ECS 132 Winter 2016 Homework 2

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1 Question 1

Continous Problem 1 in Hwk1

1) Expected value of the weight of the first item placed into Box 2

$$
\begin{aligned}
P(\text{Weight} = 1) &= [P(1, 1, 1, 1) + P(1, 1, 2) + P(1, 2, 1) + P(1, 3) + P(2, 1, 1) \\
&\quad + P(2, 2) + P(3, 2)] \cdot \frac{1}{3} \\
&= (\frac{1}{81} + \frac{1}{9} + \frac{1}{3}) \cdot \frac{3}{1} \\
&= \frac{37}{243} \\
P(\text{Weight} = 2) &= P(\text{Weight} = 1) + P(1, 1, 1, 2) + P(1, 2, 2) + P(2, 1, 2) + P(3, 2) \\
&= \frac{85}{243} \\
P(\text{Weight} = 3) &= P(\text{Weight} = 2) + P(1, 1, 3) + P(2, 3) + P(3, 3) \\
&= \frac{121}{243} \\
E(\text{Weight}) &= P(\text{Weight} = 1) + 2 \cdot P(\text{Weight} = 2) \\
&\quad + 3 \cdot P(\text{Weight} = 3) = 2.345679
\end{aligned}
$$

2) Variance of the weight

$$
\begin{aligned}
Var(\text{Weight}) &= E(W^2) - EW^2 \\
&= P(\text{Weight} = 1) + 4 \cdot P(\text{Weight} = 2) + 9 \cdot P(\text{Weight} = 3) - 2.345679^2 \\
&= 6.032922 - 5.502209 = 0.530711
\end{aligned}
$$

2 Question 2

Bus Ridership Example: Find Cov(L1,L2)

$$
\begin{aligned}
Cov(L_1, L_2) &= E(L_1 \cdot L_2) - EL_1 \cdot EL_2 \\
E(L_1) &= 0.4 + 0.2 \cdot 1 = 0.6 \\
P(L_2 = 1) &= 0.5 \cdot 0.4 + 0.4 \cdot 0.8 \cdot 0.5 + 0.4 \cdot 0.4 \cdot 0.2 \\
&+ 0.1 \cdot 0.2 \cdot 0.8 \cdot 0.5 \cdot 2 + 0.1 \cdot 0.2 \cdot 0.2 \cdot 0.4 \\
&= 0.4096 \\
P(L_2 = 2) &= 0.5 \cdot 0.1 + 0.4 \cdot 0.8 \cdot 0.4 + 0.4 \cdot 0.2 \cdot 0.1 \\
&+ 0.1 \cdot 0.2 \cdot 0.8 \cdot 0.4 \cdot 2 + 0.1 \cdot 0.2 \cdot 0.2 \cdot 0.1 + 0.1 \cdot 0.8 \cdot 0.8 \cdot 0.5 \\
&= 0.2312 \\
P(L_2 = 3) &= 0.4 \cdot 0.8 \cdot 0.1 + 0.1 \cdot 0.8 \cdot 0.8 \cdot 0.4 \\
&+ 0.1 \cdot 0.8 \cdot 0.2 \cdot 0.1 \cdot 2 \\
&= 0.0608 \\
P(L_2 = 4) &= 0.1 \cdot 0.8 \cdot 0.8 \cdot 0.1 = 0.0064 \\
P(L_2) &= 1.08 \\
P(L_1 \cdot L_2 = 1) &= P(L_1 = 1, L_2 = 1) = 0.192 \\
P(L_1 \cdot L_2 = 2) &= P(L_1 = 1, L_2 = 2) + P(L_1 = 2, L_2 = 1) = 0.1536 \\
P(L_1 \cdot L_2 = 3) &= P(L_1 = 1, L_2 = 3) = 0.0356 \\
P(L_1 \cdot L_2 = 4) &= P(L_1 = 2, L_2 = 2) = 0.0004 \\
P(L_1 \cdot L_2 = 6) &= P(L_1 = 2, L_2 = 3) = 0.0288 \\
P(L_1 \cdot L_2 = 8) &= P(L_1 = 2, L_2 = 4) = 0.0064 \\
E(L_1 \cdot L_2) &= 1 \\
Cov(L_1, L_2) &= 1 - 0.6 \cdot 1.08 = 0.352
\end{aligned}
$$

3 Question 3

Find Var(X+Y), as a function of p, q and r.

$$
\begin{aligned}
Var(X + Y) &= E[(X + Y)^2] - [E(X + Y)]^2 \\
&= E(X^2 + Y^2 - 2XY) - EX^2 - EY^2 - 2EX \cdot EY \\
&= Var(X) - Var(Y) - 2EX \cdot EY - 2E(XY) \\
&= p(1 - p) - q(1 - q) - 2pq - 2r \\
&= p - q - p^2 + q^2 - 2pq - 2r
\end{aligned}
$$

4 Question 4

Find ED4

$$
\begin{aligned}
P(D_4 = 1) &= \frac{1}{2} \\
P(D_4 = 2) &= \frac{1}{2} \\
E(D_4) &= 1
\end{aligned}
$$