> #Lab 2 Done by Bahdanau Aliaksandr 153502

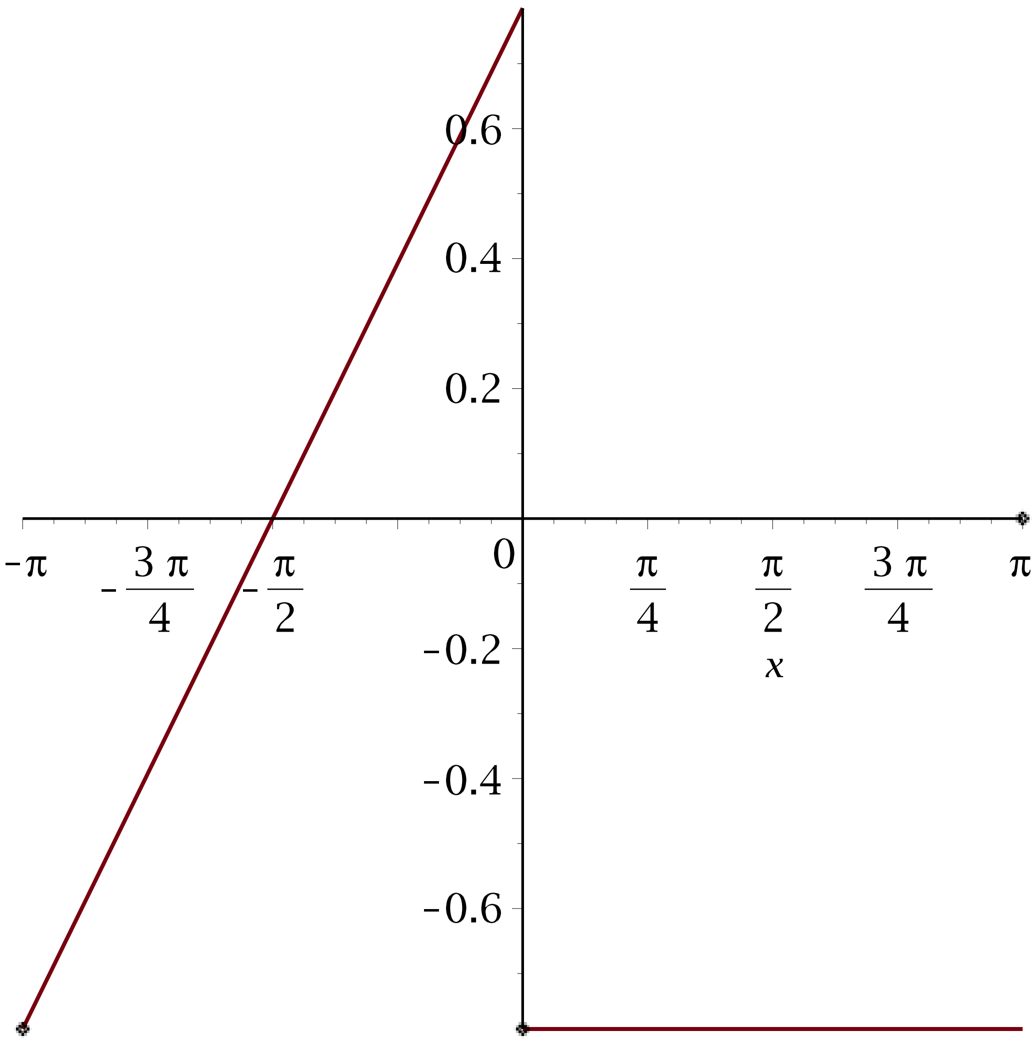
> #Var 5

> #Task 1

> $f := x \rightarrow piecewise\left(-\mathrm{Pi} \le x < 0, \frac{\mathrm{Pi}}{4} + \frac{x}{2}, 0 \le x < \mathrm{Pi}, -\frac{\mathrm{Pi}}{4}\right)$

$$f := x \rightarrow piecewise\left(-\pi \le x \textbf{ and } x < 0, \frac{1}{4}\pi + \frac{1}{2}x, 0 \le x \textbf{ and } x < \pi, -\frac{1}{4}\pi\right) \quad \textbf{(1)}$$

> $plot(f(x), x = -Pi..\mathrm{Pi}, discont = true)$



> # Solving

> $a0 := simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x), x = -\mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint$;

$$a0 := -\frac{1}{4}\pi \quad \textbf{(2)}$$

> $an := simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \cos(n \cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint$;

$$an := \frac{1}{2} \frac{(-1)^{1+n} + 1}{\pi n^2} \quad \textbf{(3)}$$

> $bn := simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \sin(n \cdot x),\ x = -\mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint$;

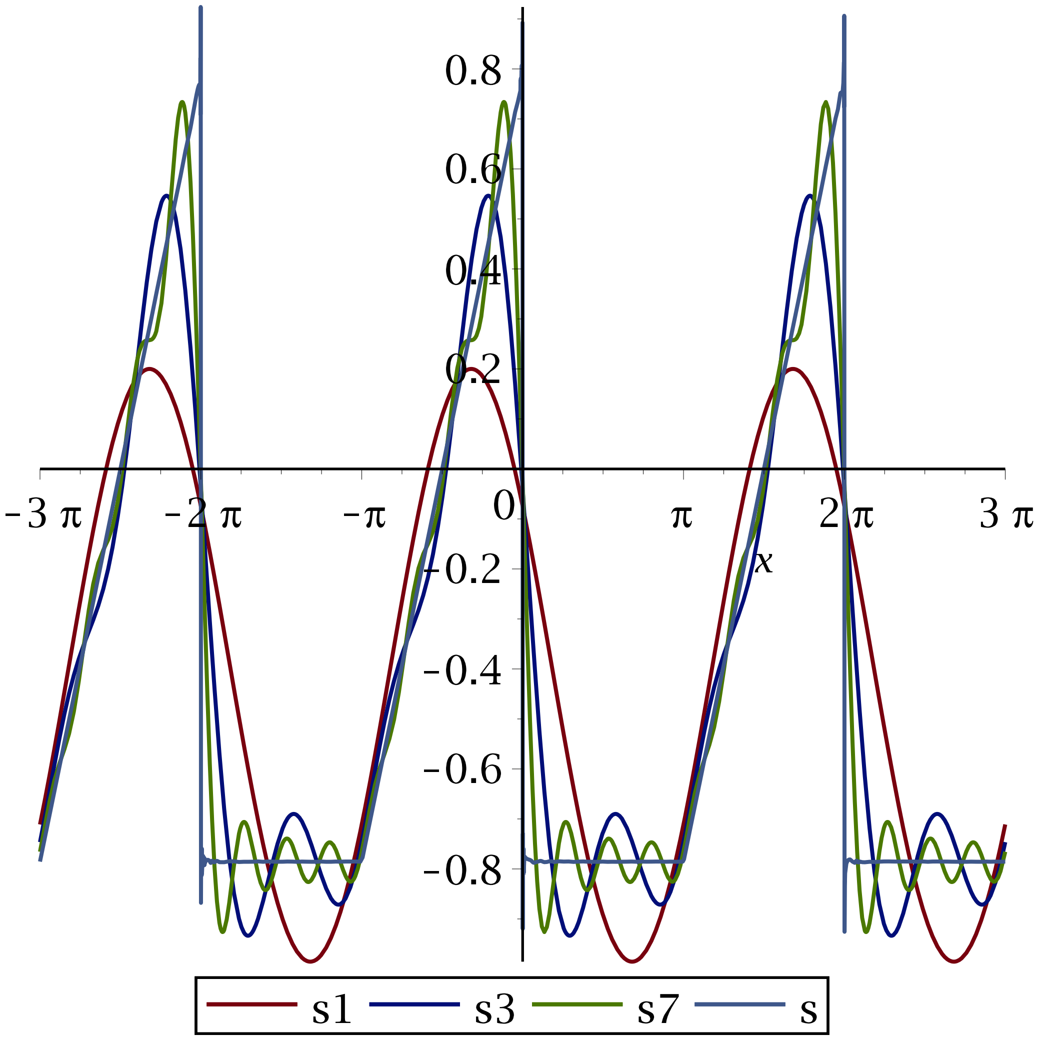
$$bn := -\frac{1}{2\,n} \tag{4}$$

> # Creating proc to get sum

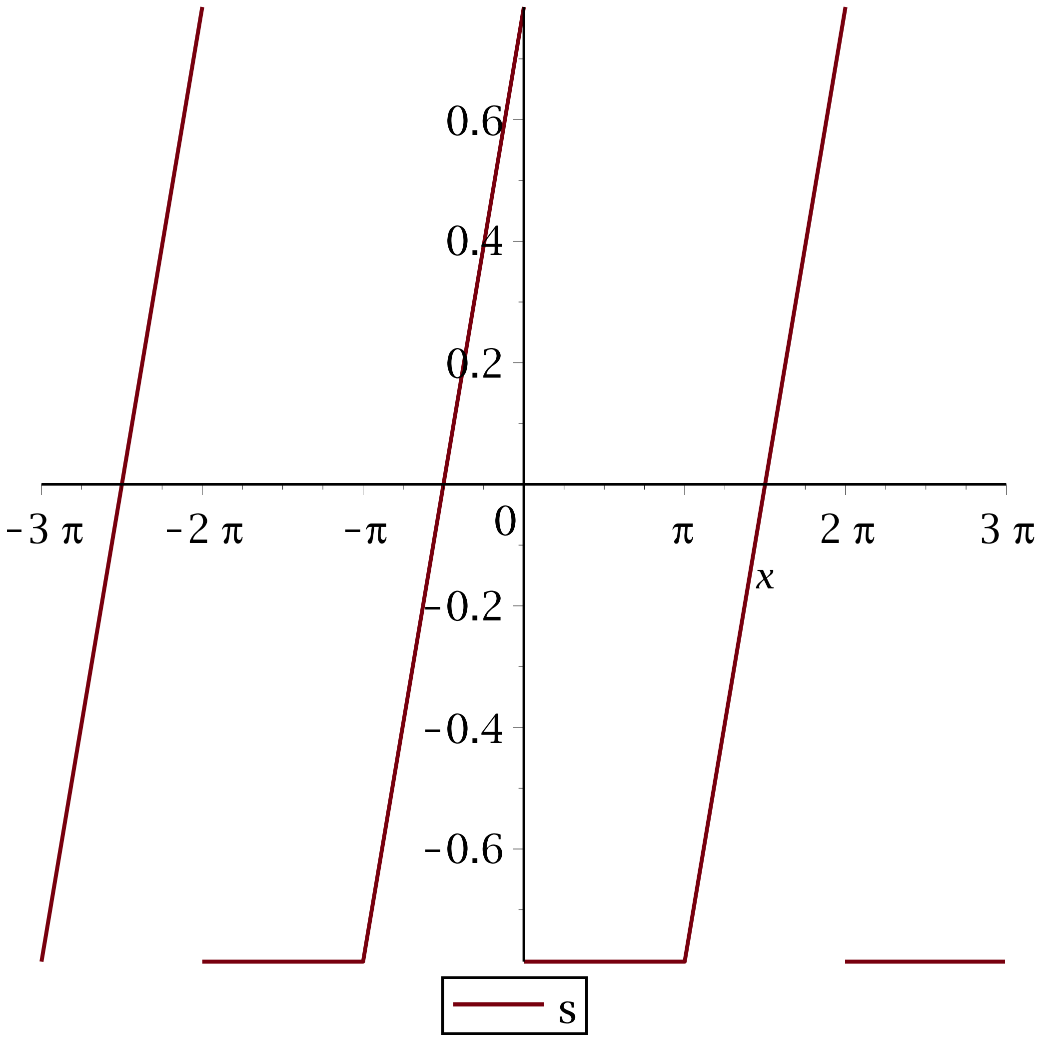
```
> FurieSum:=proc(f, k)
      local a0, an, bn, n;
      description "return Furie Sum for -Pi .. Pi";
      a0:= simplify( int( f(x), x = -π..π) / π);
      assume(n::posint);
      an:= simplify( int( f(x) * cos(n*x), x = -π..π) / π);
      bn:= simplify( int( f(x) * sin(n*x), x = -π..π) / π);
      return 1 / 2 * a0 + sum(an* cos(n*x) + bn* sin(n*x), n = 1 ..k)
  end proc
```

```
FurieSum:= proc(f, k)                                                    (5)
    local a0, an, bn, n;
    description "return Furie Sum for -Pi .. Pi";
    a0:= simplify( int( f(x), x = -π..π) / π);
    assume(n::posint);
    an:= simplify( int( f(x) * cos(n*x), x = -π..π) / π);
    bn:= simplify( int( f(x) * sin(n*x), x = -π..π) / π);
    return 1 / 2 * a0 + sum(an* cos(n*x) + bn* sin(n*x), n = 1 ..k)
end proc
```

```
> S1 = FurieSum(f, 1) :
> S3 = FurieSum(f, 3) :
> S7 = FurieSum(f, 7) :
> S = FurieSum(f, infinity) :
> plot( [FurieSum(f, 1), FurieSum(f, 3), FurieSum(f, 7), FurieSum(f, 1000) ], x = -3π
      ..3 π, legend = ["s1", "s3", "s7", "s"], discont = true)
```

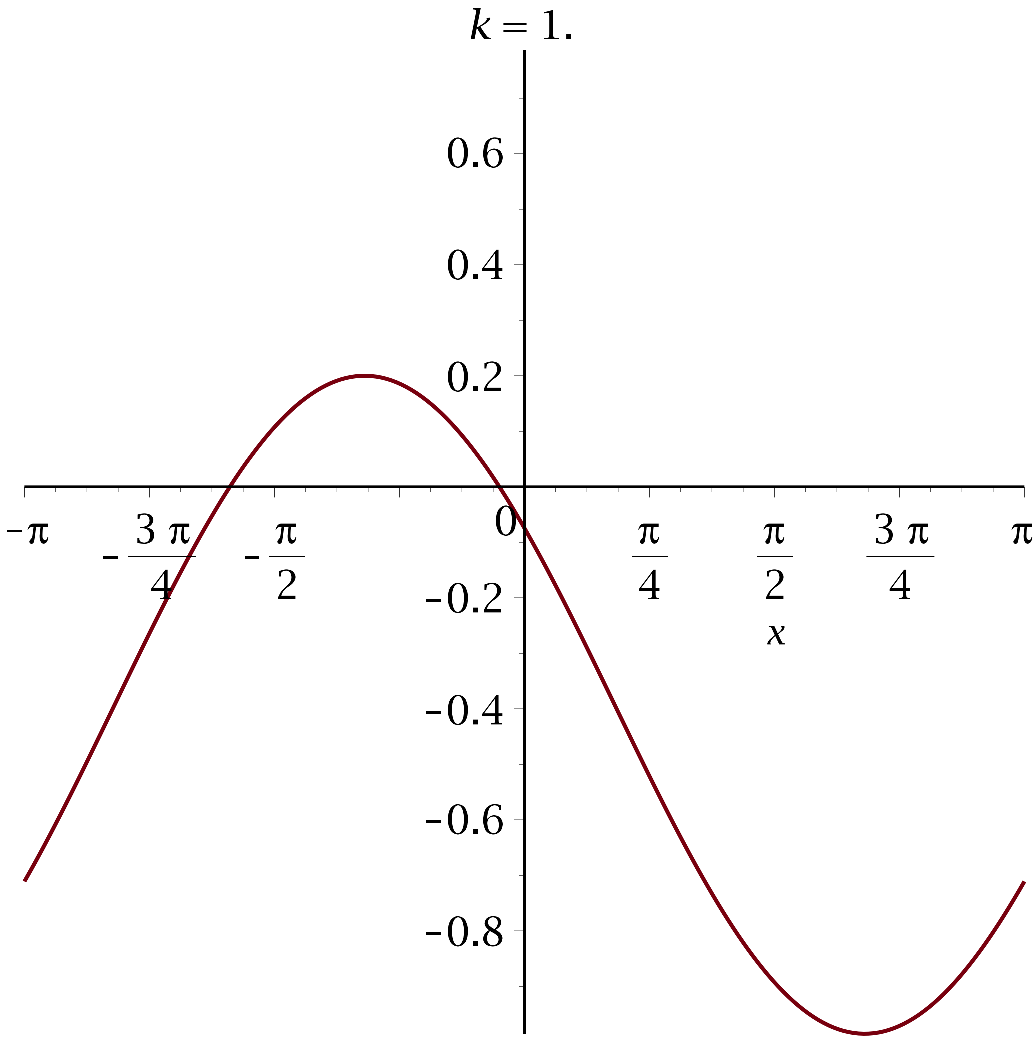


> $plot(FurieSum(f,\ infinity),\ x = -3\pi..3\pi,\ legend = ["s"],\ discont = true)$



> **#Animation**

> $plots[animate](plot, [FurieSum(f, k), x = -\mathrm{Pi}..\mathrm{Pi}], k = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10])$



