

ST 501: Homework 1

Kirill Tsarapkin

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1 Chapter 1 problems:

1. Question 1.2:

2. Two six-sided dice are thrown sequentially, and the face values that come up are recorded.

a. List the sample space:

$$\Omega = \left\{ \begin{array}{cccccc} (1, 1) & (1, 2) & (1, 3) & (1, 4) & (1, 5) & (1, 6) \\ (2, 1) & (2, 2) & (2, 3) & (2, 4) & (2, 5) & (2, 6) \\ (3, 1) & (3, 2) & (3, 3) & (3, 4) & (3, 5) & (3, 6) \\ (4, 1) & (4, 2) & (4, 3) & (4, 4) & (4, 5) & (4, 6) \\ (5, 1) & (5, 2) & (5, 3) & (5, 4) & (5, 5) & (5, 6) \\ (6, 1) & (6, 2) & (6, 3) & (6, 4) & (6, 5) & (6, 6) \end{array} \right\}. \quad (1)$$

b. List the elements that make up the following events:

(1) A = the sum of the two values is at least 5:

$$A = \{(1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

(2) B = the value of the first die is higher than the value of the second:

$$B = \{(2,1), (3,1), (4,1), (5,1), (6,1), (3,2), (4,2), (5,2), (6,2), (4,3), (5,3), (6,3), (5,4), (6,4), (6,5)\}$$

(3) C = the first value is 4:

$$C = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$$

c. List the elements of the following events:

$$(1) A \cap C = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$$

$$(2) B \cup C = \{(2,1), (3,1), (4,1), (5,1), (6,1), (3,2), (4,2), (5,2), (6,2), (4,3), (5,3), (6,3), (5,4), (6,4), (6,5), (4,4), (4,5), (4,6)\}$$

$$(3) A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \{(3,2), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3), (6,4), (6,5)\}$$

2. Question 1.5:

Let A and B be arbitrary events. Let C be the event that either A occurs or B occurs, but not both. Express C in terms of A and B using any of the basic operations of union, intersection, and complement.

Here, we need two events to occur $(A \cap B)^C$ and $(A \cup B)$, therefore,

$$C = (A \cap B)^C \cap (A \cup B) \quad (2)$$

3. Question 1.7:

Prove Bonferronis inequality: $P(A \cap B) \geq P(A) + P(B) - 1$

$$\text{Property } D, \text{ Addition Law} : P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (3)$$

$$\text{Axiom } 2 : P(A \cap B) \leq P(\Omega) \quad (4)$$

$$\text{Axiom } 1 : P(A \cap B) \leq 1 \quad (5)$$

Plugging (5) into (3) we get,

$$P(A) + P(B) - P(A \cap B) \leq 1 \rightarrow P(A) + P(B) - 1 \leq P(A \cap B) \rightarrow P(A \cap B) \geq P(A) + P(B) - 1 \quad (6)$$

4. Question 1.12:

In a game of poker, five players are each dealt 5 cards from a 52-card deck. How many ways are there to deal the cards?

$$\binom{52}{5} \binom{47}{5} \binom{42}{5} \binom{37}{5} \binom{32}{5} = \frac{52!}{5!^5(52-5 \times 5)!} = \frac{52!}{5!^5 27!} = 297686658367751290178415114240 \quad (7)$$

5. Question 1.15:

How many different meals can be made from four kinds of meat, six vegetables, and three starches if a meal consists of one selection from each group?

