

What is Mathematics, Really?

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1 Introduction

What is mathematics, really?

This is a classic deep question. Given the excellent scholarship, thought and writing of to Schwartz, Kac, Rota (and others)[2, 8, 4, 5, 3, 16, 7] I hesitate to write on this topic. However, I recently had one of those “practitioner’s epiphanies” that I really feel captures the core of the issue and quickly explains a lot about mathematics.

My current definition is:

Mathematics is the minimal environment to preserve ideas.

Notice how this differs from Rota’s definition:

Mathematics is about analogies between analogies.

Even with our short definition we can explain some odd features of mathematics and draw consequences about mathematics. In the rest of this essay I will explore ideas leading up to the statement and draw out some consequence.

2 Background

Mathematics has a reputation for being hard. Like any deep field one can really only have a deep appreciation of it after a long period of study. There are of aspects of mathematics that many people understand quickly and there are prodigies who are instantly good at mathematics. Despite the esoteric media image, mathematics is like any other high-skill field: it can be mastered by any intelligent person through hard study. There is a theory of competency that deep competency in just about any field can be achieved in around 10 years of practice [12, 17, 1, 14]. In this respect Mathematics is certainly no harder than other serious disciplines in this respect.

Mathematicians and computer scientists often express surprise and disdain that there are intelligent people with limited knowledge of mathematics. This (incorrect) attitude is

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supported by the belief that mathematics is the new literacy of our modern age. In actual fact mathematics is one field of study separate from many others.

Much of mathematics is itself about mathematical objects. This is part of why mathematics can seem like an arcane and un-approachable field to an outsider. Fortunately in asking “what is mathematics” we are asking about the meaning and consequences of mathematics as connected to the larger world. That is: we are asking what mathematics means outside of its own world- so in answering our question we can largely ignore many of the details of mathematics (especially those that are self-referring).

Taking an idea from “the selfish gene” [6] we need to look really hard at the assumption that mathematics is created by its environment. This book encouraged us to realize it is not obvious if genes and DNA are a tool people use to propagate themselves or if in fact people are an invention that genes and DNA use to propagate themselves.

This is the core of my claim that mathematics is the minimal environment to preserve ideas.

3 Explanation

The environment we call mathematics tends toward the minimum because of a ruthless concession to survival: any non minimal environment would cost more to support. Preservation of ideas is a different goal than creation of ideas, dissemination of ideas, explanation of ideas or applications of ideas.

This is compatible with the common observation that mathematics differs from philosophy in that mathematics is content-free. Philosophical objects are more expensive than corresponding mathematical objects because philosophical objects typically carry with them some attempt at interpretation.

I would explain the “unreasonable effectiveness of mathematics” [9] as being exaggerated by an observational bias in that we usually only discuss the effectiveness of ideas and techniques that make sense when copied from the mathematical environment into our world or sciences.

This view of mathematics is also compatible with the a-historical bias Gian Carlo Rota [15, 11] attributes to working mathematicians. The field of mathematics is minimal in that it prefers carrying only ideas and does not help greatly in preserving their provenance (or even helping with dissemination).

4 Common Interpretations

Some famous interpretations of of mathematics are Rota’s (mentioned before), but we can see that Rota probably was thinking more about the practitioner’s of mathematics when we see that he was paraphrasing Banach’s:

Great mathematicians see analogies between analogies.

A more philosophical/formalist view is given in Whitehead’s:

The ultimate goal of mathematics is to eliminate any need for intelligent thought.

But in my interpretation this doesn't succeed in elevating the role of formal (mechanical) methods in mathematics but merely points out the mathematics is perfectly happy to survive without us.

5 Visible Consequences

I want to outline some of the visible consequences of mathematics being a minimal environment below. These effects have been observed before- but I now think they are inevitable.

Mathematics is able to anticipate sciences and develop mathematics before it is needed (such as number theory[10] before cryptography[13]). Mathematics survives fads. Mathematics is where old ideas, procedures and algorithms go for life support or stasis until they regain popularity, it is where you should look when you are stuck.

However, and this is the big surprise to the practitioner, it may be hard to get the idea back up to "fighting weight" and use it in the real world. The idea you have found may have (temporarily) given up practicality for survival.

And this is why you need the mathematician- cracking open a book may not be good enough.

References

- [1] BERKUN, S. The myths of innovation ch04. 17.
- [2] BLANK, B. E. What is mathematics? an elementary approach to ideas and methods (review). *Notices of the AMS* (Dec 2001), 1325–1329.
- [3] BUCHBERGER, B. Mathematics and computer science: A personal view. *Seria Matematica-Informatica* (Jan 2000), 13.
- [4] COULTER, L. O. What is mathematics? toward a global view. 17.
- [5] DAVIES, E. B. What is mathematics about? 28.
- [6] DAWKINS, R. *The Selfish Gene*. Oxford University Press, September 1990.
- [7] GOLD, B. What is mathematics i: The question. 12.
- [8] GOUVEA, F. What is mathematics, really (review). *MAA Online Book Review* (Jan 1998), 4.
- [9] HAMMING, R. W. The unreasonable effectiveness of mathematics in the natural sciences. *The American Mathematical Monthly* 87, 2 (Feb 1980), 12.
- [10] HARDY, G. H. *A Mathematician's Apology*. Cambridge University Press, 1940.

- [11] KAC, M., ROTA, G.-C., SCHWARTZ, J. T., AND RENZ, P. *Discrete Thoughts: Essays on Mathematics, Science, and Philosophy*. Springer, 1993.
- [12] NORVIG, P. Teach yourself programming in ten years. 2.
- [13] RIVEST, R. L., SHAMIR, A., AND ADLEMAN, L. A method for obtaining digital signatures and public-key cryptosystems. *Communications of the ACM* 21 (1978), 120–126.
- [14] ROSS, P. E. The expert mind. *Scientific American* (Jul 2006), 5.
- [15] ROTA, G.-C. *Indiscrete Thoughts*. Birkhäuser, Boston, 1997. Edited by Fabrizio Palombi.
- [16] ROTA, G.-C. In focus... what "is" mathematics? *The Mathematics Educator* 9, 1 (Oct 2000), 7.
- [17] SPOLSKY, J. Good software takes ten years. 8.