Topological Analysis of Image Data

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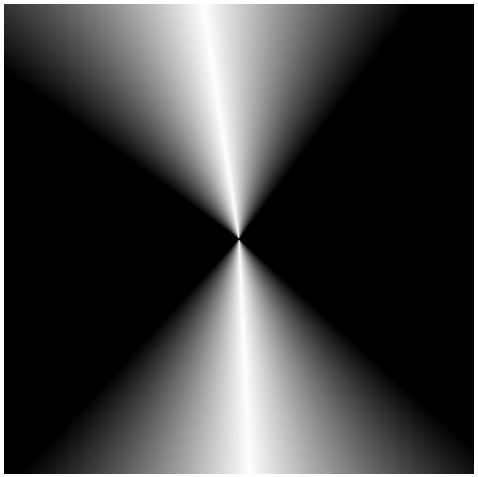
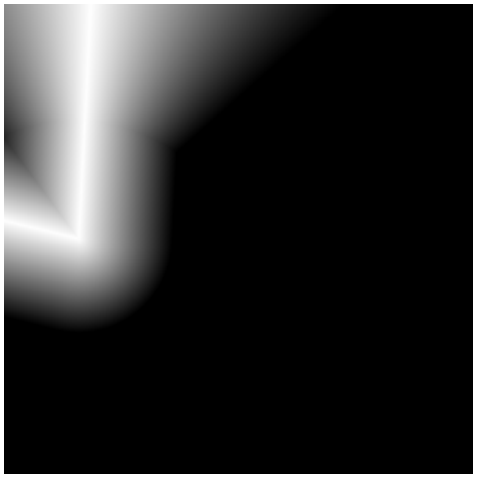
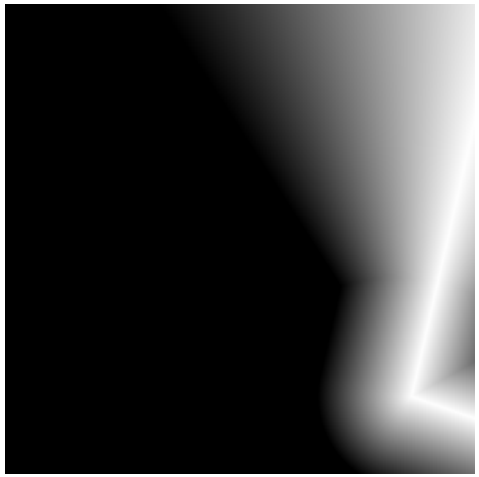
Images

Images with fixed center





Images with random center



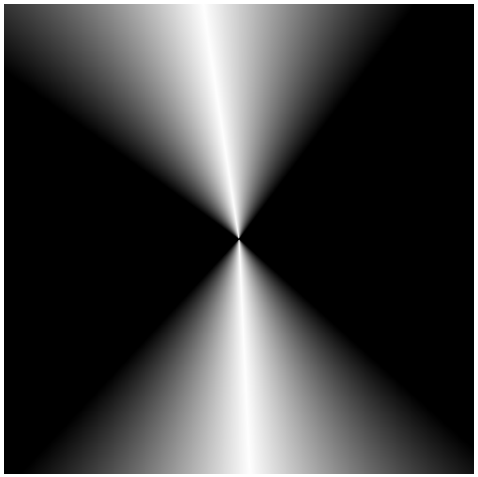
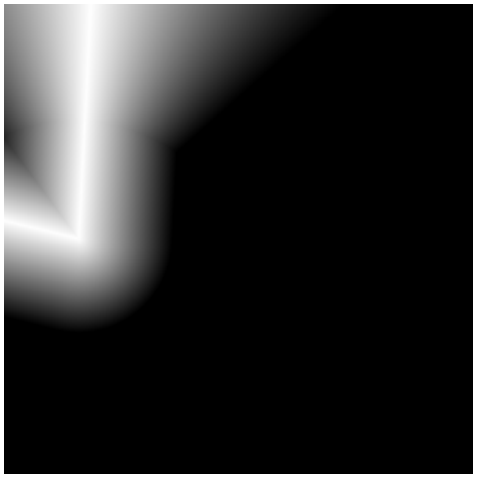
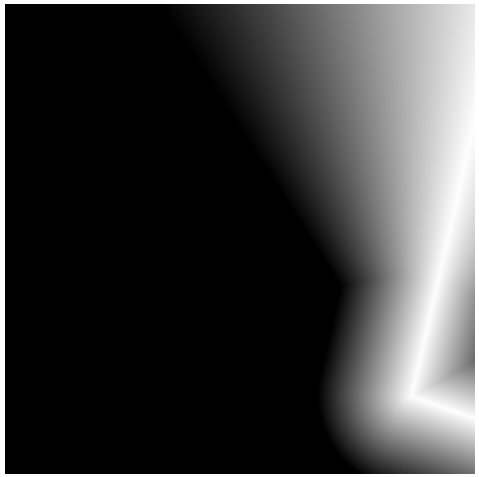
Images with semifixed center

