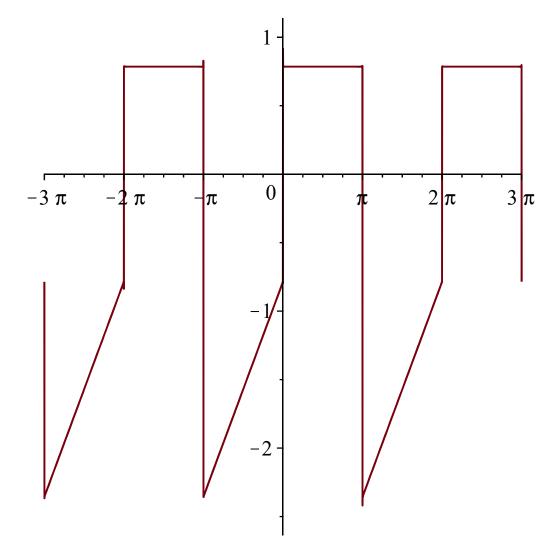
```
> #Лабораторная номер 2 Тимофеев Вариант 6
    #Задание 1
    with(plots):
> fourierseries := proc(f, x1, x2, interval) local n, l,
    a, b, s, d, count, i, m;
    l := (x2-x1)/2:
    a[0] := int(f, x = x1..x2) / l;
    a[n] := int(f * cos(n * Pi * x/l), x = x1..x2)/l;
   b[n] := int(f * sin(n * Pi * x/l), x = x1..x2)/l;
    S[k] := a[0]/2 + sum\left(a[n] * \cos\left(n * \text{Pi} * \frac{x}{l}\right) + b[n] * \sin\left(n * \text{Pi} * \frac{x}{l}\right), n = 1..k\right);
    m := 0;
    count := 60;
    for i from 0 to count do
    d[i] := plot\left(unapply\left(a[0]/2 + sum\left(a[n] * \cos\left(n * \operatorname{Pi} * \frac{x}{l}\right) + b[n] * \sin\left(n * \operatorname{Pi} * \frac{x}{l}\right), n = 1\right)\right)
         ..m, x, interval;
    m := m + 1 + \frac{m}{7};
    end do:
   return display(seq(display(d[y]), y = 0 ...count), insequence = true);
    end proc;
    coeffq := \mathbf{proc}(f, x1, x2) \mathbf{local} n, l, a, b;
    l := (x2-x1)/2;
    a[0] := int(f, x = x1..x2) / l;
    a[n] := int(f^* \cos(n * Pi * x/l), x = x1..x2)/l;
   b[n] := int(f * sin(n * Pi * x/l), x = x1..x2)/l;
   return [a[0], a[n], b[n]];
    end proc;
fourierseries := \mathbf{proc}(f, x1, x2, interval)
     local n, l, a, b, s, d, S, count, i, m;
     l := 1/2 * x^2 - 1/2 * x^1;
     a[0] := int(f, x = x1..x2)/l;
     a[n] := int(f^*\cos(n^*\pi^*x/l), x = x1..x2)/l;
     b[n] := int(f^* \sin(n^* \pi^* x/l), x = x1..x2)/l;
     S[k] := 1/2 * a[0] + sum(a[n] * cos(n * \pi * x/l) + b[n] * sin(n * \pi * x/l), n = 1..k);
```

```
m := 0;
                  count := 60;
                   for i from 0 to count do
                                    d[i] := plot(unapply(1/2 * a[0] + sum(a[n] * cos(n * \pi * x/l) + b[n] * sin(n * \pi * x/l)) + b[n] * sin(n * \pi * x/l) + b[n] * sin(n * 
                                     (l), n = 1..m), x), interval);
                                    m := 8/7 * m + 1
                   end do;
                   return plots:-display(seq(plots:-display(d[y]), y = 0 ...count), insequence = true)
 end proc
coeffq := \mathbf{proc}(f, x1, x2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                   (1)
                  local n, l, a, b;
                  l := 1/2 * x2 - 1/2 * x1;
                  a[0] := int(f, x = x1 ..x2) / l;
                  a[n] := int(f^* \cos(n^* \pi^* x/l), x = x1..x2)/l;
                  b[n] := int(f^* \sin(n^* \pi^* x/l), x = x1..x2)/l;
                  return [a[0], a[n], b[n]]
end proc
> expr := piecewise \left( -Pi \le x < 0, \frac{x}{2} - \frac{Pi}{4}, 0 \le x < Pi, \frac{Pi}{4} \right):
              fourierseries (expr, -Pi, Pi, -3·Pi..3·Pi);
```



```
> customplot := \mathbf{proc}(a0, an, bn, l, N, interval) local i, d, m, count; m := 0; count := 60; i := 0; while (i \le count and m \le N) do d[i] := plot \left(unapply\left(a0/2 + sum\left(an * \cos\left(n * \operatorname{Pi} * \frac{x}{l}\right) + bn * \sin\left(n * \operatorname{Pi} * \frac{x}{l}\right), n = 1 ...m\right), x, interval; m := m + 1 + \frac{m}{7}; i := i + 1; end do;
```

(2)

return display(seq(display(d[y]), y = 0 ... i - 1), insequence = true);

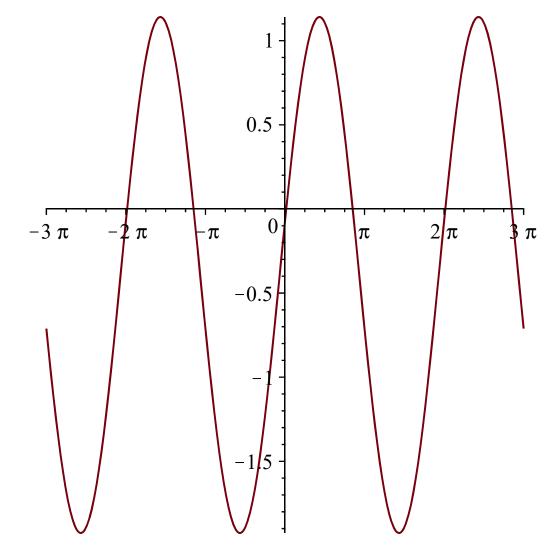
end proc;

m := 0;