> #Task 2

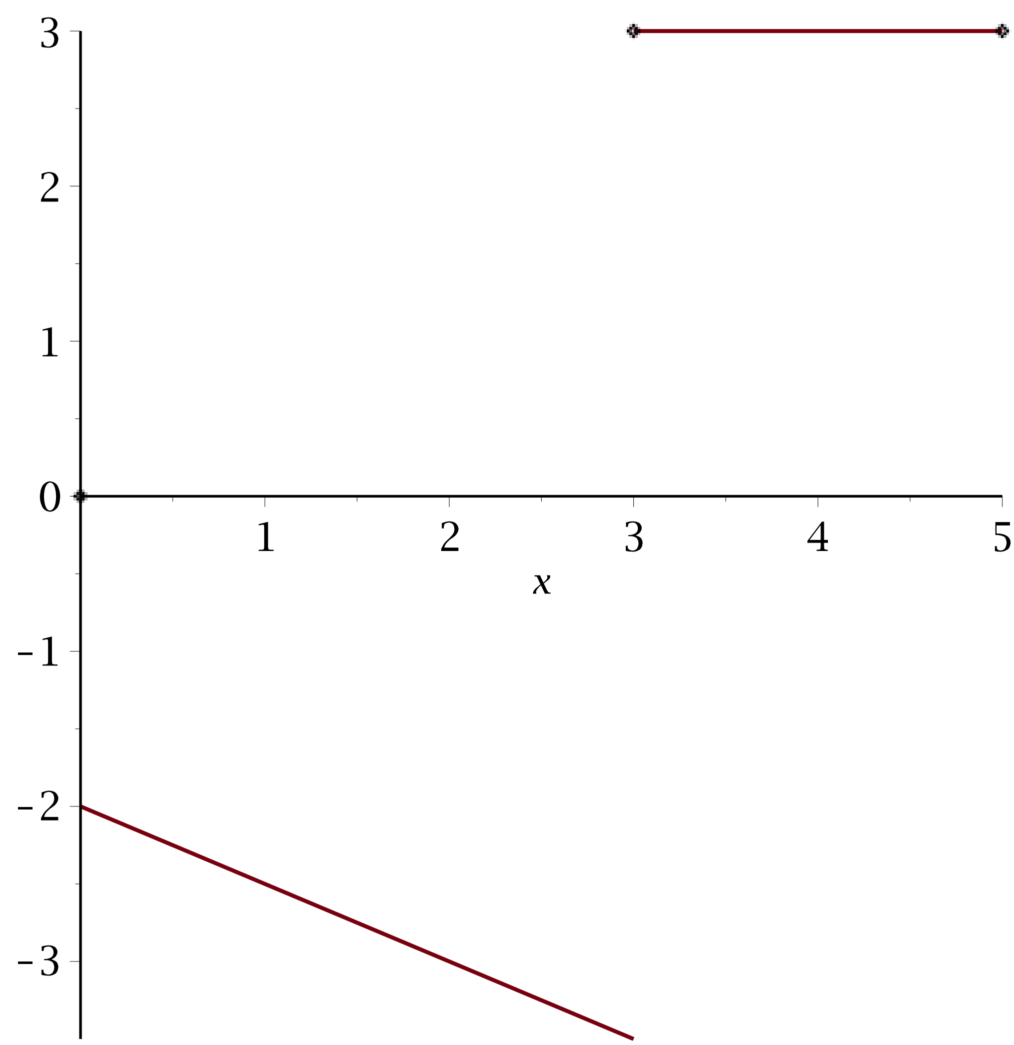
> $f := x \rightarrow piecewise\left(0 < x < 3, -\frac{1}{2} \cdot x - 2, 3 \le x \le 5, 3\right);$

$$f := x \rightarrow piecewise\left(0 < x \textbf{ and } x < 3, -\frac{1}{2}\, x - 2, 3 \le x \textbf{ and } x \le 5, 3\right) \qquad \textbf{(6)}$$

> $f(x)$

$$\begin{cases} -\frac{1}{2}\, x - 2 & 0 < x \textbf{ and } x < 3 \\ 3 & 3 \le x \textbf{ and } x \le 5 \end{cases} \qquad \textbf{(7)}$$

> $plot(f(x), x = 0..5, discont = true)$



> $l := \frac{5}{2}$:#half of a period

> $a0 := simplify\left(\frac{1}{l}\cdot int(f(x), x = 0..2\cdot l)\right);$

$$a0 := -\frac{9}{10} \tag{8}$$

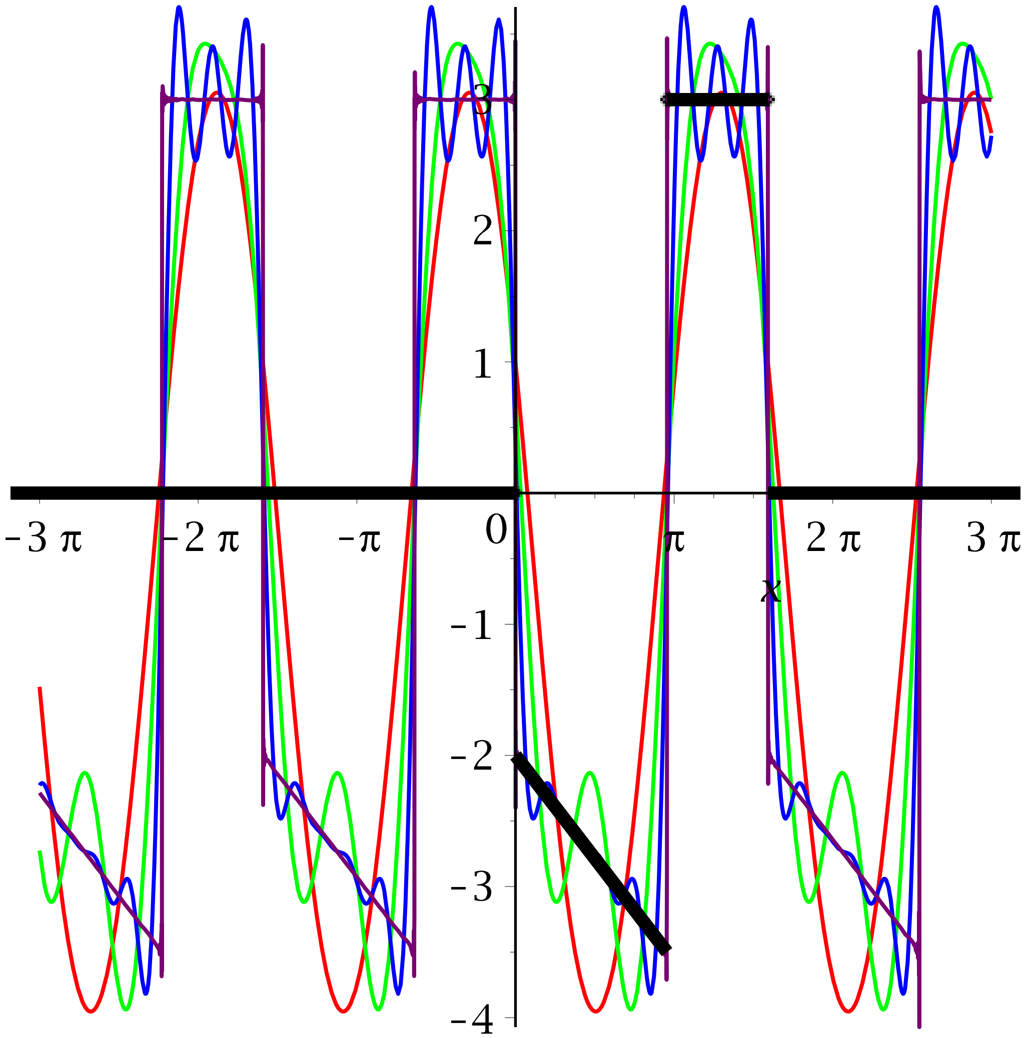
> $an := simplify\left(\frac{1}{l}\cdot int\left(f(x)\cdot\cos\left(\frac{\pi\cdot n\cdot x}{l}\right), x = 0..2\cdot l\right)\right)$ assuming $n :: posint$

$$an := -\frac{1}{4}\,\frac{26\,\pi\,n\sin\left(\frac{6}{5}\,\pi\,n\right) + 5\cos\left(\frac{6}{5}\,\pi\,n\right) - 5}{\pi^2\,n^2} \tag{9}$$

> $bn := simplify\left(\frac{1}{l}\cdot int\left(f(x)\cdot\sin\left(\frac{\pi\cdot n\cdot x}{l}\right), x = 0..2\cdot l\right)\right)$ assuming $n :: posint$

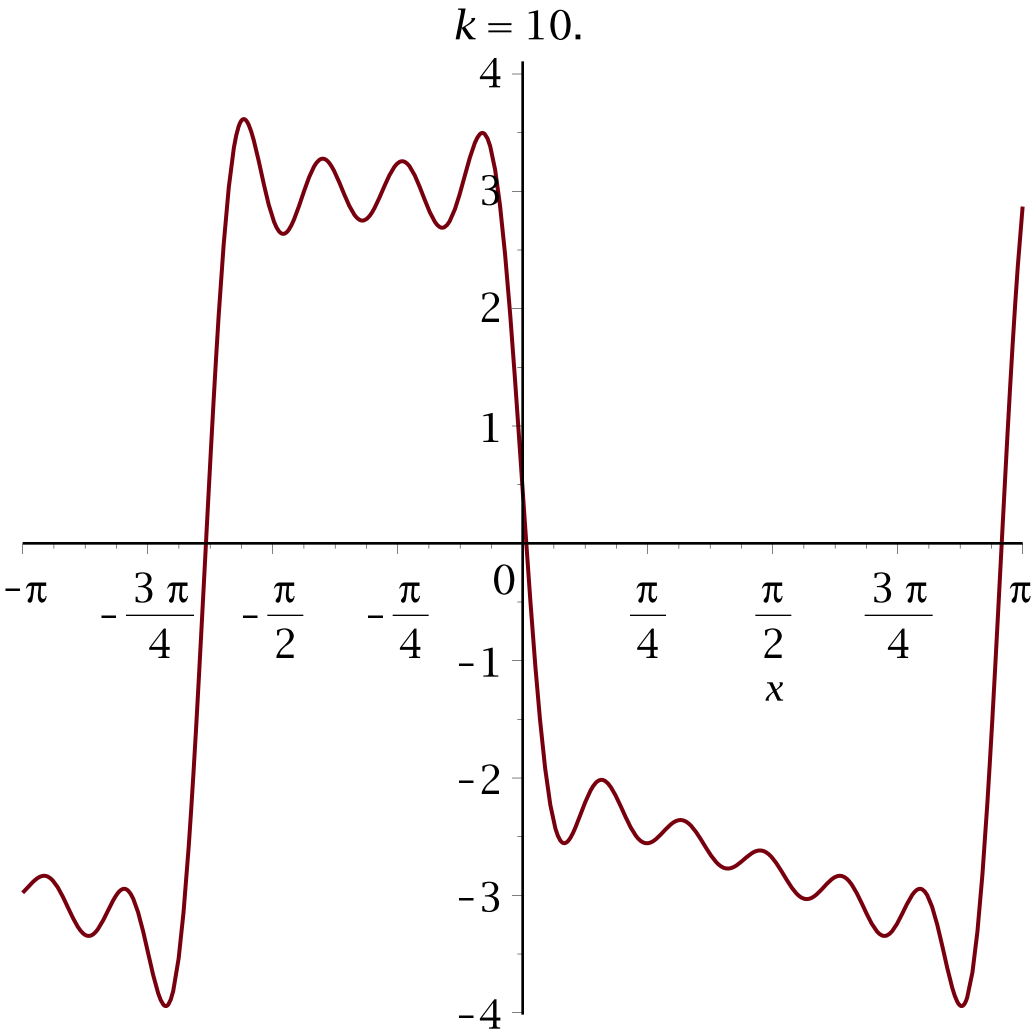
$$bn := \frac{1}{4}\,\frac{26\,\pi\,n\cos\left(\frac{6}{5}\,\pi\,n\right) - 20\,\pi\,n - 5\sin\left(\frac{6}{5}\,\pi\,n\right)}{\pi^2\,n^2} \tag{10}$$

```
> #New proc
> FurieSumNew :=proc(f, k, x1, x2)
      local  a0, an, bn, n, l;
      l:= 1 / 2 * x2 − 1 / 2 * x1;
      a0:= simplify(int(f(x), x = 0 ..2 * l) / l);
      assume(n::posint);
      an:= simplify( int( f(x) * cos(π * n * x / l), x = 0 ..2 * l) / l);
      bn:= simplify( int( f(x) * sin(π * n * x / l), x = 0 ..2 * l) / l);
      return  1 / 2 * a0 + sum( an * cos(π * n * x / l) + bn * sin(π * n * x / l), n = 1 ..k)
  end proc
FurieSumNew:= proc(f, k, x1, x2)                                              (11)
    local  a0, an, bn, n, l;
    l:= 1 / 2 * x2 − 1 / 2 * x1;
    a0:= simplify(int(f(x), x = 0 ..2 * l) / l);
    assume(n::posint);
    an:= simplify( int( f(x) * cos( n * π * x / l), x = 0 ..2 * l) / l);
    bn:= simplify( int( f(x) * sin( n * π * x / l), x = 0 ..2 * l) / l);
    return  1 / 2 * a0 + sum( an * cos( n * π * x / l) + bn * sin( n * π * x / l), n = 1 ..k)
end proc
> fur := plot( [ FurieSumNew(f, 1, 0, 5), FurieSumNew(f, 3, 0, 5),  FurieSumNew(f, 7,
      0, 5), FurieSumNew(f, 1000, 0, 5) ], x = - 3·Pi ..3·Pi, discont = true, color = [ red,
       green, blue, purple]) :
> func := plot(f(x), x = - 10 ..10, discont = true, color = black, thickness = 5) :
> plots[display](fur, func)
```



> #Animation

> *plots*[*animate*](*plot*, [*FurieSumNew*(*f*, *k*, 0, 5), *x* = -Pi..Pi], *k* = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10])

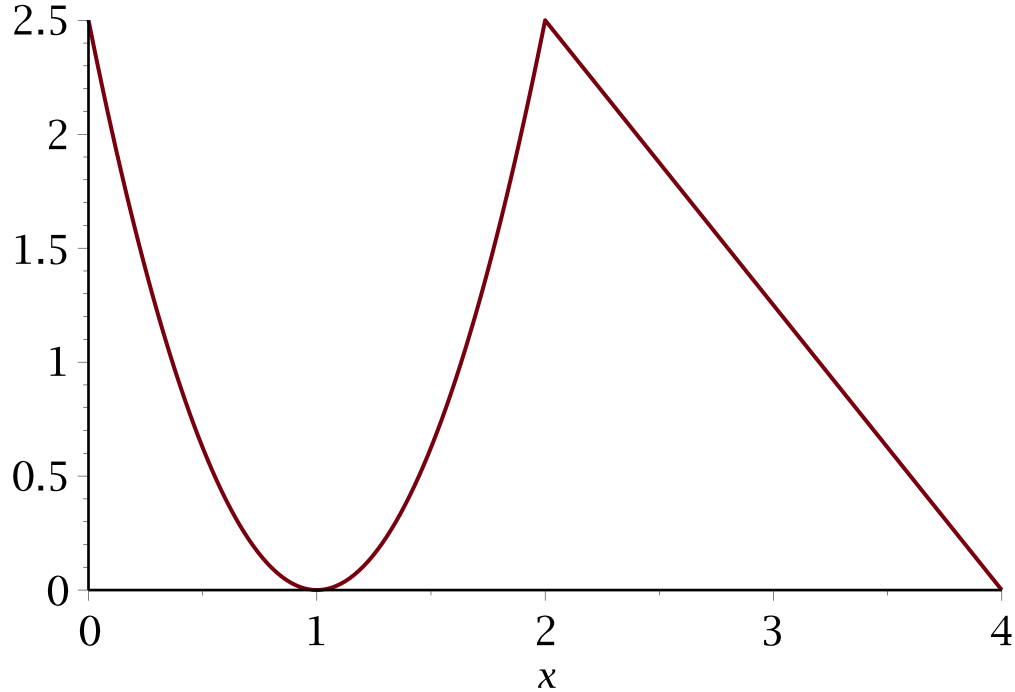


> **#Task 3**

> $f := x \rightarrow piecewise\left(0 \leq x \leq 2, \frac{5}{2} \cdot (x-1)^2, 2 < x < 4, \frac{5}{4} \cdot (4-x)\right);$

$$f := x \rightarrow piecewise\left(0 \leq x \textbf{ and } x \leq 2, \frac{5}{2}\,(x-1)^2, 2 < x \textbf{ and } x < 4, 5 - \frac{5}{4}\,x\right) \tag{12}$$

