COMP SCI 2201-7201 Algorithm and Data Structure Analysis

Asymptotic Analysis

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- ***Asymptotic analysis* or *Asymptotics* is the calculus of approximations**
  - Approximation of functions by simpler functions

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- ***Asymptotic analysis*** **or** ***Asymptotics*** **is the calculus of approximations**
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- **The term "asymptotic" can also be used more broadly to describe situations where two things approach each other or become more similar, but never quite reach a point of perfect equality.**

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Example

$$f(n) = n^2 + 4n + 7 \qquad\qquad g(n) = n^2$$

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- **All the below statements mean the same**

$$f(n) \text{ is ''asymptotically similar'' to } g(n) \text{ as } n \to \infty$$

$$f(n) \text{ is ''asymptotically same'' to } g(n) \text{ as } n \to \infty$$

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$$f(n) \sim n^2$$

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$$f(n) \text{ is "asymptotic" to } g(n) \text{ as } n \to \infty$$

$$f(n) \sim n^2$$

$\sim$ is an asymptotic notation used to convey the message that a function is asymptotically similar/same/equivalent/equal to another function

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Asymptotic Notations

- **A way to communicate the relationship between the behaviors of different functions**
  - Usually, the relationship between the *growth rates* of different functions (in computer science)

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Asymptotic Notations (Relation formulas)

$\mathbf{f(n) = O(g(n))}$ $\exists c > 0, \exists n_0 > 0$ such that $\forall n \geq n_0$ $|\mathbf{f(n)}| \leq \mathbf{cg(n)}$ Asymptotic upper bound

$\mathbf{f(n) = \Omega(g(n))}$ $\exists c > 0, \exists n_0 > 0$ such that $\forall n \geq n_0$ $|\mathbf{f(n)}| \geq \mathbf{cg(n)}$ Asymptotic lower bound

$\mathbf{f(n) = \Theta(g(n))}$ $\exists c_1, c_2 > 0, \exists n_0 > 0$ such that $\forall n \geq n_0$ $\mathbf{c_1 g(n)} \leq |\mathbf{f(n)}| \leq \mathbf{c_2 g(n)}$ Asymptotic tight bound

$\mathbf{f(n) = o(g(n))}$ $\forall \mathbf{c} > \mathbf{0}, \exists n_0 > 0$ such that $\forall n \geq n_0$ $|\mathbf{f(n)}| \leq \mathbf{cg(n)}$

upper bound that is not asymptotically tight

$\mathbf{f(n)} = \omega(\mathbf{g(n)})$ $\forall \mathbf{c} > \mathbf{0}, \exists n_0 > 0$ such that $\forall n \geq n_0$ $|\mathbf{f(n)}| \geq \mathbf{cg(n)}$

lower bound that is not asymptotically tight

$\mathbf{f(n)} \sim \mathbf{g(n)}$ $\lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} = 1$

$f(n) = O(g(n))$ can also be written as $f(n) \in O(g(n))$

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Asymptotic Notations (Limits)

$$\mathbf{f(n) = O(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq \infty$$

$$\mathbf{f(n) = \Omega(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq 0$$

$$\mathbf{f(n) = \Theta(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq 0, \infty$$

$$\mathbf{f(n) = o(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} = 0$$

$$\mathbf{f(n)} = \omega(\mathbf{g(n)}) \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} = \infty$$

$$\mathbf{f(n)} \sim \mathbf{g(n)} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} = 1$$

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How to perform asymptotic analysis

- **Identify the dominant term** $f(n) = 3n^3 + 2n^2 + 5n + 7$

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- **Drop the lower order terms** $$3n^3$$

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- **Drop the constant factors** $$n^3$$

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- **Identify the dominant term** $$f(n) = \underline{3n^3} + 2n^2 + 5n + 7$$

- **Drop the lower order terms** $$3n^3$$

- **Drop the constant factors** $$n^3$$ Let's call it $g(n) = n^3$

How to perform asymptotic analysis

- **Identify the dominant term** $f(n) = \underline{3n^3} + 2n^2 + 5n + 7$

- **Drop the lower order terms** $3n^3$

- **Drop the constant factors** n^3 Let's call it $g(n) = n^3$

- **Identify the asymptotic notations**

  Analyze $f(n)$ and $g(n)$ using relation or limits formulas

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How to perform asymptotic analysis

1. Using relation formulas

$$f(n) = 3n^3 + 2n^2 + 5n + 7 \qquad g(n) = n^3$$

$$\mathbf{f(n) = O(g(n))} \quad \exists c > 0, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad \mathbf{|f(n)| \leq cg(n)}$$

$$\mathbf{f(n) = \Omega(g(n))} \quad \exists c > 0, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad \mathbf{|f(n)| \geq cg(n)}$$

$$\mathbf{f(n) = \Theta(g(n))} \quad \exists c_1, c_2 > 0, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad \mathbf{c_1 g(n) \leq |f(n)| \leq c_2 g(n)}$$

$$\mathbf{f(n) = o(g(n))} \quad \forall \mathbf{c > 0}, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad \mathbf{|f(n)| \leq cg(n)}$$

$$\mathbf{f(n)} = \omega\mathbf{(g(n))} \quad \forall \mathbf{c > 0}, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad \mathbf{|f(n)| \geq cg(n)}$$