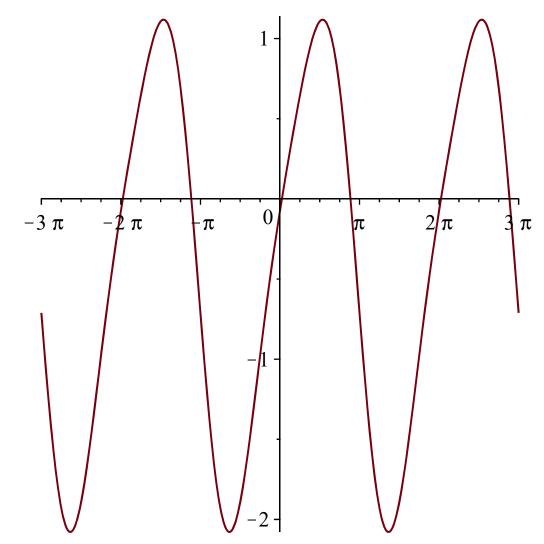
local *i*, *d*, *m*, *count*;

 $customplot := \mathbf{proc}(a0, an, bn, l, N, interval)$

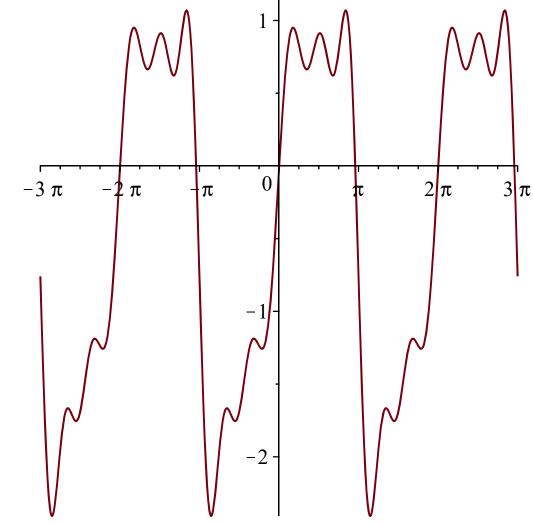
```
count := 60;
       i := 0;
       while i \le count and m \le N do
             d[i] := plot(unapply(1/2*a0 + sum(an*cos(n*\pi*x/l) + bn*sin(n*\pi*x/l), n))
             =1..m), x), interval);
             m := 8/7 * m + 1;
             i := i + 1
       end do;
       return plots:-display(seq(plots:-display(d[y]), y = 0..i - 1), insequence = true)
 end proc
> customplot\left(-\frac{\text{Pi}}{4}, \frac{1-(-1)^n}{2 \cdot \text{Pi} \cdot n^2}, \frac{1-2 \cdot (-1)^n}{2 \cdot n}, \text{Pi}, 100000, -3 \cdot \text{Pi} ... 3 \cdot \text{Pi}\right);
                                                                   1
                                                                   0
              -3\pi
                                                                                                       2\pi
                                                                                                                         3\pi
                                -2\pi
> customplot\left(-\frac{\text{Pi}}{4}, \frac{1-(-1)^n}{2 \cdot \text{Pi} \cdot n^2}, \frac{1-2 \cdot (-1)^n}{2 \cdot n}, \text{Pi}, 1, -3 \cdot \text{Pi} ..3 \cdot \text{Pi}\right);
```



>
$$customplot\left(-\frac{\text{Pi}}{4}, \frac{1-(-1)^n}{2 \cdot \text{Pi} \cdot n^2}, \frac{1-2 \cdot (-1)^n}{2 \cdot n}, \text{Pi}, 3, -3 \cdot \text{Pi} ..3 \cdot \text{Pi}\right);$$



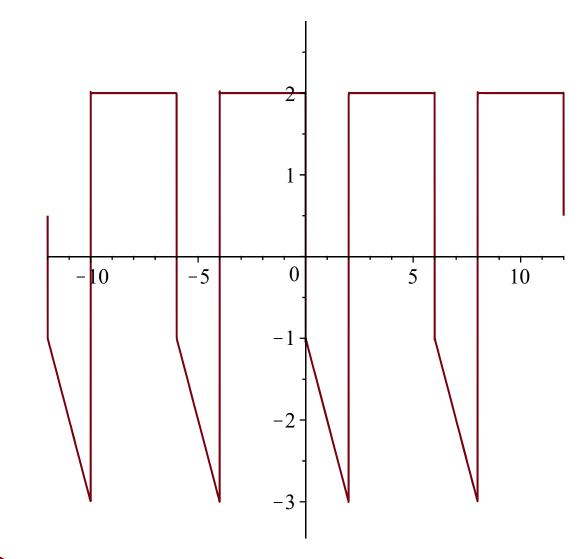
$$\overline{ } > customplot \left(-\frac{\text{Pi}}{4}, \frac{1 - (-1)^n}{2 \cdot \text{Pi} \cdot n^2}, \frac{1 - 2 \cdot (-1)^n}{2 \cdot n}, \text{Pi}, 7, -3 \cdot \text{Pi} ... 3 \cdot \text{Pi} \right);$$



$$expr2 := piecewise(0 < x < 2, -x - 1, 2 \le x \le 6, 2);$$

$$expr2 := \begin{cases} -x - 1 & 0 < x < 2 \\ 2 & 2 \le x \le 6 \end{cases}$$
(3)

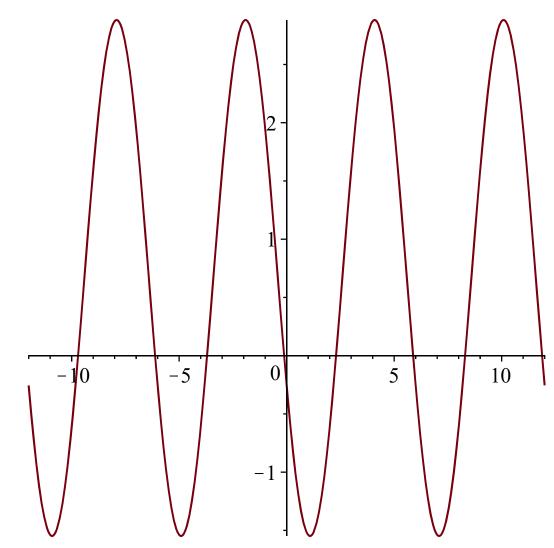
> fourierseries(expr2, 0, 6, -12 ..12);