Our Data

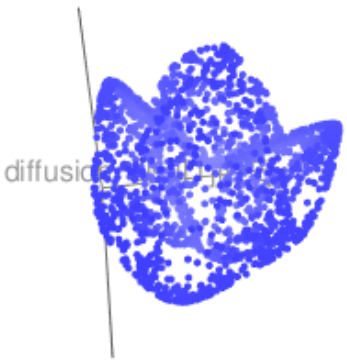
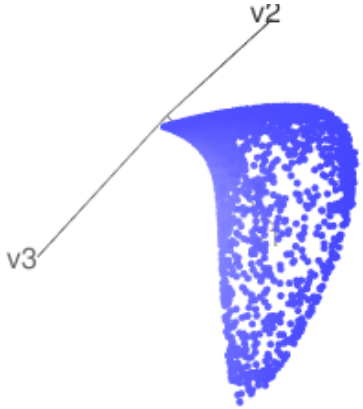
- Data is stored as an n^2-tuple, where n is the height and width, in pixels, of the grid
- Each entry in the n^2-tuple is a number between 0 and 1 representing the lightness of a particular pixel (0 is black, 1 is white)
- Brightness is assigned relative to distance from center of pixel to the lines
- Distance between two data points is taken as Euclidean distance between the two points in $\mathbb{R}^{n^2}$