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How to perform asymptotic analysis

1. Using relation formulas

$$f(n) = 3n^3 + 2n^2 + 5n + 7 \qquad g(n) = n^3$$

$$|\mathbf{f(n)}| \le \mathbf{cg(n)} \quad \text{for a } \mathbf{c > 0} \text{ and } \mathbf{n_0 > 0}$$

$$\mathbf{f(n) = O(g(n))} \quad \exists c > 0, \exists n_0 > 0 \text{ such that } \forall n \ge n_0 \quad |\mathbf{f(n)}| \le \mathbf{cg(n)}$$

$$\mathbf{f(n) = \Omega(g(n))} \quad \exists c > 0, \exists n_0 > 0 \text{ such that } \forall n \ge n_0 \quad |\mathbf{f(n)}| \ge \mathbf{cg(n)}$$

$$\mathbf{f(n) = \Theta(g(n))} \quad \exists c_1, c_2 > 0, \exists n_0 > 0 \text{ such that } \forall n \ge n_0 \quad \mathbf{c_1 g(n)} \le |\mathbf{f(n)}| \le \mathbf{c_2 g(n)}$$

$$\mathbf{f(n) = o(g(n))} \quad \forall \mathbf{c > 0}, \exists n_0 > 0 \text{ such that } \forall n \ge n_0 \quad |\mathbf{f(n)}| \le \mathbf{cg(n)}$$

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$$\mathbf{f(n) = o(g(n))} \quad \forall \mathbf{c > 0}, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad \mathbf{|f(n)| \leq cg(n)}$$

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$$\mathbf{|f(n)| \leq cg(n)} \quad \text{for a } \mathbf{c > 0} \text{ and } \mathbf{n_0 > 0}$$

$$3n^3 + 2n^2 + 5n + 7 \leq 17n^3, \quad c = 17, n_0 = 1$$

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How to perform asymptotic analysis

1. Using relation formulas

$$f(n) = 3n^3 + 2n^2 + 5n + 7 \qquad g(n) = n^3$$

$|\mathbf{f(n)}| \leq \mathbf{cg(n)}$ for a $\mathbf{c > 0}$ and $\mathbf{n_0 > 0}$

$3n^3 + 2n^2 + 5n + 7 \leq 17n^3, \quad c = 17, n_0 = 1$

$\mathbf{f(n) = O(n^3)}$ YES

$$\mathbf{f(n) = O(g(n))} \quad \exists c > 0, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad |\mathbf{f(n)}| \leq \mathbf{cg(n)}$$

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$\mathbf{|f(n)| \leq cg(n)}$ for a $\mathbf{c > 0}$ and $\mathbf{n_0 > 0}$

$$3n^3 + 2n^2 + 5n + 7 \leq 17n^3, \qquad c = 17, n_0 = 1$$

$\mathbf{f(n) = O(n^3)}$ YES

$\mathbf{|f(n)| \geq cg(n)}$ for a $\mathbf{c > 0}$ and $\mathbf{n_0 > 0}$

$$3n^3 + 2n^2 + 5n + 7 \geq 3n^3, \qquad c = 3, n_0 = 1$$

$\mathbf{f(n) = \Omega(n^3)}$ YES

How to perform asymptotic analysis

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$\mathbf{|f(n)| \geq cg(n)}$ for a $\mathbf{c > 0}$ and $\mathbf{n_0 > 0}$

$3n^3 + 2n^2 + 5n + 7 \geq 3n^3, \quad c = 3, n_0 = 1$

$\mathbf{f(n) = \Omega(n^3)}$ YES

We have already found two c's above $c_1 = 3, c_2 = 17$

$3n^3 \leq f(n) \leq 17n^3$

$\mathbf{f(n) = \Theta(n^3)}$ YES

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How to perform asymptotic analysis

1. Using relation formulas

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$$\mathbf{f(n) = \Theta(g(n))} \quad \exists c_1, c_2 > 0, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad \mathbf{c_1 g(n) \leq |f(n)| \leq c_2 g(n)}$$
$$\mathbf{f(n) = o(g(n))} \quad \forall \mathbf{c} > \mathbf{0}, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad \mathbf{|f(n)| \leq cg(n)}$$
$$\mathbf{f(n)} = \omega\mathbf{(g(n))} \quad \forall \mathbf{c} > \mathbf{0}, \exists n_0 > 0 \text{ such that } \forall n \geq n_0 \quad \mathbf{|f(n)| \geq cg(n)}$$

$\mathbf{|f(n)| \leq cg(n)}$ for a $\mathbf{c > 0}$ and $\mathbf{n_0 > 0}$

$3n^3 + 2n^2 + 5n + 7 \leq 17n^3, \quad c = 17, n_0 = 1$

$\mathbf{f(n) = O(n^3)}$ YES

$\mathbf{|f(n)| \geq cg(n)}$ for a $\mathbf{c > 0}$ and $\mathbf{n_0 > 0}$

$3n^3 + 2n^2 + 5n + 7 \geq 3n^3, \quad c = 3, n_0 = 1$

$\mathbf{f(n) = \Omega(n^3)}$ YES

We have already found two c's above $c_1 = 3, c_2 = 17$

$3n^3 \leq f(n) \leq 17n^3$

$\mathbf{f(n) = \Theta(n^3)}$ YES

$\mathbf{|f(n)| \leq cg(n)} \quad \forall \mathbf{c > 0}$

$3n^3 + 2n^2 + 5n + 7 \leq cn^3$, not for all $c > 0$

$\mathbf{f(n) = o(n^3)}$ NO

$\mathbf{|f(n)| \geq cg(n)} \quad \forall \mathbf{c > 0}$

$3n^3 + 2n^2 + 5n + 7 \geq cn^3$, not for all $c > 0$

$\mathbf{f(n)} = \omega\mathbf{(n^3)}$ NO

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$\mathbf{f(n) = O(g(n))}$ $\exists c > 0, \exists n_0 > 0$ such that $\forall n \geq n_0$ $\mathbf{|f(n)| \leq cg(n)}$