> $plot(f(x), x = 0..4, scaling = constrained)$



> $l := 2$

$$l := 2 \tag{13}$$

> $a0 := simplify\left(\frac{1}{l} \cdot int(f(x), x = 0..2 \cdot l)\right);$

$$a0 := \frac{25}{12} \tag{14}$$

> $an := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right), x = 0..2 \cdot l\right)\right)$ assuming $n :: posint$

$$an := \frac{5}{2} \, \frac{3 + 5\,(-1)^n}{\pi^2\, n^2} \tag{15}$$

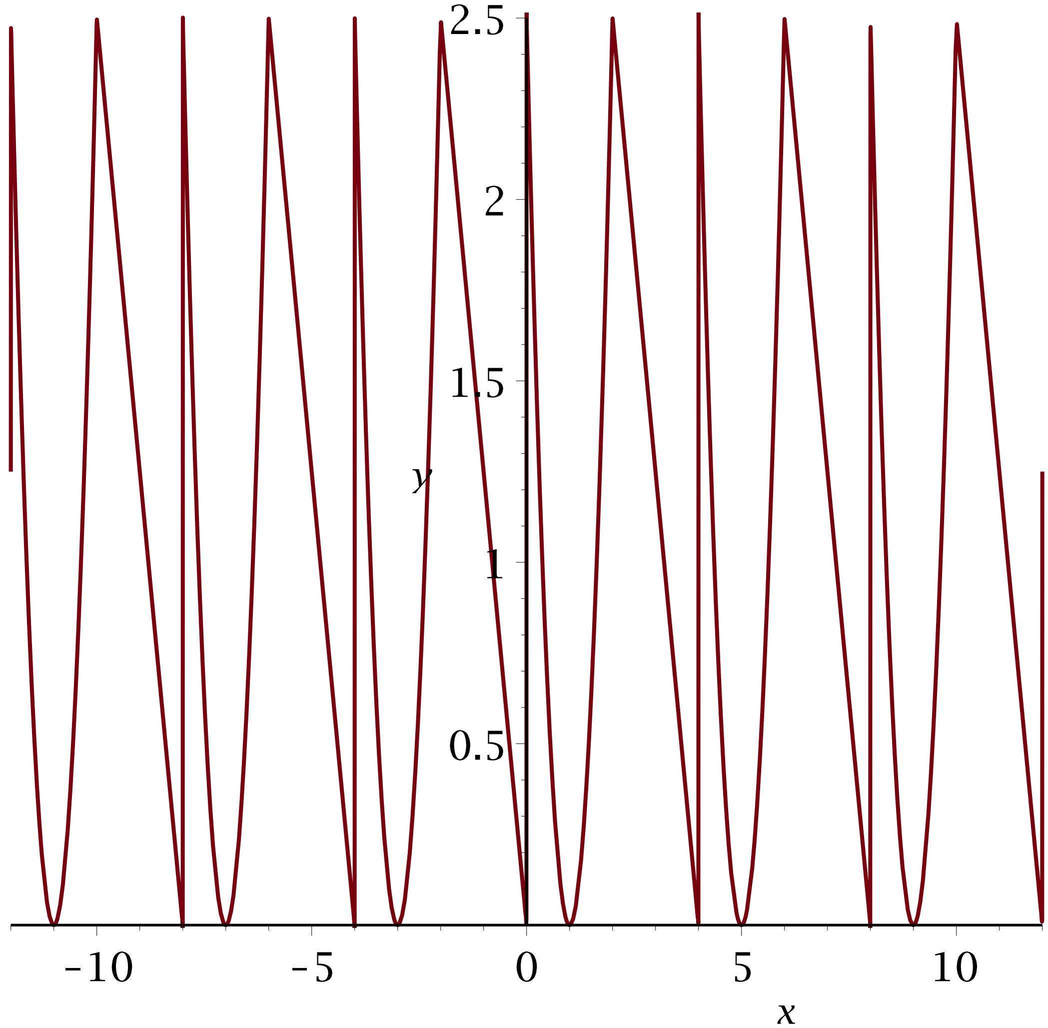
> $bn := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right), x = 0..2 \cdot l\right)\right)$ assuming $n :: posint$

$$bn := \frac{5}{2} \, \frac{\pi^2\, n^2 + 8\,(-1)^n - 8}{\pi^3\, n^3} \tag{16}$$

> $S := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1..k\right)$

$$S := k \to \frac{1}{2}\, a0 + \sum_{n=1}^{k} \left(an \cos\left(\frac{\pi\, n\, x}{l}\right) + bn \sin\left(\frac{\pi\, n\, x}{l}\right) \right) \tag{17}$$

> $plot(S(10000),\ x = -12..12,\ y = 0..2.5,\ discont = true)$



> $f(x)$

$$\begin{cases} \frac{5}{2}\,(x-1)^2 & 0 \le x \text{ and } x \le 2 \\ 5 - \frac{5}{4}\,x & 2 < x \text{ and } x < 4 \end{cases} \tag{18}$$

> $fchetn := x \to piecewise\left(-4 < x < -2,\ \frac{5}{4}\cdot(4+x), -2 \le x \le 0,\ \frac{5}{2}\cdot(-x-1)^2,\ 0 \le x \le 2,\ \frac{5}{2}\cdot(x-1)^2,\ 2 < x < 4,\ \frac{5}{4}\cdot(4-x)\right);$

#chetn func

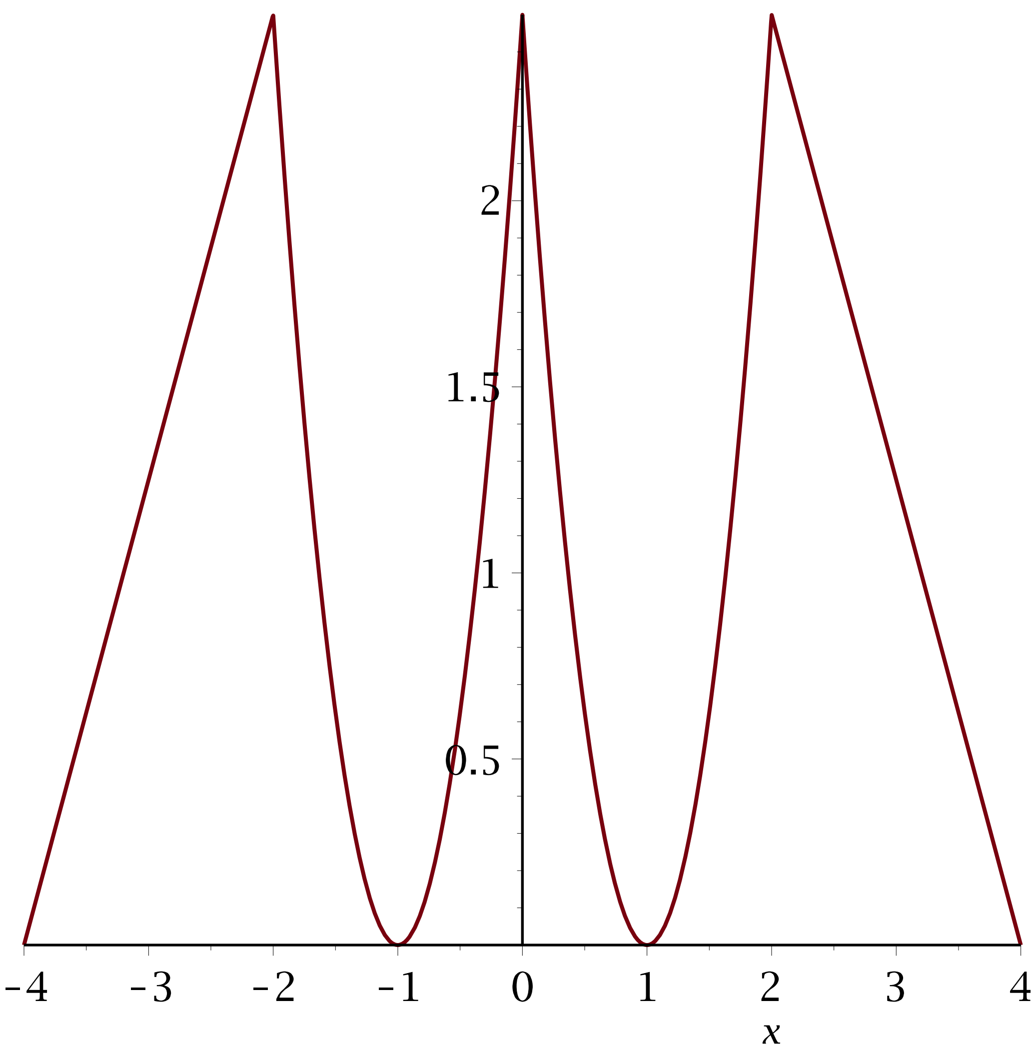
$$fchetn := x \to piecewise\left(-4 < x \text{ and } x < -2,\ 5 + \frac{5}{4}\,x,\ -2 \le x \text{ and } x \le 0,\ \frac{5}{2}\,(-x \right. \tag{19}$$

$-1)^2, 0 \le x$ **and** $x \le 2, \frac{5}{2}(x-1)^2, 2 < x$ **and** $x < 4, 5 - \frac{5}{4}x\Big)$

> $fchetn(x)$

$$\begin{cases} 5 + \frac{5}{4}x & -4 < x \textbf{ and } x < -2 \\ \frac{5}{2}(-x-1)^2 & -2 \le x \textbf{ and } x \le 0 \\ \frac{5}{2}(x-1)^2 & 0 \le x \textbf{ and } x \le 2 \\ 5 - \frac{5}{4}x & 2 < x \textbf{ and } x < 4 \end{cases} \quad (20)$$

> $plot(fchetn(x), x = -4..4)$



> $l := 4$

$$l := 4 \quad (21)$$

> $a0 := simplify\left(\frac{1}{l}\cdot int(fchetn(x),\ x=-l..l)\right);$

$$a0 := \frac{25}{12} \tag{22}$$

> $an := simplify\left(\frac{1}{l}\cdot int\left(fchetn(x)\ \cdot\cos\left(\frac{\pi\cdot n\cdot x}{l}\right),\ x=-l..l\right)\right)$ assuming $n :: posint$

$$an := \frac{10\,(-1)^{n+1}\,\pi\,n + 50\,\pi\,n\cos\left(\frac{1}{2}\,\pi\,n\right) + 40\,\pi\,n - 160\sin\left(\frac{1}{2}\,\pi\,n\right)}{\pi^3\,n^3} \tag{23}$$

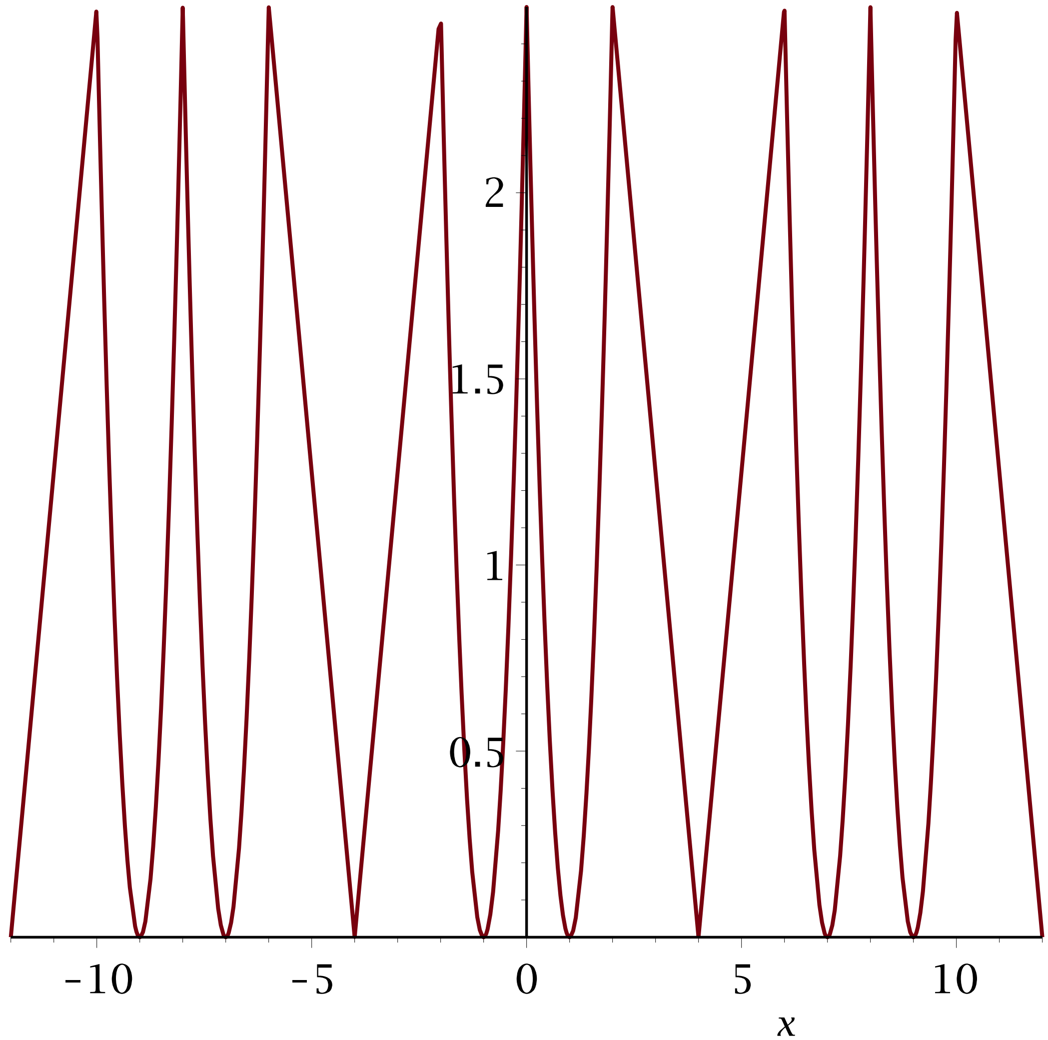
> $bn := simplify\left(\frac{1}{l}\cdot int\left(fchetn(x)\ \cdot\sin\left(\frac{\pi\cdot n\cdot x}{l}\right),\ x=-l..l\right)\right)$ assuming $n :: posint$

$$bn := 0 \tag{24}$$

> $S := k \rightarrow \frac{a0}{2} + sum\left(an\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{l}\right) + bn\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{l}\right),\ n=1..k\right)$

$$S := k \rightarrow \frac{1}{2}\,a0 + \sum_{n=1}^{k}\left(an\cos\left(\frac{\pi\,n\,x}{l}\right) + bn\sin\left(\frac{\pi\,n\,x}{l}\right)\right) \tag{25}$$

> $plot(S(10000),\ x=-12..12,\ discont = true);$

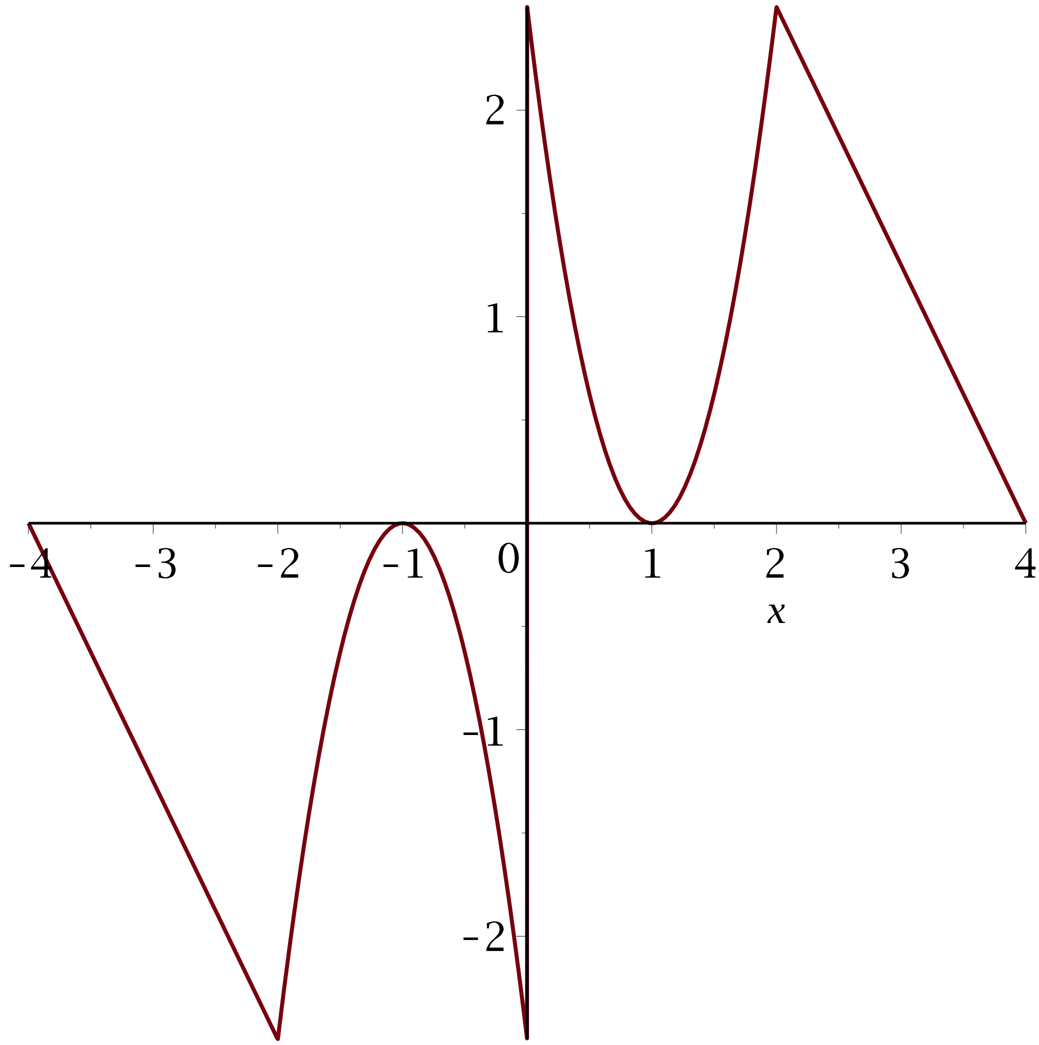


> **#nechentn func**

> $fnech := x \rightarrow piecewise\left(-4 < x < -2, -\frac{5}{4}\cdot(4+x), -2 \leq x \leq 0, -\frac{5}{2}\cdot(-x-1)^2, 0 \leq x \leq 2, \frac{5}{2}\cdot(x-1)^2, 2 < x < 4, \frac{5}{4}\cdot(4-x)\right);$

$$fnech := x \rightarrow piecewise\left(-4 < x \textbf{ and } x < -2, -5-\frac{5}{4}\,x, -2 \leq x \textbf{ and } x \leq 0, -\frac{5}{2}\,(-x-1)^2, 0 \leq x \textbf{ and } x \leq 2, \frac{5}{2}\,(x-1)^2, 2 < x \textbf{ and } x < 4, 5-\frac{5}{4}\,x\right) \quad \textbf{(26)}$$

