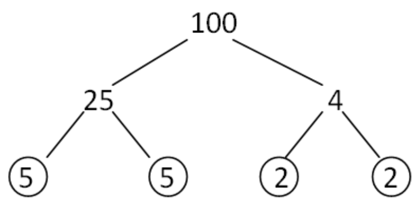
Now we have to split 10 into two factors; the only option is 10 = 2 x 5 as these are both prime numbers we will circle them both and the tree is complete.



So we have 100 = 2 x 2 x 5 x 5
Using index notation we write $100 = 2^2 \times 5^2$

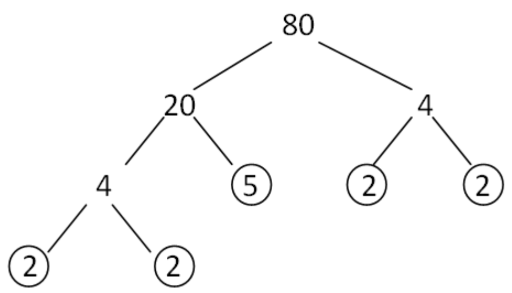
It doesn't matter which way you draw your branches or which factor pairs you chose – using different pairs will still give you the same answer as you can see below.

This time we will use 100 = 25 x 4 on the first level. On the second level we will use 25 = 5 x 5 and 4 = 2 x 2.



This gives $100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$ as before.

Example: Write 80 as a product of prime factors



$80 = 2^4 \times 5$

Activity 1.4B

Write each of the numbers as product of prime factors:

a. 18	b. 84	c.45	d. 60	e. 24
f. 195	g. 1620	h.720	i. 945	j. 10400

Highest Common Factor and Lowest Common Multiple

The **highest common factor** (HCF) of a two numbers is the largest number that is a factor of them both.
The **lowest common multiple** (LCM) of two numbers is the smallest number that is a multiple of both of them.

In order to find the HCF or LCM you first need to find the prime factor decomposition of each of the numbers – this is simply writing it as a product of prime factors without using index notation.

The HCF is the overlap of the two decompositions. The LCM is the product of the overlap *and* the remaining numbers.

Example:
Find the HCF and LCM of 80 and 100

From above we know that 80 = 2 x 2 x 2 x 2 x 5 and that 100 = 2 x 2 x 5 x 5.
Write these sums above each other so the numbers line up and circle those that match.

80 =	2	x	2	x	2	x	2	x	5		
100 =	2	x	2					x	5	x	5

HCF is the overlap of the two decompositions, that is the ones we have circled. So, here we have 2 x 2 x 5 = 20
LCM is the overlap and the remaining numbers. Here the remaining numbers, those that aren't circled, are 2 and 5 so we have 20 x 2 x 2 x 5 = 400.

So the highest common factor of 80 and 100 is 20 and the lowest common multiple is 400.

Example:
Find the HCF and the LCM of 180 and 300

Using a prime factor tree we find that
$180 = 2^2 \times 3^2 \times 5$
$300 = 2^2 \times 3 \times 5^2$

Writing these out in line with one another gives

180 =	2	x	2	x	3	x	3	x	5		
300 =	2	x	2	x	3			x	5	x	5

The highest common factor is: 2 x 2 x 3 x 5 = 60
The lowest common multiple is: 60 x 3 x 5 = 900

Activity 1.4C

Find the highest common factor and lowest common multiple of each of these pairs of numbers:

a. 240 and 180 b. 35 and 20 c. 50 and 80 d. 15 and 9

ASSIGNMENT ONE

Answers to these questions are not provided. You should send your work to your tutor for marking.
No calculators are allowed – show all of your working.

1. Calculate:
    a. -2 x 4   b. -5 - 10   c. -90 ÷ -9   d.-9 - -5   (4)

2. Calculate:
    a. 13 + 5 x 4 – 6   b. 2 x (5 + (4 – 1)) – 2   c. $\frac{5\times2+4}{15-2\times4}$ (3)

3. Calculate:
a. 0.0023 x 10   b. 12.34 x 100   c. 123.456 ÷ 10^2 d. 267.19 x 10^3 (4)

4. Find the value of $10^5 \times 10^2$ (1)

5. Is 6392 divisible by 9? How do you know?   (1)

6. Write down a multiple of 3 between 214 and 220   (1)

7. Some numbers are in the box below. Choose from these numbers to answer the questions below.

25		19		2		10
	24		49		3	
8		7		64		5

a. Write down all the square numbers.
b. Write down all the prime numbers.
c. Write down all the factors of 24.
d. Write down all the multiples of 3.   (4)

8. Write 1800 as a product of prime factors.   (2)

9. Find the highest common factor and lowest common multiple of 50 and 32   (4)

10. Find the highest common factor and lowest common multiple of 90 and 100   (4)

Total 28 marks

Chapter Two: Fractions and Decimals

2.1 Decimals

Ordering Decimals

To order decimals you have to consider the place value of each digit (see section 1.2). For example 0.1 is greater than 0.09 because 1 hundredth is greater than 9 thousandths.

The easiest way to order decimals is to put in arbitrary zeros such that they all contain the same number of digits after the decimal point and then imagine the numbers without a decimal point.

Example:
Put these numbers in order from smallest to largest
1.09 1.1 1.3 1.11 1.03 1.42 1.2

We can add zeros after the decimal point without changing the value of the number. For example, 1 is the same as 1.0. If we do this with the numbers above we have
1.09 1.10 1.30 1.11 1.03 1.42 1.20

Temporarily removing the decimal points gives
109 110 130 111 103 142 120

These are now easy to put in ascending order
103 109 110 111 120 130 142

Now all that's left to do is put the decimal points back in and remove the extra zeros
1.03 1.09 1.1 1.11 1.2 1.3 1.42

Of course you don't have to write out all of these steps individually when you're doing your own working.

