

> #Lab 3.2 Completed by Bahdanau Aliaksandr 153502

> #Variant 5

> #Task 1. Solve the equations and compare with the results obtained in maple. Plot several integral curves in the same coordinate system.

> #1.1

> restart;

> de := x = $\frac{d^2}{dx^2}(y(x)) - \cos(x) \cdot \frac{d^2}{dx^2}(y(x))$

$$de := x = \frac{d^2}{dx^2} y(x) - \cos(x) \left(\frac{d^2}{dx^2} y(x) \right) \quad (1)$$

> dsolve(de)

$$y(x) = \int \left(-\frac{x}{\tan\left(\frac{1}{2} x\right)} + 2 \ln\left(\tan\left(\frac{1}{2} x\right)\right) - \ln\left(1 + \tan\left(\frac{1}{2} x\right)^2\right) \right) dx + _C1 x + _C2 \quad (2)$$

> #Subs y'' with z, x=z-cos(z)

> x_ := z - cos(z)

$$x_ := z - \cos(z) \quad (3)$$

> dx := diff(z - cos(z), z)

$$dx := 1 + \sin(z) \quad (4)$$

> #dy1 = zdx

> y1 := int(z·dx, z)

$$y1 := \frac{1}{2} z^2 + \sin(z) - z \cos(z) \quad (5)$$

> #dy=y1dx

> y_ := int((y1 + C1)·dx, z) + C2

$$y_ := \frac{1}{2} z \cos(z)^2 - \frac{3}{4} \cos(z) \sin(z) + \frac{1}{4} z - \frac{1}{2} \cos(z) z^2 - \cos(z) C1 + \frac{1}{6} z^3 + C1 z - \cos(z) + C2 \quad (6)$$

> a, b, c := seq(subs(C2 = i, y_), i = -1..1) :

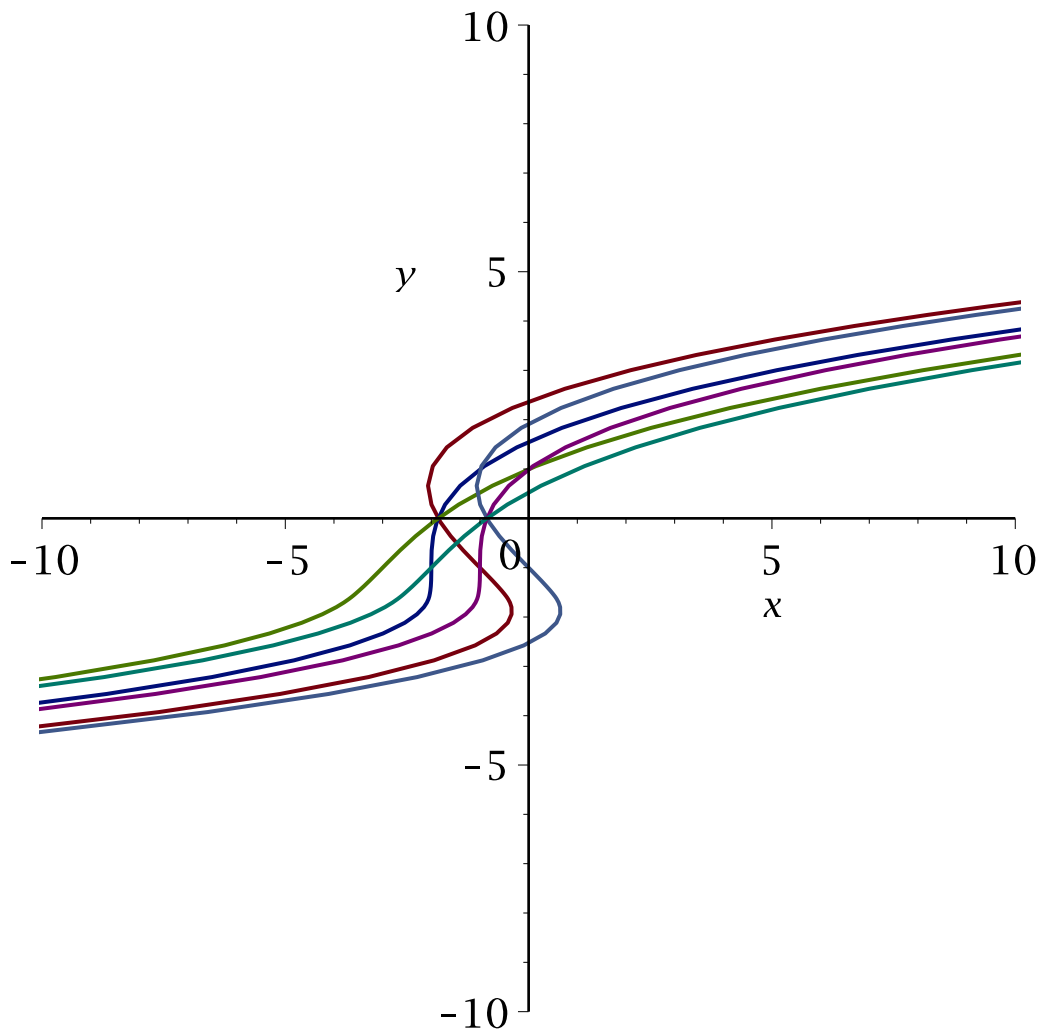
> a1, a2, a3 := seq(subs(C1 = i, a), i = -1..1) :

