ST 501: Homework 1

Kirill Tsarapkin

September 6, 2019

1 Chapter 1 problems:

1. Question 1.2:

2. Two six-sided dice are thrown sequentially, and the face values that come up are recorded.

a. List the sample space:

$$\mathbf{\Omega} = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\}. \tag{1}$$

b. List the elements that make up the following events:

(1) A = the sum of the two values is at least 5:

A = {(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

(2) B = the value of the first die is higher than the value of the second:

B = {(2,1),(3,1),(4,1),(5,1),(6,1),(3,2),(4,2),(5,2),(6,2),(4,3),(5,3),(6,3),(5,4),(6,4),(6,5)}

(3) C = the first value is 4:

C = { (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)}

c. List the elements of the following events:

(1) A$\cap C = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$

(2) $B \cup C = \{(2,1),(3,1),(4,1),(5,1),(6,1),(3,2),(4,2),(5,2),(6,2),(4,3),(5,3),(6,3),(5,4),(6,4),(6,5),(4,4),(4,5),(4,6)\}$

(3) $A\cap(B\cup C) = (A\cap B)\cup(A\cap C) = \{(3,2),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(6,1),(6,2),(6,3),(6,4),(6,5)\}$

2. Question 1.5:

   Let A and B be arbitrary events. Let C be the event that either A occurs or B occurs, but not both. Express C in terms of A and B using any of the basic operations of union, intersection, and complement.

   Here, we need two events to occur $(A \cap B)^C$ and $(A \cup B)$, therefore,

$$C = (A \cap B)^C \cap (A \cup B) \tag{2}$$

3. Question 1.7:

   Prove Bonferronis inequality: $P(A \cap B) \geq P(A) + P(B) - 1$

$$Property\ D\,,\ Addition\ Law\ :\ P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{3}$$

$$Axiom\ 2\ :\ P(A \cap B) \leq P(\Omega) \tag{4}$$

$$Axiom\ 1\ :\ P(A \cap B) \leq 1 \tag{5}$$

   Plugging (5) into (3) we get,

$$P(A) + P(B) - P(A \cap B) \leq 1 \rightarrow P(A) + P(B) - 1 \leq P(A \cap B) \rightarrow P(A \cap B) \geq P(A) + P(B) - 1 \tag{6}$$

4. Question 1.12:

   In a game of poker, five players are each dealt 5 cards from a 52-card deck. How many ways are there to deal the cards?

$$\binom{52}{5}\binom{47}{5}\binom{42}{5}\binom{37}{5}\binom{32}{5} = \frac{52!}{5!^5(52 - 5 \times 5)!} = \frac{52!}{5!^5 27!} = 2976866583677512901784151142400 \tag{7}$$

5. Question 1.15:

   How many different meals can be made from four kinds of meat, six vegetables, and three starches if a meal consists of one selection from each group?