Activity 2.1A

Put each of these lists of numbers in order from smallest to largest

a. 2.4 3.5 2.1 2.5 2.7 3.7

b. 4.6 4.2 4.87 4.8 4.9 4.1

c. 0.04 0.13 0.5 1.3 1.4 0.02

d. 0.05 0.16 0.3 0.012 0.001 0.46

e. 5.05 500.05 500.5 5.0005 5.005 50.05

Adding and Subtracting

In order to add or subtract decimals you can either use column addition/subtraction or partitioning.

If you use the column method it is done in the same way as you would if you were using integers, making sure that you line the decimal points up. Any blank spaces are assumed to be zero.

Example:

23.15 + 35.21

$$\begin{array}{r} 23.15 \\ +\,35.21 \\ \hline 58.36 \\ \hline \end{array}$$

381.28 + 12.513

$$\begin{array}{r} 381.28 \\ +\;\;12.513 \\ \hline 393.793 \\ \hline \end{array}$$

58.92 – 43.2

$$\begin{array}{r} 58.92 \\ -\,43.2 \\ \hline 15.72 \\ \hline \end{array}$$

If you use partitioning you split the number into smaller parts that are easier to deal with. For example 9.2 would be split into 9 and 0.2, you would first deal with the units then the tenths.

Example:

18.5 – 2.4 = 18.5 – 2 – 0.4
= 16.5 – 0.4
= 16.1

7.2 + 1.5 = 7.2 + 1 + 0.5
= 8.2 + 0.5
=8.7

15.2 + 10.45 = 15.2 + 10 + 0.45
= 25.2 + 0.45
= 25.65

17.3 – 12.25 = 17.3 –12 – 0.2 – 0.05
= 5.3 – 0.2 – 0.05
= 5.1 – 0.05 *(= 5.10 – 0.05)*
=5.05

Multiplying and Dividing

When multiplying decimals you remove the decimal point and treat the numbers as integers. For example 9.5 would be treated as if it were 95.
Once you have completed the multiplication count how many numbers came after the decimal point in the question and, in your answer, put the decimal point back in by counting that many places from the right.

Example:
2.5 x 1.2

Work out 25 x 12 using your preferred method, here we use partitioning:
25 x 12 = (25 x 10) + (25 x 2) = 250 + 50 = 300

In the question there were two numbers after the decimal point (one in 2.5 and one in 1.2).
Counting in two places from the right gives 3.00
So we have 2.5 x 1.2 = 3

Example:
1.11 x 2.2

Work out 111 x 22 using your preferred method, here we use the grid method:

X	100	10	1
20	2,000	200	20
2	200	20	2

2,000 + 200 + 20 + 200 + 20 + 2 = 2442

In this question there are three numbers after the decimal point (two in 1.11 and one in 2.2) so counting in three places from the right gives 2.442
So we have 1.11 x 2.2 = 2.442

In this text we use the short division method, also known as bus stop division to divide decimals. You can use long division if you prefer.
If you are dividing a decimal by an integer then you carry out the calculation as normal. If the number you are dividing by is a decimal move the point the required number of spaces in order to make it an integer then move the decimal point in the other number the same number of spaces in the same direction.

Example:
13.5 ÷ 9
Here the number we are dividing by is an integer so carry out the calculation normally.

$$\begin{array}{r|l} \multicolumn{1}{r}{} & 1\,.\,5 \\ \cline{2-2} 9 & 1^{1}3\,.\,{}^{4}5 \end{array}$$

So we have 13.5 ÷ 9 = 1.5

Example:
9.45 ÷ 1.5
Here the number we are dividing by has one number after the decimal point so we move the point one place to the right in both numbers to give:
94.5 ÷ 15

Now we can carry out the calculation as before

$$\begin{array}{r|l} \multicolumn{1}{r}{} & 6\,.\,3 \\ \cline{2-2} 15 & 9^{9}4\,.\,{}^{4}5 \end{array}$$

So we have 94.5 ÷ 15 = 6.3 which means that 9.45 ÷ 1.5 = 6.3

Activity 2.1B

1. Calculate:

a. 3.4 + 2.3	b. 3.5 + 8.4	c. 8.9 + 2.2	d. 27.3 + 4.2
e. 23.45 + 34.12	f. 28.31 + 29.31	g. 3.23 + 29.12	h. 18.2 + 12.12
i. 234.23 – 231.11	j. 72.46 – 21.23	k. 2.31 – 1.24	l. 124.28 – 12.42

2. Calculate:

a. 1.2 x 3.3	b. 2.7 x 9.2	c. 28.3 x 21.2	d. 7.29 x 2.15
e. 2.13 x 9.1	f. 2.05 x 2.12	g. 24.21 x 2.6	h. 28.49 x 1.23

i. 34.4 ÷ 4	j. 40.2 ÷ 6	k. 32.76 ÷ 5.2	l. 26.88 ÷ 3.2
m. 7.14 ÷ 2.1	n. 33.92 ÷ 5.3	o. 14.904 ÷ 4.6	p. 3.8928 ÷ 1.2

2.2 Standard Form

Standard form is used to write very large or very small numbers. In order for a number to be in standard form you need to have one non-zero number before the decimal point which is multiplied by 10 raised to a power.
To find the power you need to count how many places you want the decimal point to move.

