> #Lab 4. Done by Aliaksandr Bahdanau 153502. Var 5

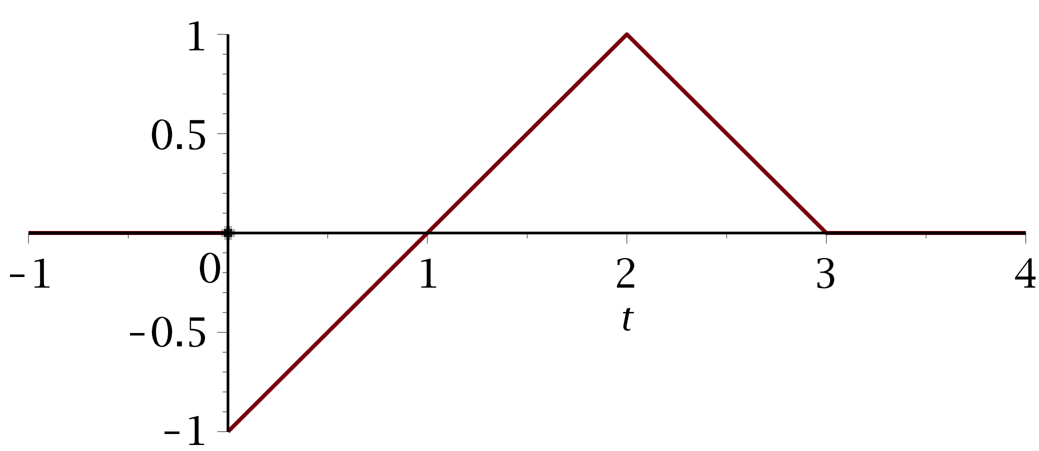
> #Task 1. According to the graph of the original function, find its imagenie Laplace. Get the answer in the Maple system and compare the results.

> $restart;$

> $f := a \to piecewise\left(t \le 0, 0, 0 < t \le 2 \cdot a, -1 + \frac{t}{a}, 2\,a < t \le 3\,a, 3 - \frac{t}{a}, 3\,a < t \le 4\,a, 0\right) : f(a)$

$$\begin{cases} 0 & t \le 0 \\ -1 + \frac{t}{a} & 0 < t \textbf{ and } t \le 2\,a \\ 3 - \frac{t}{a} & 2\,a < t \textbf{ and } t \le 3\,a \\ 0 & 3\,a < t \textbf{ and } t \le 4\,a \end{cases} \quad (1)$$

> $plot(f(1), t = -1..4, discont = true, scaling = constrained)$



> $with(inttrans):$

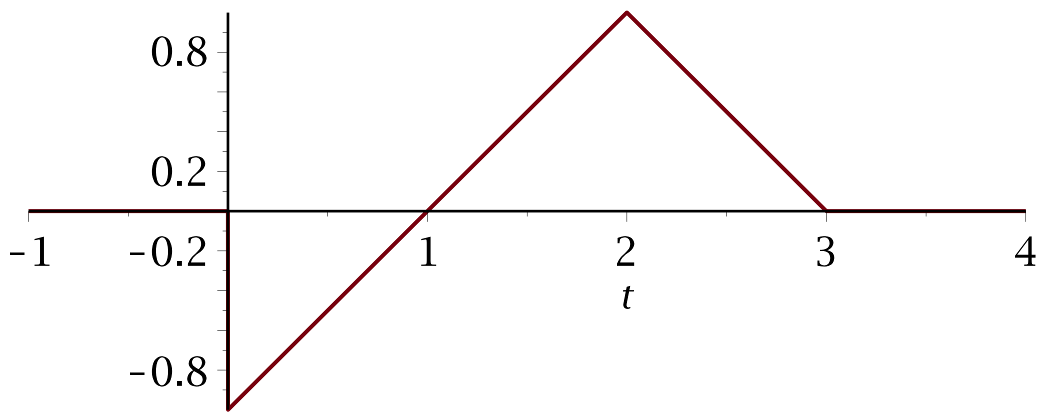
> $laplace(f(1), t, p)$

$$-\frac{1}{p} + \frac{1 + e^{-3p} - 2e^{-2p}}{p^2} \tag{2}$$

> $fp := \mathrm{Heaviside}(t) + \mathrm{Heaviside}(t)\cdot(t-2) + \mathrm{Heaviside}(t-2)\cdot(4-2t) + \mathrm{Heaviside}(t-3)\cdot(t-3)$

$$fp := \mathrm{Heaviside}(t) + \mathrm{Heaviside}(t)\,(t-2) + \mathrm{Heaviside}(t-2)\,(4-2t) + \mathrm{Heaviside}(t-3)\,(t-3) \tag{3}$$

> $plot(fp, t = -1..4, scaling = constrained)$



> **#Task 2. Find the original according to the given image "manually" and using Maple.**

> $restart:$
$with(inttrans):$

> $f := \frac{3p+2}{(p+1)(p^2+4p+5)}$

$$f := \frac{3p+2}{(p+1)(p^2+4p+5)} \tag{4}$$

> $invlaplace(f, p, t)$

$$-\frac{1}{2}\,e^{-t}+\frac{1}{2}\,e^{-2t}\,(\cos(t)+7\sin(t)) \qquad (5)$$

> #Task 3. Find a solution to the differential equation that satisfies the conditions y 0   0 and y   0   0 , using the operator method (using the Duhamel integral) and the Lagrange method. Compare the results and check them using the Maple system.

> $restart:$

> $de := diff(diff(y(t), t), t) + diff(y(t), t) = \frac{\exp(2\cdot t)}{3+\exp(t)}$

$$de := \frac{d^2}{dt^2}\,y(t) + \frac{d}{dt}\,y(t) = \frac{e^{2t}}{3+e^{t}} \qquad (6)$$

> $dsolve(de)$

$$y(t) = \frac{1}{2}\,e^{t} - 3\ln(3+e^{t}) - 9\,e^{-t}\ln(3+e^{t}) - e^{-t}\,\_C1 + \_C2 \qquad (7)$$

> #Task 4. Solve the Cauchy problem using the operator method and compare with the solution in Maple.

> $de := diff(diff(y(t), t), t) + diff(y(t), t) + y(t) = 7\exp(2\,t)$

$$de := \frac{d^2}{dt^2}\,y(t) + \frac{d}{dt}\,y(t) + y(t) = 7\,e^{2t} \qquad (8)$$

> $dsolve(\{de, y(0) = 1, y'(0) = 4\})$

$$y(t) = \frac{4}{3}\,e^{-\frac{1}{2}t}\sin\left(\frac{1}{2}\sqrt{3}\,t\right)\sqrt{3} + e^{2t} \qquad (9)$$

> # Task 5. Solve the system of differential equations using the operator method. Compare with the solution obtained in Maple.

> $dsys := \{diff(x(t), t) = 2\,x(t) + 5\,y(t), diff(y(t), t) = x(t) - 2\,y(t) + 2\}$

$$dsys := \left\{\frac{d}{dt}\,x(t) = 2\,x(t) + 5\,y(t), \frac{d}{dt}\,y(t) = x(t) - 2\,y(t) + 2\right\} \qquad (10)$$

> $dsolve(\{dsys[1], dsys[2], x(0) = -1, y(0) = 1\})$

$$\left\{x(t) = \frac{5}{9}\,e^{3t} - \frac{4}{9}\,e^{-3t} - \frac{10}{9}, y(t) = \frac{1}{9}\,e^{3t} + \frac{4}{9}\,e^{-3t} + \frac{4}{9}\right\} \qquad (11)$$

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