$$4 \times 6 \times 3 = 72 \tag{8}$$

6. Question 1.17:

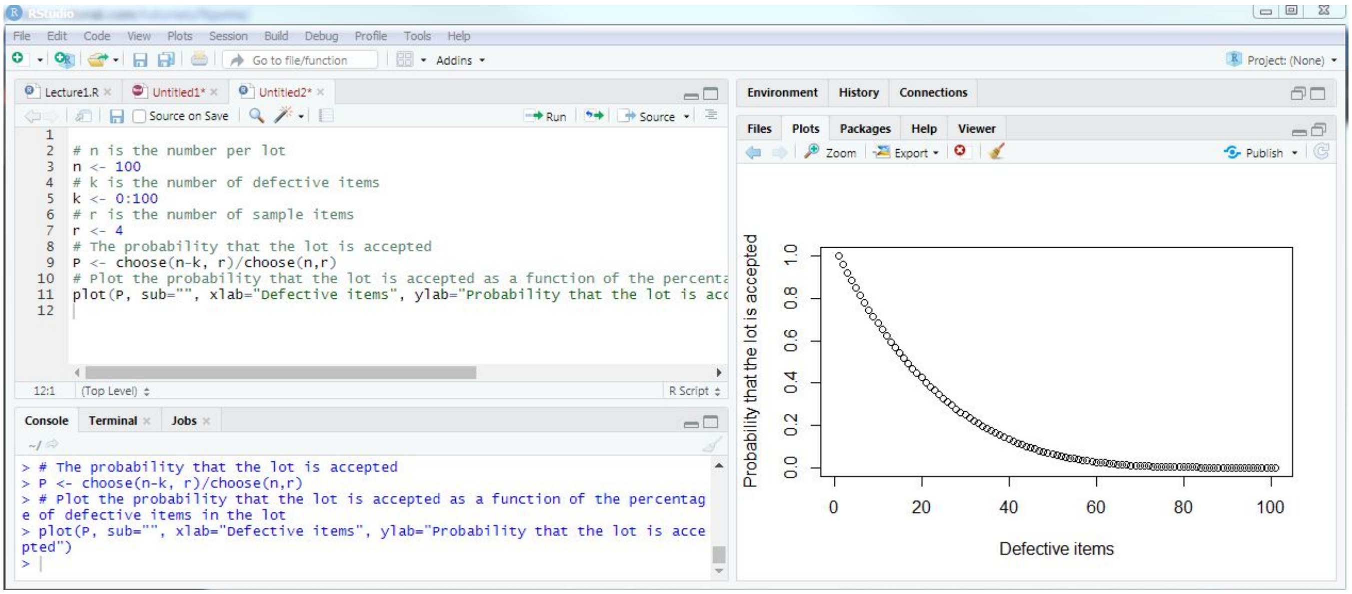
   In acceptance sampling, a purchaser samples 4 items from a lot of 100 and rejects the lot if 1 or more are defective. Graph the probability that the lot is accepted as a function of the percentage of defective items in the lot.

   The probability that the lot is accepted is:

$$P(A) = \frac{\binom{k}{m}\binom{n-k}{r-m}}{\binom{n}{r}} \tag{9}$$

   Here, m =0, n=100, r=4 and the number of defective products "k" can be between 0 and a 100, therefore

$$P(A) = \frac{\binom{n-k}{r-m}}{\binom{n}{r}} = \frac{\binom{100-k}{4}}{\binom{100}{4}} \tag{10}$$



7. Question 1.37:

   What is the coefficient of $x^2y^2z^3$ in the expansion of $(x + y + z)^7$?

The multinomial coefficient is:

$$\binom{7}{2, 2, 3} = \frac{7!}{2!2!3!} = 210 \tag{11}$$

8. Question 1.38:

   A child has six blocks, three of which are red and three of which are green. How many patterns can she make by placing them all in a line?

$$\binom{6}{3}\binom{3}{3} = \frac{6!}{3!3!} = 20 \tag{12}$$

If she is given three white blocks, how many total patterns can she make by placing all nine blocks in a line?

$$\binom{9}{3}\binom{6}{3}\binom{3}{3} = \frac{9!}{3!3!3!} = 1680 \tag{13}$$

9. Question 1.41:

   A drawer of socks contains seven black socks, eight blue socks, and nine green socks. Two socks are chosen in the dark.

a. What is the probability that they match?

$$\frac{\binom{7}{2}+\binom{8}{2}+\binom{9}{2}}{\binom{24}{2}} = \frac{85}{276} \approx 0.3080 \ or \ 30.80\% \tag{14}$$

b. What is the probability that a black pair is chosen?

$$\frac{\binom{7}{2}}{\binom{24}{2}} = \frac{7}{92} \approx 0.0761 \ or \ 7.61\% \tag{15}$$

2 Other problems:

10. Question 1

If P (A) = 1/3 and $P(B^c) = 1/4$, can A and B be disjoint? Explain

$$Property\ D\,,\ Addition\ Law\ :\ P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{16}$$

If events A and B are disjoint, then they have no elements in common, thus $P(A \cap B) = 0$, so we get

$$P(A \cup B) = P(A) + P(B) = \frac{1}{3} + \frac{3}{4} = \frac{13}{12} > 1 \tag{17}$$

However, the probability of $P(A \cup B)$ cannot be greater than 1. Therefore, we can conclude that events A and B are not disjoint.

