ECS 132 Winter 2016 Homework 3

Justin Jia, Ethan Wang, Ho Lun Sin, Yangzihao Wang

1 Math Problem A

a)

$$\begin{aligned} P(\text{5 defectives}) &= \binom{82}{5} \cdot 0.05^5 \cdot (1-0.05)^{77} \\ &= 27285336 \cdot 3.125 \cdot 10^5 \cdot 0.0192627 \\ &= 0.16424 \end{aligned}$$

b)

$$\begin{aligned} \lambda &= 82 \cdot 0.05 = 4.1 \\ k &= 5 \\ P &= \frac{e^{-4.1} \cdot 4.1^5}{5!} \end{aligned}$$

c)

$$\begin{aligned} P(D=3|X \leq 42) &= \frac{P(D=3 \text{ and } X \leq 42)}{P(X \leq 42)} \\ &= \frac{\sum_{n=1}^{42} \binom{n}{3} \cdot 0.05^3 \cdot (1-0.05)^{n-3}}{\sum_{n=1}^{42} e^{-50} \cdot \frac{50^n}{n!}} \\ &= 0.4643792984 \end{aligned}$$

2 Math Problem B

$$\begin{aligned} E((X-EX)^3) &= E((X-p)^3) \\ &= E(X^3 - p^3 + 3(p^2) \cdot x - 3p \cdot (x^2)) \\ &= E(X^3) - E(p^3) + E(3 \cdot (p^2) \cdot X) - E(3p \cdot X^2) \\ &= E(X) - p^3 + 3(p^2)E(X) - 3p \cdot E(X^2) \\ &= p - p^3 + 3p^3 - 3p^2 \\ &= 2p^3 - 3p^2 - p \end{aligned}$$

3 Math Problem C

a)

$$\begin{aligned}
&\text{Since}\\
g(E_1 + E_2 + E_3) &= g(E_1) \cdot g(E_2) \cdot g(E_3)\\
&\text{and}\\
g(E_1) = g(E_2) &= g(E_3) = b()\\
&\text{thus}\\
b() &= 0.5{\cdot}X^0 + 0.4 \cdot X^1 + 0.1 \cdot X^2\\
&\text{so}\\
P(X = 6) &= 0.001\\
P(X = 5) &= 0.012\\
P(X = 4) &= 0.063\\
P(X = 3) &= 0.184\\
P(X = 2) &= 0.315\\
P(X = 1) &= 0.1\\
P(X = 0) &= 0.125
\end{aligned}$$