$$4 \times 6 \times 3 = 72 \quad (8)$$

6. Question 1.17:

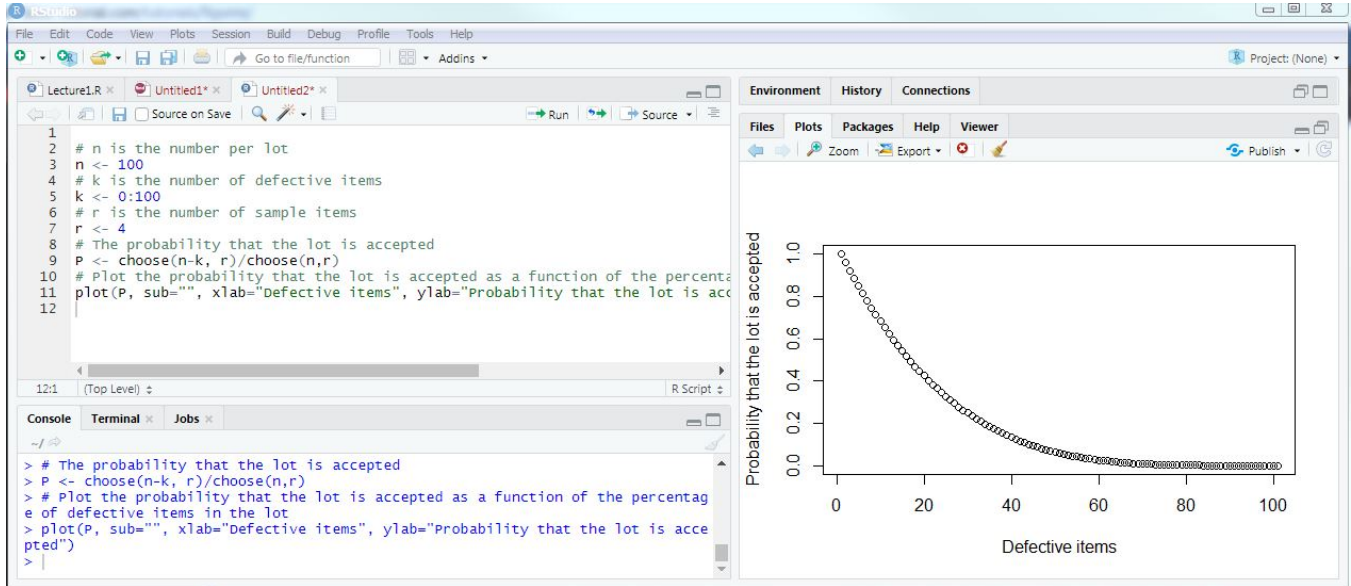
In acceptance sampling, a purchaser samples 4 items from a lot of 100 and rejects the lot if 1 or more are defective. Graph the probability that the lot is accepted as a function of the percentage of defective items in the lot.

The probability that the lot is accepted is:

$$P(A) = \frac{\binom{k}{m} \binom{n-k}{r-m}}{\binom{n}{r}} \quad (9)$$

Here, $m=0$, $n=100$, $r=4$ and the number of defective products "k" can be between 0 and a 100, therefore

$$P(A) = \frac{\binom{n-k}{r-m}}{\binom{n}{r}} = \frac{\binom{100-k}{4}}{\binom{100}{4}} \quad (10)$$



7. Question 1.37:

What is the coefficient of $x^2y^2z^3$ in the expansion of $(x + y + z)^7$?

The multinomial coefficient is:

$$\binom{7}{2, 2, 3} = \frac{7!}{2!2!3!} = 210 \quad (11)$$

8. Question 1.38:

A child has six blocks, three of which are red and three of which are green. How many patterns can she make by placing them all in a line?

$$\binom{6}{3} \binom{3}{3} = \frac{6!}{3!3!} = 20 \quad (12)$$

If she is given three white blocks, how many total patterns can she make by placing all nine blocks in a line?

$$\binom{9}{3} \binom{6}{3} \binom{3}{3} = \frac{9!}{3!3!3!} = 1680 \quad (13)$$

9. Question 1.41:

A drawer of socks contains seven black socks, eight blue socks, and nine green socks. Two socks are chosen in the dark.

a. What is the probability that they match?

