> #Lab 3.3. Done by Aliaksandr Bahdanau 153502 Var 5

> # Task 1. Explore the behavior of the phase curves of the system of equations near the rest point. Make a drawing. Determine the type of rest point from the phase portrait and eigenvalues of the system matrix. Find the general solution of the system and isolate the fundamental system solutions. Compare with the results obtained in Maple. Construct in a rectangular system Oxy1 y2 spatial curves that satisfy the given system and contain, respectively, points (0, y10 , y20) . Values y10 , y20 take the same ones that were used to build phase portrait. Compare the drawings obtained on the plane and in space. Change from a system of equations to a homogeneous differential 1st order equation with respect to the function y2 (y1) , plot its field directions in a neighborhood of a singular point. Compare with the phase portrait of the system

> *piecewise*(*diff*(*y1*(*x*), *x*) =-5· *y1*(*x*) + 2·*y2*(*x*), ``, *diff*(*y2*(*x*), *x*) =-15· *y1*(*x*) + 8 · *y2*(*x*), ``);

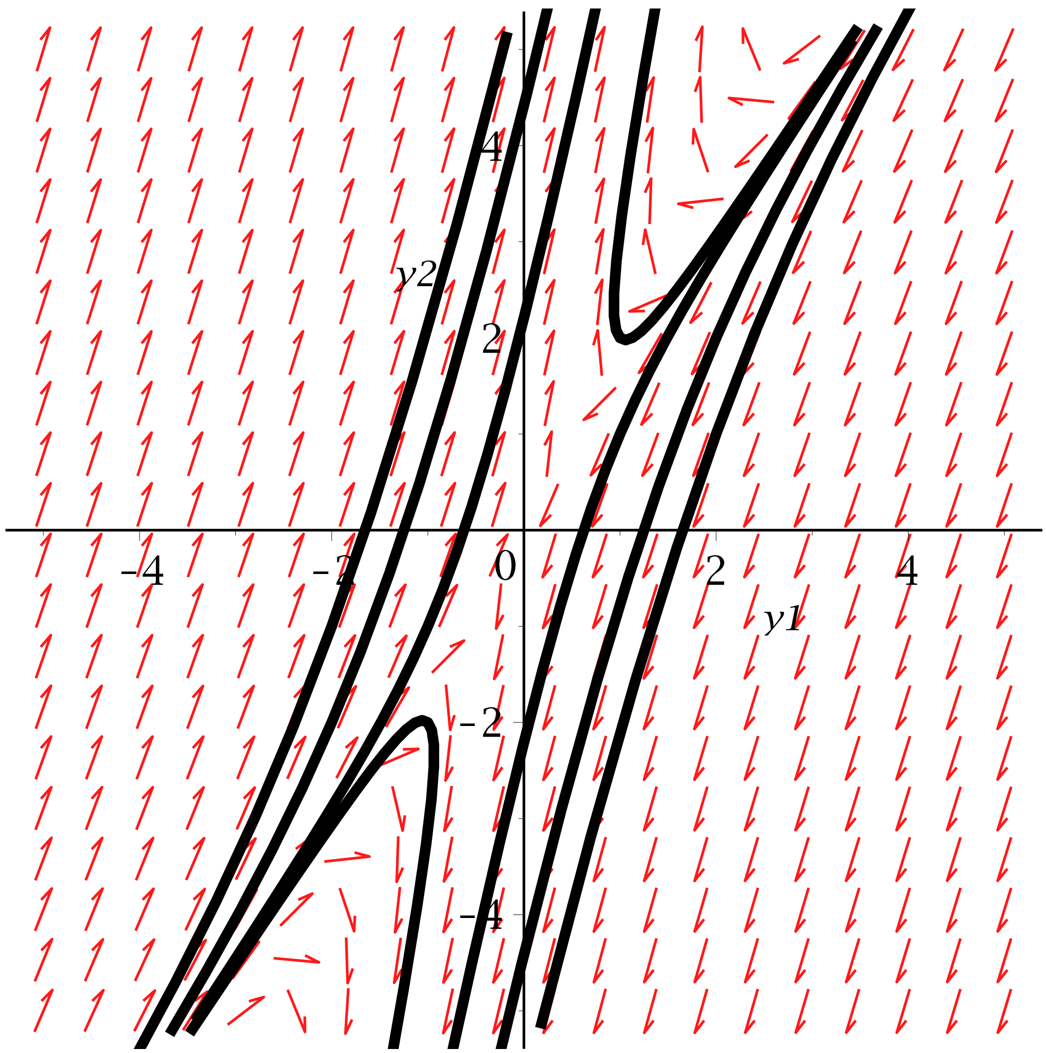
$$\begin{cases} \frac{d}{dx} y1(x) = -5\, y1(x) + 2\, y2(x) \\ \frac{d}{dx} y2(x) = -15\, y1(x) + 8\, y2(x) \end{cases} \tag{1}$$