Example:
Write each of these numbers in standard form
a. 237 b. 0.0035 c. 280 000 d. 0.000 000 23

a. 2.37×10^2
b. 3.5×10^{-3}
c. 2.8×10^5
d. 2.3×10^{-7}

In example a the point moved 2 places to the left so the power is positive 2. In example d the point moved 7 places to the right so the power is – 7.
The easiest way to see this is to write the number out and draw the "jumps" with your pencil.
The same logic is used when converting from standard form to ordinary numbers.

Example:
Write each of these as ordinary numbers
a. 2.7×10^6 b. 4.5×10^{-3}

a. 2700000 b. 0.0045

Multiplying and Dividing

To multiply numbers in standard form you multiply the numbers, ignoring the 10s, then add the powers. To divide numbers in standard form you divide the numbers then subtract the powers. Make sure your final answer only has one number before the decimal point or it isn't in standard form, you may have to move the point and adjust the power to do this.

Example:
$(2.24 \times 10^5) \times (1.35 \times 10^4) = 2.24 \times 1.35 \times 10^{(5+4)} = 3.024 \times 10^9$

Example:
$(5.29 \times 10^9) \div (2.381 \times 10^4) = (5.29 \div 1.35) \times 10^{(9-4)} = 3.919 \times 10^5$

Example:
$(6.5 \times 10^4) \times (4.2 \times 10^3) = 6.5 \times 4.2 \times 10^{(4+3)} = 27.3 \times 10^7 = 2.73 \times 10^8$

Adding and Subtracting

It is not possible to add or subtract numbers whilst they are in standard form. You must first convert them into ordinary numbers then add or subtract them as normal before converting them back to standard form.

Example:
$(1.8 \times 10^4) + (2.1 \times 10^3) = 18000 + 2100 = 20100 = 2.01 \times 10^4$

Example:
$(4.5 \times 10^5) - (2.2 \times 10^4) = 450000 - 22000 = 428000 = 4.28 \times 10^5$

Activity 2.2

1. Convert to standard form

a. 2390000	b.2800	c.0.000273	d. 18200000	e. 0.0000137
f. 19000000	g. 129.391	h. 6780000	i. 98.93	j. 0.0000042
k. 60×10^6	l. 30×10^6	m.0.5×10^5	n. 0.4×10^7	o. 247×10^5

2. Convert to ordinary numbers

a. 4.4×10^7	b. 5.2×10^3	c. 1.34×10^5	d. 1.32×10^{-3}
e. 1.1×10^{-2}	f. 2.34×10^4	g. 1.2×10^{-4}	h. 4.1×10^{-2}

3. Calculate. Write your answers in standard form.

a. $(3 \text{ x } 10^4) \text{ x } (4 \text{ x } 10^6)$	b. $(6 \text{ x } 10^{-5}) \div (3 \text{ x } 10^{-4})$
c. $(5 \text{ x } 10^6) \text{ x } (7 \text{ x } 10^9)$	d. $(7 \times 10^2) \times (2 \times 10^5)$
e. $(1 \times 10^{-2}) \times (7 \times 10^{-2})$	f. $(1 \times 10^{-4}) \times (2 \times 10^5)$

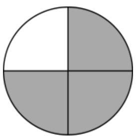
4. Calculate. Write your answers in standard form.

a. $(4.4 \times 10^2) + (1.8 \times 10^2)$	b. $(2.6 \times 10^{-6}) + (1.6 \times 10^{-6})$
c. $(5.5 \times 10^2) - (1.9 \times 10^2)$	d. $(4.9 \times 10^{-4}) - (1.3 \times 10^{-4})$
e. $(3.2 \times 10^7) + (1.7 \times 10^8)$	f. $(3.8 \times 10^7) - (1.4 \times 10^6)$

2.3 Fractions

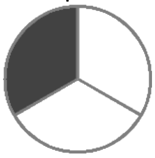
A fraction is a part of a whole when we can assume that the whole is divided into equal parts.

Example:



This shape is divided into four equal sections, three of them are shaded so we say the shape is $\frac{3}{4}$ coloured.

Example:



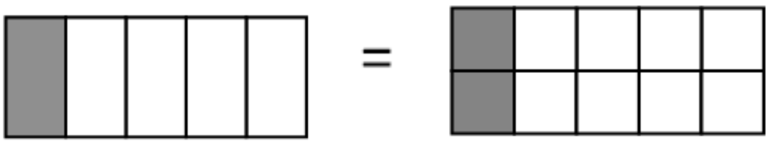
This shape is divided into three equal sections, one is coloured so we say the shape is $\frac{1}{3}$ coloured.

The number at the top of the fraction is called the **numerator**.
The number at the bottom is called the **denominator**.

Simplifying Fractions

Equivalent fractions can be found by multiplying or dividing both the numerator and the denominator by the same number.

Example:



The first diagram is $\frac{1}{5}$ shaded. The second is $\frac{2}{10}$ shaded. You can see from the diagrams that these are equivalent but, when working with fractions, we usually avoid using diagrams. Instead you can see that the second fraction is the result of multiplying the numerator and denominator of the first fraction by 2.

$$\frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}$$

The **simplest form** of a fraction is the result of dividing both the numerator and denominator by all of their common factors.

Example:
Write $\frac{5}{10}$ in its simplest form

Both the numerator and denominator are multiples of 5 so we divide them both by this.

$$\frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

Sometimes it is not easy to spot the highest common factor immediately. You can divide by any common factor to simplify the fraction then continue dividing by other common factors until you reach the simplest form.

Example:
Write $\frac{18}{60}$ in its lowest terms

Note: lowest terms means the same as simplest form.

