ECS 132 Winter 2016 Homework 1

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1 Question 1

1) There are 3 items in Box 1.

$$\begin{aligned}
P(3 \text{ items in box } 1) &= P(W_1 = 1 \text{ and } W_2 = 1 \text{ and } W_3 = 1 \text{ and } W_4 \neq 1) \\
&\quad + P(W_1 = 2 \text{ and } W_2 = 1 \text{ and } W_3 = 1) \\
&\quad + P(W_1 = 1 \text{ and } W_2 = 2 \text{ and } W_3 = 1) \\
&\quad + P(W_1 = 1 \text{ and } W_2 = 1 \text{ and } W_3 = 2) \\
&= P(W_1 = 1 \text{ and } W_2 = 1 \text{ and } W_3 = 1 \text{ and } W_4 \neq 1) \\
&\quad + 3 \cdot P(W_a = 2 \text{ and } W_b = 1 \text{ and } W_c = 1) \\
&= (\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3}) + 3 \cdot (\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}) \\
&= 0.135802469
\end{aligned}$$

2) The total weight in Box 1 is under 4.

$$\begin{aligned}
P(\text{box 1 total weight} < 4) &= P(W_1 + W_2 + W_3 < 4 \text{ and } W_1 + W_2 + W_3 + W_4 > 4) \\
&\quad + P(W_1 + W_2 < 4 \text{ and } W_1 + W_2 + W_3 > 4) \\
&\quad + P(W_1 < 4 \text{ and } W_1 + W_2 > 4) \\
&= P(W_1 + W_2 + W_3 = 3) \cdot P(W_4 > 1) \\
&\quad + P(W_1 + W_2 = 3) \cdot P(W_3 > 1) + P(W_1 + W_2 = 2) \cdot P(W_3 > 2) \\
&\quad + P(W_1 = 3) \cdot P(W_2 > 1) + P(W_1 = 2) \cdot P(W_2 > 2) \\
&= (\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}) \cdot \frac{2}{3} + (2 \cdot \frac{1}{3} \cdot \frac{1}{3}) \cdot \frac{2}{3} + (\frac{1}{3} \cdot \frac{1}{3}) \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{3} \\
&= 0.543209877
\end{aligned}$$

3) The weight of the first item placed into Box 2 is 1.

$$\begin{aligned}
P(\text{weight of first item in } 2 = 1) &= P(W_n = 1 | W_{box1} = 4) \\
&= P(w_{box1} = 4 \text{ and } W_n = 1) \\
&= (1 - P(w_{box1} < 4)) \cdot P(W_n = 1) \\
&= (1 - 0.543209877)(\frac{1}{3}) \\
&= 0.152263374
\end{aligned}$$

4) Given that the weight of the first item placed into Box 2 is 1, the probability that the first item in Box 1 was 1.

$$
\begin{aligned}
P(W_{box1,1} = 1|W_{box2,1} = 1) &= P(W_1 = 1|W_{box1} = 4) \\
&= \frac{P(W_1 = 1 \text{ and } W_{box1} = 4)}{P(W_{box1} = 4)} \\
&= \frac{P(W_1 = 1 \text{ and } (W_2 + W_3 + W_4 = 3 \text{ or } W_2 + W_3 = 3 \text{ or } W_2 = 3))}{1 - P(w_{box1} < 4)} \\
&= \frac{\frac{1}{3} \cdot (\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3})}{1 - 0.543209877} \\
&= 0.432432433
\end{aligned}
$$

2 Question 2

1) Jack is at square 0.

$$
\begin{aligned}
P(\text{Jack at } 0) &= P(R_{jack} = 6) \\
&= P(R_1 = 6 or (R_1 = 1 and R_2 = 5)) \\
&= P(\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) \\
&= 0.194444444
\end{aligned}
$$

2) Jill has overtaken Jack.

$$
\begin{aligned}
P(\text{Jill overtakes}) &= P(R_{Jill} > R_{Jack} + 2) \\
&= P(R_{Jill} = 5 \text{ and } R_{Jack} = 2) + P(R_{Jill} = 6 \text{ and } R_{Jack} < 4) + P(R_{Jill} = 7 \text{ and } R_{Jack} < 5) \\
&\quad + P(R_{Jill} = 8 \text{ and } R_{Jack} < 6) + P(R_{Jill} = 9 \text{ and } R_{Jack} < 7) \\
&= (\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) \cdot (\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) \cdot 2(\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) \\
&\quad + \frac{1}{36} \cdot 3(\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) + \frac{1}{36} \cdot 4(\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) + \frac{1}{36} \cdot 5(\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) \\
&= 0.17824074
\end{aligned}
$$

3) Neither Jack nor Jill had a bonus roll, if we are told that he is at the same square as Jill.

$$
\begin{aligned}
P(\text{No bouns roll}) &= P(R_{Jill} \text{ and } R_{Jack} \text{ has no bonus} \mid R_{Jill} = R_{Jack} + 2) \\
&= \frac{P(R_{Jill} = R_{Jack} + 2, R_{Jill} \neq 3 \text{ and } R_{Jack} \neq 1)}{P(R_{Jill} = R_{Jack} + 2)} \\
&= \frac{P(R_{Jill1} = 4, R_{Jack1} = 2) + P(R_{Jill1} = 5, R_{Jack1} = 3) + P(R_{Jill1} = 6, R_{Jack1} = 4)}{3(\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) \cdot (\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) + 2(\frac{1}{6} \cdot \frac{1}{6}) \cdot (\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}) + \frac{1}{36} \cdot \frac{1}{36}} \\
&= \frac{\frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}}{\frac{1}{8}} \\
&= 0.666666667
\end{aligned}
$$