$$\frac{\binom{7}{2} + \binom{8}{2} + \binom{9}{2}}{\binom{24}{2}} = \frac{85}{276} \approx 0.3080 \text{ or } 30.80\% \quad (14)$$

b. What is the probability that a black pair is chosen?

$$\frac{\binom{7}{2}}{\binom{24}{2}} = \frac{7}{92} \approx 0.0761 \text{ or } 7.61\% \quad (15)$$

2 Other problems:

10. Question 1

If $P(A) = 1/3$ and $P(B^c) = 1/4$, can A and B be disjoint? Explain

$$\text{Property D, Addition Law : } P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (16)$$

If events A and B are disjoint, then they have no elements in common, thus $P(A \cap B) = 0$, so we get

$$P(A \cup B) = P(A) + P(B) = \frac{1}{3} + \frac{3}{4} = \frac{13}{12} > 1 \quad (17)$$

However, the probability of $P(A \cup B)$ cannot be greater than 1. Therefore, we can conclude that events A and B are not disjoint.

11. Question 2

(a) If $2r$ shoes are chosen at random ($2r < n$), the probability that there will be no matching pair in the sample is

$$P(\text{non-match}) = \frac{\binom{n}{2r} 2^{2r}}{\binom{2n}{2r}} \quad (18)$$

Give an explanation for where this equation comes from.

Answer: In the case above, there are $\binom{n}{2r}$ possible ways to get different pairs and there are 2^{2r} possible ways to pick one left or right shoe from those pairs. There are $\binom{2n}{2r}$ equally possible ways to choose $2r$ shoes from $2n$ shoes. Therefore, we get equation (18).

(b) This part should be completed in R. Suppose we have 5 pairs of shoes (10 total shoes). Label them L1, R1, L2, R2, ..., L5, R5. Using R, find all the possible selections of shoes if 2 shoes are selected. Similarly, do so for 4 shoes being selected.

```

1 # Store our shoes in a vector called s
2
3 s <- c("L1","R1","L2","R2","L3","R3","L4","R4","L5","R5")
4
5 # All the possible selections of shoes if 2 shoes are selected
6
7 combn(s,2)
8
9 # All the possible selections of shoes if 4 shoes are selected
10
11 combn(s,4)
12
13