Here you may notice that both 18 and 60 are multiples of 6, in this case you would do

$$\frac{18}{60} = \frac{18 \div 6}{60 \div 6} = \frac{3}{10}$$

However you may have only noticed that both numbers are even meaning that they are both divisible by 2 so you would do

$$\frac{18}{60} = \frac{18 \div 2}{60 \div 2} = \frac{9}{30}$$

So we have a simplified fraction but it is not yet in its lowest terms because both 9 and 30 are multiples of three. The next step would then be

$$\frac{9}{20} = \frac{9 \div 3}{30 \div 3} = \frac{3}{10}$$

As you can see this slightly longer, but arguably easier, method gives the same result.

<u>Improper Fractions and Mixed Numbers</u>

An **improper fraction** is one in which the numerator is higher than the denominator, for example, $\frac{5}{4}$. (A **proper fraction** is one in which the numerator is lower than the denominator.) These are converted to **mixed numbers** which have one integer and one fraction, for example, $1\frac{1}{4}$.

Converting Improper Fractions to Mixed Numbers

- Divide the numerator by the denominator.
- Write the answer as the integer.
- Write the remainder as the new numerator.
- The denominator does not change.

Example:
Convert $\frac{7}{3}$ to a mixed number.

7 ÷ 3 = 2 remainder 1

So $\frac{7}{3} = 2\frac{1}{3}$

Example:
Convert $\frac{11}{4}$ to a mixed number.

11 ÷ 4 = 2 remainder 3

So we have $\frac{11}{4} = 2\frac{3}{4}$

Converting Mixed Numbers to Improper Fractions

- Multiply the integer by the denominator then add this to the numerator.
- Write your answer to the above calculation as the new numerator.
- The denominator does not change.

Example:
Convert $9\frac{1}{10}$ to an improper fraction.

9 x 10 = 90
90 + 1 = 91

So $9\frac{1}{10} = \frac{91}{10}$

Example:
Convert $6\frac{2}{5}$ to an improper fraction.

6 x 5 = 30
30 + 2 = 32

So $6\frac{2}{5} = \frac{32}{5}$

<u>Ordering Fractions</u>

To order fractions you first need to ensure that they have a common denominator. They are then ordered by using the numerators.

Example:
Put these fractions in order from smallest to largest

$\frac{1}{2} \quad \frac{3}{4} \quad \frac{2}{3} \quad \frac{1}{12}$

We need to convert each of these to equivalent fractions so that they each have the same denominator. In this case each denominator is a factor of 12 so that is the number we will change it to.
In each fraction we need to multiply the numerator and denominator by an appropriate number in order to change the denominator to 12.

$\frac{1}{2} = \frac{1\times6}{2\times6} = \frac{6}{12} \qquad \frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12} \qquad \frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$

So the fractions we have now are

$\frac{6}{12} \quad \frac{9}{12} \quad \frac{8}{12} \quad \frac{1}{12}$

It is easy to see that these can be placed in increasing order as follows

$\frac{1}{12} \quad \frac{6}{12} \quad \frac{8}{12} \quad \frac{9}{12}$

When writing the final answer you should use the fractions originally given in the question:

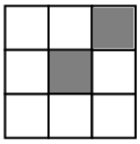
$\frac{1}{12}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$

Activity 2.3 A

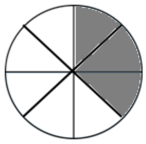
1. What fraction is shaded?

a.

b.



c.



2. Write each of these fractions in their simplest forms

a. $\frac{6}{10}$ b. $\frac{7}{14}$ c. $\frac{15}{20}$ d. $\frac{16}{32}$ e. $\frac{10}{100}$

f. $\frac{9}{99}$ g. $\frac{24}{40}$ h. $\frac{64}{144}$ i. $\frac{42}{48}$ j. $\frac{21}{28}$

3. Convert each of these improper fractions to mixed numbers

a. $\frac{11}{7}$ b. $\frac{14}{3}$ c. $\frac{22}{5}$ d. $\frac{8}{5}$ e. $\frac{9}{8}$

f. $\frac{6}{2}$ g. $\frac{19}{5}$ h. $\frac{21}{13}$ i. $\frac{7}{2}$ j. $\frac{21}{21}$

4. Convert each of these mixed numbers to improper fractions

a. $4\frac{3}{10}$ b. $2\frac{1}{4}$ c. $5\frac{3}{7}$ d. $3\frac{1}{8}$ e. $8\frac{2}{3}$

f. $9\frac{3}{4}$ g. $10\frac{11}{12}$ h. $1\frac{13}{14}$ i. $6\frac{5}{6}$ j. $8\frac{10}{11}$

5. Put each list of fractions in order from smallest to largest

a. $\frac{2}{5}$ $\frac{3}{10}$ $\frac{1}{2}$ $\frac{4}{5}$

b. $\frac{1}{6}$ $\frac{2}{3}$ $\frac{5}{12}$ $\frac{1}{2}$

c. $\frac{2}{25}$ $\frac{3}{5}$ $\frac{1}{10}$ $\frac{13}{50}$

Adding and Subtracting

You can only add and subtract fractions if they have a common denominator. As with ordering fractions the first thing you must do is convert them to equivalent fractions such that they share a denominator.

Once they have a common denominator you add or subtract the numerators only, the denominator stays the same, and then simplify your answer as far as possible.

Example:
$$\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}=\frac{5}{7}$$

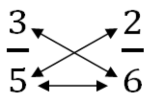
Example:
$$\frac{1}{2}+\frac{1}{4}$$

Two is a factor of four so we can double both of the numbers in this fraction so that they both have four as a denominator.
$$\frac{1}{2}+\frac{1}{4} = \frac{1\times 2}{2\times 2}+\frac{1}{4} = \frac{2}{4}+\frac{1}{4} = \frac{3}{4}$$

When the common denominator isn’t obvious we multiply in a cross fashion as shown in the diagram in the following example. Numbers at either end of each line are multiplied together.