> $plot(fnech(x), x = -4..4)$



> $a0 := simplify\left(\frac{1}{l} \cdot int(fnech(x), x = -l..l)\right);$

$$a0 := 0 \tag{27}$$

> $an := simplify\left(\frac{1}{l} \cdot int\left(fnech(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right), x = -l..l\right)\right)$ assuming $n :: posint$

$$an := 0 \tag{28}$$

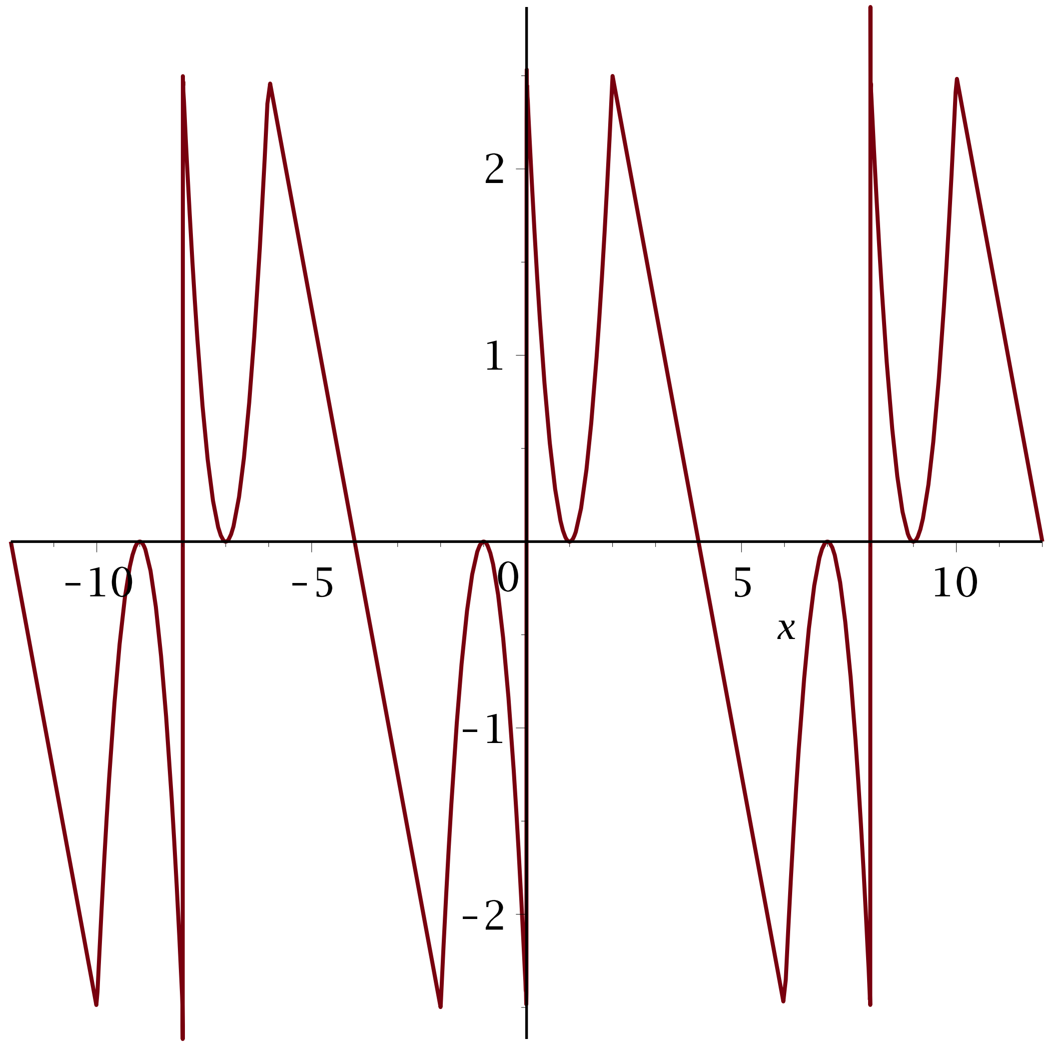
> $bn := simplify\left(\frac{1}{l} \cdot int\left(fnech(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right), x = -l..l\right)\right)$ assuming $n :: posint$

$$bn := \frac{5\,\pi^2\,n^2 + 50\,\pi\,n \sin\left(\frac{1}{2}\,\pi\,n\right) + 160\cos\left(\frac{1}{2}\,\pi\,n\right) - 160}{\pi^3\,n^3} \tag{29}$$

> $S := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1..k\right)$

$$S := k \rightarrow \frac{1}{2}\,a0 + \sum_{n=1}^{k}\left(an \cos\left(\frac{\pi\,n\,x}{l}\right) + bn \sin\left(\frac{\pi\,n\,x}{l}\right)\right) \tag{30}$$

> $plot(S(10000), x = -12..12, discont = true);$



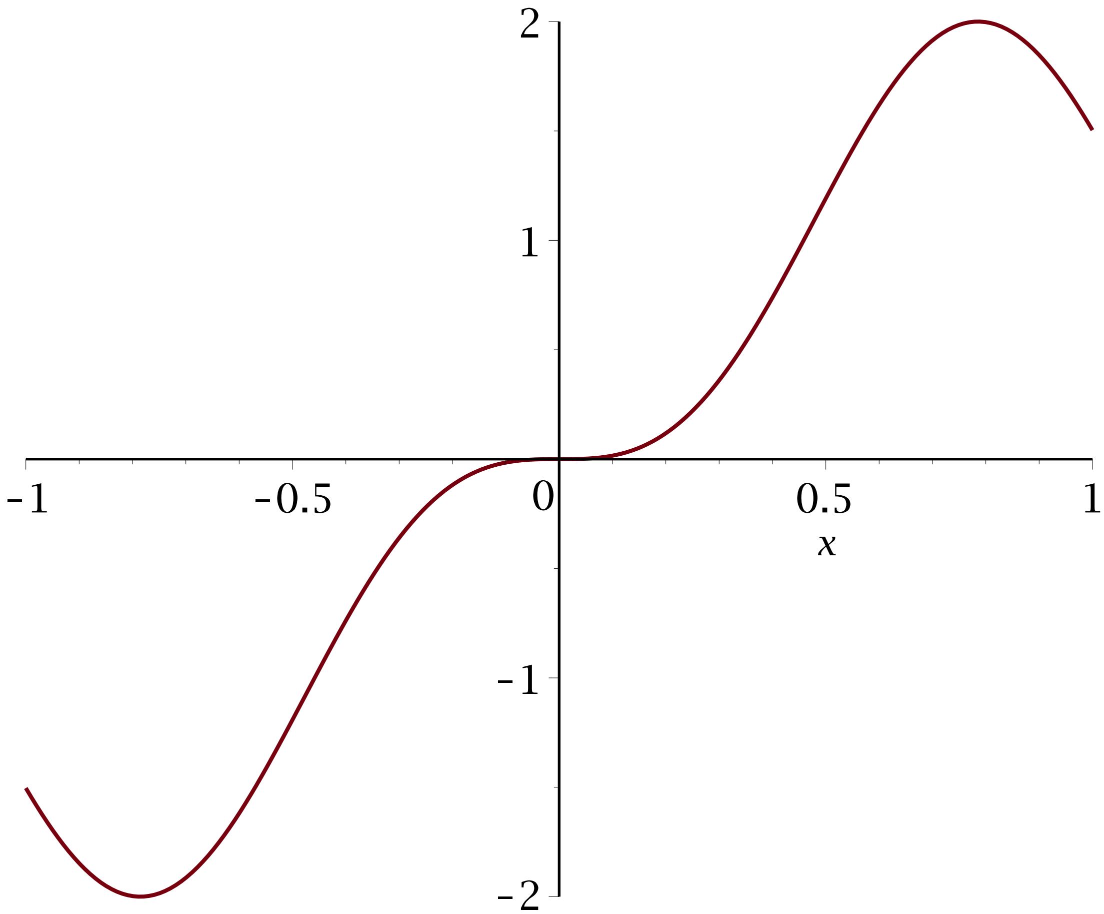
> #Task 4

> $f := 2 \cdot \sin^3(2 \cdot x)$

$$f := 2 \sin(2\,x)^3 \qquad (31)$$

> $funcplot := plot(f, x = -1..1)$

$funcplot :=$



> $with(orthopoly)$

$$[G, H, L, P, T, U] \tag{32}$$

> **for** n **from** 0 **to** 7 **do** $c[n] := \dfrac{\int_{-1}^{1} f\cdot P(n, x)\,dx}{\int_{-1}^{1} P(n, x)^2 dx}$**; end do** #Lejandr coef

$$c_0 := 0$$

$$c_1 := -\sin(2)^2\cos(2) - 2\cos(2) + \frac{1}{6}\sin(2)^3 + \sin(2)$$

$$c_2 := 0$$

$$c_3 := -\frac{49}{36}\sin(2)^2\cos(2) + \frac{133}{9}\cos(2) + \frac{469}{216}\sin(2)^3 + \frac{77}{18}\sin(2)$$

$$c_4 := 0$$

$$c_5 := -\frac{6215}{48}\sin(2) - \frac{6721}{24}\cos(2) + \frac{715}{288}\sin(2)^3 + \frac{209}{48}\sin(2)^2\cos(2)$$

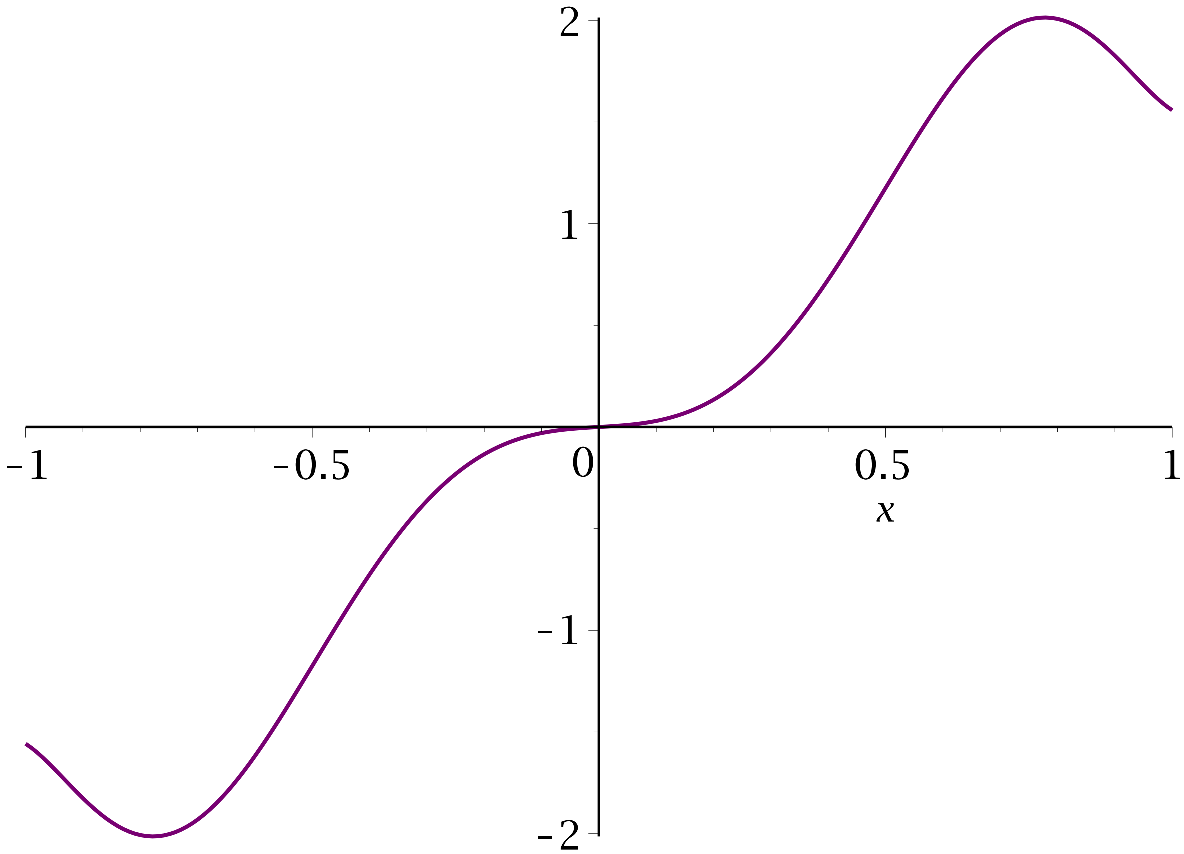
$$c_6 := 0$$

(33)

$$c_7 := -\frac{123305}{10368}\sin(2)^3 + \frac{2499805}{432}\sin(2) + \frac{681785}{54}\cos(2) - \frac{8395}{1728}\sin(2)^2\cos(2) \quad \textbf{(33)}$$

> $lej := plot(add(c[n]\cdot P(n, x), n = 0..7), x = -1..1, color = purple)$

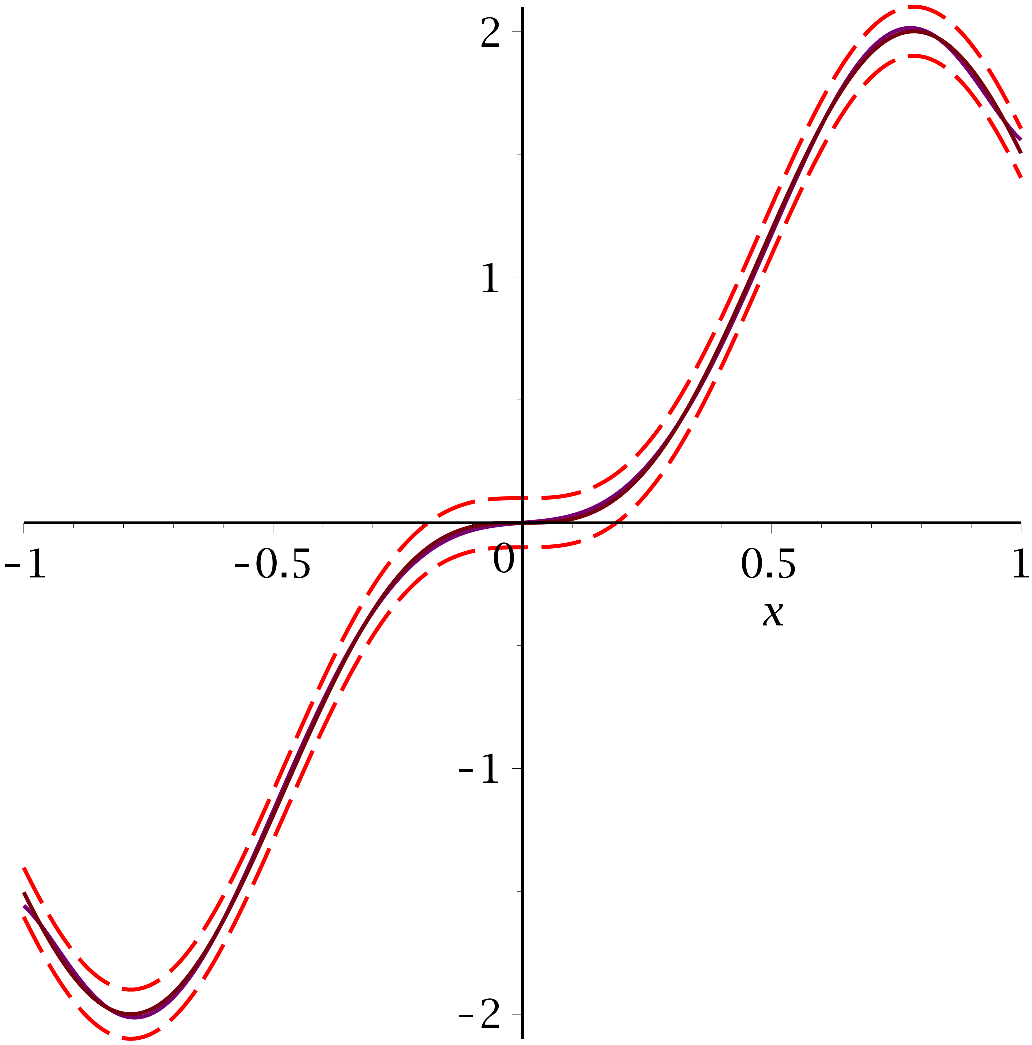
$lej :=$



> $f1 := plot(f + 0.1, x = -1..1, linestyle = dash, color = red)$:

