



1. For each given recurrence relation, find the first five terms, derive the closed-form solution, and check it by substituting it back to the recurrence relation.
 - (a) $a_n = a_{n-1} + n$ with $a_0 = 2$
 - (b) $a_n = 2a_{n-1} + 2$ with $a_0 = 1$
 - (c) $a_n = 3a_{n-1} + 2^n$ with $a_0 = 5$
 - (d) $a_n = 4a_{n-1} + 5a_{n-2}$ with $a_0 = 1, a_1 = 17$
 - (e) $a_n = 4a_{n-1} - 4a_{n-2}$ with $a_0 = 3, a_1 = 11$
 - (f) $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ with $a_{0,1,2} = 3, 2, 6$
2. Solve the following recurrences by applying the [Master theorem](#). For the cases where the Master theorem does not apply, use the [Akra–Bazzi method](#). In cases where neither of these two theorems apply, explain why and solve the recurrence relation by closely examining the recursion tree. Solutions must be in the form $T(n) \in \Theta(\dots)$.
 - (a) $T(n) = 2T(n/2) + n$
 - (b) $T(n) = T(3n/4) + T(n/4) + n$
 - (c) $T(n) = 3T(n/2) + n$
 - (d) $T(n) = 2T(n/2) + n/\log n$
 - (e) $T(n) = 6T(n/3) + n^2 \log n$
 - (f) $T(n) = T(3n/4) + n \log n$
 - (g) $T(n) = T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + n$
 - (h) $T(n) = T(n/2) + T(n/4) + 1$
 - (i) $T(n) = T(n/2) + T(n/3) + T(n/6) + n$
 - (j) $T(n) = 2T(n/3) + 2T(2n/3) + n$
 - (k) $T(n) = \sqrt{2n}T(\sqrt{2n}) + \sqrt{n}$
 - (l) $T(n) = \sqrt{2n}T(\sqrt{2n}) + n$
3. Consider a recurrence relation $a_n = 2a_{n-1} + 2a_{n-2}$ with $a_0 = a_1 = 2$. Solve it (i.e. find a closed formula) and show how it can be used to estimate the value of $\sqrt{3}$ (hint: observe $\lim_{n \rightarrow \infty} a_n/a_{n-1}$). After that, devise an algorithm for constructing a recurrence relation with integer coefficients and initial conditions that can be used to estimate the square root \sqrt{k} of a given integer k .
4. Find a closed formula for the n -th term of the sequence with generating function $\frac{3x}{1-4x} + \frac{1}{1-x}$.
5. Given the generating function $G(x) = \frac{5x^2+2x+1}{(1-x)^3}$, decompose it into partial fractions and find the sequence that it represents.
6. Pell–Lucas numbers are defined by $Q_0 = Q_1 = 2$ and $Q_n = 2Q_{n-1} + Q_{n-2}$ for $n \geq 2$. Derive the corresponding generating function and find a closed formula for the n -th Pell–Lucas number.
7. For each given recurrence relation, derive the corresponding generating function and find a closed formula for the n -th term of the sequence.
 - (a) $a_n = 2a_{n-1} - a_{n-2}$ with $a_0 = 3, a_1 = 5$
 - (b) $a_n = a_{n-1} + a_{n-2} - a_{n-3}$ with $a_0 = 1, a_1 = 1, a_2 = 5$
 - (c) $a_n = a_{n-1} + n$ with $a_0 = 0$
 - (d) $a_n = a_{n-1} + 2a_{n-2} + 2^n$ with $a_0 = 2, a_1 = 1$
8. Find the number of non-negative integer solutions to the Diophantine equation $3x + 5y = 100$ using generating functions.
9. Consider a $2n$ -digit ticket number to be “lucky” if the sum of its first n digits is equal to the sum of its last n digits. Each digit (including the first one!) in a number can take value from 0 to 9. For example, a 6-digit ticket 345 264 is lucky since $3 + 4 + 5 = 2 + 6 + 4$.
 - (a) Find the number of lucky 6-digit and 8-digit tickets.
 - (b) Find the generating function for the number of $2n$ -digit lucky tickets.
 - (c) Find a closed formula for the number of $2n$ -digit lucky tickets.