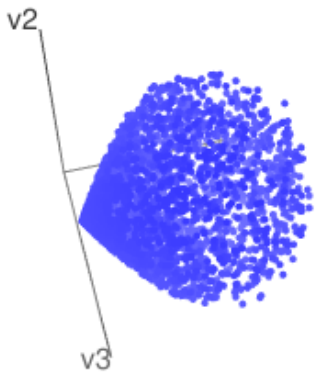
Diffusion map

Diffusion: Neat nonlinear dimension reduction algorithm which preserves topology

Generating 4000 images, applying diffusion map



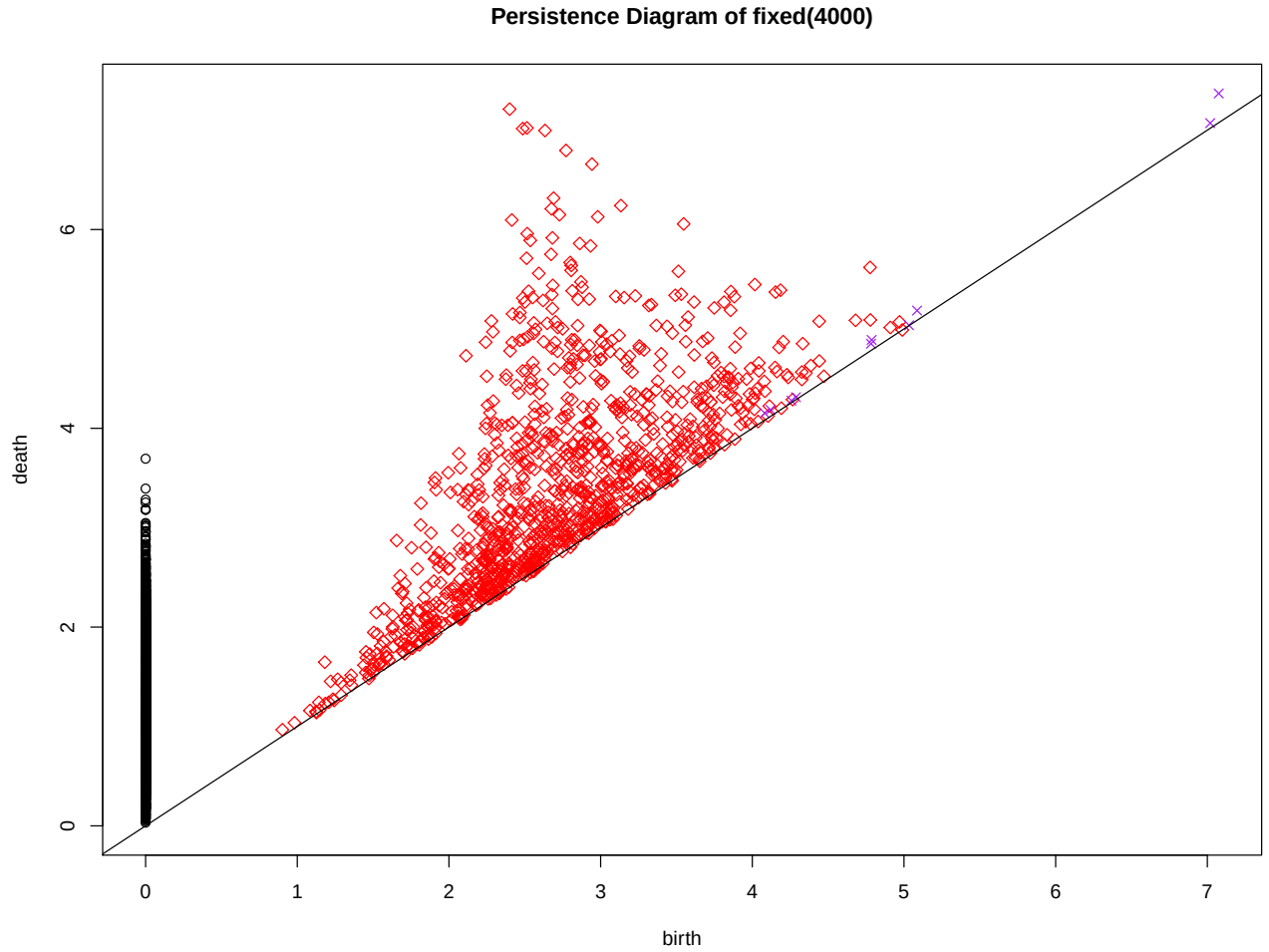
Fixed center case

We expect our space to be that of a Möbius strip:

- There are two degrees of freedom:
  - The angle to the bisector (from 0) and the angle between the lines, in which case if the angle between the lines are 180°, there are two choices of bisectors
  - or the angle to each line from 0, where there is an equivalence when the angles are switched
- A Möbius strip has the same homology as S^1 ($\beta_0 = \beta_1 = 1$, β_i=0 for all other i)