$\mathbf{f(n) = \Omega(g(n))}$ $\exists c > 0, \exists n_0 > 0$ such that $\forall n \geq n_0$ $\mathbf{|f(n)| \geq cg(n)}$

$\mathbf{f(n) = \Theta(g(n))}$ $\exists c_1, c_2 > 0, \exists n_0 > 0$ such that $\forall n \geq n_0$ $\mathbf{c_1 g(n) \leq |f(n)| \leq c_2 g(n)}$

$\mathbf{f(n) = o(g(n))}$ $\mathbf{\forall c > 0}, \exists n_0 > 0$ such that $\forall n \geq n_0$ $\mathbf{|f(n)| \leq cg(n)}$

$\mathbf{f(n)} = \omega\mathbf{(g(n))}$ $\mathbf{\forall c > 0}, \exists n_0 > 0$ such that $\forall n \geq n_0$ $\mathbf{|f(n)| \geq cg(n)}$

$\mathbf{|f(n)| \leq cg(n)}$ for a $\mathbf{c > 0}$ and $\mathbf{n_0 > 0}$

$3n^3 + 2n^2 + 5n + 7 \leq 17n^3, \quad c = 17, n_0 = 1$

$\mathbf{f(n) = O(n^3)}$ YES

$\mathbf{|f(n)| \geq cg(n)}$ for a $\mathbf{c > 0}$ and $\mathbf{n_0 > 0}$

$3n^3 + 2n^2 + 5n + 7 \geq 3n^3, \quad c = 3, n_0 = 1$

$\mathbf{f(n) = \Omega(n^3)}$ YES

We have already found two c's above $c_1 = 3, c_2 = 17$

$$3n^3 \leq f(n) \leq 17n^3$$

$\mathbf{f(n) = \Theta(n^3)}$ YES

$\mathbf{|f(n)| \leq cg(n)}$ $\mathbf{\forall c > 0}$

$3n^3 + 2n^2 + 5n + 7 \leq cn^3$, not for all $c > 0$

$\mathbf{f(n) = o(n^3)}$ NO

$\mathbf{|f(n)| \geq cg(n)}$ $\mathbf{\forall c > 0}$

$3n^3 + 2n^2 + 5n + 7 \geq cn^3$, not for all $c > 0$

$\mathbf{f(n)} = \omega\mathbf{(n^3)}$ NO

If a function $f(n)$ is both $O(g(n))$ and $\Omega(g(n))$, then $f(n)$ is also $\Theta(g(n))$

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How to perform asymptotic analysis

1. Using limits formulas

$$f(n) = 3n^3 + 2n^2 + 5n + 7 \qquad g(n) = n^3$$

$\mathbf{f(n) = O(g(n))}$ $\quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq \infty$

$\mathbf{f(n) = \Omega(g(n))}$ $\quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq 0$

$\mathbf{f(n) = \Theta(g(n))}$ $\quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq 0, \infty$

$\mathbf{f(n) = o(g(n))}$ $\quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} = 0$

$\mathbf{f(n)} = \omega(\mathbf{g(n)})$ $\quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} = \infty$

$\mathbf{f(n) \sim g(n)}$ $\quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} = 1$

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$$f(n) = 3n^3 + 2n^2 + 5n + 7 \qquad g(n) = n^3$$

$$\lim_{n\to\infty} \frac{3n^3+2n^2+5n+7}{n^3}$$

How to perform asymptotic analysis

1. Using limits formulas

$$f(n) = 3n^3 + 2n^2 + 5n + 7 \qquad g(n) = n^3$$

$$\lim_{n\to\infty} \frac{3n^3+2n^2+5n+7}{n^3} = \frac{\infty}{\infty} \qquad \text{indeterminate form}$$

$$\mathbf{f(n) = O(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq \infty$$

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1. Using limits formulas

$$f(n) = 3n^3 + 2n^2 + 5n + 7 \qquad g(n) = n^3$$

$$\lim_{n\to\infty} \frac{3n^3+2n^2+5n+7}{n^3} = \frac{\infty}{\infty}$$ indeterminate form

To get out of indeterminate situation, take the first derivate and then apply the limits

Keep taking the derivative until you get out of the indeterminate situation

$$\mathbf{f(n) = O(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq \infty$$

$$\mathbf{f(n) = \Omega(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq 0$$

$$\mathbf{f(n) = \Theta(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq 0, \infty$$

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$$\mathbf{f(n)} \sim \mathbf{g(n)} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} = 1$$

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How to perform asymptotic analysis