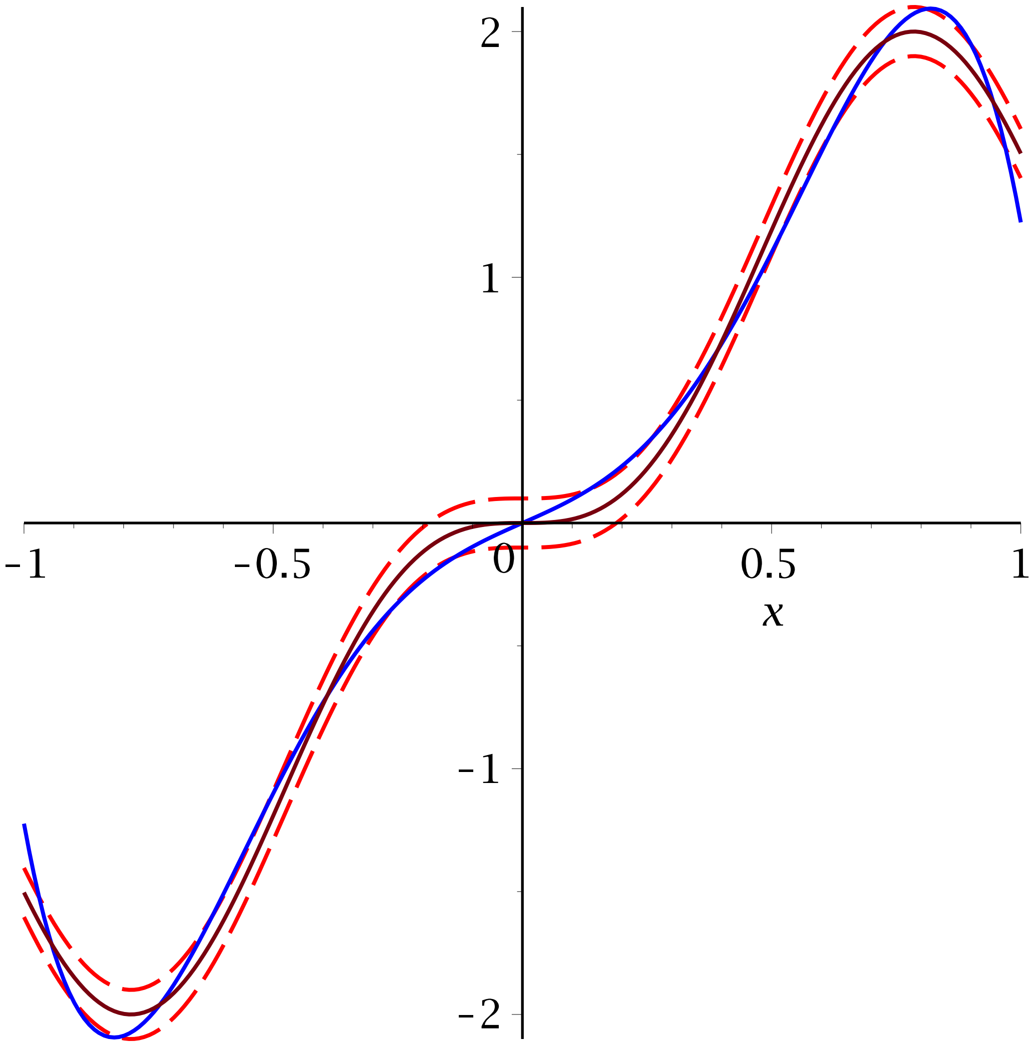
> $f2 := plot(f - 0.1, x = -1..1, linestyle = dash, color = red)$:

> $plots[display]([f1, f2, lej, funcplot])$



> $nmin := plot(add(c[n] \cdot P(n, x), n = 0..6), x = -1..1, color = blue):$

> $plots[display](f1, f2, nmin, funcplot)$ **#how we can see, when n is 6, function deviates by more than 0.1**



> **for** n **from** 0 **to** 7 **do** $c[n] := \frac{2}{\pi}\int_{-1}^{1} \frac{f \cdot T(n, x)}{\sqrt{1-x^2}}\, dx$; **end do** #chebish coef

$$c_0 := 0$$

$$c_1 := \frac{2\left(\int_{-1}^{1} \frac{2 \sin(2x)^3 x}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

$$c_2 := 0$$

$$c_3 := \frac{2\left(\int_{-1}^{1} \frac{2 \sin(2x)^3 (4x^3 - 3x)}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

$$c_4 := 0$$

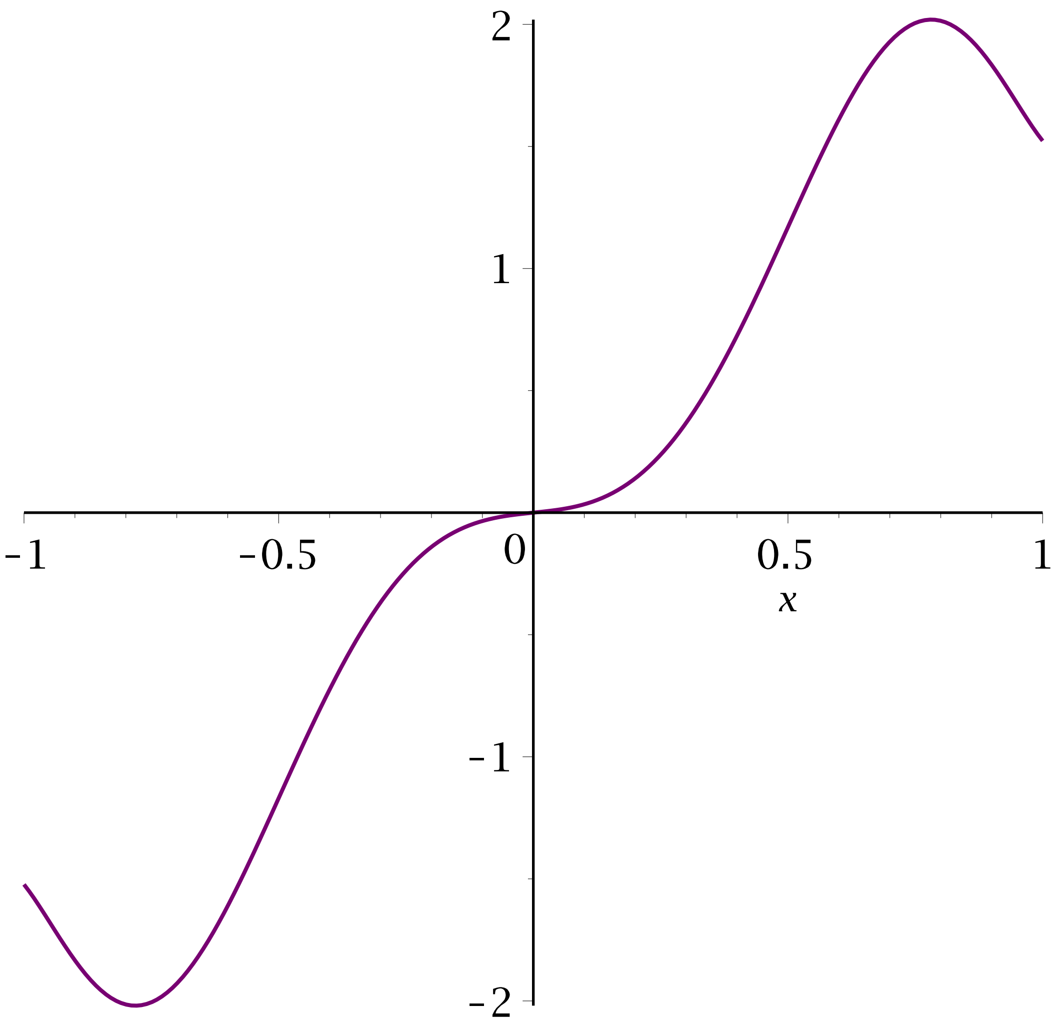
$$c_5 := \frac{2\left(\int_{-1}^{1} \frac{2\sin(2x)^3\,(16x^5 - 20x^3 + 5x)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_6 := 0$$

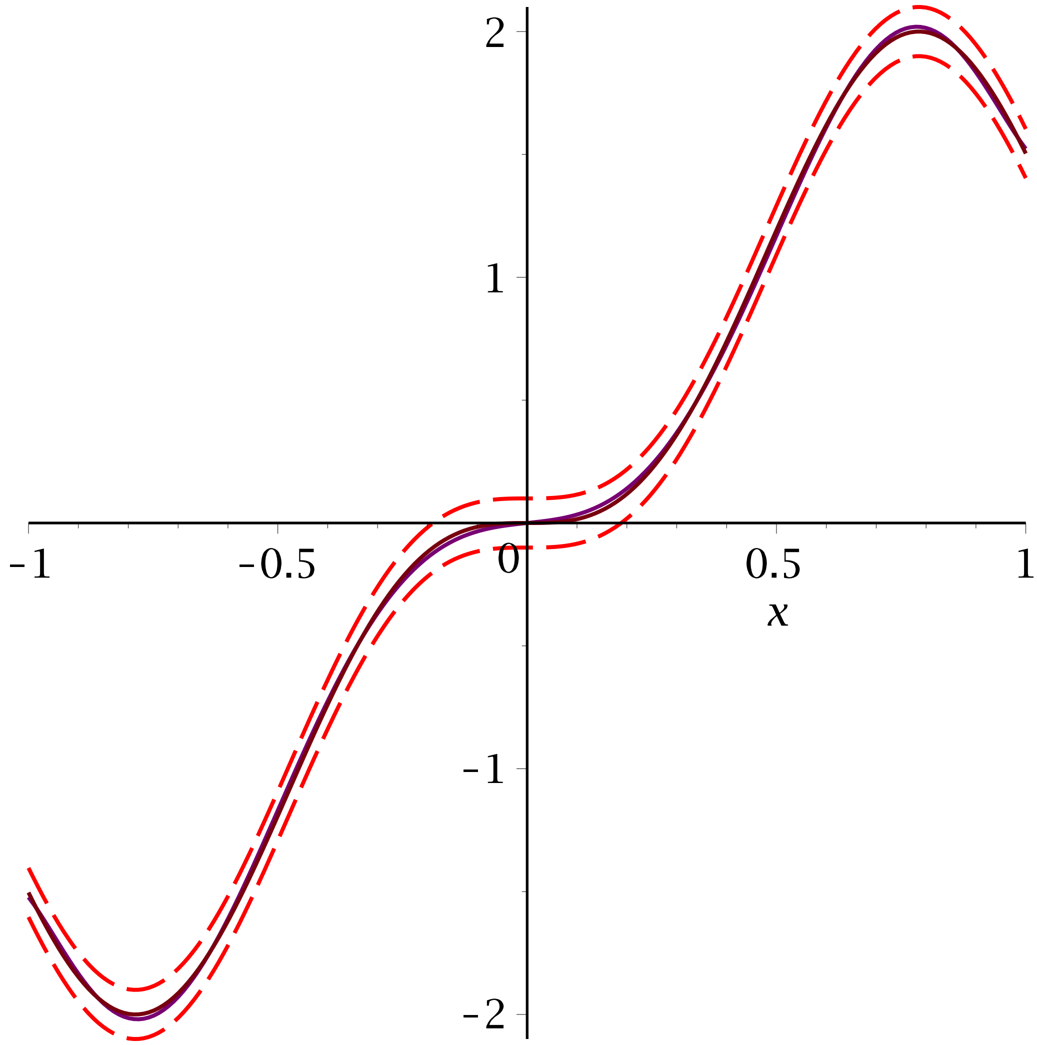
$$c_7 := \frac{2\left(\int_{-1}^{1} \frac{2\sin(2x)^3\,(64x^7 - 112x^5 + 56x^3 - 7x)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi} \tag{34}$$

> cheb := $plot\left(\frac{c[0]}{2} + add(c[n] \cdot T(n, x), n = 1..7), x = -1..1, color = purple\right)$

cheb :=

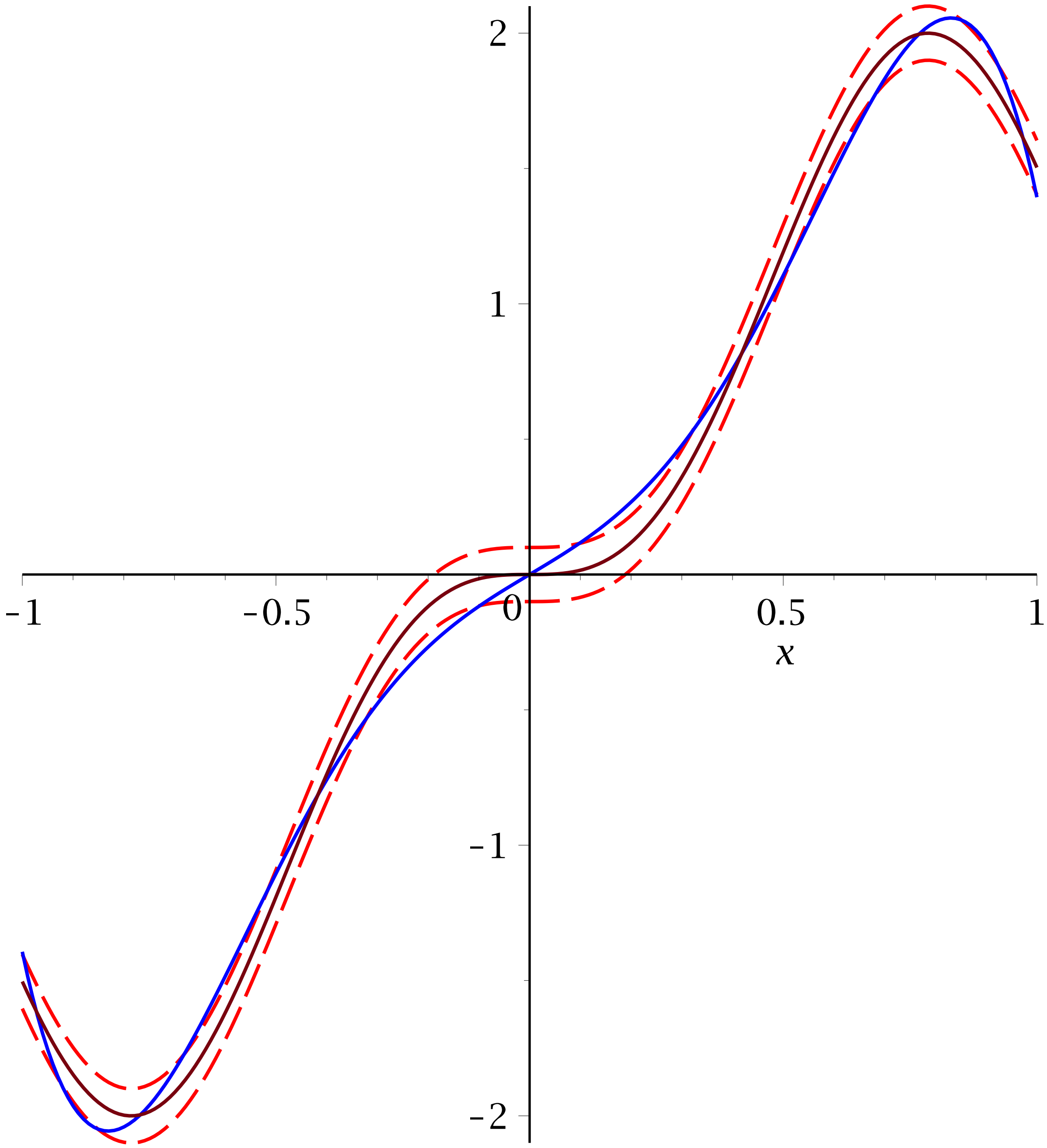


> $plots[display](f1, f2, cheb, funcplot)$



> $nmin := plot\left(\frac{c[0]}{2} + add(c[n]\cdot T(n, x), n = 1..6), x = -1..1, color = blue\right):$

> $plots[display](f1, f2, nmin, funcplot)$ **#how we can see, when n is 6, function deviates by more than 0.1**



> **#lets check Fouriers coefs, func is odd, so we can find only Bn**

> $bn := simplify(int(\mathrm{f}\cdot\sin(\mathrm{Pi}\cdot nn\cdot x), x = -1..1))$ assuming $nn :: posint$