> $sys := \left\{ \frac{d}{dx} y1(x) = -5\, y1(x) + 2\, y2(x), \frac{d}{dx} y2(x) = -15\, y1(x) + 8\, y2(x) \right\}$

$$sys := \left\{ \frac{d}{dx} y1(x) = -5\, y1(x) + 2\, y2(x), \frac{d}{dx} y2(x) = -15\, y1(x) + 8\, y2(x) \right\} \tag{2}$$

> *with*(*DETools*) : *with*(*LinearAlgebra*) :

> *phaseportrait*([sys[1], *sys*[2]], [*y1*, *y2*], *x* =-5 ..5, [[0, 1, 1], [0, 1, 2], [0, 2, 1], [0, -1,-1], [0,-1,-2], [0,-2,-1], [0, 2, 2], [0,-2,-2]], *y1* =-5 ..5, *y2* =-5 ..5, *stepsize* = 0.05, *linecolor* = *black*, *thickness* = 4)



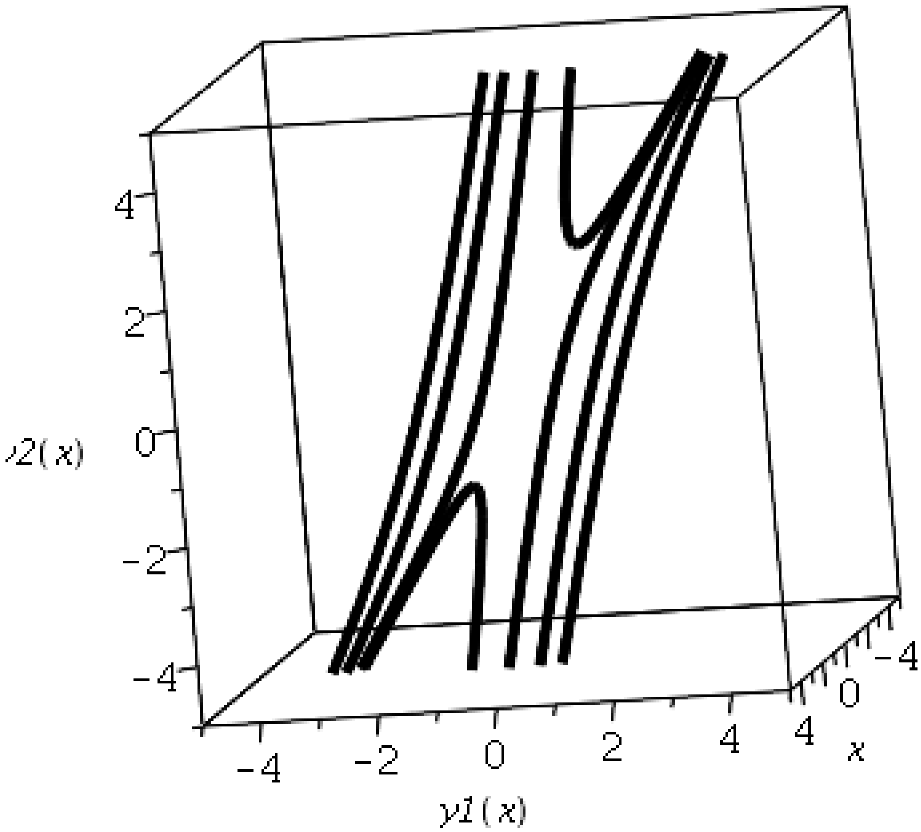
> #rest point is seddle

> #lets solve DE

> *dsolve*(*sys*)