1. Using limits formulas

$$\mathbf{f(n) = O(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq \infty$$

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$$f(n) = 3n^3 + 2n^2 + 5n + 7 \qquad g(n) = n^3$$

$$\lim_{n\to\infty} \frac{3n^3+2n^2+5n+7}{n^3} = \frac{\infty}{\infty} \qquad \text{indeterminate form}$$

$$\left.\begin{array}{l}\text{To get out of indeterminate situation, take} \\ \text{the first derivate and then apply the limits} \\ \text{Keep taking the derivative until} \\ \text{you get out of the indeterminate situation}\end{array}\right\} \text{L'Hôpital's rule for } \frac{\infty}{\infty} \text{ and } \frac{0}{0} \text{ indeterminate forms}$$

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How to perform asymptotic analysis

1. Using limits formulas

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To get out of indeterminate situation, take the first derivate and then apply the limits
Keep taking the derivative until you get out of the indeterminate situation

L'Hôpital's rule for $\frac{\infty}{\infty}$ and $\frac{0}{0}$ indeterminate forms

$$\lim_{n\to\infty} \frac{9n^2+4n+5}{3n^2}$$

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How to perform asymptotic analysis

1. Using limits formulas

$\mathbf{f(n) = O(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq \infty$

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To get out of indeterminate situation, take the first derivate and then apply the limits
Keep taking the derivative until you get out of the indeterminate situation

L'Hôpital's rule for $\frac{\infty}{\infty}$ and $\frac{0}{0}$ indeterminate forms

$$\lim_{n\to\infty} \frac{9n^2+4n+5}{3n^2}$$

$$\lim_{n\to\infty} \frac{18n+4}{6n}$$

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How to perform asymptotic analysis

1. Using limits formulas

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L'Hôpital's rule for $\frac{\infty}{\infty}$ and $\frac{0}{0}$ indeterminate forms

$$\lim_{n\to\infty} \frac{9n^2+4n+5}{3n^2}$$

$$\lim_{n\to\infty} \frac{18n+4}{6n}$$

$$\lim_{n\to\infty} \frac{18}{6} = 3$$

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How to perform asymptotic analysis

1. Using limits formulas

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$\lim_{n\to\infty} \frac{9n^2+4n+5}{3n^2}$

$\mathbf{f(n) = O(n^3)}$ YES

$\lim_{n\to\infty} \frac{18n+4}{6n}$

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$\mathbf{f(n) = \Omega(n^3)}$ YES

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$\mathbf{f(n) = O(n^3)}$ YES

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$\mathbf{f(n) = \Theta(n^3)}$ YES

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How to perform asymptotic analysis

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$\mathbf{f(n)} = \mathbf{o(n^3)}$ NO