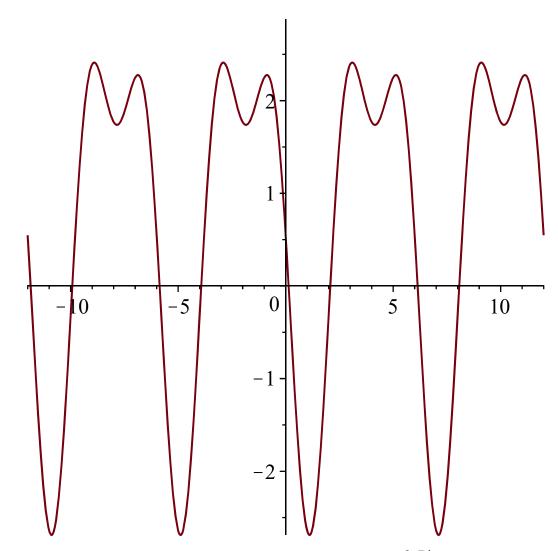
$$\begin{bmatrix}
\frac{4}{3}, -\frac{3\left(\pi n \sin\left(\frac{2\pi n}{3}\right) + \cos\left(\frac{2\pi n}{3}\right) - 1\right)}{\pi^{2}n^{2}} \\
+\frac{2\left(2\sin(\pi n)\cos(\pi n) - \sin\left(\frac{2\pi n}{3}\right)\right)}{\pi n}, \frac{3\pi n \cos\left(\frac{2\pi n}{3}\right) - \pi n - 3\sin\left(\frac{2\pi n}{3}\right)}{\pi^{2}n^{2}} \\
-\frac{2\left(2\cos(\pi n)^{2} - \cos\left(\frac{2\pi n}{3}\right) - 1\right)}{\pi n}
\end{bmatrix}$$

$$-\frac{\sin\left(\frac{2\cdot\operatorname{Pi}\cdot n}{3}\right)}{\frac{\operatorname{Pi}\cdot n}{3}} + 5\cdot\cos\left(\frac{2\cdot\operatorname{Pi}\cdot n}{3}\right) - 3, 100000, -12...12;$$

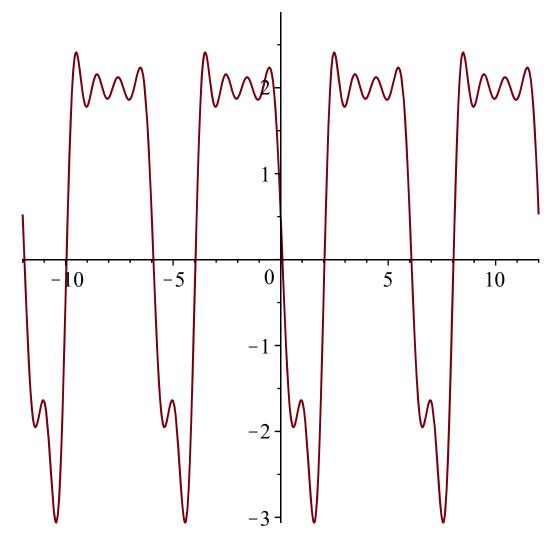
$$= \operatorname{customplot}\left(\frac{4}{3}, \frac{1}{\operatorname{Pi} \cdot n} \cdot \left(-5 \cdot \sin\left(\frac{2 \cdot \operatorname{Pi} \cdot n}{3}\right) + \frac{3}{\operatorname{Pi} \cdot n} - \frac{3 \cdot \cos\left(\frac{2 \cdot \operatorname{Pi} \cdot n}{3}\right)}{\operatorname{Pi} \cdot n}\right), \frac{1}{\operatorname{Pi} \cdot n} \cdot \left(\frac{\sin\left(\frac{2 \cdot \operatorname{Pi} \cdot n}{3}\right)}{\frac{\operatorname{Pi} \cdot n}{3}} + 5 \cdot \cos\left(\frac{2 \cdot \operatorname{Pi} \cdot n}{3}\right) - 3\right), 3, 1, -12 ...12\right);$$



$$= \frac{1}{1 - \frac{1}{1$$



$$> customplot \left(\frac{4}{3}, \frac{1}{\text{Pi} \cdot n} \cdot \left(-5 \cdot \sin\left(\frac{2 \cdot \text{Pi} \cdot n}{3}\right) + \frac{3}{\text{Pi} \cdot n} - \frac{3 \cdot \cos\left(\frac{2 \cdot \text{Pi} \cdot n}{3}\right)}{\text{Pi} \cdot n} \right), \frac{1}{\text{Pi} \cdot n} \cdot \left(-\frac{\sin\left(\frac{2 \cdot \text{Pi} \cdot n}{3}\right)}{\frac{\text{Pi} \cdot n}{3}} + 5 \cdot \cos\left(\frac{2 \cdot \text{Pi} \cdot n}{3}\right) - 3 \right), 3, 7, -12 ...12 \right);$$



$$\Rightarrow$$
 expr := piecewise $(0 < x < 2, -2 \cdot (x-1)^2, 2 \le x \le 4, x-4)$:

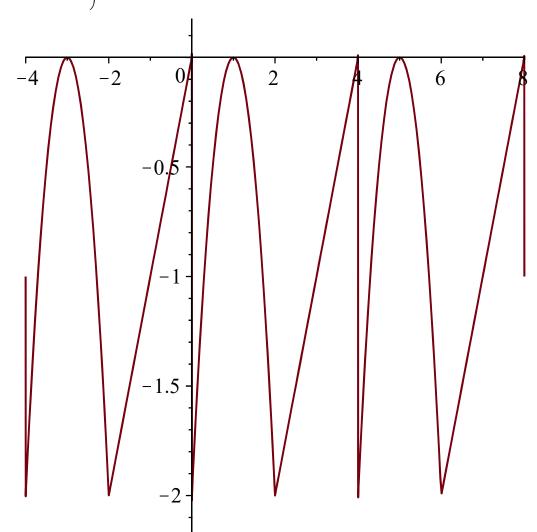
(5)