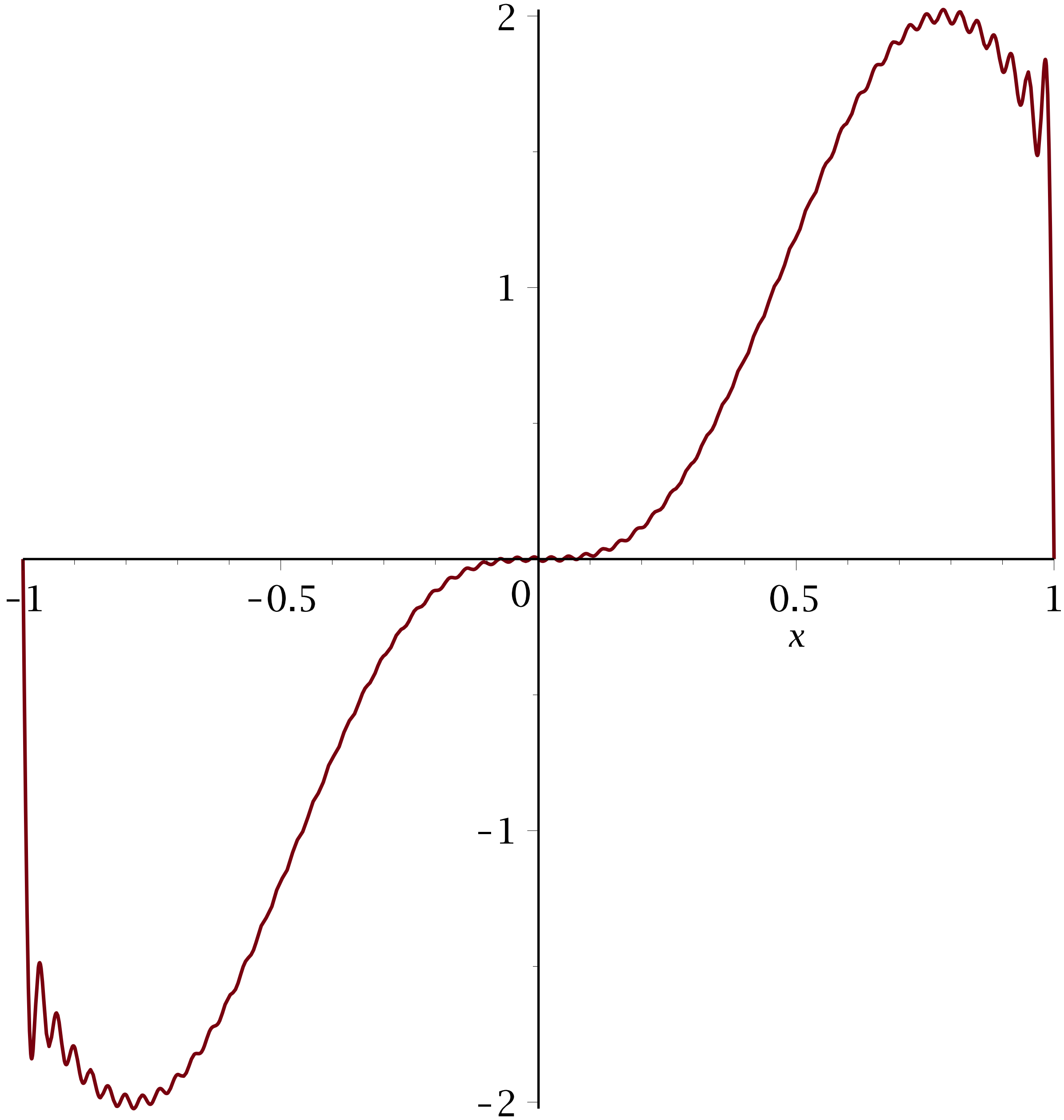
$$bn := -\frac{\pi\, nn\, (-1)^{nn} \left(3\,\pi^2\, nn^2 \sin(2) - \pi^2\, nn^2 \sin(6) - 108 \sin(2) + 4 \sin(6)\right)}{\pi^4\, nn^4 - 40\,\pi^2\, nn^2 + 144} \quad \textbf{(35)}$$

> $Sm := \mathrm{k} \rightarrow sum(bn\cdot\sin(\pi\cdot\mathrm{nn}\cdot\mathrm{x}), nn = 1..k)$

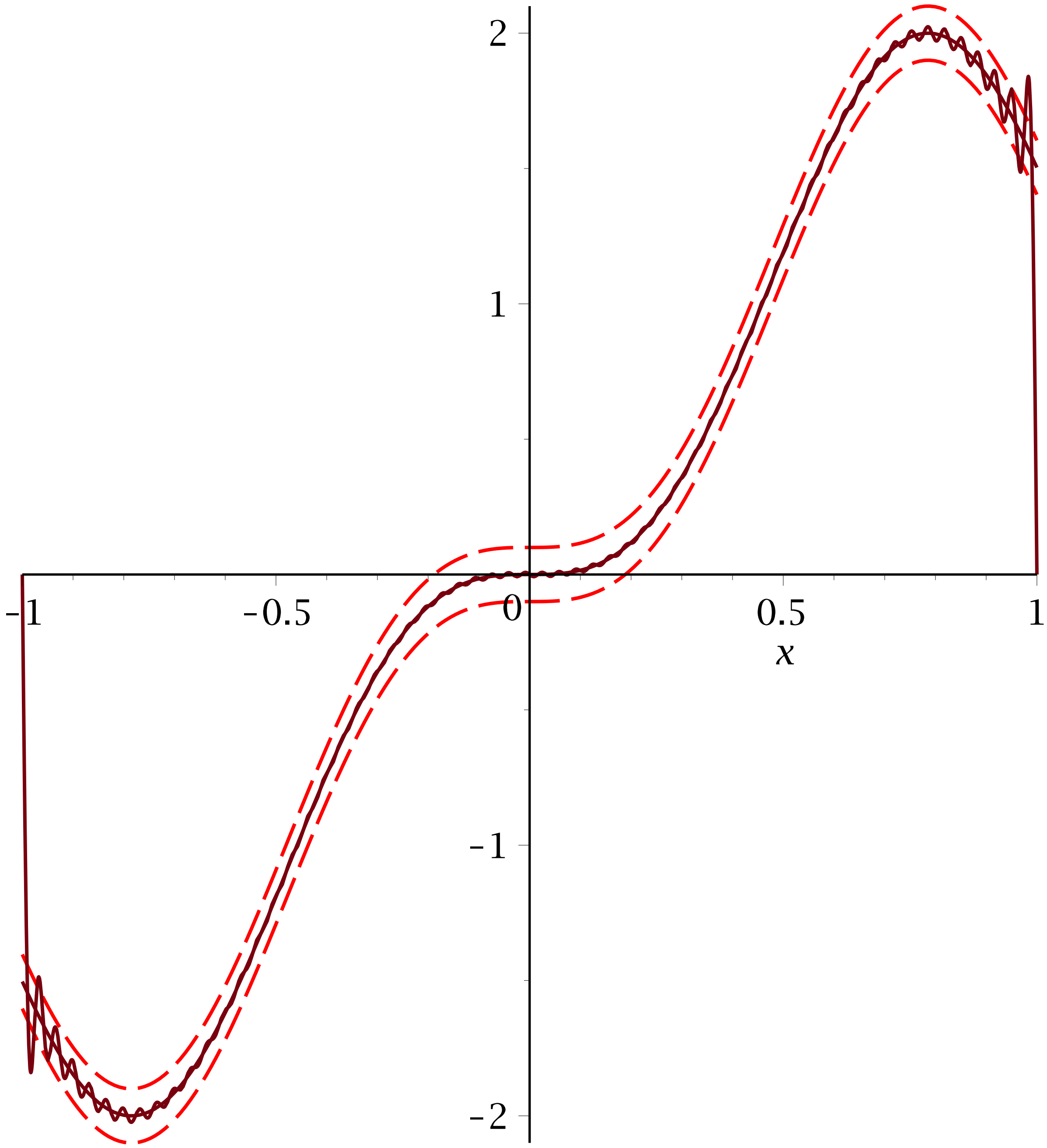
$$Sm := k \rightarrow \sum_{nn=1}^{k} bn \sin(\pi\, nn\, x) \quad \textbf{(36)}$$

> $fur := plot(Sm(60), x = -1..1, discont = true)$

$fur :=$



> *plots*[*display*](*f1*, *f2*, *fur*, *funcplot*)



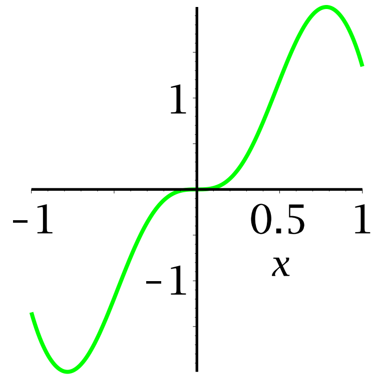
> #how we can see, when n is 60, function deviates by more than 0.1

>

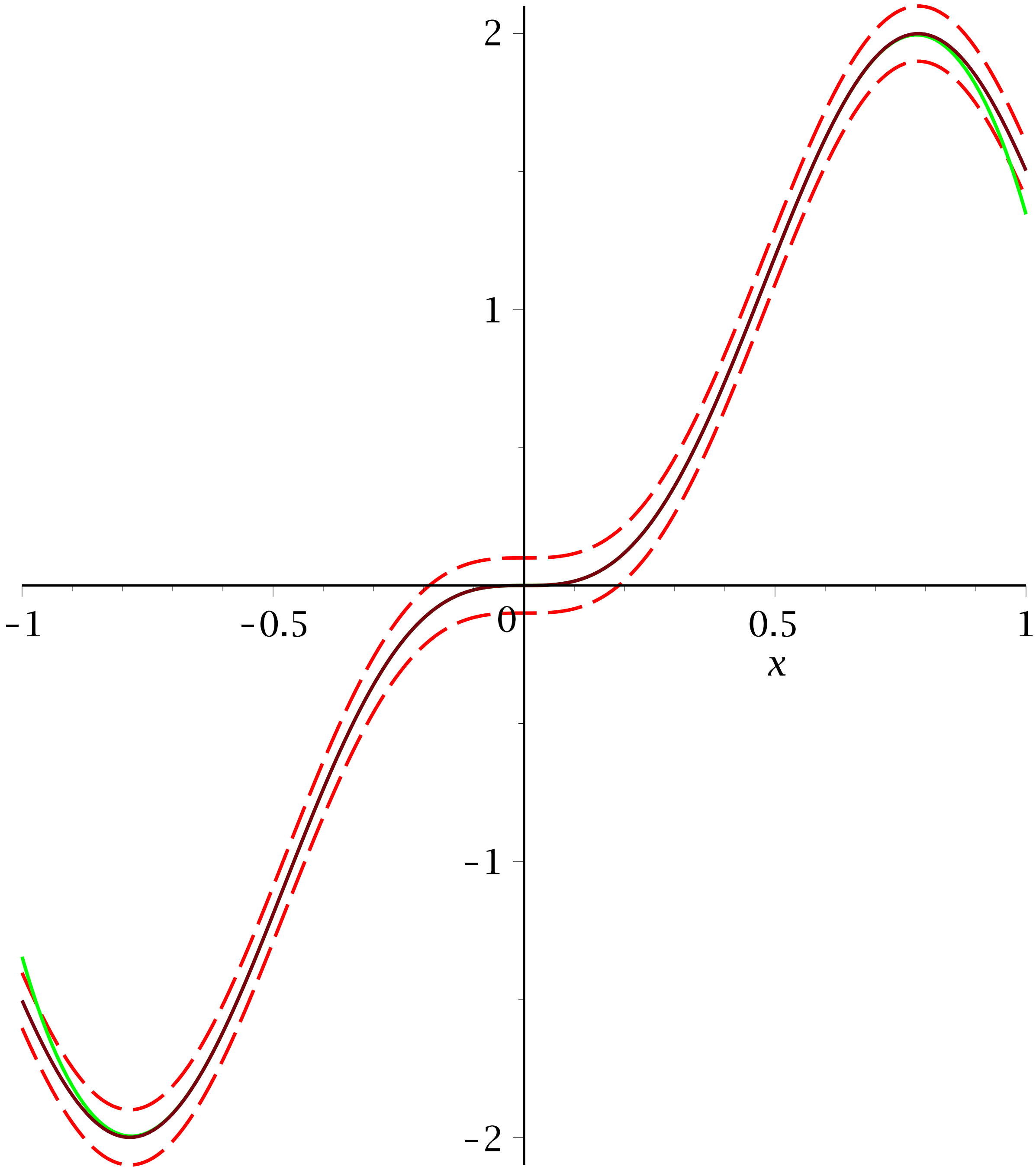
> $St := convert(taylor(f, x = 0, 14), polynom)$

$$St := 16\,x^3 - 32\,x^5 + \frac{416}{15}\,x^7 - \frac{2624}{189}\,x^9 + \frac{21472}{4725}\,x^{11} - \frac{4672}{4455}\,x^{13} \quad \textbf{(37)}$$

> $StF := plot(St, x = -1\,..1, color = green)$

$StF :=$ 

> $plots[display](f1, f2, StF, funcplot)$



> # how we can see, when n is 14, function deviates by more than 0.1
> # Lejandr and Chebishevs polynomes are more accurate than taylors and fouriers

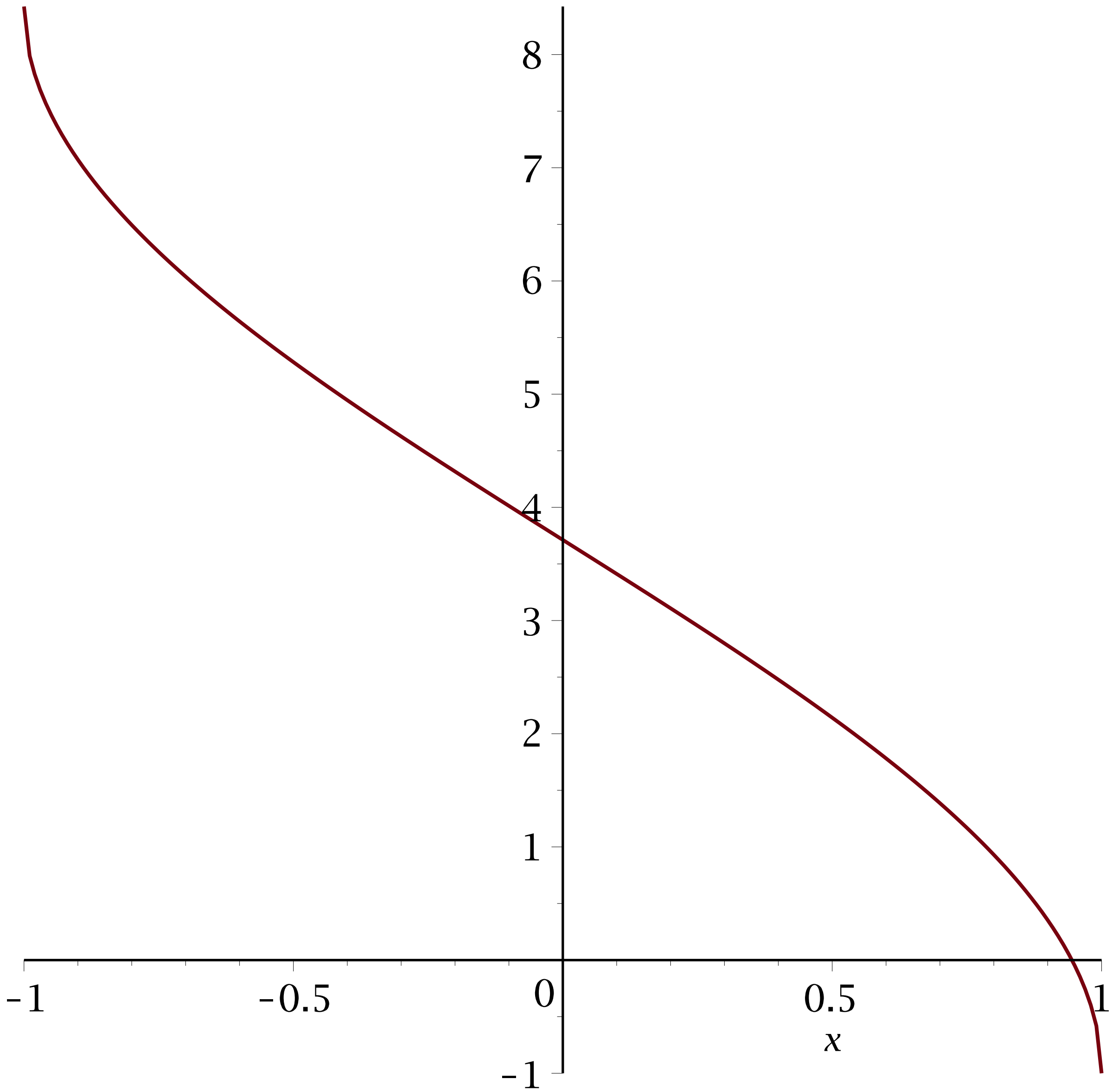
> *restart*

> $f := 3 \cdot \arccos(x) - 1$

$$f := 3 \arccos(x) - 1 \quad (38)$$

> $funcplot := plot(f, x = -1 .. 1)$

$funcplot :=$



> *with*(*orthopoly*)

$$[G, H, L, P, T, U] \tag{39}$$

> **for** n **from** 0 **to** 7 **do** $c[n] := \dfrac{\int_{-1}^{1} f \cdot P(n, x)\, dx}{\int_{-1}^{1} P(n, x)^2 dx}$; **end do** **#Lejandr coef**

