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lab 3.2

var. 2

Task 1: Решите уравнения и сравните с результатами, полученными в Maple. Постройте в одной системе координат несколько интегральных кривых.

$$> de := x = \frac{d^2}{dx^2} (y(x)) + \ln\left(\frac{d^2}{dx^2} (y(x))\right)$$

$$de := x = \frac{d^2}{dx^2} y(x) + \ln\left(\frac{d^2}{dx^2} y(x)\right) \quad (1)$$

> dsolve(de)

$$y(x) = \frac{\text{LambertW}(e^x)^3}{6} + \frac{3 \text{LambertW}(e^x)^2}{4} + \text{LambertW}(e^x) + _C1 x + _C2 \quad (2)$$

> x_ := z + ln(z)

$$x_ := z + \ln(z) \quad (3)$$

> dx := diff(x_, z)

$$dx := 1 + \frac{1}{z} \quad (4)$$

> y1 := int(z·dx, z)

$$y1 := z + \frac{1}{2} z^2 \quad (5)$$

> y_ := int((y1 + C1)dx, z) + C2

$$y_ := \frac{z^3}{6} + C1 z + \frac{3z^2}{4} + z + C1 \ln(z) + C2 \quad (6)$$

> a, b, c := seq(subs(C2=i, y_), i=-1..1) :

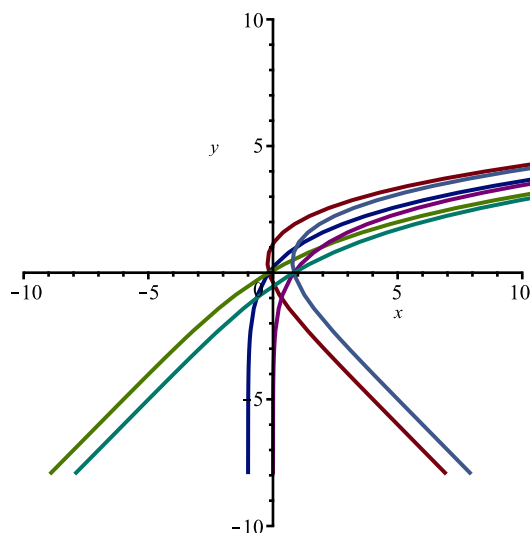
> a1, a2, a3 := seq(subs(C1=i, a), i=-1..1) :

b1, b2, b3 := seq(subs(C1=i, b), i=-1..1) :

c1, c2, c3 := seq(subs(C1=i, c), i=-1..1) :

len := z=-20..20 :

> plot([[a1, x_, len], [a2, x_, len], [a3, x_, len], [b1, x_, len], [b2, x_, len], [b3, x_, len]], x=-10..10, y=-10..10)



Task 1.2

```
> restart;
```

```
> de := (x^2 + 1) * ( y * d^2 (y(x)) / dx^2 - ( d (y(x)) / dx )^2 ) = 2 * x * y * y'
```

$$de := (x^2 + 1) \left(y(x) \left(\frac{d^2}{dx^2} y(x) \right) - \left(\frac{d}{dx} y(x) \right)^2 \right) = 2 x y(x) \left(\frac{d}{dx} y(x) \right) \quad (7)$$

```
> y_ := dsolve(de)
```

$$y_ := y(x) = e^{\frac{C1 x^3}{3}} e^{-C1 x} _C2 \quad (8)$$

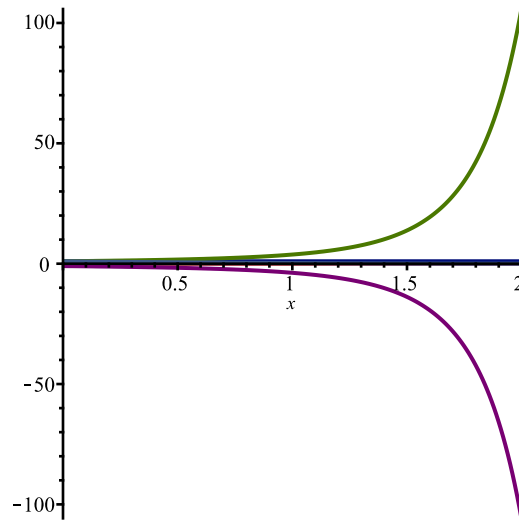
```
> a, b, c := seq(subs(_C2 = i, y_), i = -1 .. 1) :
```

```
> a1, a2, a3 := seq(subs(_C1 = i, a), i = -1 .. 1) :
```

```
b1, b2, b3 := seq(subs(_C1 = i, b), i = -1 .. 1) :
```

```
c1, c2, c3 := seq(subs(_C1 = i, c), i = -1 .. 1) :
```

```
> plot([rhs(b2), rhs(c2), rhs(c3), rhs(c1), rhs(a3)], x = 0 .. 2)
```