b1, b2, b3 := seq(subs(C1 = i, b), i = -1..1) :

c1, c2, c3 := seq(subs(C1 = i, c), i = -1..1) :

len := z = -20..20 :

> plot([[a1, x_, len], [a2, x_, len], [a3, x_, len], [b1, x_, len], [b2, x_, len], [b3, x_, len], x = -10..10, y = -10..10)



> **#Task 1.2**

> restart;

> $de := \text{diff}(\text{diff}(y(x), x), x) \cdot y(x) - \text{diff}(y(x), x)^2 = y(x) \cdot \text{diff}(y(x), x) \cdot \tanh(x)$

$$de := \left(\frac{d^2}{dx^2} y(x) \right) y(x) - \left(\frac{d}{dx} y(x) \right)^2 = y(x) \left(\frac{d}{dx} y(x) \right) \tanh(x) \quad (7)$$

> $y_- := \text{dsolve}(de)$

$$y_- := y(x) = e^{\frac{1}{2} e^x - C1} - C2 e^{-\frac{1}{2} \frac{-C1}{e^x}} \quad (8)$$

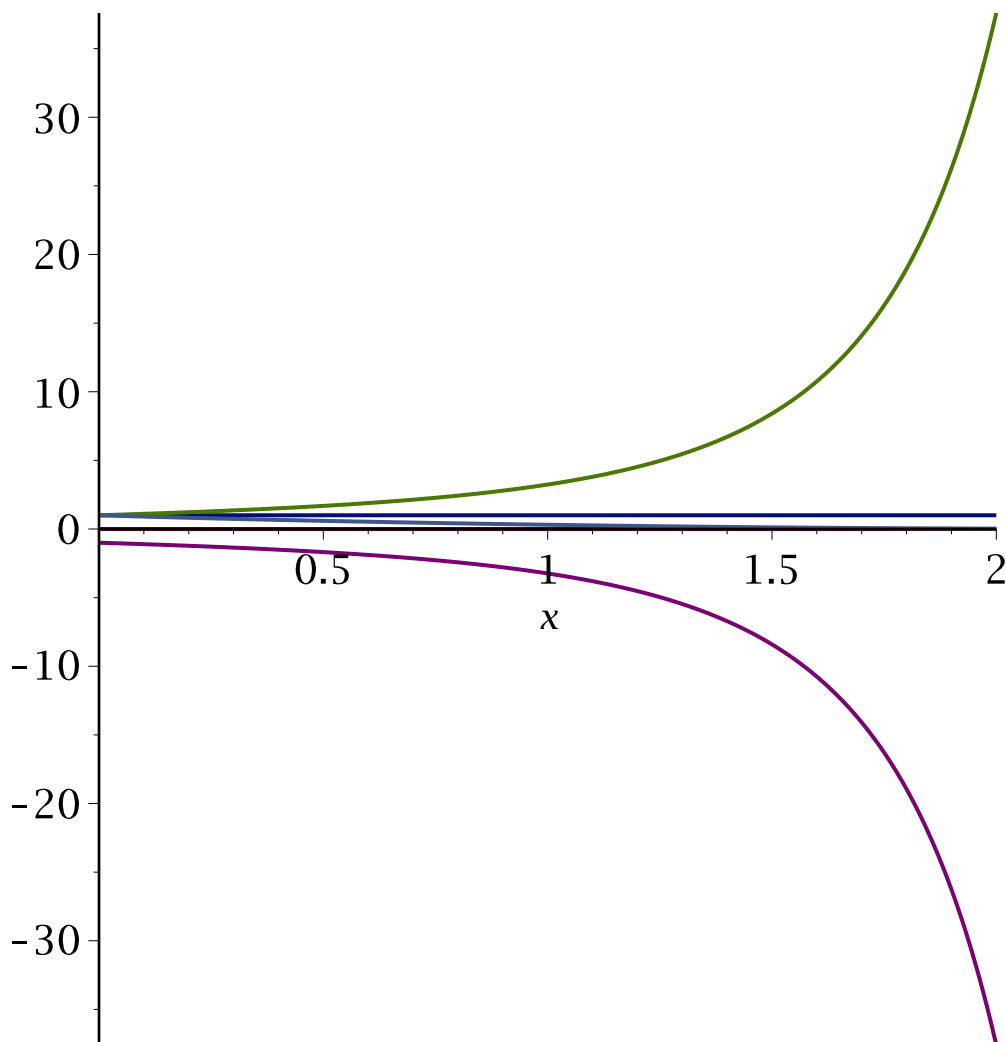
> $a, b, c := \text{seq}(\text{subs}(_C2 = i, y_-), i = -1..1) :$

