

1. For each given regular expression  $P$ , construct a DFA (Deterministic Finite Automaton), and find the number of accepted word of length at most 5, i.e. the size of the set  $\mathcal{L}' = \{w \in \mathcal{L}(P) \mid |w| \leq 5\}$ . For “any” ( $.$ ) and “negative” ( $[\wedge \dots]$ ) matches, assume that the alphabet is  $\Sigma = \{a, b, c, d\}$ .

- (a)  $P_1 = ab^*$  (c)  $P_3 = [\wedge cd]^+ c\{3\}$  (e)  $P_5 = d(a|bc)^*$   
(b)  $P_2 = a+b?c$  (d)  $P_4 = [\wedge a](. | ddd)?$  (f)  $P_6 = ((a|ab)[cd])\{2\}$

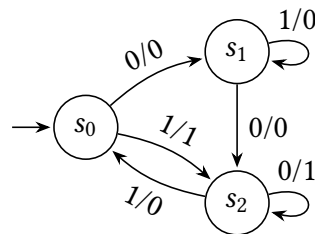
2. Describe the set of strings defined by each of these sets of productions in EBNF (extended Backus-Naur form).

- (a)  $\langle string \rangle ::= \langle L \rangle + \langle D \rangle? \langle L \rangle +$   
 $\langle L \rangle ::= a | b | c$   
 $\langle D \rangle ::= 0 | 1$   
 (b)  $\langle string \rangle ::= \langle sign \rangle? \langle N \rangle$   
 $\langle sign \rangle ::= '+' | '-'$   
 $\langle N \rangle ::= \langle D \rangle (\langle D \rangle | 0)^*$   
 $\langle D \rangle ::= 1 | \dots | 9$   
 (c)  $\langle string \rangle ::= \langle L \rangle^* (\langle D \rangle +)? \langle L \rangle^*$   
 $\langle L \rangle ::= x | y$   
 $\langle D \rangle ::= 0 | 1$   
 (d)  $\langle string \rangle ::= \langle C \rangle \langle R \rangle^*$   
 $\langle C \rangle ::= a | \dots | z | A | \dots | Z$   
 $\langle D \rangle ::= 0 | \dots | 9$   
 $\langle R \rangle ::= \langle C \rangle | \langle D \rangle | '-'$

3. Let  $\mathcal{G} = \langle V, T, S, P \rangle$  be the phrase-structure grammar with vocabulary  $V = \{A, S\}$ , terminal symbols  $T = \{0, 1\}$ , start symbol  $S = S$ , and set of productions  $P: S \rightarrow 1S, S \rightarrow 00A, A \rightarrow 0A, A \rightarrow 0$ .

- (a) Show that 111000 belongs to the language generated by  $\mathcal{G}$ .  
 (b) Show that 11001 does not belong to the language generated by  $\mathcal{G}$ .  
 (c) What is the language generated by  $\mathcal{G}$ ?

4. Find the output generated from the input string 01110 for each of the following Mealy machines.



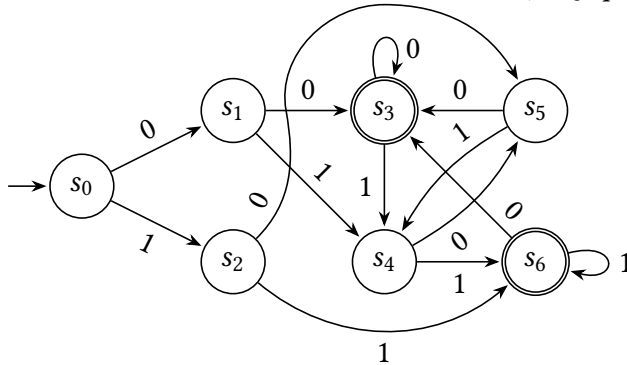
5. Construct a Moore machine for each of the following descriptions.

- (a) Determine the residue modulo 3 of the input treated as a binary number. For example, for input  $\varepsilon$  (which corresponds to “value” 0) the residue is 0; 101 (5 in decimal) has residue 2; and 1010 (value 10) has residue 1.  
 (b) Output the residue modulo 5 of the input from  $\{0, 1, 2\}^*$  treated as a ternary (base 3) number.  
 (c) Output  $A$  if the binary input ends with 101; output  $B$  if it ends with 110; otherwise output  $C$ .

6. Show that regular languages are *closed* under the following operations.

- (a) Union, that is, if  $L_1$  and  $L_2$  are regular languages, then  $L_1 \cup L_2$  is also regular.  
 (b) Concatenation, that is, if  $L_1$  and  $L_2$  are regular languages, then  $L_1 \cdot L_2$  is also regular.  
 (c) Kleene star, that is, if  $L$  is a regular language, then  $L^*$  is also regular.  
 (d) Complement, that is, if  $L$  is a regular language, then  $\bar{L} = \Sigma^* - L$  is also regular.  
 (e) Intersection, that is, if  $L_1$  and  $L_2$  are regular languages, then  $L_1 \cap L_2$  is also regular.