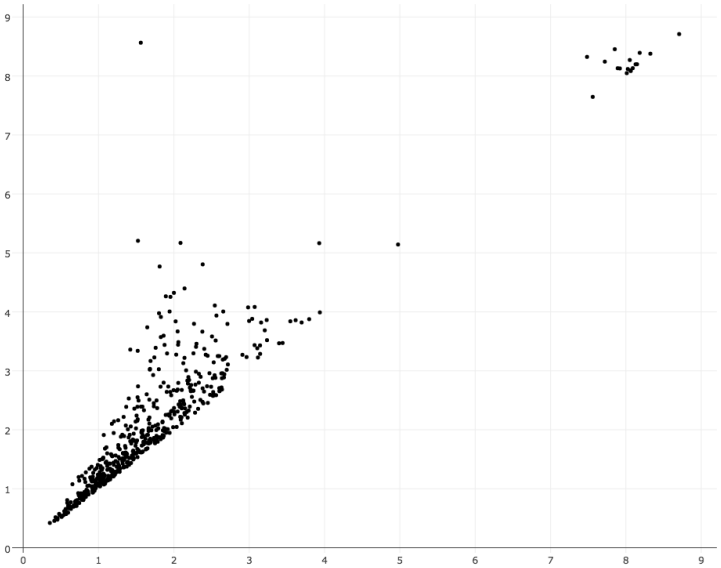
Persistence Diagram

Persistence diagram of 4000 points without diffusion

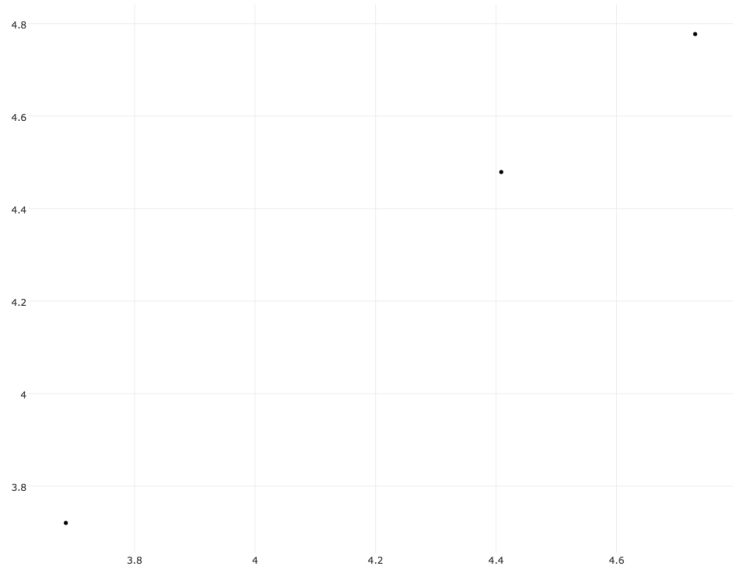


Persistence Diagram

Persistence diagram of 2000 points with diffusion

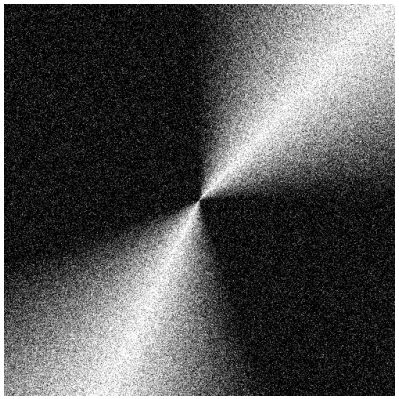


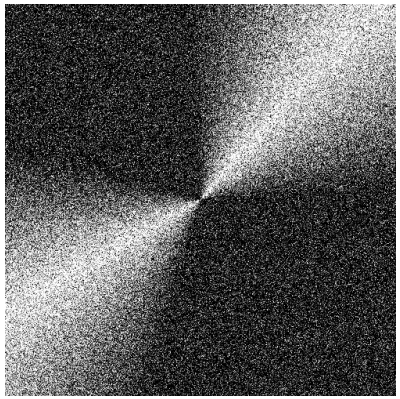
Dimension=1

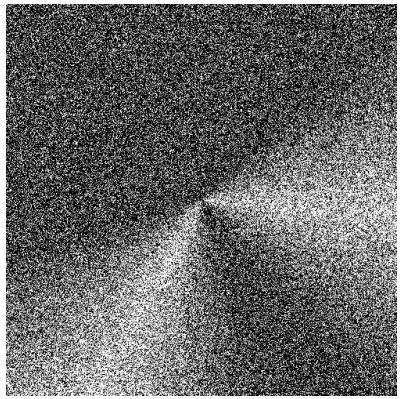


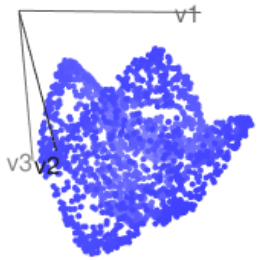
Dimension=2

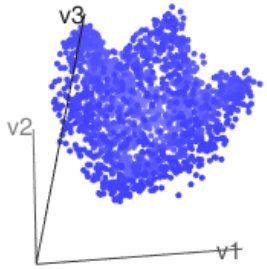
What if we include noise?