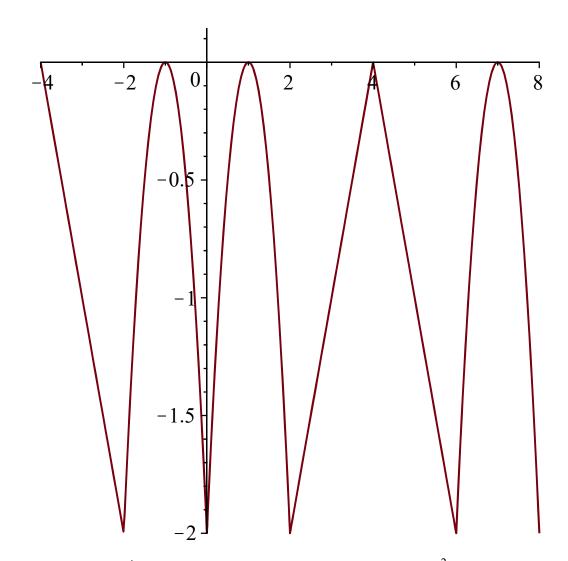
$$+\frac{2\left(n\pi\sin(n\pi)+2\cos(n\pi)^2-\cos(n\pi)-1\right)}{n^2\pi^2},$$

$$\frac{2 \left(n^2 \pi^2 \cos(n \pi) - n^2 \pi^2 - 4 n \pi \sin(n \pi) - 8 \cos(n \pi) + 8\right)}{n^3 \pi^3}$$

$$-\frac{2 \left(n \pi \cos(n \pi) - 2 \sin(n \pi) \cos(n \pi) + \sin(n \pi)\right)}{n^2 \pi^2}$$

>
$$customplot\left(-\frac{5}{3}, \frac{-2\cdot\left(5\cdot(-1)^{n}+3\right)}{\text{Pi}^{2}\cdot n^{2}}, \left((-1)^{n}-1\right)\cdot\left(\frac{2}{\text{Pi}\cdot n}-\frac{16}{\left(\text{Pi}\cdot n\right)^{3}}\right)-\frac{2\cdot(-1)^{n}}{\text{Pi}\cdot n}, 2,$$



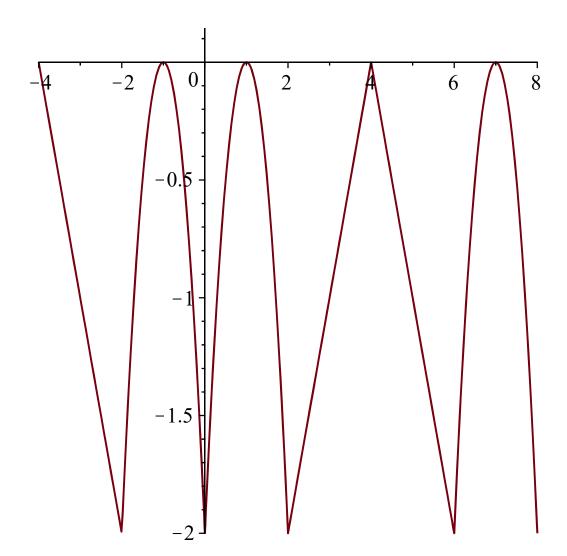


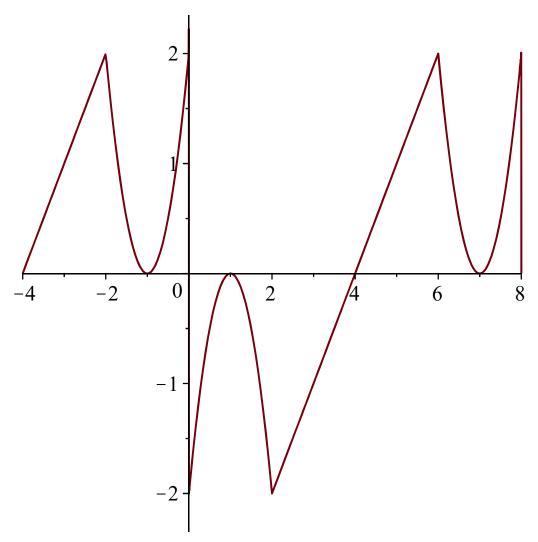
> $expr3b := piecewise(-4 \le x \le -2, -x - 4, -2 < x < 0, -2 \cdot (x + 1)^2, 0 < x < 2, -2 \cdot (x - 1)^2, 2 \le x \le 4, x - 4);$ coeffq(expr3b, -4, 4);

$$expr3b := \begin{cases} -x - 4 & -4 \le x \le -2 \\ -2(x+1)^2 & -2 < x < 0 \\ -2(x-1)^2 & 0 < x < 2 \\ x - 4 & 2 \le x \le 4 \end{cases}$$

$$\left[-\frac{5}{3}, \frac{4\left(n\pi\sin\left(\frac{n\pi}{2}\right) + 2\cos(n\pi) - 2\cos\left(\frac{n\pi}{2}\right)\right)}{n^2\pi^2} - \frac{4\left(n^2\pi^2\sin\left(\frac{n\pi}{2}\right) + 8n\pi\cos\left(\frac{n\pi}{2}\right) + 8n\pi - 32\sin\left(\frac{n\pi}{2}\right)\right)}{n^3\pi^3}, 0\right]$$
(6)

> fourierseries(expr3b, -4, 4, -4 ..8);





> #Задание 4

#Функция для ряда с многочленами Чебышева with(orthopoly);

cheb := proc(f, count) local c, m, i, d;

$$T[0] := 1;$$
 $T[1] := x;$

for i from 0 to count do

$$T[i+2] := 2 \cdot x \cdot T[i+1] - T[i] :$$

$T[i] = \cos(n \cdot \arccos(x));$
 $T[i+2] := simplify(T[i+2]);$
 $c[i] := int\left(\frac{f \cdot T[i]}{\operatorname{sqrt}(1-x^2)}, x = -1 ...1\right);$

norm $f[i] := int\left(\frac{T[i] \cdot T[i]}{\operatorname{sqrt}(1-x^2)}, x = -1 ...1\right) :$

end do:

return $add\left(\frac{c[n] \cdot T[n]}{norm f[n]}, n = 0 ...count\right)$

end proc;