> $a1, a2, a3 := \text{seq}(\text{subs}(_C1 = i, a), i = -1..1) :$

$b1, b2, b3 := \text{seq}(\text{subs}(_C1 = i, b), i = -1..1) :$

$c1, c2, c3 := \text{seq}(\text{subs}(_C1 = i, c), i = -1..1) :$

> $\text{plot}([\text{rhs}(b2), \text{rhs}(c2), \text{rhs}(c3), \text{rhs}(c1), \text{rhs}(a3)], x = 0..2)$



> #Task 1.3

> restart;

> solutions := dsolve($\text{diff}(y(x), x) = x \cdot \text{diff}(\text{diff}(y(x), x), x) - \frac{1}{10} \cdot \text{diff}(\text{diff}(y(x), x), x)^{10}$)

$$\begin{aligned}
 \text{solutions} := y(x) &= \frac{81}{190} x^{19/9} + _C1, y(x) = \frac{81}{380} I x^{19/9} (-\sqrt{3} + I) + _C1, y(x) \\
 &= \frac{81}{380} I x^{19/9} (\sqrt{3} + I) + _C1, y(x) = -\frac{81}{190} x^{19/9} \left(-I \cos\left(\frac{7}{18} \pi\right) \right. \\
 &\quad \left. + \cos\left(\frac{1}{9} \pi\right) \right) + _C1, y(x) = -\frac{81}{190} x^{19/9} \left(I \cos\left(\frac{7}{18} \pi\right) + \cos\left(\frac{1}{9} \pi\right) \right) + _C1, \\
 y(x) &= \frac{81}{190} x^{19/9} \left(\cos\left(\frac{2}{9} \pi\right) - I \cos\left(\frac{5}{18} \pi\right) \right) + _C1, y(x) \\
 &= \frac{81}{190} x^{19/9} \left(\cos\left(\frac{2}{9} \pi\right) + I \cos\left(\frac{5}{18} \pi\right) \right) + _C1, y(x) = \frac{81}{190} x^{19/9} \left(\cos\left(\frac{4}{9} \pi\right) \right.
 \end{aligned}
 \tag{9}$$

$$-I \cos\left(\frac{1}{18} \pi\right) + _C1, y(x) = \frac{81}{190} x^{19/9} \left(\cos\left(\frac{4}{9} \pi\right) + I \cos\left(\frac{1}{18} \pi\right)\right) + _C1,$$

