

$$\frac{1}{3} + C_2 = -x + C_1$$

$$\Rightarrow y = \ln(\sec(x+c)) - \frac{x^2}{2} + C_1$$

MATH 383L

(A) $y'' - ty = 0, y(0) = 1, y'(0) = 0$

(1) $x_1 = y, x_2 = y'$

(2) $x_2' - tx_1 = 0$

(3) $x_2' = tx_1, x_1' = x_2$

(4) $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ t & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ IC $\rightarrow y(0) = 1, y'(0) = 0$

(B) $2y'' - 17y' + 5y = 0, y(0) = 1, y'(0) = -\frac{1}{2}$

(1) $x_1 = y, x_2 = y'$

(2) $2x_2' - 17x_2 + 5x_1 = 0$

(3) $2x_2' = 17x_2 - 5x_1 \rightarrow x_2' = \frac{17}{2}x_2 - \frac{5}{2}x_1$

$x_1' = x_2$

(4) $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{5}{2} & \frac{17}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ IC $\rightarrow y(0) = 1, y'(0) = -\frac{1}{2}$

(C) $t^2 y'' - 2ty' + 5y = 0, y(0) = 0, y'(0) = 1$

(1) $x_1 = y, x_2 = y'$

(2) $t^2 x_2' - 2tx_2 + 5x_1 = 0$

(3) $t^2 x_2' = 2tx_2 - 5x_1 \rightarrow x_2' = \frac{2}{t}x_2 - \frac{5}{t^2}x_1$ and $x_1' = x_2$

(4) $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{5}{t^2} & \frac{2}{t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ IC $\rightarrow y(0) = 0, y'(0) = 1$

(D) $y'' + 12ty'' - 6y' = t^2 - t, y(0) = 0$

(1) $x_1 = y, x_2 = y', x_3 = y''$

(2) $x_3' + 12tx_3 - 6x_2 = t^2 - t$

(3) $x_3' = t^2 - t + 6x_2 - 12tx_3$

$x_2' = x_3$ and $x_1' = x_2$

(4) $\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 6 & -12t \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ t^2 - t \end{bmatrix}$ IC $\rightarrow y(0) = 0, y'(0) = 0, y''(0) = \pi$