$\mathbf{f(n)} = \omega\mathbf{(n^3)}$ NO

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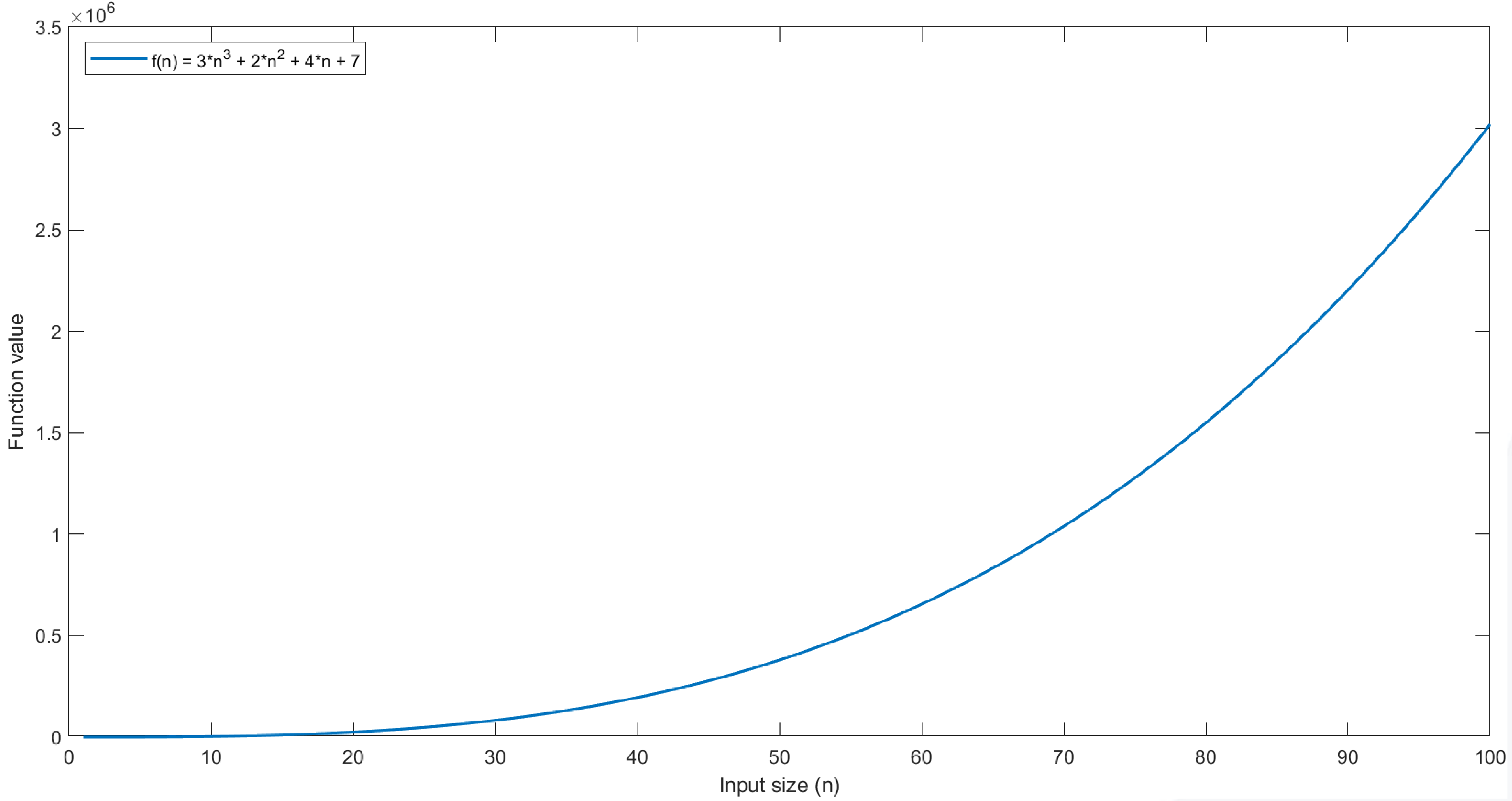
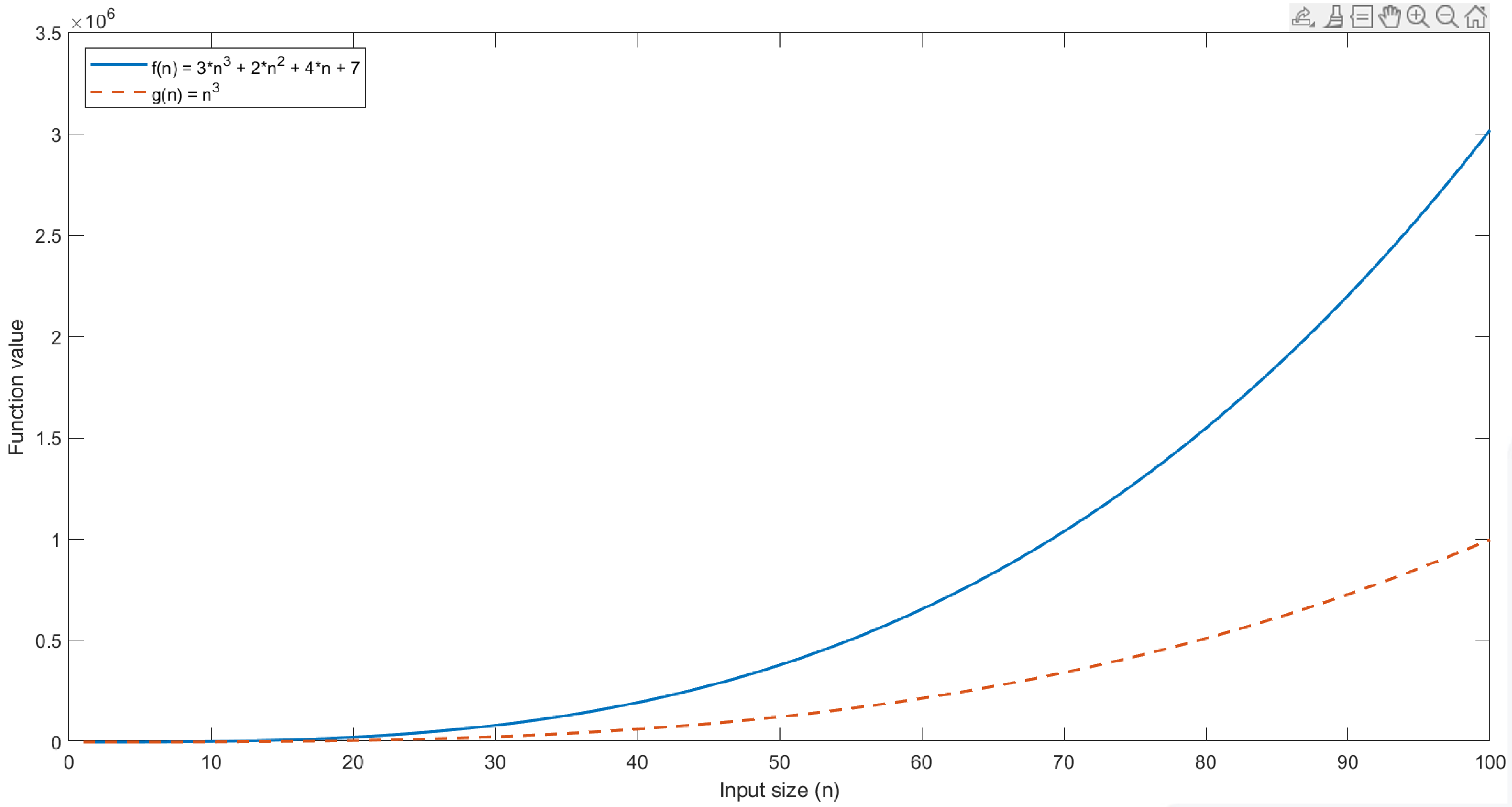
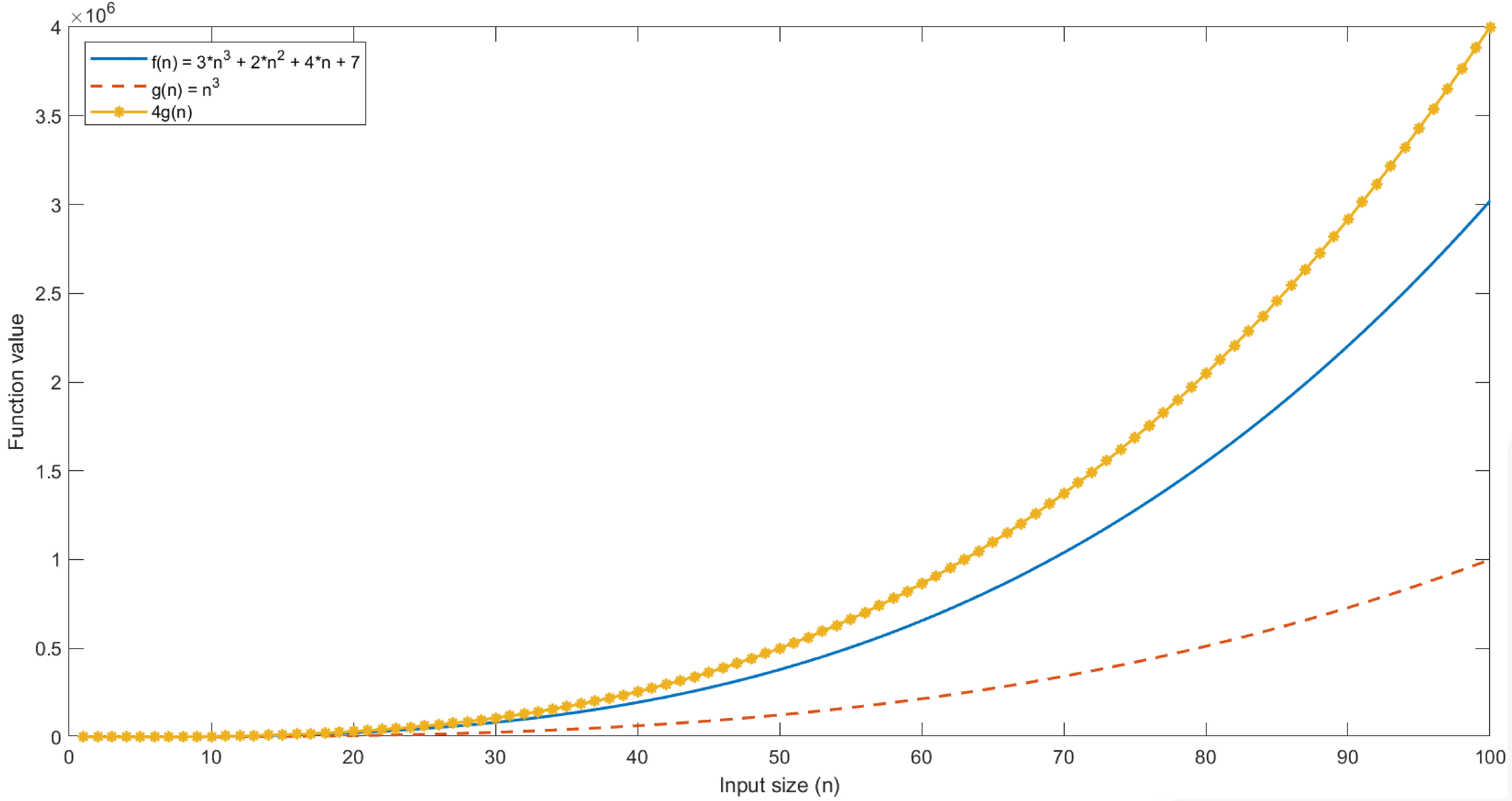
Visual inspection (O)