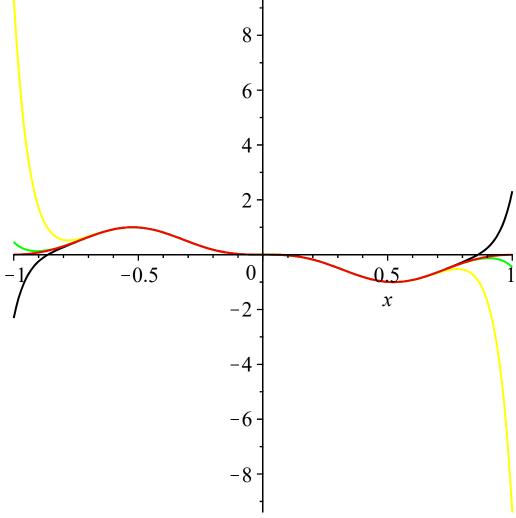
```
Warning, `T` is implicitly declared local to procedure `cheb`
Warning, `normf` is implicitly declared local to procedure
`cheb`
                                     [G, H, L, P, T, U]
Warning, `T` is implicitly declared local to procedure `cheb`
Warning, `normf` is implicitly declared local to procedure
`cheb`
cheb := \mathbf{proc}(f, count)
                                                                                               (7)
   local c, m, i, d, T, normf;
    T[0] := 1;
    T[1] := x;
    for i from 0 to count do
        T[i+2] := 2 * x * T[i+1] - T[i];
        T[i+2] := simplify(T[i+2]);
       c[i] := int(f^*T[i]/sqrt(-x^2 + 1), x = -1..1);
       normf[i] := int(T[i] * T[i] / sqrt(-x^2 + 1), x = -1..1)
    end do:
    return add(c[n] * T[n]/normf[n], n = 0..count)
end proc
> lej := proc(f, count)
   Pn[0] := 1;
   for i from 0 to count do
   Pn[i+1] := \frac{diff((x^2-1)^{i+1}, x\$(i+1))}{2^{i+1} \cdot (i+1)!};
   c[i] := int(f \cdot Pn[i], x = -1..1);
   normf[i] := int(Pn[i] \cdot Pn[i], x = -1..1);
   end do:
  return unapply \left(add\left(\frac{c[n] \cdot Pn[n]}{normf[n]}, n = 0 ... count\right), x\right), -1 ...1;
   end proc;
rning, `Pn` is implicitly declared local to procedure `lej`
Warning, `i` is implicitly declared local to procedure `lej`
Warning, `c` is implicitly declared local to procedure `lej` Warning, `normf` is implicitly declared local to procedure `lej`
lej := \mathbf{proc}(f, count)
                                                                                               (8)
    local Pn, i, c, normf;
   Pn[0] := 1;
    for i from 0 to count do
       Pn[i+1] := diff((x^2-1)^i, x^2i+1)/(2^i+1) * factorial(i+1);
       c[i] := int(f*Pn[i], x = -1..1);
       normf[i] := int(Pn[i] * Pn[i], x = -1..1)
    end do;
    return unapply(add(c[n]*Pn[n]/normf[n], n = 0...count), x), -1...1
end proc
> with(orthopoly);
```

```
f := (\sin(-3x))^3;
    d[1] := plot(lej(f, 1), x = -1 ...1, color = yellow) :
    d[3] := plot(lej(f, 3), x = -1 ...1, color = black):
   d[5] := plot(lej(f, 5), x = -1 ..1, color = green) : d[7] := plot(lej(f, 7), x = -1 ..1, color = blue) : display([d[1], d[3], d[5], d[7], plot(f, x = -1 ..1, color = red)]);
                                                   [G, H, L, P, T, U]
                                                     f := -\sin(3x)^3
                                                           1
                                                       0.5
                                  -0.5
                                                                                     0.5
                                                            0
                                                     -0.5
\rightarrow Maclor := proc(f, count) local y, n;
    n := 0;
    for n from 1 to count do
    y[n] := \frac{(unapply(diff(f, x\$n), x))(0)}{\cdot x^n} \cdot x^n;
    end do;
    y[0] := unapply(f, x)(0);
    return sum(y[nI], nI = 0...count);
    end proc;
Maclor := \mathbf{proc}(f, count)
                                                                                                                                  (9)
     local y, n;
     n := 0;
```

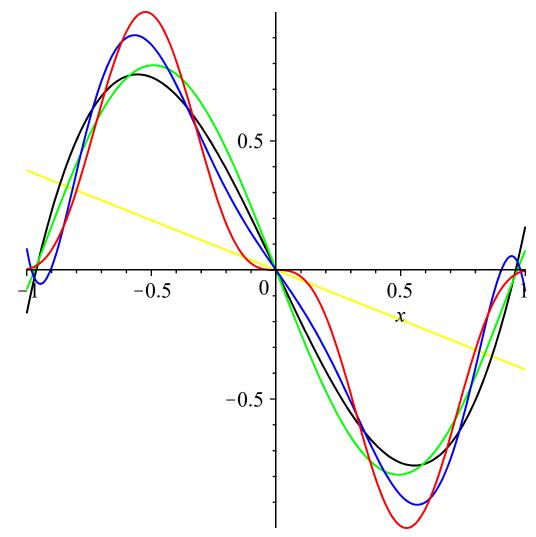
```
for n to count do y[n] := unapply(diff (f, x$n), x) (0) * x^n/factorial(n) end do;
y[0] := unapply(f, x) (0);
return sum(y[n1], n1 = 0 ...count)

end proc

> d[1] := plot(Maclor(f, 15), x = -1 ..1, color = yellow) :
d[3] := plot(Maclor(f, 17), x = -1 ..1, color = black) :
d[5] := plot(Maclor(f, 20), x = -1 ..1, color = green) :
d[7] := plot(Maclor(f, 60), x = -1 ..1, color = blue) :
display([d[1], d[3], d[5], d[7], plot(f, x = -1 ..1, color = red)]);
```



```
> d[1] := plot(cheb(f, 2), x =-1 ..1, color = yellow) :
d[3] := plot(cheb(f, 4), x =-1 ..1, color = black) :
d[5] := plot(cheb(f, 6), x =-1 ..1, color = green) :
d[7] := plot(cheb(f, 8), x =-1 ..1, color = blue) :
display([d[1], d[3], d[5], d[7], plot(f, x =-1 ..1, color = red)]);
```



