ECS 132 Final Course Project

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1. Problem A

We use users' average ratings as random samples to compute the approximate 95% confidence interval for: the population mean rating by men, by women, and the difference between the two means. We show the results in Table 3.

	From	To
approx. CI for E(ratings by men)	3.561174	3.616033
approx. CI for E(ratings by women)	3.556519	3.617838
approx. CI for E(rating diffs by gender)	-0.03971356	0.04256391

Table 1: Approximate 95% confidence interval for various means.

From Table 3 we observe that the center of approximate 95% confidence interval of the rating differences by gender is close to 0. Let the null hypothesis be that "the male and female population means are equal". We form a significance test of this hypothesis. We show the output of our significant test in the following code snapshot:

```
Two Sample t-test

data: a and b
t = 0.0446, df = 941, p-value = 0.9645
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.06134628  0.06419663
sample estimates:
mean of x mean of y
 3.588604  3.587179
```

The 0.9645 p-value shows no significant evidence to reject the null hypothesis, thus we should accept the hypothesis that the male and female population means are equal. Figure 1 further corroborates our conclusion.

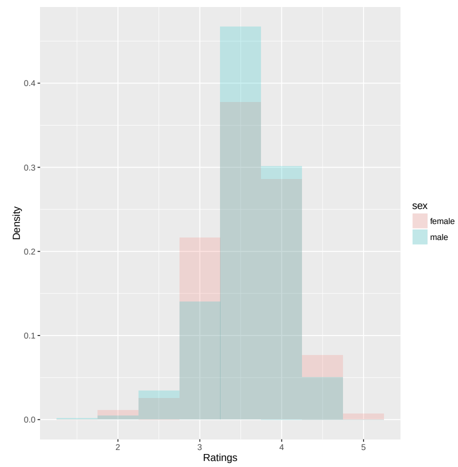


Figure 1: Histograms of the male and female ratings.

We show the approximate 95% confidence interval for the difference between the population mean number of ratings by men and women, as well as the approximate 95% confidence interval for the population proportion of users who are men in Table 2

	From	To
approx. CI for E(rating number diffs by gender)	7.505438	25.59478
approx. CI for proportion of male users	0.6815512	0.7394457

Table 2: Approximate 95% confidence interval for mean diff and proportion.

We use a linear model to estimate the population regression function in which we predict rating from age and gender. The summary output is shown in the following code snapshot:

```
Call:
lm(formula = data$avg.rating ~ data$age + data$gender)

Residuals:
     Min       1Q   Median       3Q      Max
-2.06903 -0.25972  0.03078  0.27967  1.34615
```

```
Coefficients:
              Estimate  Std. Error  t value  Pr(>|t|)
(Intercept)  3.4725821  0.0482655   71.947   < 2e-16 ***
data$age     0.0033891  0.0011860    2.858   0.00436 **
data$gender  0.0002862  0.0318670    0.009   0.99284
---
Signif. codes:  0  ***  0.001  **  0.01  *  0.05
  .  0.1      1

Residual standard error: 0.4438 on 940 degrees of freedom
Multiple R-squared:  0.008615,  Adjusted R-squared:  0.006505
F-statistic: 4.084 on 2 and 940 DF,  p-value: 0.01714
```

The confidence interval of β_1 (age coefficient) is from 0.001064581 to 0.01714. From the output of $summary(lmout)$ we observe that its p-value is 0.00436, which means the difference between null hypothesis (β_1 is 0) and our sample results is significant enough for us to reject the null hypothesis.

The approximate 95% confidence interval for population mean rating of women of age 28 is from 3.513127 to 3.621827. We show the visualization of our linear model in Figure 2.