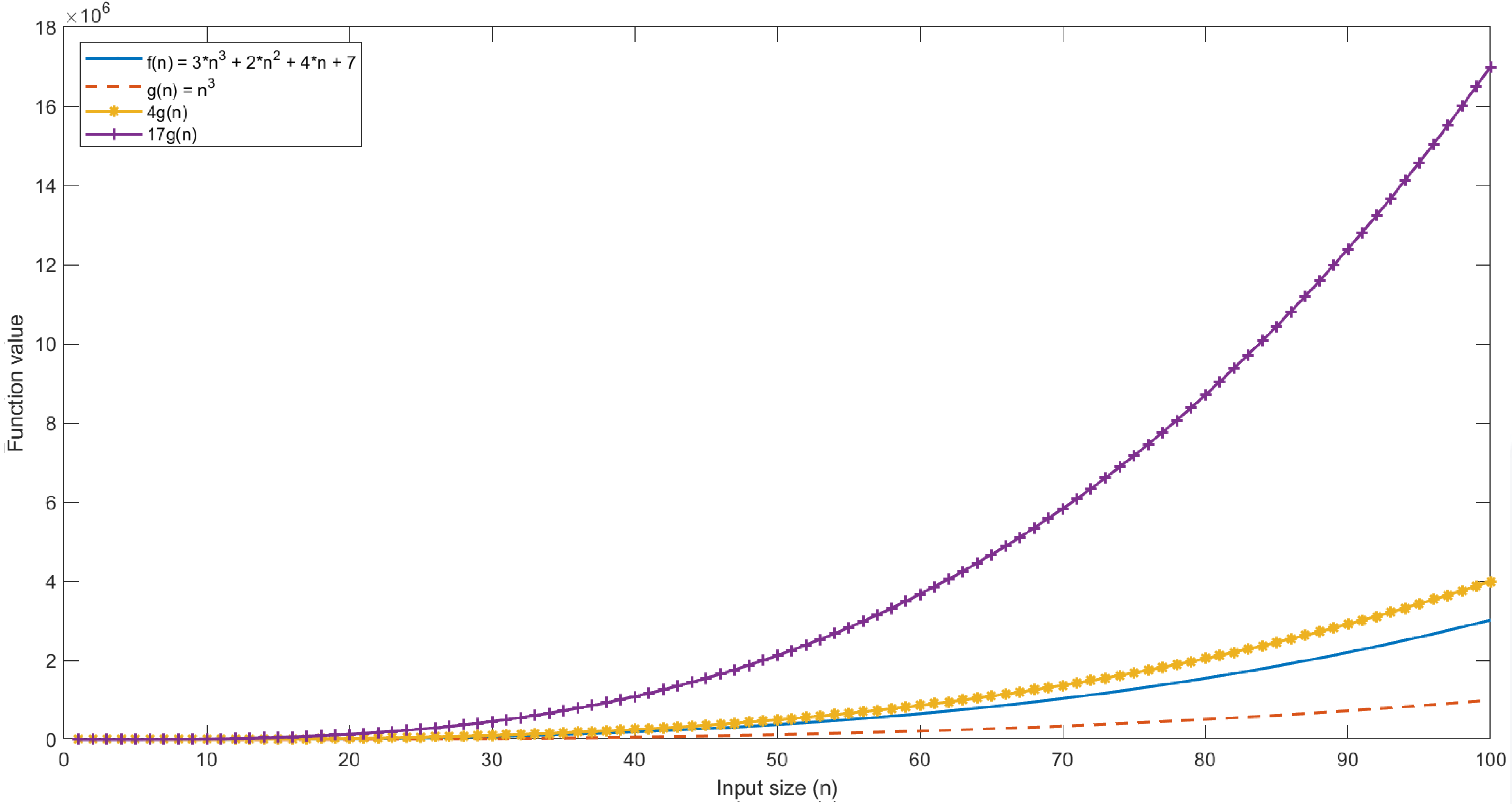
Visual inspection (O)