Example:
$$\frac{3}{5}+\frac{2}{6}$$

First draw the lines as shown below, don’t worry if the diagram looks complicated at first glance, all you have to do is draw a cross then a line connecting the denominators.

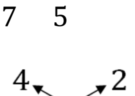


We multiply the denominators together. We also multiply the numbers at either end of the diagonal arrows (3x6 and 2x5) these remain as the numerators.

$$\frac{3}{5}+\frac{2}{6}=\frac{3\times 6+2\times 5}{5\times 6}=\frac{18+10}{30}=\frac{28}{30}$$

Now all that remains is to simplify the answer
$$\frac{28}{30}=\frac{14}{15}$$

Example:
$$\frac{4}{7}-\frac{2}{5}$$



$$\frac{4}{7}-\frac{2}{5}=\frac{4\times 5-2\times 7}{7\times 5}=\frac{20-14}{35}=\frac{6}{35}$$

To add and subtract mixed numbers first convert to improper fractions then continue as before – remembering to simplify and convert back to mixed numbers at the end.

Example:
$$1\frac{2}{3}+2\frac{3}{4} = \frac{5}{3}+\frac{11}{4} = \frac{5\times4+11\times3}{3\times4} = \frac{20+33}{12} = \frac{53}{12} = 4\frac{5}{12}$$

Activity 2.3B

1. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $\frac{3}{7}+\frac{3}{7}$ b. $\frac{3}{5}+\frac{1}{5}$ c. $\frac{5}{9}-\frac{4}{9}$ d. $\frac{3}{10}+\frac{7}{10}$ e. $\frac{10}{13}-\frac{2}{13}$

2. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $\frac{4}{5}+\frac{2}{10}$ b. $\frac{1}{6}+\frac{2}{3}$ c. $\frac{7}{10}-\frac{2}{5}$ d. $\frac{5}{9}-\frac{1}{3}$ e. $\frac{1}{2}-\frac{1}{4}$

3. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $\frac{1}{3}+\frac{1}{4}$ b. $\frac{3}{4}+\frac{3}{5}$ c. $\frac{4}{5}-\frac{2}{7}$ d. $\frac{3}{8}+\frac{1}{3}$ e. $\frac{5}{9}+\frac{2}{5}$

   f. $\frac{8}{10}-\frac{3}{8}$ g. $\frac{9}{10}-\frac{3}{13}$ h. $\frac{8}{9}-\frac{2}{3}$ i. $\frac{2}{3}+\frac{4}{5}$ j. $\frac{4}{7}+\frac{3}{8}$

4. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $1\frac{1}{4}+2\frac{1}{5}$ b. $3\frac{3}{7}-2\frac{1}{8}$ c. $6\frac{1}{2}+1\frac{2}{5}$ d. $4\frac{2}{5}+5\frac{1}{3}$ e. $2\frac{2}{3}-1\frac{3}{4}$

   f. $1\frac{2}{5}+2$ g. $\frac{6}{7}+2\frac{3}{4}$ h. $3\frac{4}{5}-\frac{7}{10}$ i. $4\frac{3}{15}+\frac{1}{2}$ j. $4\frac{5}{7}+5\frac{2}{5}$

<u>Multiplying</u>

Multiplication is, arguably, the easiest operation with fractions. Multiply the numerators, multiply the denominators then simplify your answer where possible.

Example:
$$\frac{4}{5}\times\frac{2}{3}=\frac{4\times2}{5\times3}=\frac{8}{15}$$

Example:
$$\frac{7}{10}\times\frac{5}{6}=\frac{7\times5}{10\times6}=\frac{35}{60}=\frac{7}{12}$$

In order to multiply mixed numbers you must first convert them to improper fractions before continuing as before.

As with the other operations in order to divide mixed numbers you must first convert them to improper fractions before continuing as before.

Example:
$3\frac{1}{2} \div 1\frac{1}{3} = \frac{7}{2} \div \frac{4}{3} = \frac{7}{2} \times \frac{3}{4} = \frac{21}{8} = 2\frac{5}{8}$

Activity 2.3 C

1. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $\frac{4}{5} \times \frac{1}{3}$ b. $\frac{3}{4} \times \frac{1}{2}$ c. $\frac{6}{7} \times \frac{7}{9}$ d. $\frac{8}{11} \times \frac{5}{6}$ e. $\frac{5}{7} \times \frac{6}{7}$ f. $\frac{1}{2} \times \frac{4}{9}$

2. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $2\frac{3}{4} \times 1\frac{2}{3}$ b. $1\frac{2}{3} \times 3\frac{3}{5}$ c. $1\frac{2}{7} \times 4\frac{2}{5}$ d. $5\frac{3}{5} \times 4\frac{6}{7}$ e. $\frac{5}{6} \times 10\frac{2}{11}$ f. $1\frac{5}{7} \times 2\frac{4}{7}$

3. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $\frac{1}{10} \times 3$ b. $\frac{4}{5} \times 4$ c. $10 \times \frac{5}{11}$ d. $\frac{5}{7} \times 9$ e. $11 \times \frac{5}{6}$ f. $\frac{1}{4} \times 12$

4. Write the reciprocal of each of these numbers.
   a. $\frac{2}{9}$ b. $\frac{13}{27}$ c. $\frac{1}{17}$ d. $\frac{1}{99}$ e. 5 f. 8

5. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $\frac{2}{3} \div \frac{5}{6}$ b. $\frac{1}{11} \div \frac{5}{7}$ c. $\frac{1}{2} \div \frac{11}{12}$ d. $\frac{2}{11} \div \frac{6}{11}$ e. $\frac{2}{5} \div \frac{12}{14}$ f. $\frac{3}{5} \div \frac{4}{5}$

6. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $4\frac{2}{5} \div 3\frac{4}{5}$ b. $1\frac{2}{9} \div 3\frac{5}{8}$ c. $2\frac{3}{5} \div \frac{3}{10}$ d. $1\frac{3}{10} \div 1\frac{4}{15}$ e. $2\frac{7}{10} \div 3\frac{4}{9}$ f. $\frac{6}{11} \div 2\frac{3}{5}$

7. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $\frac{3}{4} \div 5$ b. $4 \div \frac{2}{3}$ c. $7 \div \frac{1}{4}$ d. $\frac{5}{6} \div 8$ e. $\frac{1}{10} \div 9$ f. $1 \div \frac{2}{7}$

8. Calculate the following. Write all of your answers as proper fractions or mixed numbers in their simplest form.
   a. $5 \div 1\frac{3}{4}$ b. $2\frac{4}{5} \div 2$ c. $9 \div 4\frac{3}{5}$ d. $10 \div 2\frac{1}{4}$ e. $\frac{3}{4} \times \frac{1}{3} \times \frac{2}{5}$ f. $\frac{2}{3} \times \frac{3}{10} \times 6$

<u>Fraction of a Number</u>

There are two ways to find a fraction of an integer, both of which are illustrated in the examples below.

Example:
$$1\frac{2}{3} \times 2\frac{3}{4} = \frac{5}{3} \times \frac{11}{4} = \frac{5 \times 11}{3 \times 4} = \frac{55}{12} = 4\frac{7}{12}$$

To multiply a fraction by an integer you can either write the integer as a fraction then multiply

Example:
$$\frac{3}{4} \times 3 = \frac{3}{4} \times \frac{3}{1} = \frac{3 \times 3}{4 \times 1} = \frac{9}{4} = 2\frac{1}{4}$$

Or you can multiply only the numerator by the integer

Example:
$$\frac{3}{4} \times 3 = \frac{3 \times 3}{4} = \frac{9}{4} = 2\frac{1}{4}$$

As you can see both of these methods give the same answer, it is up to you to decide which one you prefer.

<u>Dividing</u>

In order to divide by a fraction you need to multiply by it's **reciprocal**. The reciprocal is, essentially, the fraction turned upside down so you will often hear dividing fractions referred to as "flipping upside down and multiplying."

Example:

a. The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$

b. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

c. The reciprocal of $\frac{1}{4}$ is $\frac{4}{1} = 4$ *(remember that $\frac{4}{1}$ means 4÷1)*

d. The reciprocal of 7 is $\frac{1}{7}$

Example:
$$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$$

The same logic is applied when dividing with fractions and integers as shown in the examples below.

Example:
$$3 \div \frac{2}{5} = 3 \times \frac{5}{2} = \frac{3 \times 5}{2} = \frac{15}{2} = 7\frac{1}{2}$$

Example:
$$\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$$

The first option is to multiply the fraction by the integer and then simplify the answer.

Example: Find $\frac{2}{3}$ of 9

The calculation is $\frac{2}{3} \times 9 = \frac{2\times9}{3} = \frac{18}{3} = 6$

The second option is to divide by the denominator then multiply by the numerator. Both methods give you the same answer as illustrated below.

Example: Find $\frac{2}{3}$ of 9

First divide by the denominator $9 \div 3 = 3$
Then multiply by the numerator $3 \times 2 = 6$

Example:
I have £50, I spend $\frac{2}{5}$ of this amount, how much do I have left?

$$\frac{2}{5} \times 50 = \frac{100}{5} = 20$$

Or

50 ÷ 5 = 10
10 × 2 = 20

So there is £20 left.

Activity 2.3 D

1. Find $\frac{3}{10}$ of 60

2. Find $\frac{1}{5}$ of 40

3. A man is 110 years old. If his brother is $\frac{9}{10}$ of his age, how old is his brother?

4. Textbooks cost £35. Elizabeth has $\frac{2}{7}$ of this amount, how much more does she need?

5. Daisy gets £5 pocket money a week. She spends $\frac{2}{5}$ of it on stickers, $\frac{3}{10}$ of it on sweets and saves the rest. How much will she have saved after 5 weeks?

6. David gets paid £70. He owes his sister half of this amount. If he pays her $\frac{1}{7}$ of what he owes her, how much will he have left?

7. There are 600 students in a school. 150 of these are in year 8, in its simplest form, what fraction are not in year 8?

8. Abbie shares a bag of sweets with her friends. She gives Amy $\frac{2}{5}$ of the sweets and Elliot $\frac{1}{6}$ of the sweets. If she had 30 to begin with, how many does she have left?

2.4 Fractions Decimals and Percentages

Converting Between Fractions and Decimals

A decimal is another way of writing a fraction and you need to know how to convert between them.

Remember that a fraction means the numerator divided by the denominator. If you can convert the fraction to an equivalent fraction with 10, 100, 1000... as the denominator it becomes easier to divide to find the decimal.