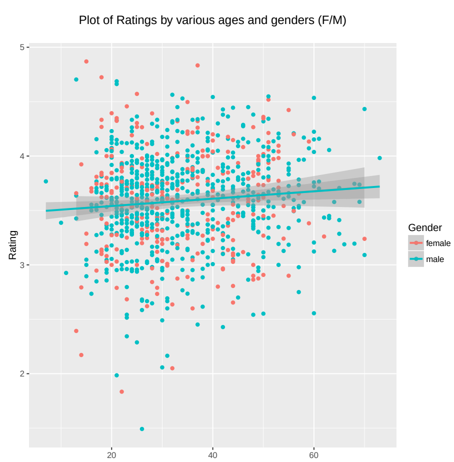
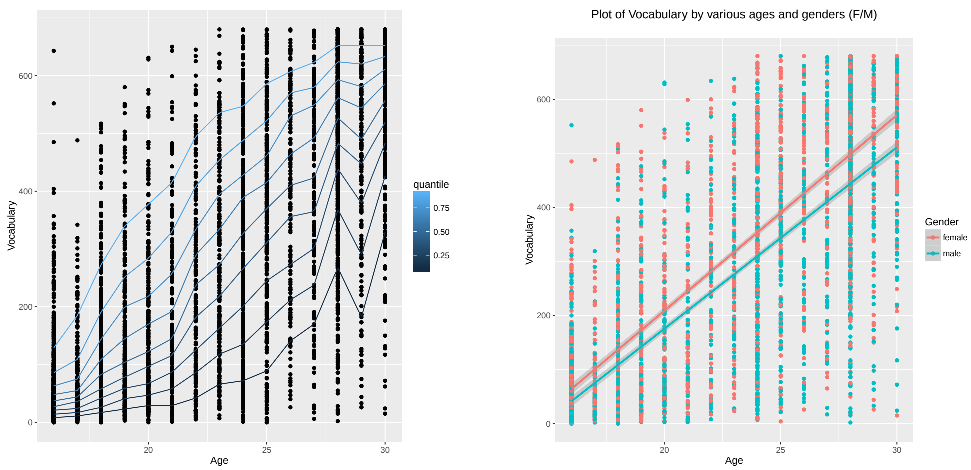


Figure 2: Linear model of ratings and age and gender.



(a) Quantile graph of vocabulary changes over age increase.

(b) Linear model of ratings and age and gender.

2. Problem B

In this section we analyze the Wordbank dataset. More specifically, we form a linear regression model from a description point of view, by showing the relationship between vocabulary size and age (in months), birth order, ethnicity, sex and mon's education. Figure 3a shows the quantile of vocabulary size at different age. We observe a clear direct proportional relationship between age and vocabulary size. We also observe such relationship in Figure 3b, which shows the linear regression model of vocabulary size with age. It also reveals that female children have larger mean vocabulary size than male children, at all ages, regardless of other factors.

Category	Sample Size
Gender	Male: 2091 Female: 1981
Race	Asian: 71 Hispanic: 133 Black: 231 White: 2228 Other: 100
Education	Graduate: 577 Some Graduate: 157 College: 850 Some College: 601 Secondary: 433 Some Secondary: 128 Primary: 8
Birth Order	First: 1423 Second: 920 Third: 284 Fourth: 92 Fifth: 20 Sixth: 10 Seventh: 4 Eighth: 1

Table 3: Sample size in terms of gender and race.

To find the more detailed relationship, we perform the linear regression prediction for vocabulary size using all the variables, including age, gender, ethnicity, birth order, and mother's education level. In our first linear model, we convert all the categorical variables (gender, ethnicity, birth order, and mother's education level) into dummy variables, and use R's lm function to achieve our model. By making all the variables dummy variables, we can obtain the first model with Multiple R-squared value being 0.5151. From the model, age and gender has the lowest P values which are less than 2e-16. Small P value shows that it is more likely to fit the data. Each unit increase in age (one month) will cause 33.3164 increase in the mean value of vocabulary size. In general, male children will have a 48.6006 decrease of mean value of vocabulary size. Followed by age and gender, comes the factor of being white, or having a mother with secondary education, or college education. However, the P value for those variables are significantly larger which are around 0.0005. Thus the results shows that both birth order and graduate education level has no impact on the vocabulary size.