$$c_0 := \frac{3}{2}\pi - 1$$

$$c_1 := -\frac{9}{8}\pi$$

$$c_2 := 0$$

$$c_3 := -\frac{21}{128}\pi$$

$$c_4 := 0$$

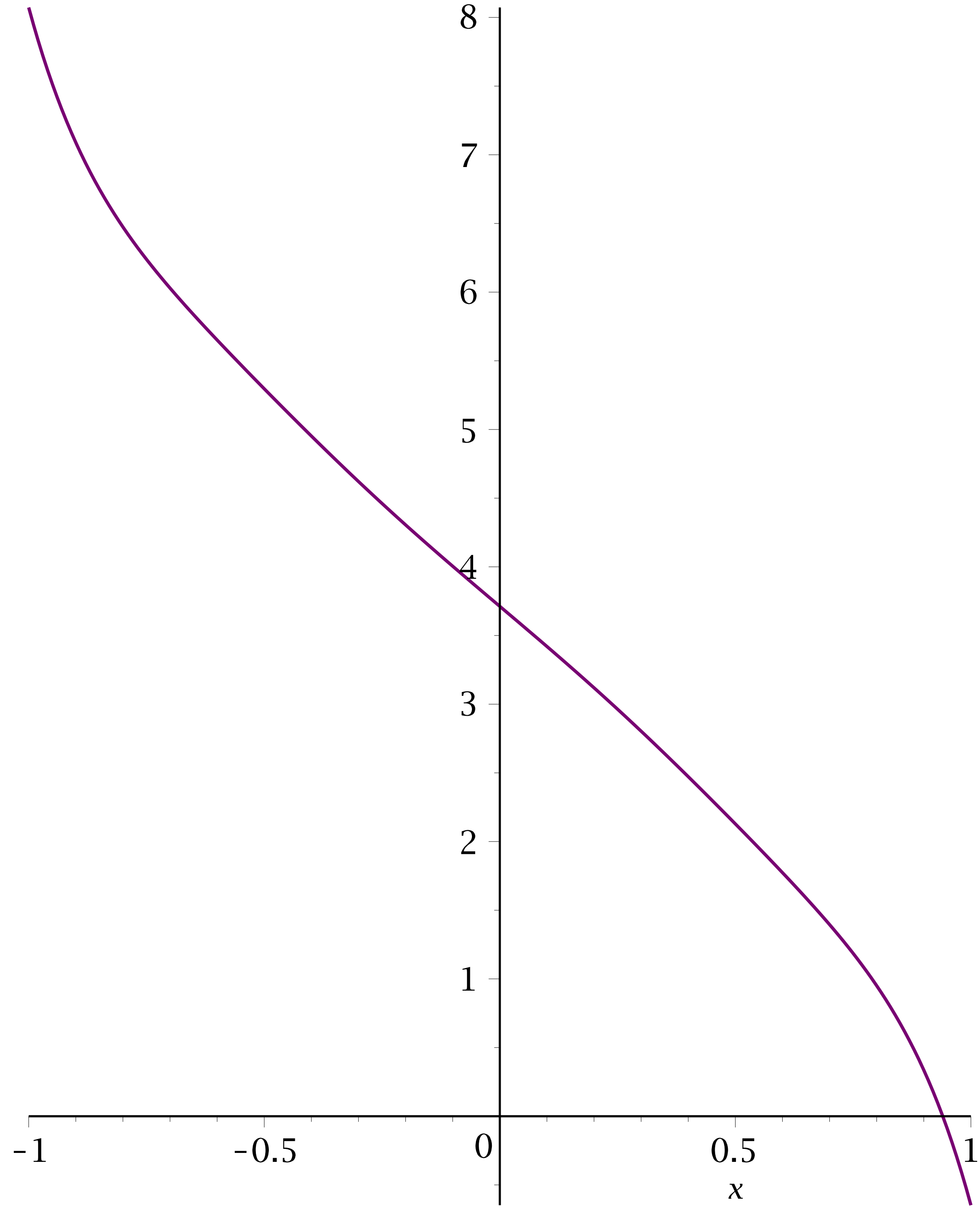
$$c_5 := -\frac{33}{512}\pi$$

$$c_6 := 0$$

$$c_7 := -\frac{1125}{32768}\pi \tag{40}$$

> $lej := plot(add(c[n]\cdot P(n, x), n = 0..7), x = -1..1, color = purple)$

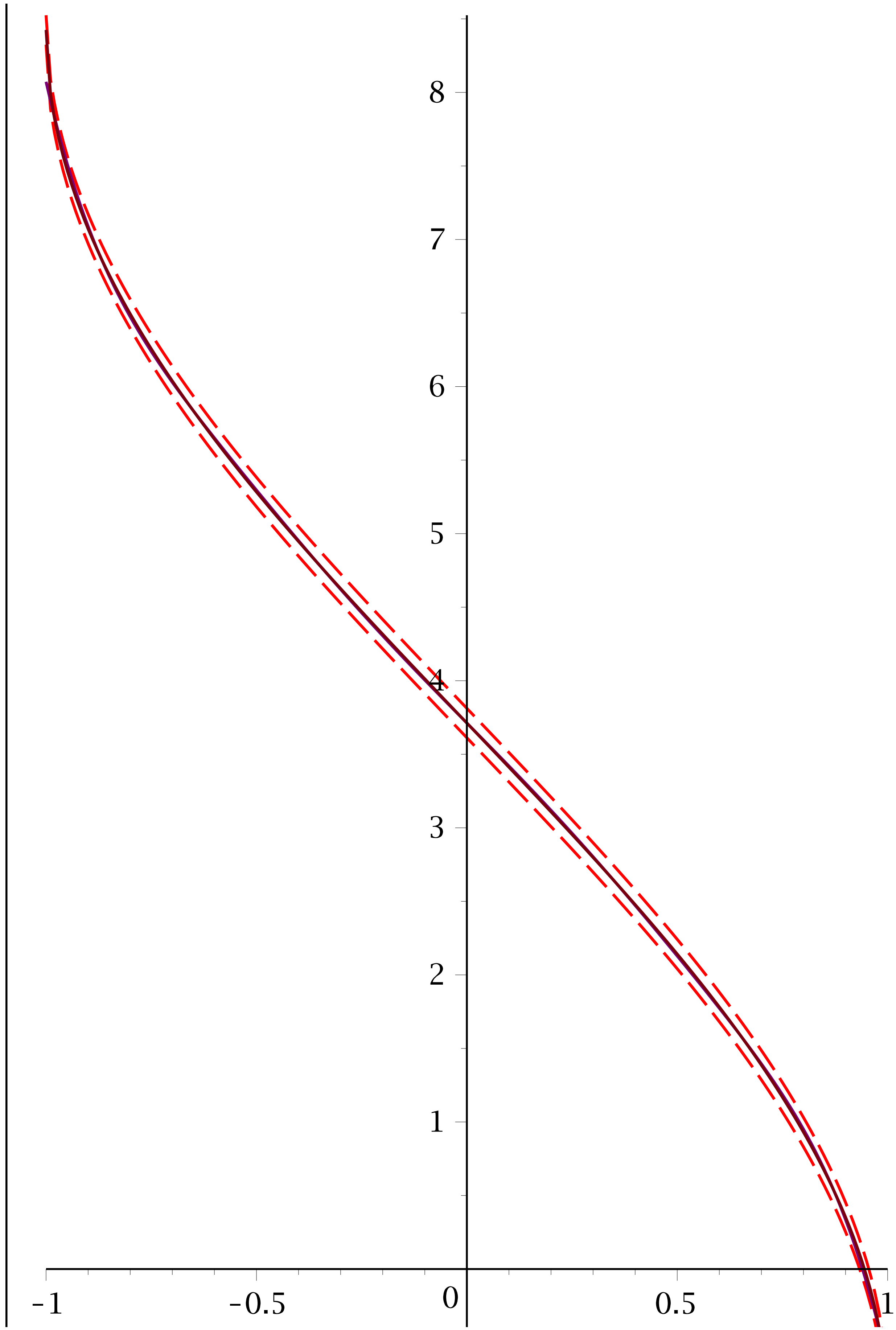
$lej :=$



> $f1 := plot(f + 0.1, x = -1..1, linestyle = dash, color = red)$:

> $f2 := plot(f - 0.1, x = -1..1, linestyle = dash, color = red)$:

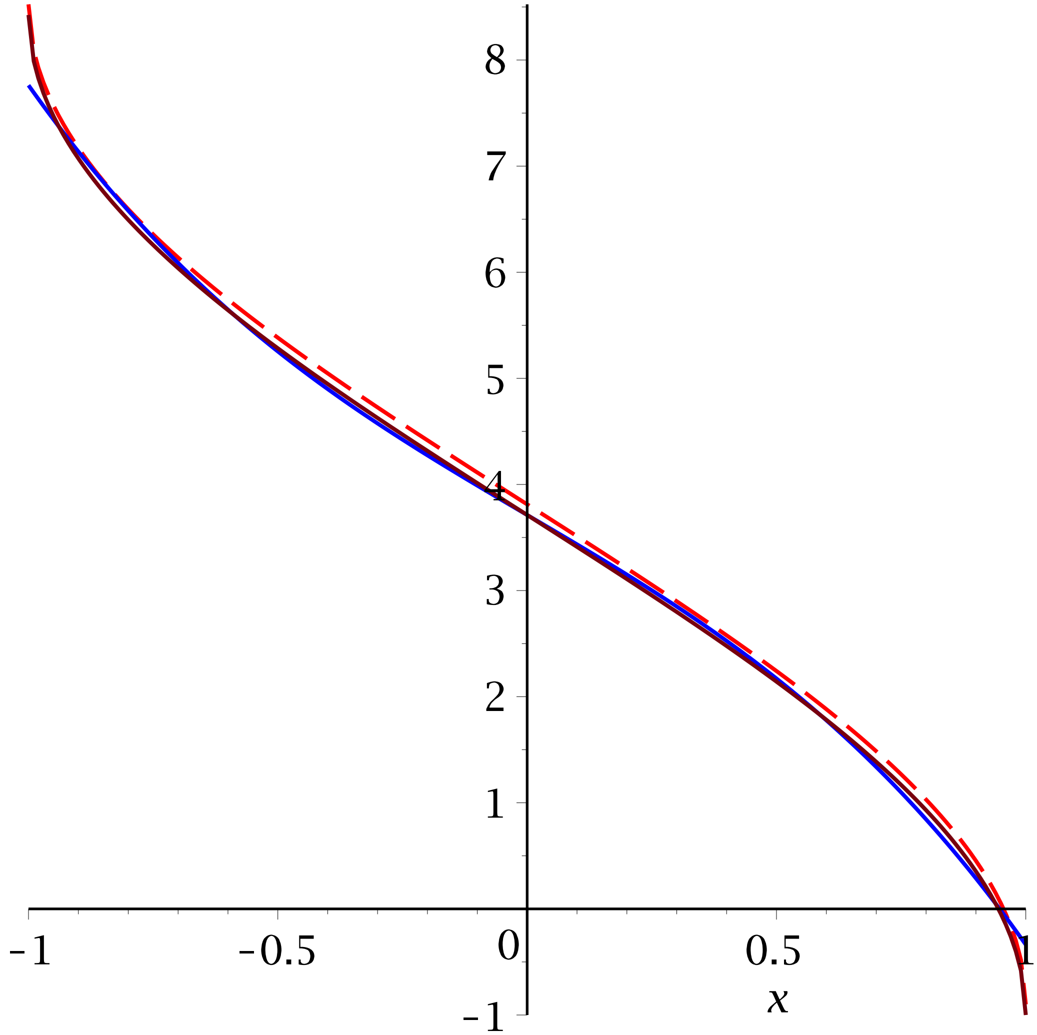
> $plots[display]([f1, f2, lej, funcplot])$



8
7
6
5
4
3
2
1
-1
-0.5
0
0.5
1

> $nmin := plot(add(c[n]\cdot P(n, x), n = 0..4), x = -1..1, color = blue)$:

> $plots[display](f1, f2, nmin, funcplot)$ **#how we can see, when n is 4, function deviates by more than 0.1**

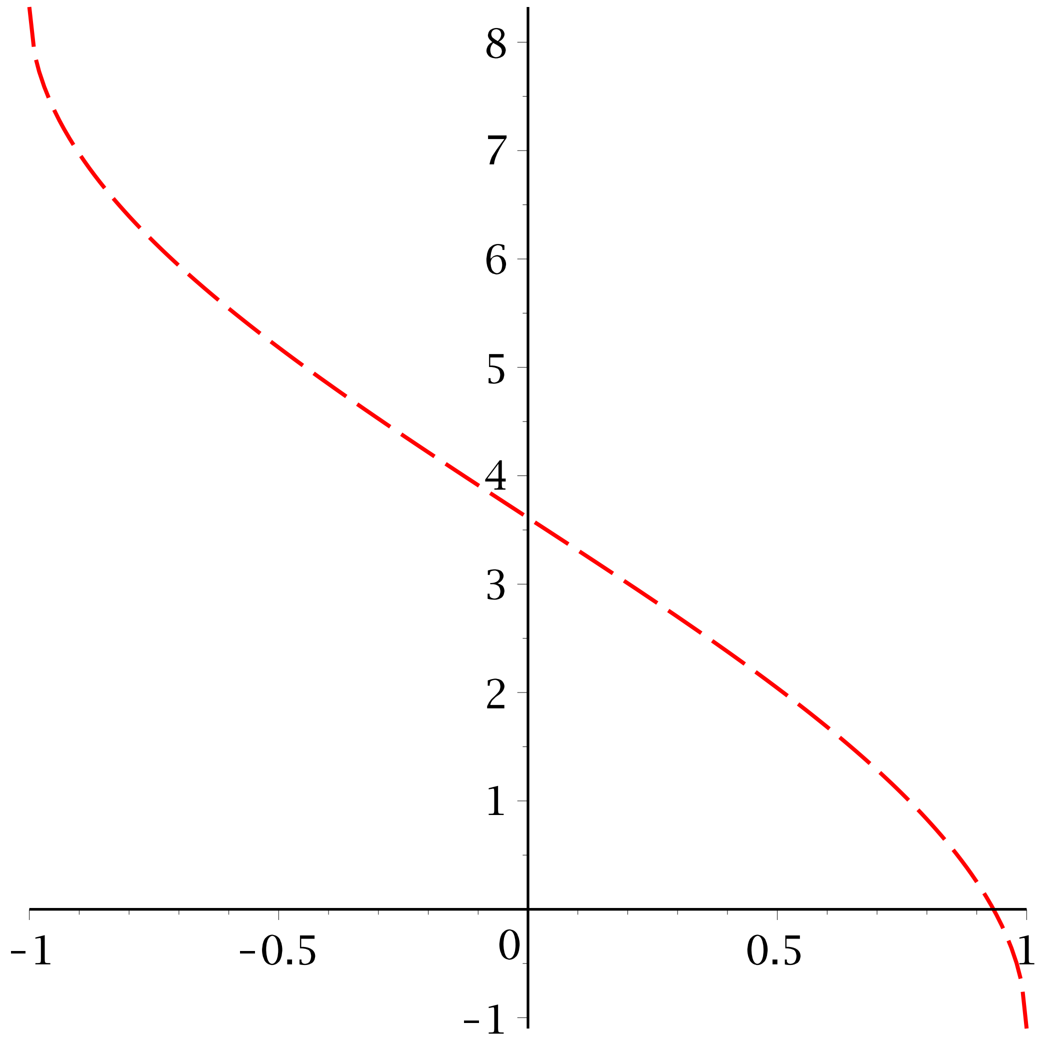


> **for** n **from** 0 **to** 7 **do** $c[n] := \frac{2}{\pi}\int_{-1}^{1} \frac{f\cdot T(n, x)}{\sqrt{1-x^2}}\,dx$; **end do** **#chebish coef**