```

8:1 (Top Level) R Script

Console

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[3,] "L4" "L4" "L4" "R4" "R4" "L5" "R3" "R3" "R3" "R3" "L4" "L4" "L4" "R4" "R4" "L5"
[4,] "R4" "L5" "R5" "R5" "R5" "L4" "R4" "L5" "R5" "R4" "L5" "R5" "R5" "R5" "R5" "R5"
[1,] [1,31] [1,32] [1,33] [1,34] [1,35] [1,36] [1,37] [1,38] [1,39] [1,40] [1,41] [1,42] [1,43] [1,44] [1,45] [1,46]
[2,] "R1" "R1" "R1" "R1" "R1" "R1" "R1" "R1" "R1" "R1" "L2" "L2" "L2" "L2" "L2" "L2"
[3,] "R3" "R3" "R3" "R3" "R3" "R3" "R3" "R3" "R3" "R3" "L4" "L4" "L4" "L4" "L4" "L4"
[4,] "L4" "L4" "L4" "R4" "R4" "L5" "R4" "L5" "L5" "L3" "L3" "L3" "L3" "L3" "L3" "L3"
[1,] "R4" "L5" "R5" "R5" "R5" "L5" "R5" "R5" "R5" "R5" "R3" "L4" "R4" "L5" "R5" "L4"
[2,] [1,47] [1,48] [1,49] [1,50] [1,51] [1,52] [1,53] [1,54] [1,55] [1,56] [1,57] [1,58] [1,59] [1,60] [1,61] [1,62]
[3,] "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2"
[4,] "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "L3" "L3" "L3" "L3" "L3" "L3"
[1,] "R3" "R3" "R3" "L4" "L4" "L4" "R4" "R4" "L5" "R3" "R3" "R3" "L4" "L4" "L4" "L4"
[2,] "R4" "L5" "R5" "R4" "L5" "R5" "R5" "R5" "R5" "R5" "L4" "L5" "R4" "L5" "R5" "R5"
[3,] [1,63] [1,64] [1,65] [1,66] [1,67] [1,68] [1,69] [1,70] [1,71] [1,72] [1,73] [1,74] [1,75] [1,76] [1,77] [1,78]
[4,] "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2" "L2"
[1,] "L3" "L3" "L3" "R3" "R3" "R3" "R3" "R3" "R3" "R3" "L4" "L4" "L4" "L4" "L4" "L4"
[2,] "R4" "R4" "L5" "L4" "L4" "L4" "R4" "R4" "R4" "R4" "L5" "R4" "L5" "R3" "R3" "R3"
[3,] "L5" "R5" "R5" "R4" "L5" "R5" "R5" "R5" "R5" "R5" "L5" "R5" "R5" "R5" "R5" "R5"
[4,] [1,79] [1,80] [1,81] [1,82] [1,83] [1,84] [1,85] [1,86] [1,87] [1,88] [1,89] [1,90] [1,91] [1,92] [1,93] [1,94]
[1,] "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2" "R2"
[2,] "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "R3" "R3" "R3" "R3" "L4" "L4"
[3,] "R3" "L4" "L4" "L4" "R4" "R4" "L5" "L4" "L4" "L4" "R4" "R4" "L5" "R4" "R4" "L5"
[4,] "R5" "R4" "L5" "R5" "L5" "R5" "R5" "R5" "R5" "R5" "L5" "R5" "R5" "R5" "R5" "R5"
[1,] [1,195] [1,196] [1,197] [1,198] [1,199] [1,200] [1,201] [1,202] [1,203] [1,204] [1,205] [1,206] [1,207] [1,208] [1,209] [1,210]
[2,] "L2" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L3" "L4"
[3,] "R4" "R3" "R3" "R3" "R3" "R3" "R3" "R3" "R3" "R3" "L4" "L4" "L4" "L4" "L4" "L4"
[4,] "L5" "L4" "L4" "L4" "R4" "R4" "L5" "R4" "R4" "L5" "R4" "R4" "L5" "R4" "L5" "L5"
[1,] "R5" "R4" "L5" "R5" "L5" "R5" "R5" "R5" "R5" "R5" "L5" "R5" "R5" "R5" "R5" "R5"
> # Store our shoes in a vector called s
>
> s <- c("L1","R1","L2","R2","L3","R3","L4","R4","L5","R5")
>
> # All the possible selections of shoes if 2 shoes are selected
>
> combn(s,2)
[1,] [1,] [2,] [3,] [4,] [5,] [6,] [7,] [8,] [9,] [10,] [11,] [12,] [13,] [14,] [15,] [16,] [17,] [18,] [19,] [20,] [21,]
[2,] "L1" "L1" "L1" "L1" "L1" "L1" "L1" "L1" "L1" "R1" "R1" "R1" "R1" "R1" "R1" "R1" "L2" "L2" "L2" "L2"
[1,] [22,] [23,] [24,] [25,] [26,] [27,] [28,] [29,] [30,] [31,] [32,] [33,] [34,] [35,] [36,] [37,] [38,] [39,] [40,]
[2,] "L2" "L2" "L2" "R2" "R2" "R2" "R2" "R2" "R2" "L3" "L3" "L3" "L3" "L3" "L3" "R3" "R3" "R3" "R3" "L4"
[1,] [41,] [42,] [43,] [44,] [45,]
[2,] "R4" "L5" "R5" "L3" "R3" "L4" "R4" "L5" "R5" "R3" "L4" "R4" "L5" "R5" "L4" "R4" "L5" "R5" "R4"
[1,] [46,] [47,] [48,] [49,] [50,]
[2,] "L4" "L4" "R4" "R4" "L5"
[1,] [51,] [52,] [53,] [54,] [55,]
[2,] "L5" "R5" "L5" "R5" "R5"

```

(c) This part should be completed in R. For each case, use R to find the probability that you do not select a pair (using the enumerated possibilities, not the formula from (a)). Check that each of these matches the probability you found in part (a).