> fourierseries2 := $\operatorname{proc}(f, count) \operatorname{local} n, l$,

$$a, b;$$
 $l := (1 - (-1))/2;$
 $a[0] := int(f, x = -1..1)/l;$
 $a[nl] := int(f^* \cos(nl * Pi * x/l), x = -1..1)/l;$
 $b[nl] := int(f^* \sin(nl * Pi * x/l), x = -1..1)/l;$
 $return a[0]/2 + sum(a[nl] * \cos(nl * Pi * \frac{x}{l}) + b[nl] * \sin(nl * Pi * \frac{x}{l}), nl = 1..count);$

end proc;

fourierseries2 :=
$$\operatorname{proc}(f, count)$$

local n, l, a, b ;
 $l := 1;$
 $a[0] := \operatorname{int}(f, x = -1..1)/l;$
 $a[nl] := \operatorname{int}(f^* \cos(nl * \operatorname{Pi} * x/l), x = -1..1)/l;$
 $b[nl] := \operatorname{int}(f^* \sin(nl * \operatorname{Pi} * x/l), x = -1..1)/l;$

```
return 1/2 * a[0] + sum(a[nl] * cos(nl * Pi * x/l) + b[nl] * sin(nl * Pi * x/l), nl = 1
...count)
```

end proc

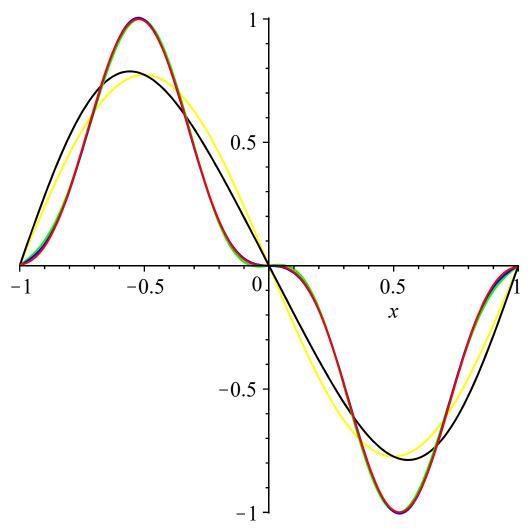
```
> d[1] := plot(fourierseries2(f, 1), x = -1..1, color = yellow) :

d[3] := plot(fourierseries2(f, 2), x = -1..1, color = black) :

d[5] := plot(fourierseries2(f, 3), x = -1..1, color = green) :

d[7] := plot(fourierseries2(f, 4), x = -1..1, color = blue) :

d[7] := plot(fourierseries2(f, 4), x = -1..1, color = blue) :
```

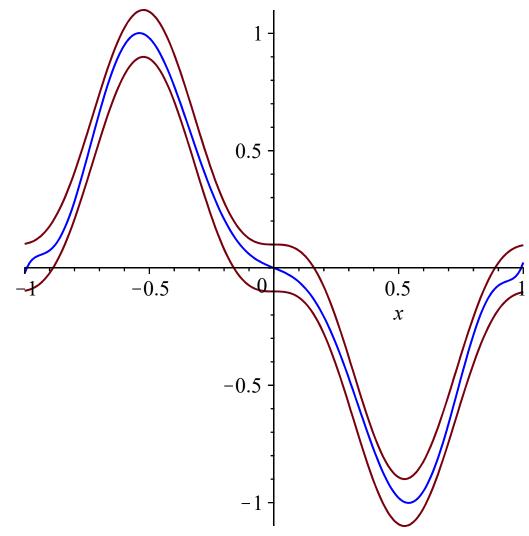


```
> fmin := plot(f - 0.1, x = -1..1):

fmax := plot(f + 0.1, x = -1..1):

pl := plot(cheb(f, 9), x = -1..1, colour = blue):

display([fmin, fmax, pl]);
```

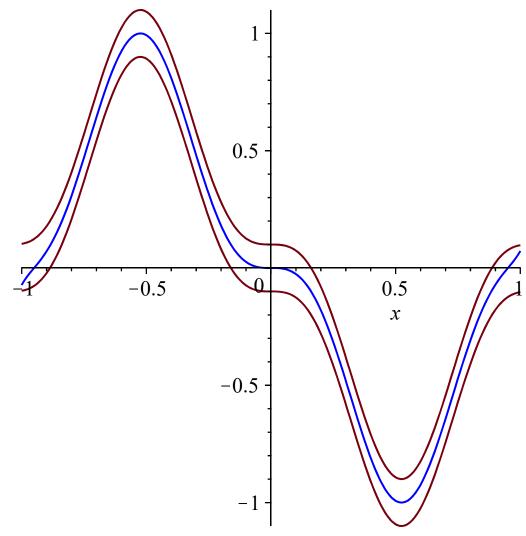


```
> fmin := plot(f - 0.1, x = -1..1):

fmax := plot(f + 0.1, x = -1..1):

pl := plot(Maclor(f, 21), x = -1..1, colour = blue):

display([fmin, fmax, pl]);
```

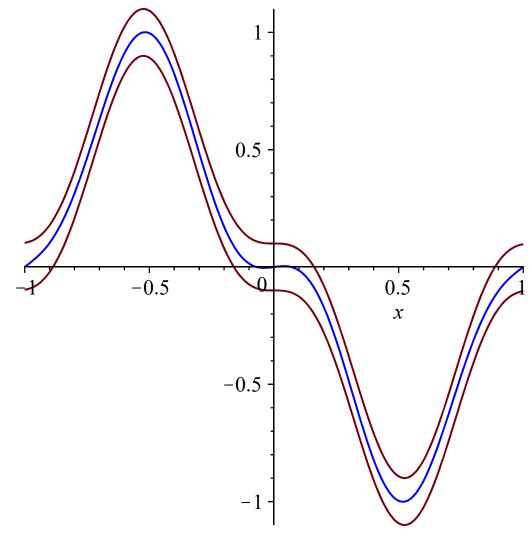


```
> fmin := plot(f - 0.1, x = -1..1):

fmax := plot(f + 0.1, x = -1..1):

pl := plot(fourierseries2(f, 3), x = -1..1, colour = blue):

display([fmin, fmax, pl]);
```