7. Determine whether the following languages are regular or not. For non-regular languages, use Pumping lemma to prove that they are not regular. For each regular language, provide a regular expression and construct an  $\varepsilon$ -NFA.
- (a)  $L_1 = \{w \in \{0, 1\}^* \mid \text{length of } w \text{ is odd}\}$
  - (b)  $L_2 = \{0^n 1^n \mid n \in \mathbb{N}\}$
  - (c)  $L_3 = \{w \in \{0, 1\}^* \mid w \text{ contains an even number of 1s}\}$
  - (d)  $L_4 = \{1^{n^2} \mid n \in \mathbb{N}\}$
8. Consider a finite-state automaton  $M = (\Sigma, Q, q_0, F, \delta)$  and a non-negative integer  $k$ . Let  $R_k$  be the relation on the set of states of  $M$  such that  $s R_k t$  if and only if for every input string  $w \in \Sigma^*$  with  $|w| \leq k$ ,  $\delta(s, w)$  and  $\delta(t, w)$  are both final states or both not final states. Furthermore, let  $R^*$  be the relation on the set of states of  $M$  such that  $s R^* t$  if and only if for every input string  $w \in \Sigma^*$ , regardless of length,  $\delta(s, w)$  and  $\delta(t, w)$  are both final states or both not final states.
- (a) Show that for every nonnegative integer  $k$ ,  $R_k$  is an equivalence relation on  $S$ .  
Two states  $s$  and  $t$  are called  $k$ -equivalent if  $s R_k t$ .
  - (b) Show that  $R^*$  is an equivalence relation on  $S$ .  
Two states  $s$  and  $t$  are called  $*$ -equivalent if  $s R^* t$ .
  - (c) Show that if two states  $s$  and  $t$  are  $k$ -equivalent ( $k > 0$ ), then they are also  $(k - 1)$ -equivalent.
  - (d) Show that the equivalence classes of  $R_k$  are a *refinement* of the equivalence classes of  $R_{k-1}$ .
  - (e) Show that if two states  $s$  and  $t$  are  $k$ -equivalent for every non-negative integer  $k$ , then they are  $*$ -equivalent.
  - (f) Show that all states in a given  $R^*$ -equivalence class are final or all are not final.
  - (g) Show that if two states  $s$  and  $t$  are  $*$ -equivalent, then  $\delta(s, a)$  and  $\delta(t, a)$  are also  $*$ -equivalent for all  $a \in \Sigma$ .
9. Consider the finite-state automaton  $M = (\Sigma, Q, q_0, F, \delta)$  depicted below.



- (a) Find the  $k$ -equivalence classes of  $M$  for  $k = 0, 1, 2, 3$ .
- (b) Find the  $*$ -equivalence classes of  $M$ .
- (c) Construct the quotient automaton  $\overline{M}$  of  $M$ .  
  - The quotient automaton  $\overline{M}$  of the deterministic finite-state automaton  $M = (\Sigma, S, s_0, F, \delta)$  is the finite state automaton  $\overline{M} = (\Sigma, \overline{S}, [s_0]_{R^*}, \overline{F}, \overline{\delta})$ , where the set of states  $\overline{S}$  is the set of  $R^*$ -equivalence classes of  $S$ ; the transition function  $\overline{\delta}$  is defined by  $\overline{\delta}([s]_{R^*}, a) = [\delta(s, a)]_{R^*}$  for all states  $[s]_{R^*}$  of  $\overline{M}$  and input symbols  $a \in \Sigma$ ; and  $\overline{F}$  is the set consisting of  $R^*$ -equivalence classes of final states of  $M$ .

10. Solve the following regex crosswords<sup>1</sup>. Fill each cell with a single ASCII character (an uppercase letter, a digit, a punctuation mark, or a space). Each row/column, when read left to right or top to bottom must match the regular expression(s) given for that row/column.

EP|IP|EF  
[<sup>^</sup>SPEAK]<sub>+</sub>  
HE|LL|O<sub>+</sub>  
[PLEASE]<sub>+</sub>


(FY|F|RG)<sub>+</sub>  
[NODE]<sub>+</sub>  
(.)<sub>1</sub>[IF]<sub>+</sub>  
(YE|OT)K  
(FI|A)<sub>+</sub>  
(Y|F)(.)\2[DAF]\1  
(U|O|I)\*T[FRO]<sub>+</sub>  
[KANE]\*[GIN]\*


(D|FU|UF)<sub>+</sub>  
(FO|A|R)\*  
(N|A)\*  
[<sup>^</sup>NRU](NO|ON)  
[RUNT]\*  
O.\*[HAT]  
(.)\*DO\1


[<sup>^</sup>MCI]<sub>+</sub>  
(TM|BF)  
.A  
(CAT|A-T)<sub>+</sub>  
[MA\-\sE]<sub>+</sub>  
[<sup>^</sup>KI\sP]<sub>+</sub>  
(M|APS|EA)\*  
[AI][E\s]  
[A\-\Z]<sub>+</sub>  
[\sT\-\M]<sub>+</sub>


(.)\1(.)\2  
[<sup>^</sup>PU\sH]<sub>+</sub>  
[<sup>^</sup>C\sOU]<sub>+</sub>  
.[LUH]<sub>+</sub>  
(P|K)[<sup>^</sup>U]<sub>+</sub>  
.\*C+[TIF]  
(NO|ONE|ION)\*  
.\*L<sub>+</sub>  
[PUF\s]\*  
[TIC]\*  
[NOI\sE]<sub>+</sub>


(EM|FE)(IT|IP)  
[<sup>^</sup>P]I(IT|ME)  
.\*E.\*  
(TS|PE|KE)\*  
(EP|ST)\*  
T[A-Z]\*  
.M.T  
.\*P.[S-X]<sub>+</sub>


[PIF]<sub>+</sub>  
.\*[OWE]\*  
(TN|LF|TF)\*

<sup>1</sup> Credits: <https://regexcrossword.com>