$$c_0 := \frac{2\left(\frac{3}{2}\pi^2 - \pi\right)}{\pi}$$

$$c_1 := -\frac{12}{\pi}$$

$$c_2 := 0$$



$c_3 := -\frac{4}{3\pi}$

$$c_4 := 0$$

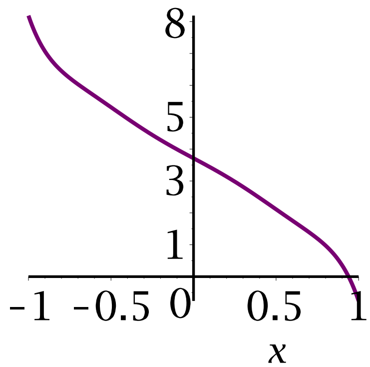
$$c_5 := -\frac{12}{25\pi}$$

$$c_6 := 0$$

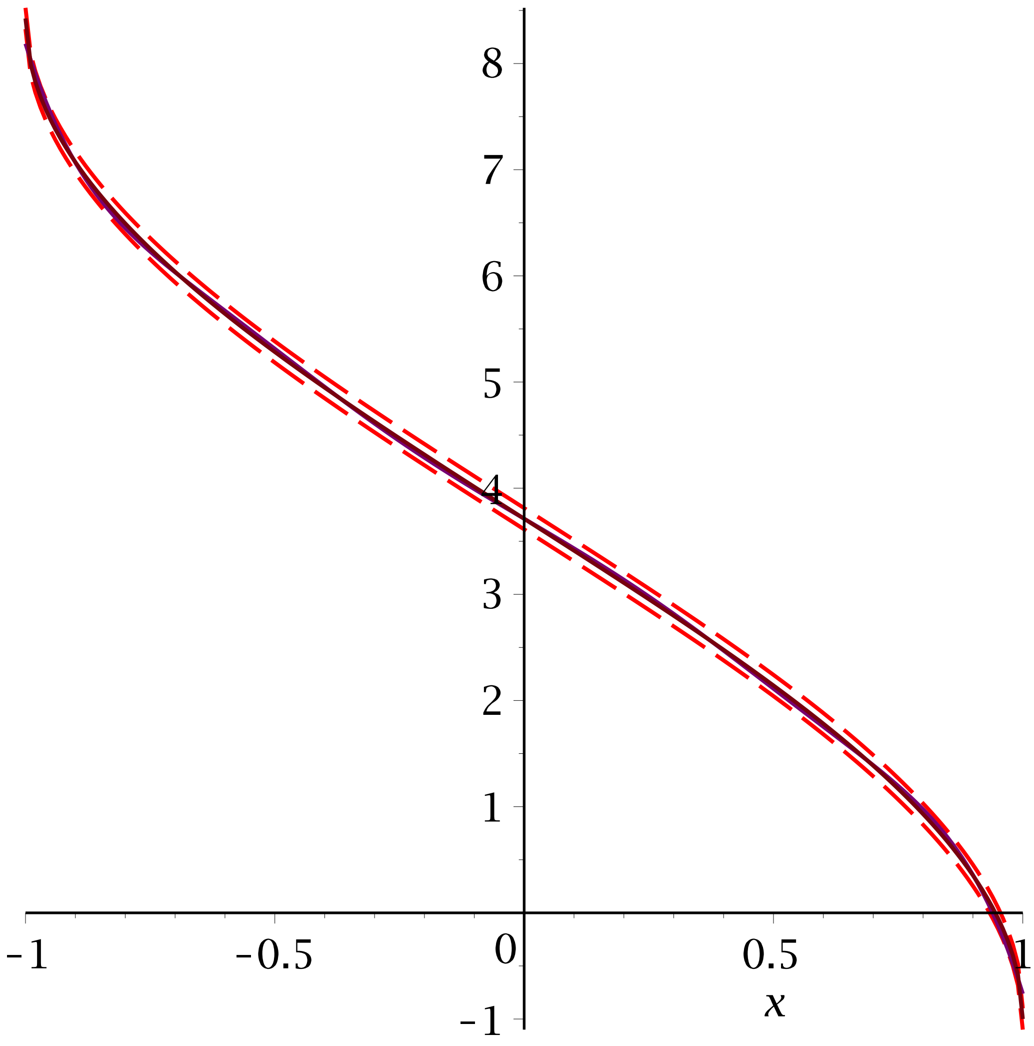
$$c_7 := -\frac{12}{49\pi} \tag{41}$$

> $\mathrm{cheb} := plot\left(\frac{c[0]}{2} + add(c[n]\cdot T(n, x), n = 1..3), x = -1..1, color = purple\right)$

cheb :=

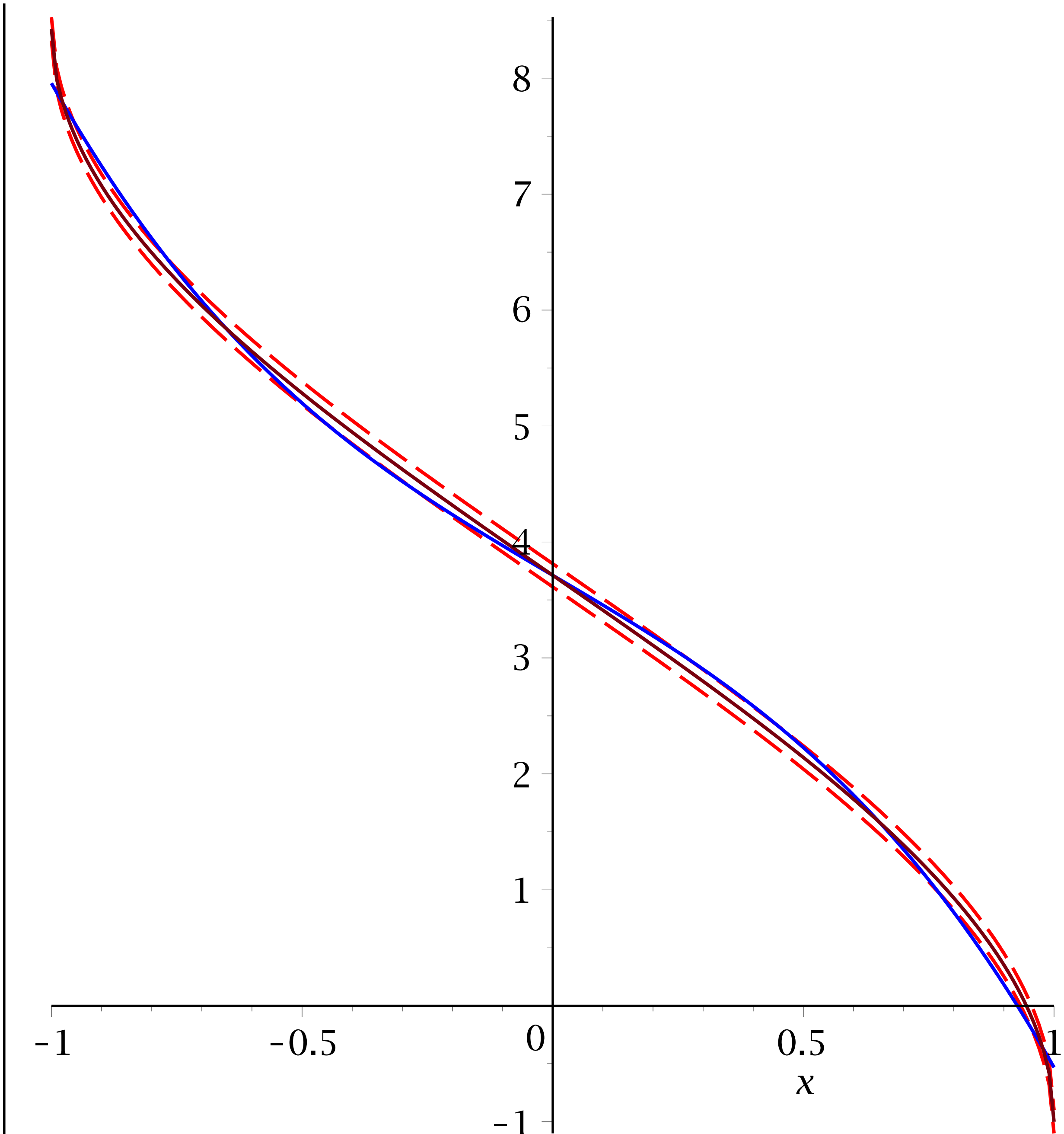


> $plots[display](f1, f2, cheb, funcplot)$



> $nmin := plot\left(\frac{c[0]}{2} + add(c[n] \cdot T(n, x), n = 1..3), x = -1..1, color = blue\right):$

> $plots[display](f1, f2, nmin, funcplot)$ **#how we can see, when n is 3, function deviates by more than 0.1**



> #lets check Fouriers coefs

> $a0 := simplify(int(f, x = -1..1))$

$$a0 := 3\pi - 2 \tag{42}$$

> $an := simplify(int(\mathrm{f} \cdot \cos(\mathrm{Pi} \cdot nn \cdot x), x = -1..1))$ assuming $nn :: posint$

$$an := 0 \tag{43}$$

> $bn := simplify(int(\mathrm{f} \cdot \sin(\mathrm{Pi} \cdot nn \cdot x), x = -1..1))$ assuming $nn :: posint$

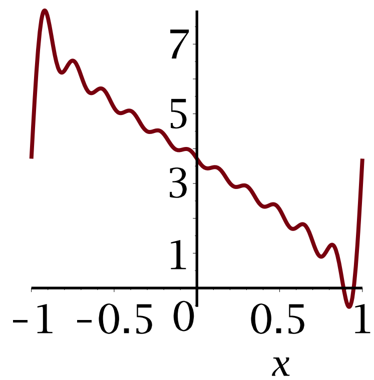
$$bn := \int_{-1}^{1} (3 \arccos(x) - 1) \sin(\pi\, nn\, x)\, \mathrm{d}x \tag{44}$$

> $Sm := \mathrm{k} \rightarrow \frac{a0}{2} + sum(bn \cdot \sin(\pi \cdot \mathrm{nn} \cdot \mathrm{x}), nn = 1..k)$

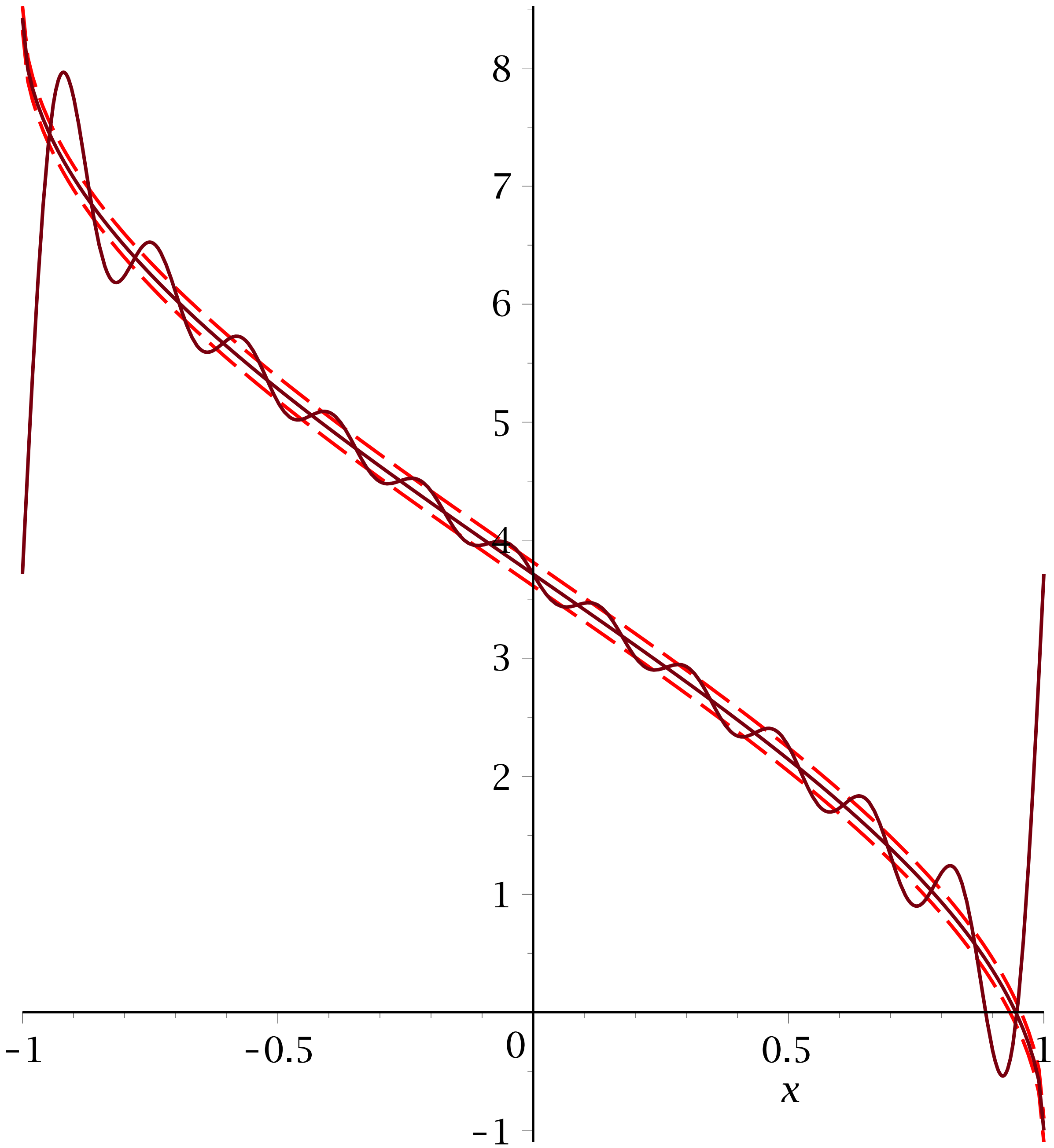
$$Sm := k \rightarrow \frac{1}{2}\, a0 + \sum_{nn=1}^{k} bn \sin(\pi\, nn\, x) \qquad \textbf{(45)}$$

> $fur := plot(\mathrm{Sm}(11),\ x = -1\,..1,\ discont = true)$

$fur :=$



> $plots[display](f1,\ f2,\ fur,\ funcplot)$



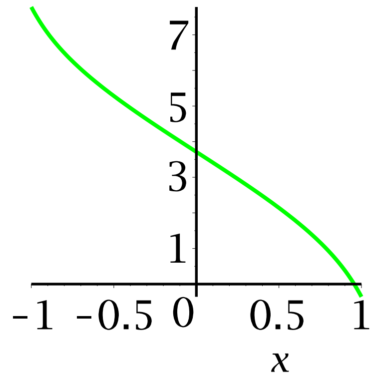
> #how we can see, when n is 11, function deviates by more than 0.1, cant experiment more, this takes a lot of time

> $St := convert(taylor(f, x=0, 14), polynom)$

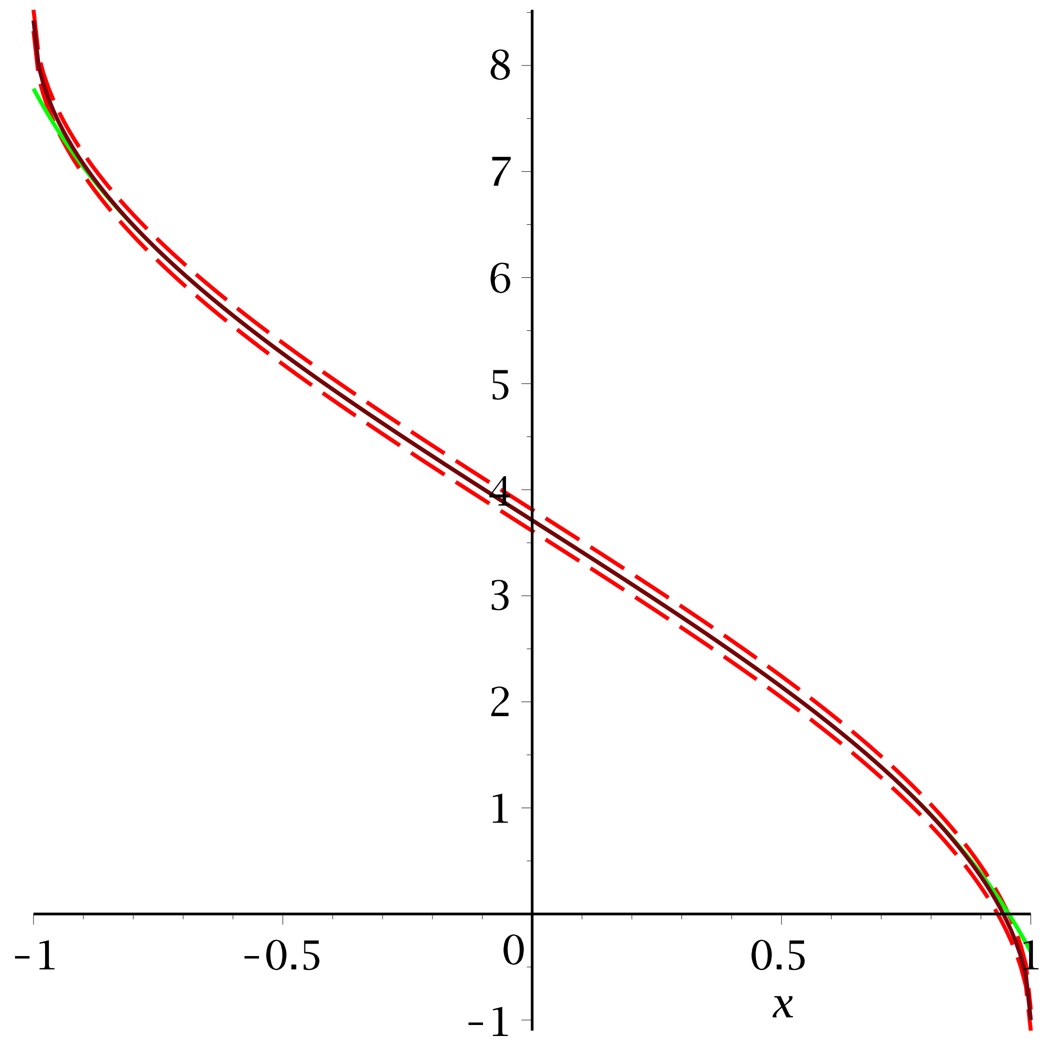
$$St := \frac{3}{2}\pi - 1 - 3x - \frac{1}{2}x^3 - \frac{9}{40}x^5 - \frac{15}{112}x^7 - \frac{35}{384}x^9 - \frac{189}{2816}x^{11} - \frac{693}{13312}x^{13} \quad \textbf{(46)}$$

> $StF := plot(St, x=-1..1, color=green)$

StF :=



> *plots*[*display*](*f1*, *f2*, *StF*, *funcplot*)



> **# how we can see, when n is 14, function deviates by more than 0.1**
> **# Lejandr and Chebishevs polynomes are more accurate than taylors and fouriers**

> *#DONE*