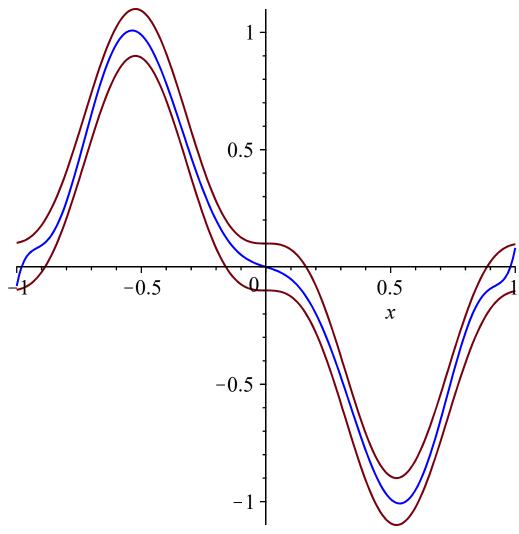


```
> fmin := plot(f - 0.1, x = -1..1):

fmax := plot(f + 0.1, x = -1..1):

pl := plot(lej(f, 9), x = -1..1, colour = blue):

display([fmin, fmax, pl]);
```



```
plej := plot(lej(f, 9), x = -1 ...1, colour = blue):

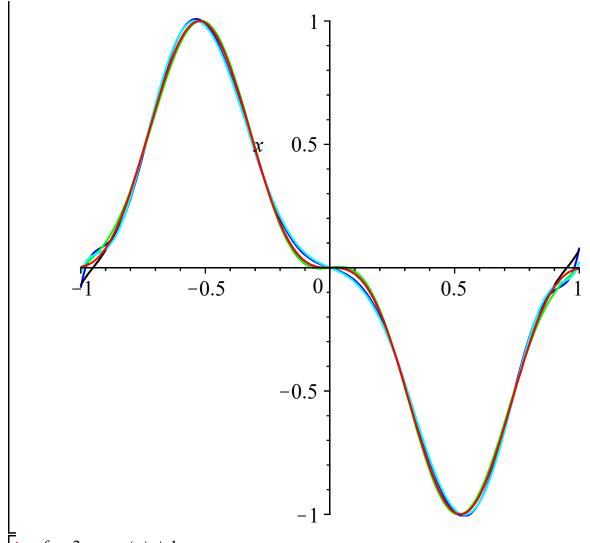
pf := plot(fourierseries2(f, 3), x = -1 ...1, colour = green):

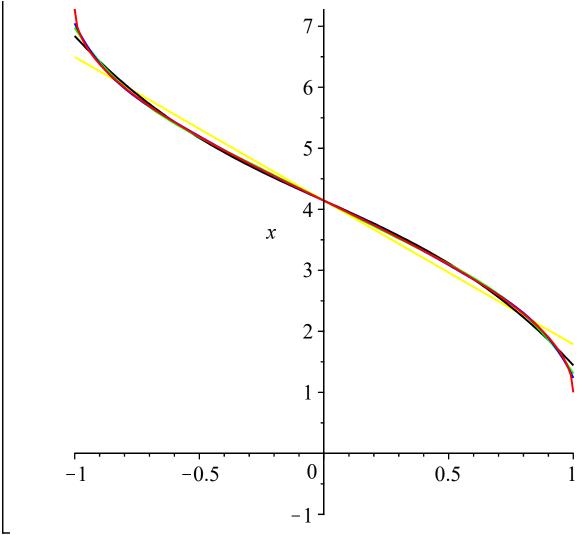
pm := plot(Maclor(f, 21), x = -1 ...1, colour = black):

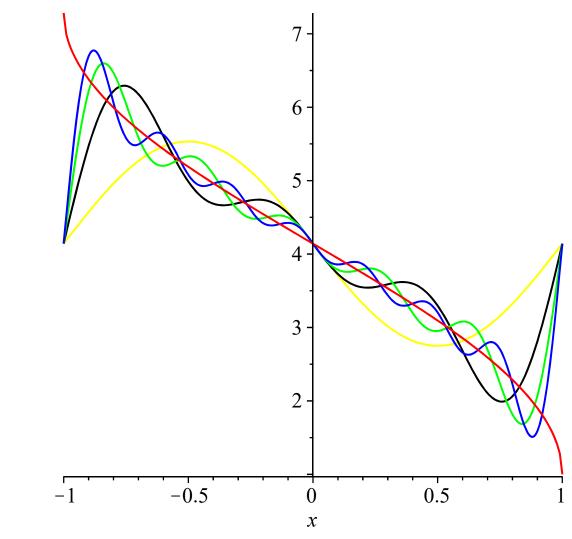
pch := plot(cheb(f, 9), x = -1 ...1, colour = cyan):

pfx := plot(f, x = -1 ...1, color = red):

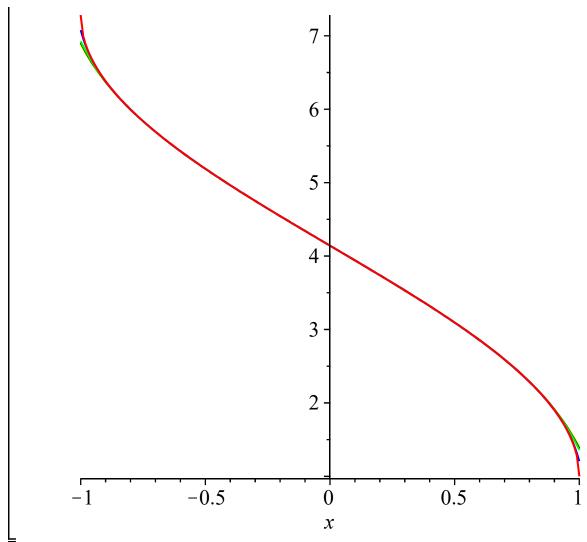
display([plej, pf, pm, pch, pfx]);
```







```
 d[1] := plot(Maclor(f, 15), x = -1..1, color = yellow) : \\ d[3] := plot(Maclor(f, 17), x = -1..1, color = black) : \\ d[5] := plot(Maclor(f, 20), x = -1..1, color = green) : \\ d[7] := plot(Maclor(f, 60), x = -1..1, color = blue) : \\ display([d[1], d[3], d[5], d[7], plot(f, x = -1..1, color = red)]);
```