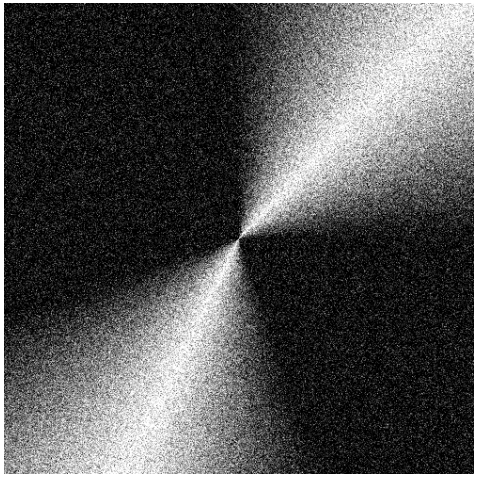


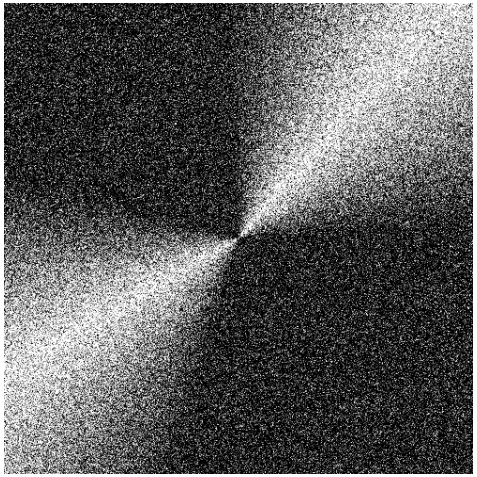


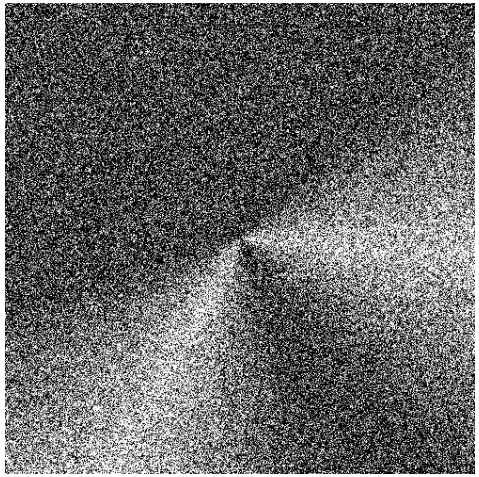


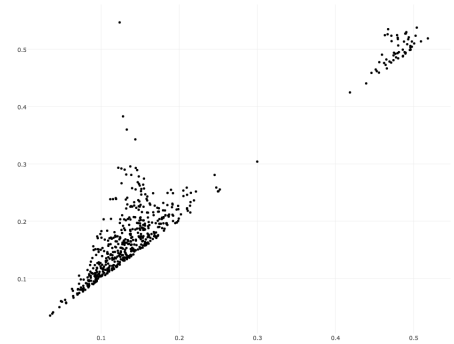


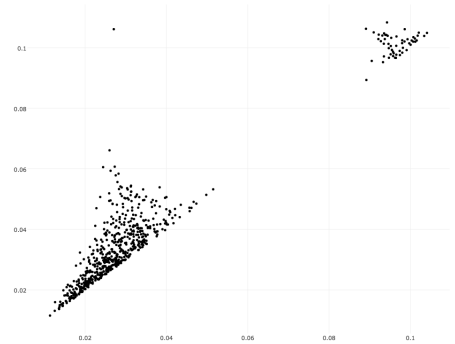
Noise:continue