Example:

$$\frac{7}{10} = 0.7$$

$$\frac{12}{10} = 1.2$$

$$\frac{17}{100} = 0.17$$

$$\frac{325}{100} = 3.25$$

Example:
Write $\frac{3}{5}$ as a decimal

First we change the denominator to a ten by multiplying both the numerator and the denominator by 2.

$$\frac{3}{5} = \frac{6}{10} = 0.6$$

Example:
Write $\frac{6}{25}$ as a decimal

With this fraction it is easier to change the denominator to 100 by multiplying both the numerator and denominator by 4.

$$\frac{6}{25} = \frac{24}{100} = 0.24$$

To convert from a decimal to a fraction you put the numbers after the decimal place over 10, 100, 1000... as appropriate and then simplify. The number of digits after the decimal point is the number of zeros you need following the one in the denominator.

Example:
Write 0.4 as a fraction

$$0.4 = \frac{4}{10} = \frac{2}{5}$$

Example:
Write 0.225 as a fraction

$$0.225 = \frac{225}{1000} = \frac{9}{40}$$

There are some common fractions for which you should learn the decimal equivalents.

$\frac{1}{2} = 0.5$ $\frac{1}{4} = 0.25$ $\frac{3}{4} = 0.75$ $\frac{1}{8} = 0.125$

$\frac{1}{3} = 0.33\dot{3}$ $\frac{2}{3} = 0.66\dot{6}$

The dot above the last number in $0.33\dot{3}$ and $0.66\dot{6}$ means recurring. $0.33\dot{3}$ is read as "zero point three three three recurring". This means that the number continues repeating forever, it is usually sufficient to write these numbers to just two or three decimal places. (**Decimal places** refers to the number of digits after the decimal point, this will be revisited in the next chapter).

If you are converting from a mixed number to a decimal the integer remains the same and you convert just the fraction part to a decimal.

Example:
Convert $3\frac{4}{5}$ to a decimal

$\frac{4}{5} = \frac{8}{10} = 0.8$ so we have $3\frac{4}{5} = 3.8$

Example:
Convert $6\frac{2}{8}$ to a decimal

Here we can either use $\frac{1}{8} = 0.125$ so $\frac{2}{8} = 0.125 \times 2 = 0.25$
Or $\frac{2}{8} = \frac{1}{4} = 0.25$

So we have $6\frac{2}{8} = 6.25$

Example:
Convert 1.3 to a fraction

$0.3 = \frac{3}{10}$ so $1.3 = 1\frac{3}{10}$

Example:
Convert 10.5 to a fraction

We know that $0.5 = \frac{1}{2}$ so $10.5 = 10\frac{1}{2}$

Converting Between Fractions and Percentages

Note: How to use percentages will be covered in chapter 11, here we deal only with converting

Percentage means parts per one hundred, it is equivalent to a fraction with a denominator of 100. Therefore to convert from a percentage to a fraction you write the percentage as the numerator, with a denominator of 100, and simplify.
To convert from a fraction to a percentage you first convert it to a fraction with a denominator of 100, the new numerator is then the percentage.

Example:
Write 60% as a fraction

$$60\% = \frac{60}{100} = \frac{3}{5}$$

Example:
Write $\frac{9}{10}$ as a percentage

$$\frac{9}{10} = \frac{90}{100} = 90\%$$

There are some common percentages for which you should recognise the fraction equivalent.

$$25\% = \frac{1}{4} \qquad 50\% = \frac{1}{2} \qquad 75\% = \frac{3}{4}$$

Converting Between Percentages and Decimals

Since we know that percentages can be written as fractions with a denominator of 100 you convert from this to a decimal.
To convert from decimals to percentages you first write the number as a fraction and then convert as above.

Example:
Write 80% as a decimal

$80\% = \frac{80}{100} = 0.8$ *Remember that the extra zero in 0.80 is irrelevant so 0.80 is written as 0.8*

Example:
Write 0.3 as a percentage

$$0.3 = \frac{3}{10} = \frac{30}{100} = 30\%$$

Recurring Decimals

Converting between recurring decimals and fractions requires the use of basic algebra. Work through this section now and try to follow the logic however, if you find the algebra particularly difficult to understand, have another go at this section after you have completed chapter 4.

The method is to set the decimal equal to x then multiply by a multiple of 10. Subtract the two values and divide to give a fraction.
If there is one number recurring, such as $0.33\dot{3}$ you use $10x$, if there are two numbers recurring such as $0.4545\dot{4}\dot{5}$ you use $100x$, if there are three numbers recurring such as $0.789789\dot{7}8\dot{9}$ you use $1000x$ and so on. Don't worry if this doesn't make sense just yet! Work through the following examples.

Example:
Convert $0.\dot{5}$ to a fraction

We set this equal to x and, since there is only one number recurring, we multiply by 10.
In algebra the notation for letters multiplied by numbers is simply to write them next to one another.
$x = 0.55555 \ldots..$
$10x = 5.5555 \ldots.$

Subtracting gives:

$$\begin{array}{r} 10x = 5.55555\ldots \\ \underline{-\quad x = 0.55555\ldots} \\ 9x = 5 \end{array}$$

So we have $9x = 5$
Dividing both sides by 9 gives
$x = \frac{5}{9}$

Example:
Convert $0.\dot{3}\dot{4}$ to a fraction

Let $x = 0.343434 \ldots$
Here we will use $100x$ because there are two numbers recurring.

$$\begin{array}{r} 100x = 34.343434\ldots \\ \underline{-\quad x = \ \ 0.343434\ldots} \\ 99x = 34 \end{array}$$

Dividing by 99 gives
$x = \frac{34}{99}$

Sometimes there are a few numbers after the decimal point before the recurring pattern starts. To deal with these you should multiply by an appropriate multiple of 10 initially such that only the recurring pattern is after the point. This is illustrated in the example below.

Example:
Convert $0.1\dot{4}\dot{7}$ to a fraction.