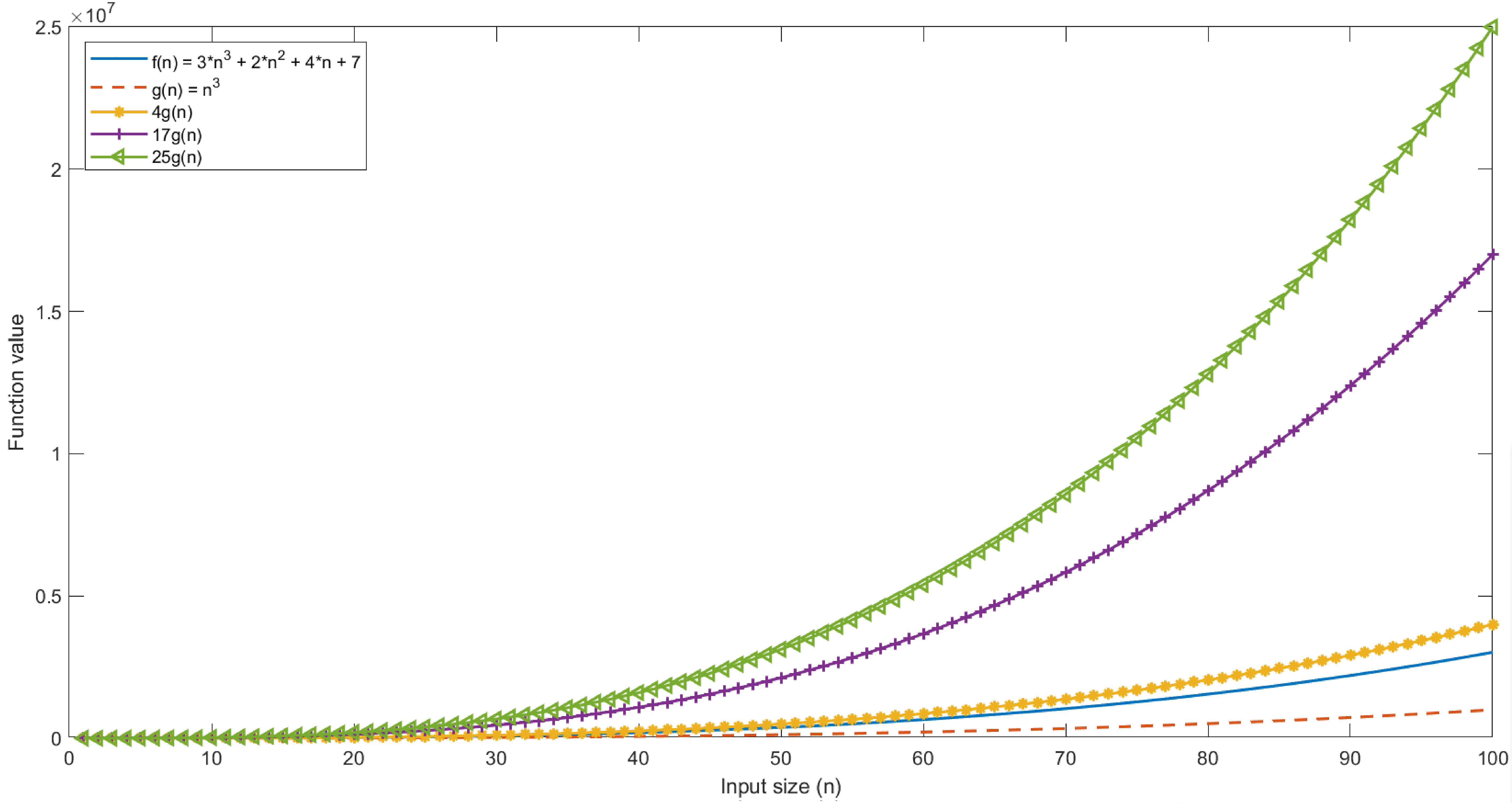
Visual inspection (O)