Task 1.3

```
> restart;
```

```
> de := d (y(x)) / dx = x * d^2 (y(x)) / dx^2 - exp( d^2 (y(x)) / dx^2 )
```

$$de := \frac{d}{dx} y(x) = x \left(\frac{d^2}{dx^2} y(x) \right) - e^{\frac{d^2}{dx^2} y(x)} \quad (9)$$

```
> y_ := dsolve(de)
```

$$y_ := y(x) = \frac{C1 x^2}{2} + \text{RootOf}(-\text{LambertW}(-e^{-C1} e^{-Z}) + _Z) x + _C2 \quad (10)$$

```
> del := u(x) = x * d (u(x)) / dx - exp( d (u(x)) / dx )
```

$$del := u(x) = x \left(\frac{d}{dx} u(x) \right) - e^{\frac{d}{dx} u(x)} \quad (11)$$

```
> u_ := dsolve(del);
```

$$u_ := u(x) = x \ln(x) - x, u(x) = _C1 x - e^{-CI} \quad (12)$$

$$> de2 := \frac{d}{dx} (y(x)) = _C1 x - e^{-CI}$$

$$de2 := \frac{d}{dx} y(x) = _C1 x - e^{-CI} \quad (13)$$

$$> y_ := dsolve(de2);$$

$$y_ := y(x) = \frac{_C1 x^2}{2} - e^{-CI} x + _C2 \quad (14)$$

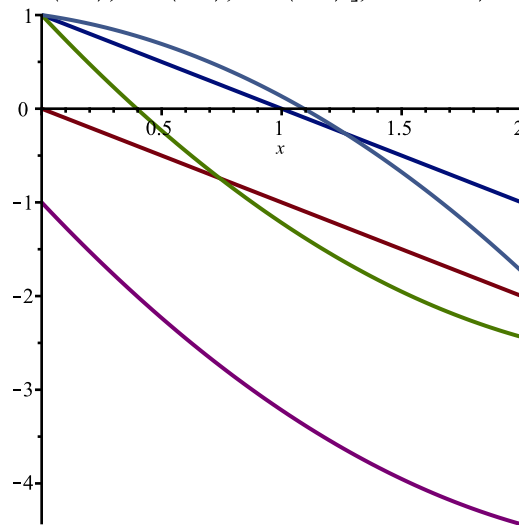
$$> a, b, c := seq(subs(_C2 = i, y_), i = -1 .. 1) :$$

$$> a1, a2, a3 := seq(subs(_C1 = i, a), i = -1 .. 1) :$$

$$b1, b2, b3 := seq(subs(_C1 = i, b), i = -1 .. 1) :$$

$$c1, c2, c3 := seq(subs(_C1 = i, c), i = -1 .. 1) :$$

$$> plot([rhs(b2), rhs(c2), rhs(c3), rhs(c1), rhs(a3)], x = 0 .. 2)$$



Task 1.4

$$> restart;$$

$$> de := \frac{d^2}{dx^2} (y(x)) = 2 \left(\frac{\frac{d}{dx} (y(x))}{x} - \frac{y(x)}{x^2} \right) + \frac{1}{x^2} \cdot \exp\left(\frac{1}{x}\right)$$

$$de := \frac{d^2}{dx^2} y(x) = \frac{2 \left(\frac{d}{dx} y(x) \right)}{x} - \frac{2 y(x)}{x^2} + \frac{e^{\frac{1}{x}}}{x^2} \quad (15)$$

$$> y_ := dsolve(de)$$

$$y_ := y(x) = e^{\frac{1}{x}} x^2 + _C2 x^2 + _C1 x \quad (16)$$

$$> simplify(y_)$$

$$y(x) = x \left(e^{\frac{1}{x}} x + _C2 x + _C1 \right) \quad (17)$$

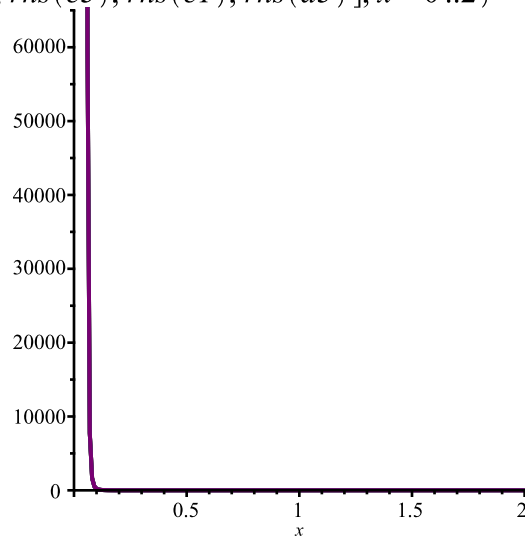
$$> a, b, c := seq(subs(_C2 = i, y_), i = -1 .. 1) :$$

$$> a1, a2, a3 := seq(subs(_C1 = i, a), i = -1 .. 1) :$$

$$b1, b2, b3 := seq(subs(_C1 = i, b), i = -1 .. 1) :$$

```
c1, c2, c3 := seq(subs(_C1=i, c), i=-1..1) :
```

```
> plot([rhs(b2), rhs(c2), rhs(c3), rhs(c1), rhs(a3)], x=0..2)
```



Task 2: Найдите общее решение уравнения и сравните с результатом, полученным в системе Maple.

```
> restart
```

```
> de := x * (d^3/dx^3)(y(x)) + (d^2/dx^2)(y(x)) = 1
```

$$de := x \left(\frac{d^3}{dx^3} y(x) \right) + \frac{d^2}{dx^2} y(x) = 1 \quad (18)$$

```
> dsolve(de)
```

$$y(x) = \frac{x^2}{2} + \ln(x) x_C1 - x_C1 +_C2 x +_C3 \quad (19)$$

Task 3: Найдите общее решение дифференциального уравнения

```
> restart;
```

```
> de := (d^2/dx^2)(y(x)) - 4 * (d/dx)(y(x)) + 4 * y(x) = -exp(2*x) * sin(6*x)
```

$$de := \frac{d^2}{dx^2} y(x) - 4 \frac{d}{dx} y(x) + 4 y(x) = -e^{2x} \sin(6x) \quad (20)$$

```
> dsolve(de)
```

$$y(x) = e^{2x} _C2 + e^{2x} x_C1 + \frac{e^{2x} \sin(6x)}{36} \quad (21)$$