$$\left\{y1(x) = \_C1\,\mathrm{e}^{5\,x} + \_C2\,\mathrm{e}^{-2\,x},\ y2(x) = 5\,\_C1\,\mathrm{e}^{5\,x} + \frac{3}{2}\,\_C2\,\mathrm{e}^{-2\,x}\right\} \quad \textbf{(3)}$$

> *DEplot3d*([sys[1], *sys*[2]], [*y1*, *y2*], *x* = -5 ..5, [[0, 1, 1], [0, 1, 2], [0, 2, 1], [0, -1, -1], [0, -1, -2], [0, -2, -1], [0, 2, 2], [0, -2, -2]], *y1* = -5 ..5, *y2* = -5 ..5, *stepsize* = 0.05, *linecolor* = *black*, *thickness* = 4)



> # Phaseportrait and 3d plot are the same

> $diff(y2(y1), y1) = \frac{-15 \cdot y1 + 8 \cdot y2(y1)}{-5 \cdot y1 + 2 \cdot y2(y1)}$

$$\frac{d}{dy1}\, y2(y1) = \frac{-15\, y1 + 8\, y2(y1)}{-5\, y1 + 2\, y2(y1)} \qquad (4)$$

> # Singular point is (0, 0)

> $DEplot\left(\left[\frac{d}{dy1}\, y2(y1) = \frac{-15\, y1 + 8\, y2(y1)}{-5\, y1 + 2\, y2(y1)}\right], y2(y1), y1 = -5..5, y2(y1) = -5..5, [y2(1) = 1, y2(1) = 2, y2(2) = 1, y2(-1) = -1, y2(-2) = -1, y2(-1) = -2, y2(2) = 2, y2(-2) = -2], linecolor = black, thickness = 4\right)$

Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further left of .93443577, probably a singularity
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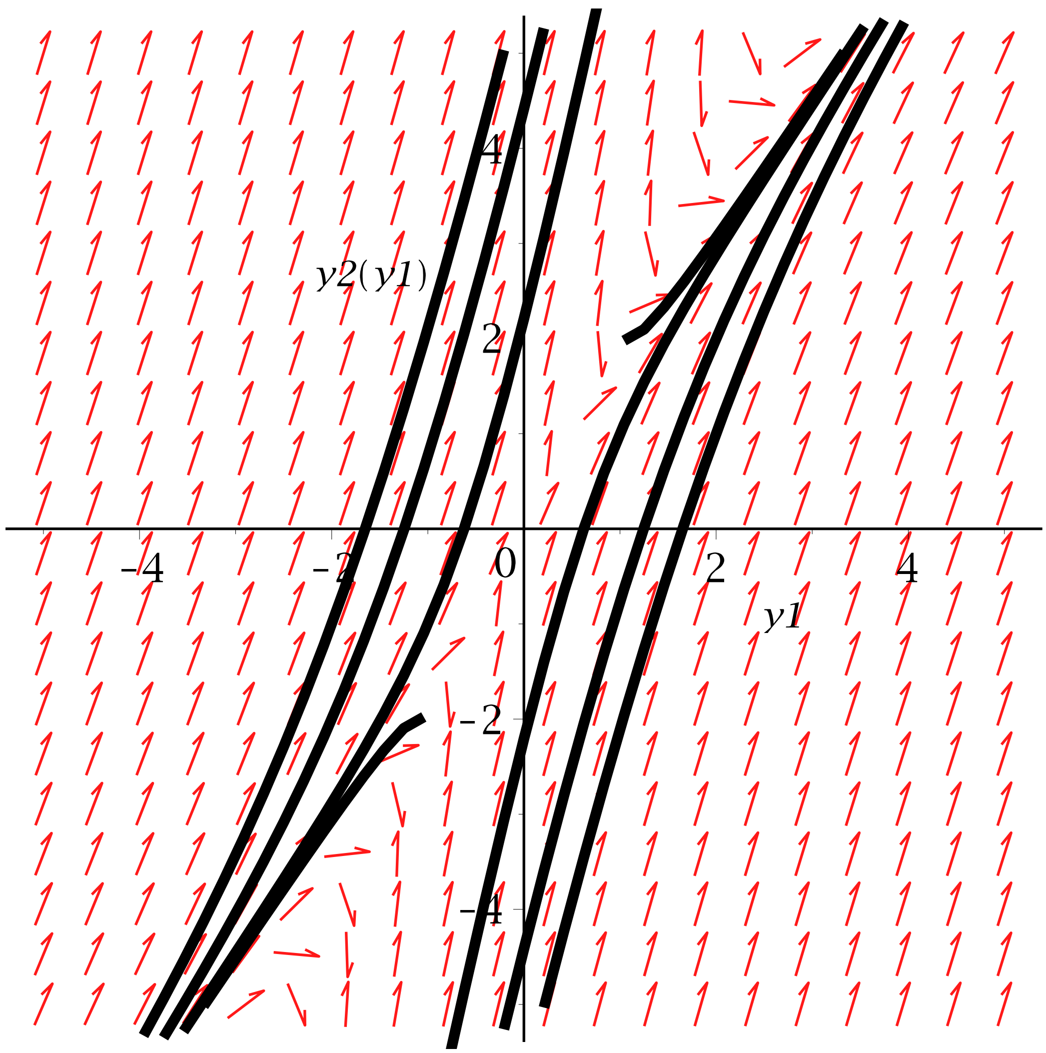