```

RStudio
File Edit Code View Plots Session Build Debug Profile Tools Help
Go to file/function
Addins
Untitled1* hwl1.R* Untitled2*
Source on Save Run Source
1 # I'm not sure how to filter character strings in R
2 # maybe index them first? I don't know, I'm an R newb.
3
4 # Lets do part (b) by storing integers in our vector
5 # to represent the shoes in a vector N
6 N <- c(1,1,2,2,3,3,4,4,5,5)
7 # Case 1(c)- Store the possible selections of shoes if 2 shoes are selected in df3
8 df3 <- combn(N,2)
9 # Case 1(c) - Number of shoe matches
10 match <- sum(df3[,1]==df3[,2])
11 # Case 1(c) - calculate the probability that you do not select a pair
12 prob1 <- 100*(1-match/choose(10,2))
13 print(prob1)
14
15
16 # Case 2(c)- Store the possible selections of shoes if 4 shoes are selected in df4
17 df4 <- combn(N,4)
18 # Case 2(c) - Number of shoe matches
19 match <- df4[,1]==df4[,2,]
20 match2 <- df4[,3]==df4[,4,]
21 match3 <- sum(match==match2)
22 # Case 2(c) - Calculate the probability that you do not select a pair
23 prob2 <- 100*(1-(match3)/choose(10,4))
24 print(prob2)
25
26
27 # Now using equation from part (a), if we use similar values
28 # for n and r
29
30 # For Case 1(c)
31 n <- 5
32 r <- 1
33 prob3 <- (choose(n,2*r)*2^(2*r))/(choose(2*n, 2*r))
34 print(prob3*100)
35 # For Case 2(c)
36 n2 <- 5
37 r2 <- 2
38 prob4 <- (choose(n2,2*r2)*2^(2*r2))/(choose(2*n2, 2*r2))
39 print(prob4*100)
40
41

```

```
Console Terminal
> # I'm not sure how to filter character strings in R
> # maybe index them first? I don't know, I'm an R newb.
>
> # Lets do part (b) by storing integers in our vector
> # to represent the shoes in a vector N
> N <- c(1,1,2,2,3,3,4,4,5,5)
> # Case 1(c)- store the possible selections of shoes if 2 shoes are selected in df3
> df3 <- combn(N,2)
> # Case 1(c) - Number of shoe matches
> match <- sum(df3[1,]==df3[2,])
> # Case 1(c) - Calculate the probability that you do not select a pair
> prob1 <- 100*(1-match/choose(10,2))
> print(prob1)
[1] 88.88889
>
>
> # Case 2(c)- store the possible selections of shoes if 4 shoes are selected in df4
> df4 <- combn(N,4)
> # Case 2(c) - Number of shoe matches
> match <- df4[1,]==df4[2,]
> match2 <- df4[3,]==df4[4,]
> match3 <- sum(match==match2)
> # Case 2(c) - Calculate the probability that you do not select a pair
> prob2 <- 100*(1-(match3)/choose(10,4))
> print(prob2)
[1] 38.09524
>
>
> # Now using equation from part (a), if we use similar values
> # for n and r
>
> # For Case 1(c)
> n <- 5
> r <- 1
> prob3 <- (choose(n,2*r)*2^(2*r))/(choose(2*n, 2*r))
> print(prob3*100)
[1] 88.88889
> # For Case 2(c)
> n2 <- 5
> r2 <- 2
> prob4 <- (choose(n2,2*r2)*2^(2*r2))/(choose(2*n2, 2*r2))
> print(prob4*100)
[1] 38.09524
>
|
```