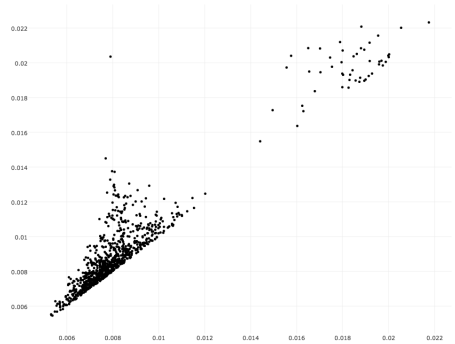




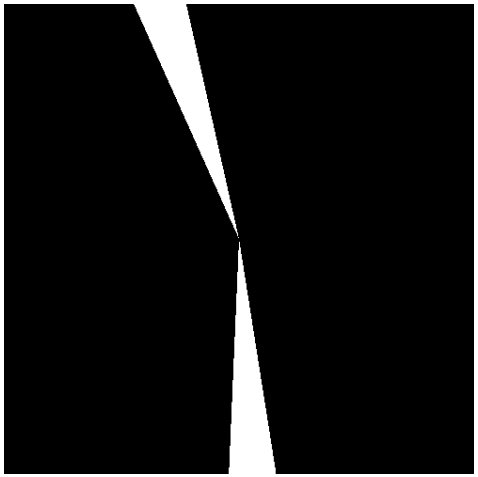


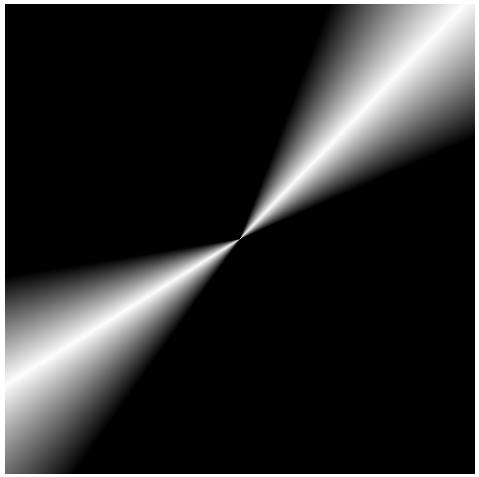


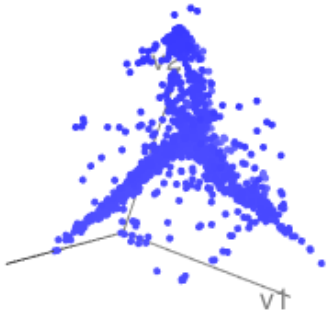


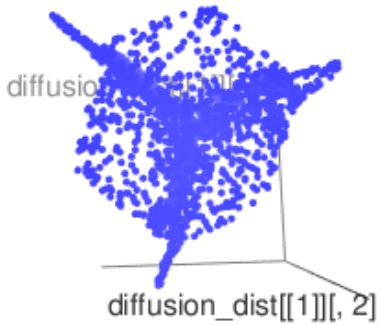


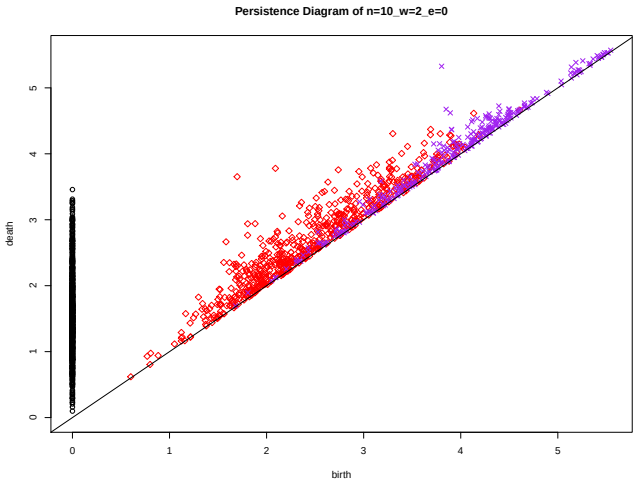