> #Task 2. Solve the system of equations using the elimination method and compare the result with the answer obtained in Maple.

> $dsys := \{diff(y1(t), t) = 9\cdot y1(t) + 2\cdot y2(t), diff(y2(t), t) = 6\cdot y1(t) + 5\cdot y2(t)\}$

$$dsys := \left\{\frac{\mathrm{d}}{\mathrm{d}t}\, y1(t) = 9\, y1(t) + 2\, y2(t),\ \frac{\mathrm{d}}{\mathrm{d}t}\, y2(t) = 6\, y1(t) + 5\, y2(t)\right\} \quad \textbf{(5)}$$

> $dsolve(dsys)$

$$\{y1(t) = \_C1\,\mathrm{e}^{3\,t} + \_C2\,\mathrm{e}^{11\,t},\ y2(t) = -3\,\_C1\,\mathrm{e}^{3\,t} + \_C2\,\mathrm{e}^{11\,t}\} \quad \textbf{(6)}$$

> #Task 3. Solve the Cauchy problem using the Lagrange and D'Alembert methods. Compare with the result obtained in Maple. Make a drawing.

> $dsys := \left\{diff(x(t), t) = 3\cdot x(t) + 2\cdot y(t), diff(y(t), t) = \frac{5}{2}\cdot x(t) - y(t) + 2\right\}$

$$dsys := \left\{\frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = 3\, x(t) + 2\, y(t),\ \frac{\mathrm{d}}{\mathrm{d}t}\, y(t) = \frac{5}{2}\, x(t) - y(t) + 2\right\} \quad \textbf{(7)}$$

> $ds := dsolve(dsys)$

$$ds := \left\{x(t) = \mathrm{e}^{4\,t}\,\_C2 + \mathrm{e}^{-2\,t}\,\_C1 - \frac{1}{2},\ y(t) = \frac{1}{2}\,\mathrm{e}^{4\,t}\,\_C2 - \frac{5}{2}\,\mathrm{e}^{-2\,t}\,\_C1 + \frac{3}{4}\right\} \quad \textbf{(8)}$$

> $with(DEtools):$

> $DEplot3d(\mathrm{dsys}, [x(t), y(t)], t = 0..1, [[x(0) = 0, y(0) = 1]])$