ECS 132 Homework#4

Bowei Jia, Ethan Wang, Alan Sin, Yangzihao Wang

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Problem 1

Part a

Extra Credit: We used fill = TRUE to ignore empty fields.

Part b

One user can review many films and one film can be reviewed by many users. It is a many-to-many relationship. By keeping user data and model data separate, and maintain another user-to-movie relationship table, they can keep the file size as small as possible without duplicating entities. That's why they use a relational database.

Problem 2

$$
\begin{aligned}
E(L_2) &= E(L_1 - A_2 + B_2) \\
&= E(L_1) - E(A_2) + E(B_2) \\
&= (0.4 \times 1 + 0.1 \times 2) \times 2 - E(A_2) \\
E(A_2) &= \sum_c P(L_1 = c) E(A_2 | L_1 = c) \\
&= 0 + 0.4E(A_2 | L_1 = 1) + 0.1E(A_2 | L_1 = 2) \\
&= 0.4 \times 0.2 \times 1 + 0.1 \times (0.2 \times 0.8 \times \binom{2}{1} \times 1 + 0.2 \times 0.2 \times 2) \\
&= 0.12 \\
E(L_2) &= 1.2 - 0.12 = 1.08
\end{aligned}
$$

Problem 3

Part a

First define the initial state matrix P:

$$P = \begin{array}{c@{}c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} & \\ \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

Reasoning:

At state 1: There is equally chance ($\frac{1}{3}$) to transfer to state 2,3,4.

At state 2: It is impossible to go to state 1 or 2. When next item weight is 1 or 3 (with probability $\frac{2}{3}$), it will go to state 3, otherwise (next item weight is 2) it goes to state 4 (probability $\frac{1}{3}$).

At state 3: If next item weight is 1 (probability $\frac{1}{3}$), goes to state 4; If next item weight is 2 (probability $\frac{1}{3}$), goes to state 2; If next item weight is 3 (probability $\frac{1}{3}$), goes to state 3.

At state 4: There is equally chance ($\frac{1}{3}$) to transfer to state 1,2,3.

Using R function **findp1** to solve, we can get the stationary distribution π of this chain:

$$\begin{aligned} \pi &= (0.08333333, 0.25000000, 0.41666667, 0.25000000) \\ &= (\frac{1}{12}, \frac{1}{4}, \frac{5}{12}, \frac{1}{4}) \end{aligned}$$

Part b

Let the weight of the first item placed in each box is WF. Then:

$$\begin{aligned} E(WF) &= P(WF = 1) \times 1 + P(WF = 2) \times 2 + P(WF = 3) \times 3 \\ &= 0.25 \times \frac{1}{3} + (0.4166667 \times \frac{1}{3} + 0.25 \times \frac{1}{3}) \times 2 + (0.4166667 \times \frac{1}{3} + 0.25 \times \frac{1}{3} \times 2) \times 3 \\ &= 1.4444445 \end{aligned}$$

Reasoning:

WF=1: This happens when current box weight is 4 and next item weight is 1 (probability is $0.25 \times \frac{1}{3}$);

WF=2: This happens when current box weight is either 3 or 4 and next item weight is 2 (probability is $0.4166667 \times \frac{1}{3} + 0.25 \times \frac{1}{3}$);

WF=3: This happens when current box weight is either 2, 3, or 4 and next item weight is 3 (probability is $0.4166667 \times \frac{1}{3} + 0.25 \times \frac{1}{3} \times 2$);