

## ECS 132 Homework#4

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### Problem 2

$$\begin{aligned}E(L_2) &= E(L_1 - A_2 + B_2) \\&= E(L_1) - E(A_2) + E(B_2) \\&= (0.4 \times 1 + 0.1 \times 2) \times 2 - E(A_2) \\E(A_2) &= \sum_c P(L_1 = c) E(A_2 | L_1 = c) \\&= 0 + 0.4 E(A_2 | L_1 = 1) + 0.1 E(A_2 | L_1 = 2) \\&= 0.4 \times 0.2 \times 1 + 0.1 \times (0.2 \times 0.8 \times \binom{2}{1} \times 1 + 0.2 \times 0.2 \times 2) \\&= 0.12 \\E(L_2) &= 1.2 - 0.12 = 1.08\end{aligned}$$

### Problem 3

#### Part a

First define the initial state matrix  $P$ :

$$P = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

Reasoning:

At state 1: There is equally chance ( $\frac{1}{3}$ ) to transfer to state 2,3,4.

At state 2: It is impossible to go to state 1 or 2. When next item weight is 1 or 3 (with probability  $\frac{2}{3}$ ), it will go to state 3, otherwise (next item weight is 2) it goes to state 4 (probability  $\frac{1}{3}$ ).

At state 3: If next item weight is 1 (probability  $\frac{1}{3}$ ), goes to state 4; If next item weight is 2 (probability  $\frac{1}{3}$ ), goes to state 2; If next item weight is 3 (probability  $\frac{1}{3}$ ), goes to state 3.

At state 4: There is equally chance ( $\frac{1}{3}$ ) to transfer to state 1,2,3.

Using R function **findp1** to solve, we can get the stationary distribution  $\pi$  of this chain:

$$\begin{aligned}\pi &= (0.08333333, 0.25000000, 0.41666667, 0.25000000) \\ &= \left(\frac{1}{12}, \frac{1}{4}, \frac{5}{12}, \frac{1}{4}\right)\end{aligned}$$

## Part b

Let the weight of the first item placed in each box is  $WF$ . Then:

$$\begin{aligned}E(WF) &= P(WF = 1) \times 1 + P(WF = 2) \times 2 + P(WF = 3) \times 3 \\ &= 0.25 \times \frac{1}{3} + (0.4166667 \times \frac{1}{3} + 0.25 \times \frac{1}{3}) \times 2 + (0.4166667 \times \frac{1}{3} + 0.25 \times \frac{1}{3} \times 2) \times 3 \\ &= 1.4444445\end{aligned}$$

Reasoning:

WF=1: This happens when current box weight is 4 and next item weight is 1 (probability is  $0.25 \times \frac{1}{3}$ );

WF=2: This happens when current box weight is either 3 or 4 and next item weight is 2 (probability is  $0.4166667 \times \frac{1}{3} + 0.25 \times \frac{1}{3}$ );

WF=3: This happens when current box weight is either 2, 3, or 4 and next item weight is 3 (probability is  $0.4166667 \times \frac{1}{3} + 0.25 \times \frac{1}{3} \times 2$ );