In order to improve the model, only age and gender were used as predictors. As the model turns out, it has a Multiple R-squared value being 0.6095 which shows that the model is better than the previous one with a Multiple R-squared value being 0.515. However, with the sample size being 5498, a maximum size of 74 predictor variables can be used, so more tests were done to see if a better model can be obtained.

A further analysis of linear model of vocabulary size using age and gender, only race, only education level, and only birth order, further proves that birth order has no impact on the vocabulary size. Specifically, the race factor, as shown in the third model with a multiple R-squared being 0.005969 which is very small. As a result, the race factor is not selected as a predictor. On the other hand, we observe the factor of being white, which has a small p value being 0.0086, and discover that the sample population is over 2000. As the sample size grows, any data can become significant, so a small p-value does not make being white or not a good predictor. Then with a model formed around the order of the child's birth, a multiple R-squared value of 0.01739 is obtained which is still rather small. For education level, it has even smaller adjusted R-squared value than birth order (0.004584 vs. 0.01488). Moreover, every single factor in this model has a P-value over 0.1 which showed that this model is not a good fit either. The linear model with all the above factors excluded and uses only age and gender achieves a highest adjusted R-squared value of 0.6093.

Based on the above reasoning, we finalize our linear model with only gender and age. The linear model is presented as follows:

$$\begin{aligned}
\bar{V} &= b_0 + b_1 Age + b_2 IsMale \\
b_0 &= -484.1307 \\
b_1 &= 34.8168 \\
b_2 &= -39.6868 \\
IsMale &= \{1, 0\}
\end{aligned}$$

We conclude that age (in month) significantly predicted positive vocabulary size, $b_1 = 34.8168$, $t(4096) = 79.239$, $p < 2e - 16$; gender (whether the child is male or not) significantly predicted negative vocabulary size, $b_2 = -39.6868$, $t(4096) = -9.208$, $p < 2e-16$. Together age and gender explained a large portion of variance in vocabulary sizes, $R^2 = .6093, F(2, 4096) = 3176, p < 2.2e - 16$. One month of age growth will contribute to 34.8168 increase of vocabulary size, being a male child will contribute to 39.6868 decrease of vocabulary size.