11. Question 2

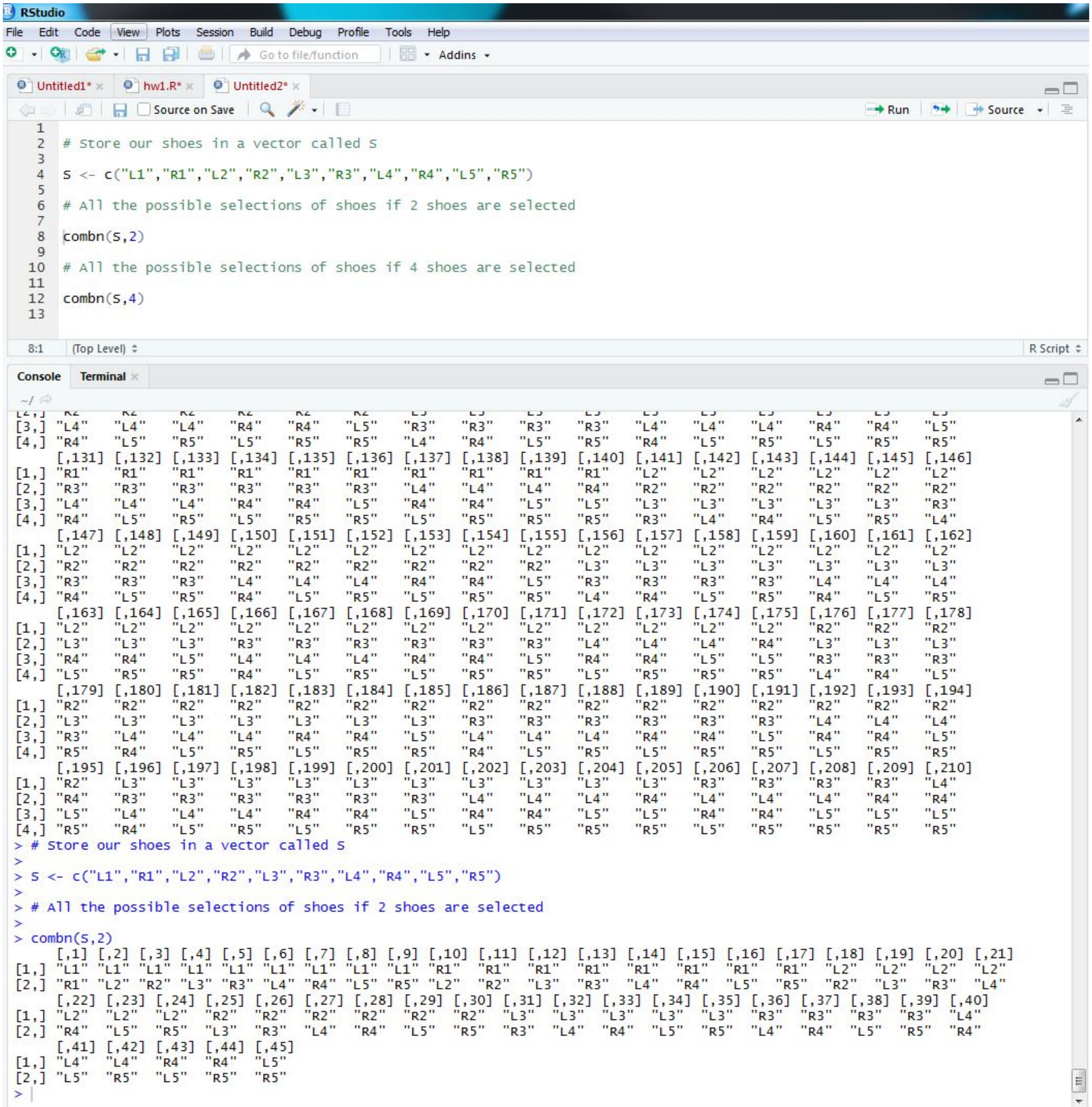
(a) If 2r shoes are chosen at random ($2r < n$), the probability that there will be no matching pair in the sample is

$$P(non - match) = \frac{\binom{n}{2r}2^{2r}}{\binom{2n}{2r}} \tag{18}$$

Give an explanation for where this equation comes from.