Let $x = 0.14747474\ldots$
As the 1 is "in the way" we will use $10x = 1.4747474..$
Now there are 2 numbers in the repeating pattern so we need to multiply by 100. As it is already multiplied by 10 this becomes 10 x 100 = 1000

$$\begin{array}{rl} 1000x = & 147.474747\ldots \\ \underline{-\quad 10x =} & \underline{\quad 1.47474\ldots} \\ 990x = & 146 \end{array}$$

Dividing both sides by 990 (and then simplifying) gives
$x = \frac{146}{990} = \frac{73}{495}$

Activity 2.4

1.Convert each of these to fractions in their simplest form

a. 0.1 b. 0.5 c. 0.25 d. 0.85 e. 0.4 f. 0.65

g. 1.2 h. 5.6 i. 10.9 j. 2.25 k. 9.1 l. 11.75

2. Convert each of these to decimals

a. $\frac{1}{3}$ b. $\frac{1}{2}$ c. $1\frac{1}{4}$ d. $\frac{9}{5}$ e. $\frac{12}{50}$ f. $3\frac{3}{4}$

g. $\frac{3}{8}$ h. $10\frac{3}{10}$ i. $\frac{7}{2}$ j. $6\frac{6}{25}$ k. $\frac{12}{250}$ l. $4\frac{2}{3}$

3. Convert each of these to fractions in their simplest form

a.65% b. 75% c. 80% d. 25% e. 90% f. 10%

4. Convert each of these to percentages

a. $\frac{3}{10}$ b. $\frac{7}{10}$ c. $\frac{3}{25}$ d. $\frac{3}{100}$ e. $\frac{2}{5}$ f. $\frac{6}{50}$

5. Convert each of these to decimals

a. 85% b. 5% c. 62% d. 12% e. 40% f. 1%

6. Convert each of these to percentages

a. 0.5 b. 0.44 c. 0.3 d. 0.32 e. 0.2 f. 0.1

7. Write each of these lists of numbers in order from smallest to largest.

a. $0.5,\ \frac{1}{5},\ \frac{7}{10},\ 80\%,\ 0.3$

b. $\frac{3}{5},\ 0.25,\ \frac{3}{2},\ 10\%,\ 0.15$

c. $25\%,\ \frac{53}{10},\ 5.4,\ \frac{2}{10},\ 5\frac{1}{5}$

8. Convert each of these recurring decimals to fractions

a. $0.\dot{4}$ b. $0.\dot{8}$ c. $0.\dot{2}\dot{7}$ d. $0.\dot{7}\dot{1}$ e. $0.\dot{4}5\dot{2}$

f. $0.\dot{5}2\dot{1}$ g. $0.\dot{3}22\dot{1}$ h. $0.2\dot{3}$ i. $0.4\dot{3}\dot{5}$ j. $0.32\dot{1}$

9. Write each of these proportions as fractions

a. 8 out of 10 b. 7 out of 18 c. 12p out of £1.44 d. 5minutes out of an hour

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ASSIGNMENT TWO

Answers to these questions are not provided. You should send your work to your tutor for marking. No calculators are allowed – show all of your working.

1. Put this list of numbers in order from smallest to largest: 0.4 0.41 0.04 0.42 0.004 0.45 (1)

2. Calculate:

a. 4.32 + 9.415 b. 2.34 - 1.45 c. 24.29 + 5.216 d. 95.53 – 89.431

e. 1.2 x 0.5 f. 0.04 x 1.3 g. 10.5 ÷ 1.5 h. 13.64 ÷ 2.2 (8)

3. For each journey a taxi firm charges £1.53 per mile. Find the cost for an 18 mile journey. (1)

4. To fix a computer Kevin charges £56.80 for the first hour and £42.50 for each extra hour. He charged Graham £269.30. How many hours work did he do? (2)

5. Write in standard form a. 0.000029 b. 2537.28 c. 13.5×10^4 (3)

6. Write as ordinary numbers a. 9.5×10^4 b. 5.324×10^{-4} (2)

7. Calculate the following. Write your answers in standard form.

a. $(6.4 \times 10^{-4}) + (7.1 \times 10^{-3})$ b. $(3.3 \times 10^3) - (7.4 \times 10^2)$

c. $(8.4 \times 10^9) \div (4.2 \times 10^5)$ d. $(2.5 \times 10^5) \times (5 \times 10^4)$ (8)

8. There are 1.25×10^6 copies of a magazine printed, each containing 18 sheets of paper. How many sheets of paper needed to print all of the magazines? Give your answer in standard form. (2)

9. Calculate the following. Write your answers in their simplest form.

a. $6 \div \frac{1}{5}$ b. $\frac{4}{5} + \frac{2}{6}$ c. $3\frac{4}{7} - 2\frac{2}{8}$ d. $4\frac{2}{9} \times 2\frac{1}{4}$ e. $\frac{3}{4} \div \frac{2}{5}$ (10)

10. Bens dog Jasper eats $\frac{2}{3}$ of a can of dog food every day. How many cans of dog food does he eat in a week? Give your answer as a mixed number. (2)

11. Susan weighs 120kg and decides to go on a diet. In the first month she loses $\frac{1}{6}$ of her weight. In the second month she loses $\frac{1}{5}$ of her remaining weight. What does she weigh now? (3)

12. In a class of 50 students, 30 of them play tennis. What percentage is this? (2)

13. Complete the table

Fraction (Simplest Form)	Decimal	Percentage
$^1/_2$	0.5	50%
		85%
$^3/_5$		

(4)

14. Write $0.\dot{2}1\dot{3}$ and $0.1\dot{4}\dot{8}$ as fractions (6)

Total 54 marks