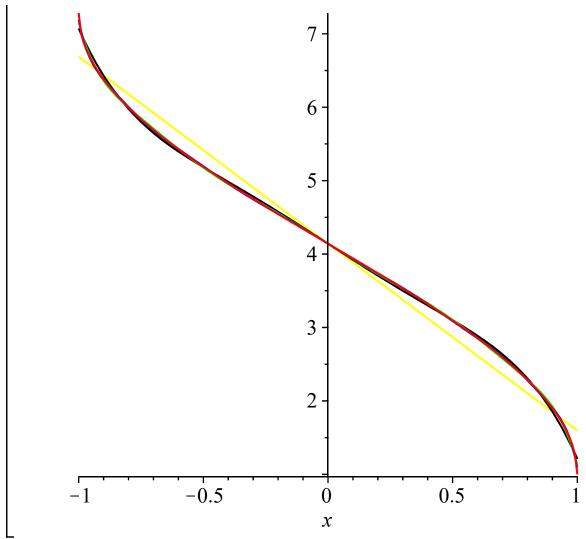
```
> d[1] := plot(cheb(f, 2), x = -1..1, color = yellow) :

d[3] := plot(cheb(f, 6), x = -1..1, color = black) :

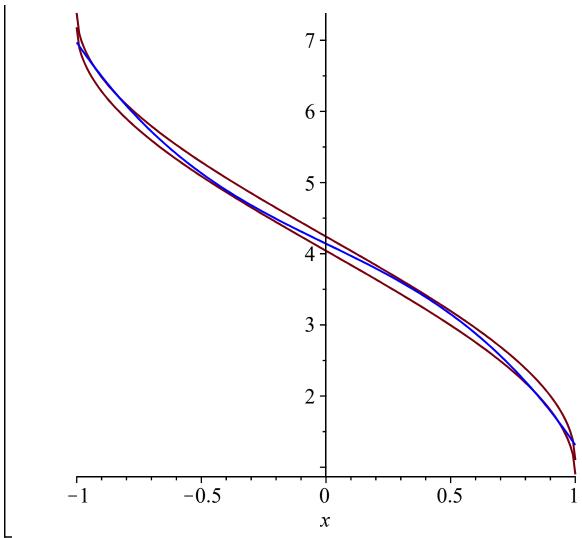
d[5] := plot(cheb(f, 10), x = -1..1, color = green) :

d[7] := plot(cheb(f, 14), x = -1..1, color = blue) :

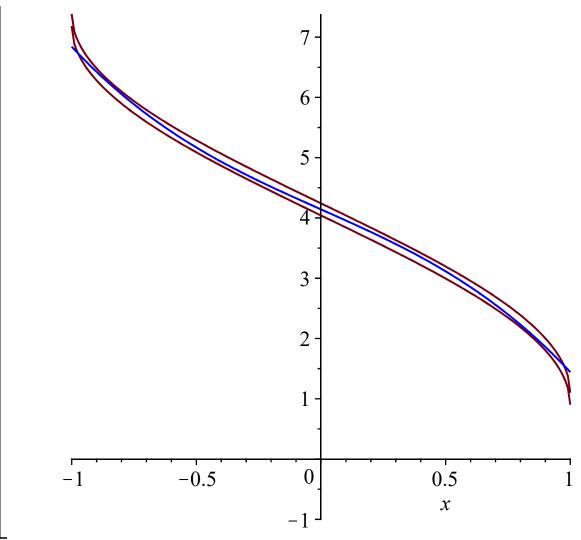
d[7] := plot(cheb(f, 14), x = -1..1, color = blue) :
```

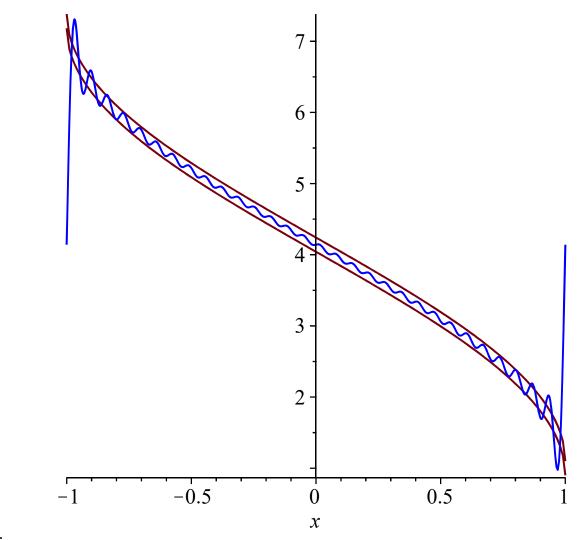


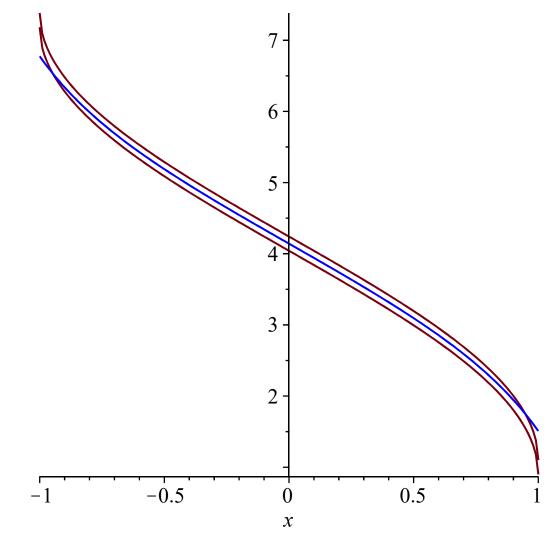
```
| fmin := plot(f - 0.1, x = -1..1) :
fmax := plot(f + 0.1, x = -1..1) :
pl := plot(cheb(f, 3), x = -1..1, colour = blue) :
display([fmin, fmax, pl]);
```



```
| fmin := plot(f - 0.1, x = -1..1) :
fmax := plot(f + 0.1, x = -1..1) :
pl := plot(lej(f, 3), x = -1..1, colour = blue) :
display([fmin, fmax, pl]);
```







```
> plej := plot(lej(f, 3), x = -1 ...1, colour = blue) :

pf := plot(fourierseries2(f, 30), x = -1 ...1, colour = green) :

pm := plot(Maclor(f, 10), x = -1 ...1, colour = black) :

pch := plot(cheb(f, 3), x = -1 ...1, colour = cyan) :

pfx := plot(f, x = -1 ...1, color = red) :

display([plej, pf, pm, pch, pfx]);
```