$$y(x) = \frac{1}{2} \text{RootOf}(_Z^{10} + 10 _C1) x^2 + _C1 x + _C2$$

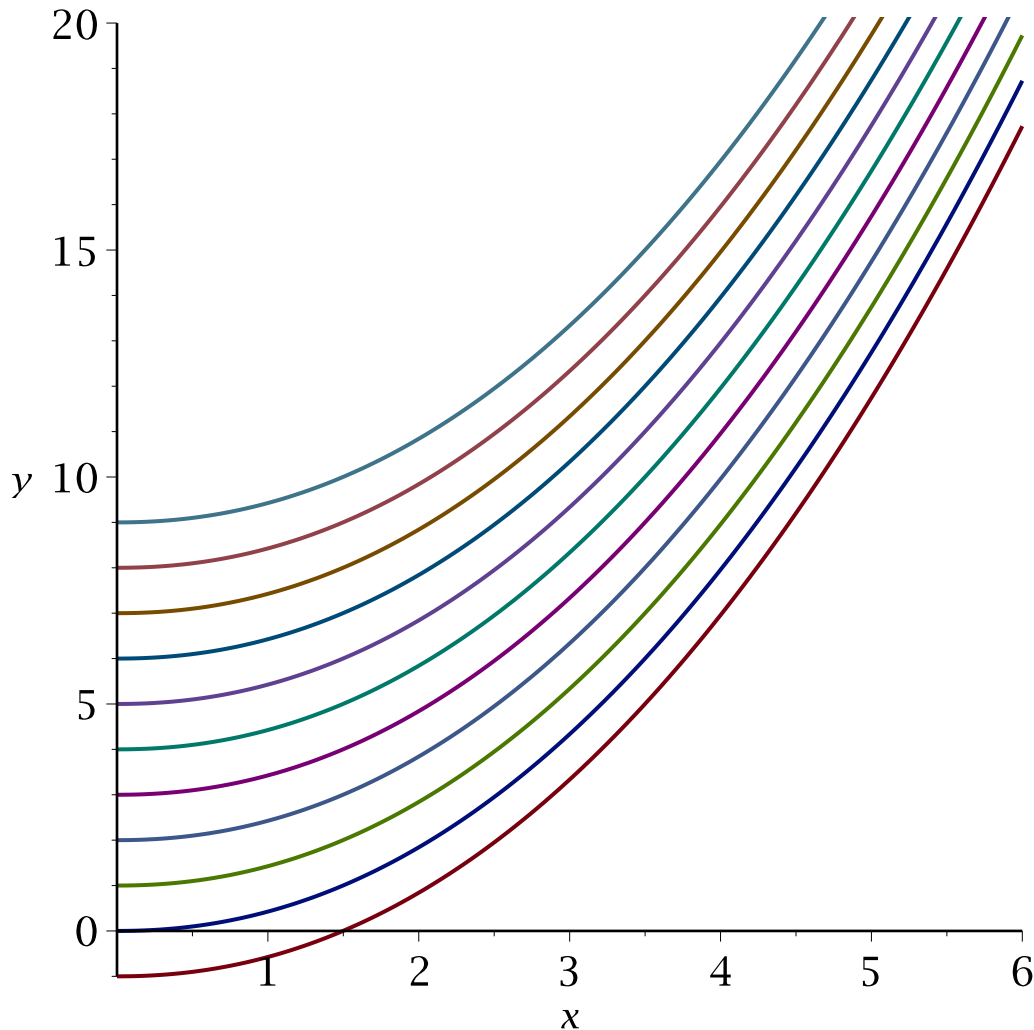
> # All solitions, except of first are not real

> $y_- := \text{rhs}(\text{solutions}[1])$

$$y_- := \frac{81}{190} x^{19/9} + _C1$$

(10)

> $\text{plot}([\text{seq}(\text{subs}(_C1 = i, y_-), i = -1..9)], x = 0..6, y = -1..20)$



> #Task 1.4

> restart;

> $de := \text{diff}(\text{diff}(y(x), x), x) = 2 \cdot \left(\frac{\text{diff}(y(x), x)}{x} - \frac{y(x)}{x^2}\right) + \frac{1}{x^2} \cdot \cos\left(\frac{1}{x}\right)$