Answer: In the case above, there are $\binom{n}{2r}$ possible ways to get different pairs and there are 2^{2r} possible ways to pick one left or right shoe from those pairs. There are $\binom{2n}{2r}$ equally possible ways to choose 2r shoes from 2n shoes.Therefore, we get equation (18).

(b) This part should be completed in R. Suppose we have 5 pairs of shoes (10 total shoes). Label them L1, R1, L2, R2, ...,L5, R5. Using R, find all the possible selections of shoes if 2 shoes are selected. Similarly, do so for 4 shoes being selected.



(c) This part should be completed in R. For each case, use R to find the probability that you do not select a pair (using the enumerated possibilities, not the formula from (a)). Check that each of these matches the probability you found in part (a).

```r
# I'm not sure how to filter character strings in R
# maybe index them first? I don't know, I'm an R newb.

# Lets do part (b) by storing integers in our vector
# to respresent the shoes in a vector N
N <- c(1,1,2,2,3,3,4,4,5,5)
# Case 1(c)- Store the possible selections of shoes if 2 shoes are selected in df3
df3 <- combn(N,2)
# Case 1(c) - Number of shoe matches
match <- sum(df3[1,]==df3[2,])
# Case 1(c) - Calculate the probability that you do not select a pair
prob1 <- 100*(1-match/choose(10,2))
print(prob1)


# Case 2(c)- Store the possible selections of shoes if 4 shoes are selected in df4
df4 <- combn(N,4)
# Case 2(c) - Number of shoe matches
match <- df4[1,]==df4[2,]
match2 <- df4[3,]==df4[4,]
match3 <- sum(match==match2)
# Case 2(c) - Calculate the probability that you do not select a pair
prob2 <- 100*(1-(match3)/choose(10,4))
print(prob2)


# Now using equation from part (a), if we use similar values
# for n and r

# For Case 1(c)
n <- 5
r <- 1
prob3 <- (choose(n,2*r)*2^(2*r))/(choose(2*n, 2*r))
print(prob3*100)
# For Case 2(c)
n2 <- 5
r2 <- 2
prob4 <- (choose(n2,2*r2)*2^(2*r2))/(choose(2*n2, 2*r2))
print(prob4*100)
```

Console Terminal

~/

```
> # I'm not sure how to filter character strings in R
> # maybe index them first? I don't know, I'm an R newb.
>
> # Lets do part (b) by storing integers in our vector
> # to respresent the shoes in a vector N
> N <- c(1,1,2,2,3,3,4,4,5,5)
> # Case 1(c)- Store the possible selections of shoes if 2 shoes are selected in df3
> df3 <- combn(N,2)
> # Case 1(c) - Number of shoe matches
> match <- sum(df3[1,]==df3[2,])
> # Case 1(c) - Calculate the probability that you do not select a pair
> prob1 <- 100*(1-match/choose(10,2))
> print(prob1)
[1] 88.88889
>
>
> # Case 2(c)- Store the possible selections of shoes if 4 shoes are selected in df4
> df4 <- combn(N,4)
> # Case 2(c) - Number of shoe matches
> match <- df4[1,]==df4[2,]
> match2 <- df4[3,]==df4[4,]
> match3 <- sum(match==match2)
> # Case 2(c) - Calculate the probability that you do not select a pair
> prob2 <- 100*(1-(match3)/choose(10,4))
> print(prob2)
[1] 38.09524
>
>
> # Now using equation from part (a), if we use similar values
> # for n and r
>
> # For Case 1(c)
> n <- 5
> r <- 1
> prob3 <- (choose(n,2*r)*2^(2*r))/(choose(2*n, 2*r))
> print(prob3*100)
[1] 88.88889
> # For Case 2(c)
> n2 <- 5
> r2 <- 2
> prob4 <- (choose(n2,2*r2)*2^(2*r2))/(choose(2*n2, 2*r2))
> print(prob4*100)
[1] 38.09524
>
```