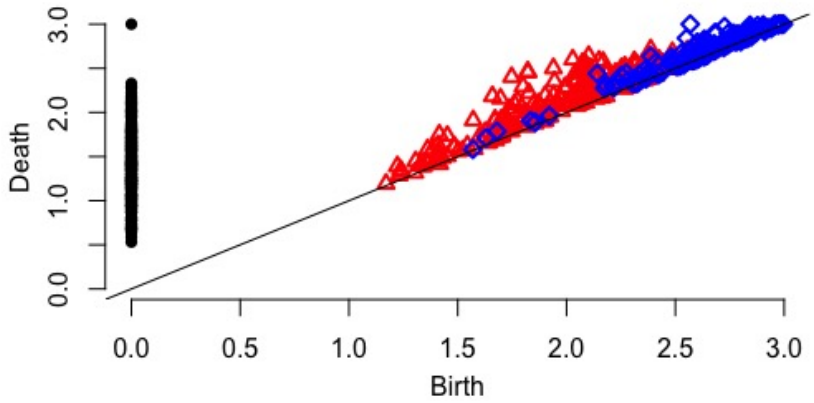
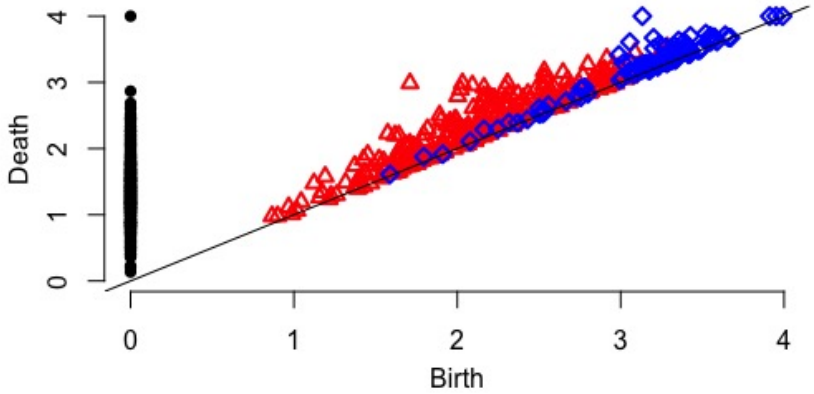
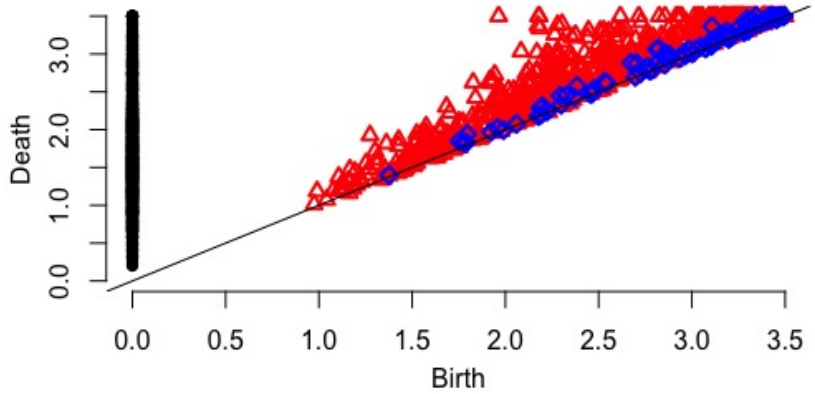
Importance of fade









Persistence Diagrams

The following were generated without a fixed point.