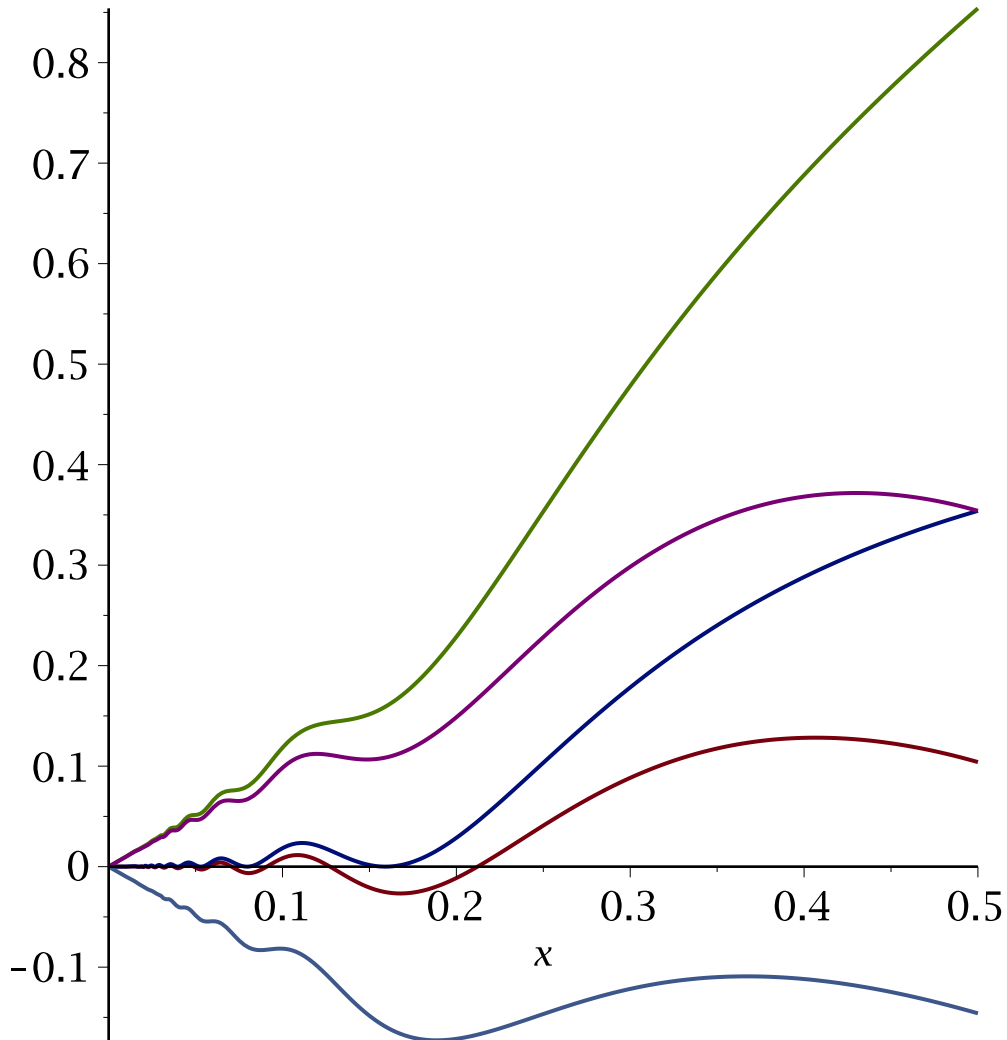
$$de := \frac{d^2}{dx^2} y(x) = \frac{2 \left(\frac{d}{dx} y(x)\right)}{x} - \frac{2 y(x)}{x^2} + \frac{\cos\left(\frac{1}{x}\right)}{x^2}$$

(11)

> $y_- := \text{dsolve}(de)$

$$y_{-} := y(x) = -x^2 \cos\left(\frac{1}{x}\right) + x^2 _C2 + x _C1 \quad (12)$$

```
> a, b, c := seq(subs(_C2 = i, y_), i = -1..1):
> a1, a2, a3 := seq(subs(_C1 = i, a), i = -1..1):
  b1, b2, b3 := seq(subs(_C1 = i, b), i = -1..1):
  c1, c2, c3 := seq(subs(_C1 = i, c), i = -1..1):
> plot([rhs(b2), rhs(c2), rhs(c3), rhs(c1), rhs(a3)], x = 0..0.5)
```



> #Task 2. Find the general solution of the equation and compare with the result obtained in the Maple system.

```
> restart;
```

```
> de := tan(x) · diff(diff(y(x), x), x) - diff(y(x), x) + 1/sin(x) = 0
```

$$de := \tan(x) \left(\frac{d^2}{dx^2} y(x) \right) - \left(\frac{d}{dx} y(x) \right) + \frac{1}{\sin(x)} = 0 \quad (13)$$

```
> dsolve(de)
```

$$y(x) = _C1 \cos(x) + \frac{1}{2} \ln(\csc(x) - \cot(x)) + _C2 \quad (14)$$

|> **#Task 3. Find the general solution of the differential equation.**

|> *restart*;

|> $de := \text{diff}(\text{diff}(y(x), x), x) + 2 \cdot \text{diff}(y(x), x) + 5 \cdot y(x) = -\sin(2 \cdot x)$

$$de := \frac{d^2}{dx^2} y(x) + 2 \left(\frac{d}{dx} y(x) \right) + 5 y(x) = -\sin(2 x) \quad (15)$$

|> *dsolve*(*de*)

$$y(x) = e^{-x} \sin(2 x) _C2 + e^{-x} \cos(2 x) _C1 - \frac{1}{17} \sin(2 x) + \frac{4}{17} \cos(2 x) \quad (16)$$