Appendix: Linear Model Outputs for Problem B

```
## Vocab ~ Age + Gender + Race + Education + Birth Order

Residuals:
    Min      1Q  Median      3Q     Max
-464.46  -96.28   -4.51   94.00  471.77

Coefficients: (2 not defined because of singularities)
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)           -583.1965   145.0625  -4.020 5.97e-05 ***
x$age                   33.3164     0.6432  51.800  < 2e-16 ***
x$is_male              -48.6006     5.4930  -8.848  < 2e-16 ***
x$is_asian              38.3629    22.7837   1.684 0.092338 .
x$is_black              54.0626    17.3605   3.114 0.001864 **
x$is_hispanic          -23.1461    19.1766  -1.207 0.227539
x$is_white              53.0242    15.3290   3.459 0.000550 ***
x$is_primary          -106.3800    51.5485  -2.064 0.039142 *
```

```
x$is_some_secondary  -49.9880     15.0552   -3.320 0.000911 ***
x$is_secondary       -21.4568      9.3609   -2.292 0.021971 *
x$is_some_college    -29.5289      8.4764   -3.484 0.000502 ***
x$is_college         -17.9607      7.7583   -2.315 0.020686 *
x$is_some_graduate    -8.1601     12.9772   -0.629 0.529532
x$is_graduate              NA          NA       NA       NA
x$is_first          115.7452    143.7341    0.805 0.420733
x$is_second          90.1358    143.7723    0.627 0.530755
x$is_third           66.9958    143.9505    0.465 0.641676
x$is_fourth          81.6032    144.4836    0.565 0.572262
x$is_fifth           40.9479    147.1721    0.278 0.780856
x$is_sixth           -2.1739    150.7379   -0.014 0.988494
x$is_seventh        -63.5342    160.6141   -0.396 0.692453
x$is_eighth                NA          NA       NA       NA
---
Signif. codes:  0 *** 0.001 ** 0.01 *
0.05 . 0.1   1

Residual standard error: 143.2 on 2721 degrees of freedom
  (2757 observations deleted due to missingness)
Multiple R-squared:  0.5151,    Adjusted R-squared:  0.5117
F-statistic: 152.1 on 19 and 2721 DF,  p-value: < 2.2e-16

## Age + Gender

Call:
lm(formula = x$vocab ~ x$age + x$is_male)

Residuals:
    Min      1Q  Median      3Q     Max
-545.37  -76.50   -5.42   91.22  518.75

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -484.1307    10.3192 -46.915   <2e-16 ***
x$age         34.8168     0.4394  79.239   <2e-16 ***
x$is_male    -39.6868     4.3099  -9.208   <2e-16 ***
---
```

```
Signif. codes:  0  ***  0.001  **  0.01  *
0.05  .  0.1     1

Residual standard error: 137.5 on 4069 degrees of freedom
  (1426 observations deleted due to missingness)
Multiple R-squared:  0.6095,    Adjusted R-squared:  0.6093
F-statistic:  3176 on 2 and 4069 DF,  p-value: < 2.2e-16

## Race

Residuals:
    Min      1Q  Median      3Q     Max
-280.32 -191.63  -33.32  178.68  446.58

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    225.420     20.424  11.037   <2e-16 ***
x$is_asian      38.524     31.697   1.215   0.2243
x$is_black      58.329     24.449   2.386   0.0171 *
x$is_hispanic   -2.172     27.033  -0.080   0.9360
x$is_white      54.900     20.878   2.630   0.0086 **
---
Signif. codes:  0  ***  0.001  **  0.01  *
0.05  .  0.1     1

Residual standard error: 204.2 on 2758 degrees of freedom
  (2735 observations deleted due to missingness)
Multiple R-squared:  0.005969,  Adjusted R-squared:  0.004527
F-statistic:  4.14 on 4 and 2758 DF,  p-value: 0.002397

## Education

Residuals:
    Min      1Q  Median      3Q     Max
-287.14 -190.48  -36.48  178.78  433.95

Coefficients: (1 not defined because of singularities)
                          Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)             289.1352     8.5078   33.985  < 2e-16 ***
x$is_primary           -144.6352    72.7530   -1.988 0.046907 *
x$is_some_secondary     -66.0883    19.9668   -3.310 0.000945 ***
x$is_secondary           -2.9366    12.9938   -0.226 0.821219
x$is_some_college       -26.6593    11.9111   -2.238 0.025289 *
x$is_college            -12.6599    11.0235   -1.148 0.250886
x$is_some_graduate        0.4126    18.3957    0.022 0.982108
x$is_graduate                 NA         NA       NA       NA
---
Signif. codes:  0   ***   0.001   **   0.01   *
0.05    .    0.1        1

Residual standard error: 204.4 on 2747 degrees of freedom
  (2744 observations deleted due to missingness)
Multiple R-squared:  0.006753,  Adjusted R-squared:  0.004584
F-statistic: 3.113 on 6 and 2747 DF,  p-value: 0.004842

## Birth Order

Residuals:
    Min      1Q  Median      3Q     Max
-296.44 -189.49  -33.92  175.31  454.51

Coefficients: (1 not defined because of singularities)
             Estimate Std. Error t value Pr(>|t|)
(Intercept)       4.0      203.2   0.020    0.984
x$is_first      292.4      203.3   1.438    0.150
x$is_second     260.0      203.3   1.279    0.201
x$is_third      218.5      203.6   1.073    0.283
x$is_fourth     248.7      204.3   1.217    0.224
x$is_fifth      198.9      208.2   0.955    0.339
x$is_sixth      195.4      213.1   0.917    0.359
x$is_seventh     59.5      227.2   0.262    0.793
x$is_eighth        NA         NA      NA       NA

Residual standard error: 203.2 on 2746 degrees of freedom
  (2744 observations deleted due to missingness)
Multiple R-squared:  0.01739,   Adjusted R-squared:  0.01488
```

```
F-statistic: 6.942 on 7 and 2746 DF,  p-value: 3.238e-08
```