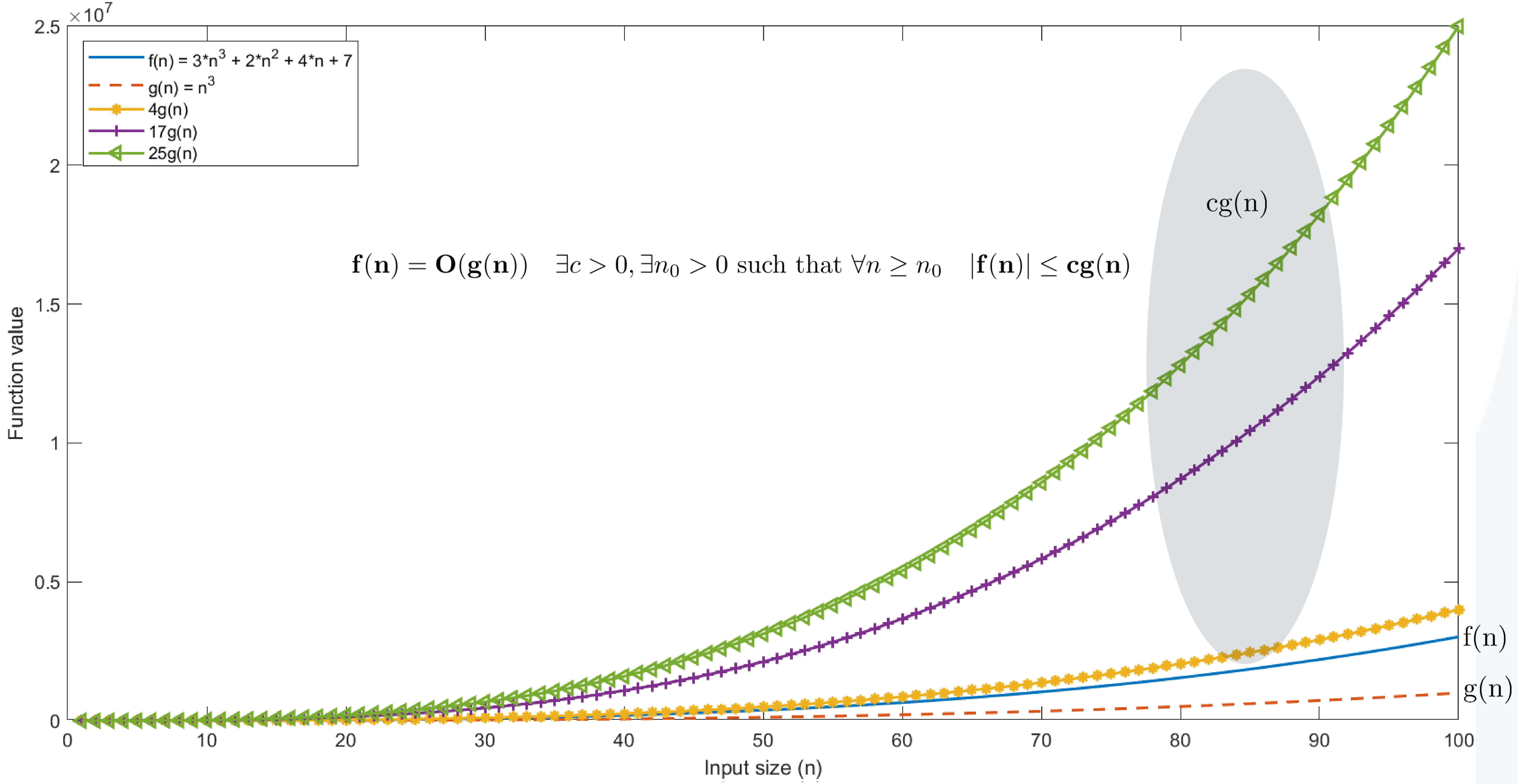
Visual inspection(O)



Visual inspection (O)



Visual inspection (O)



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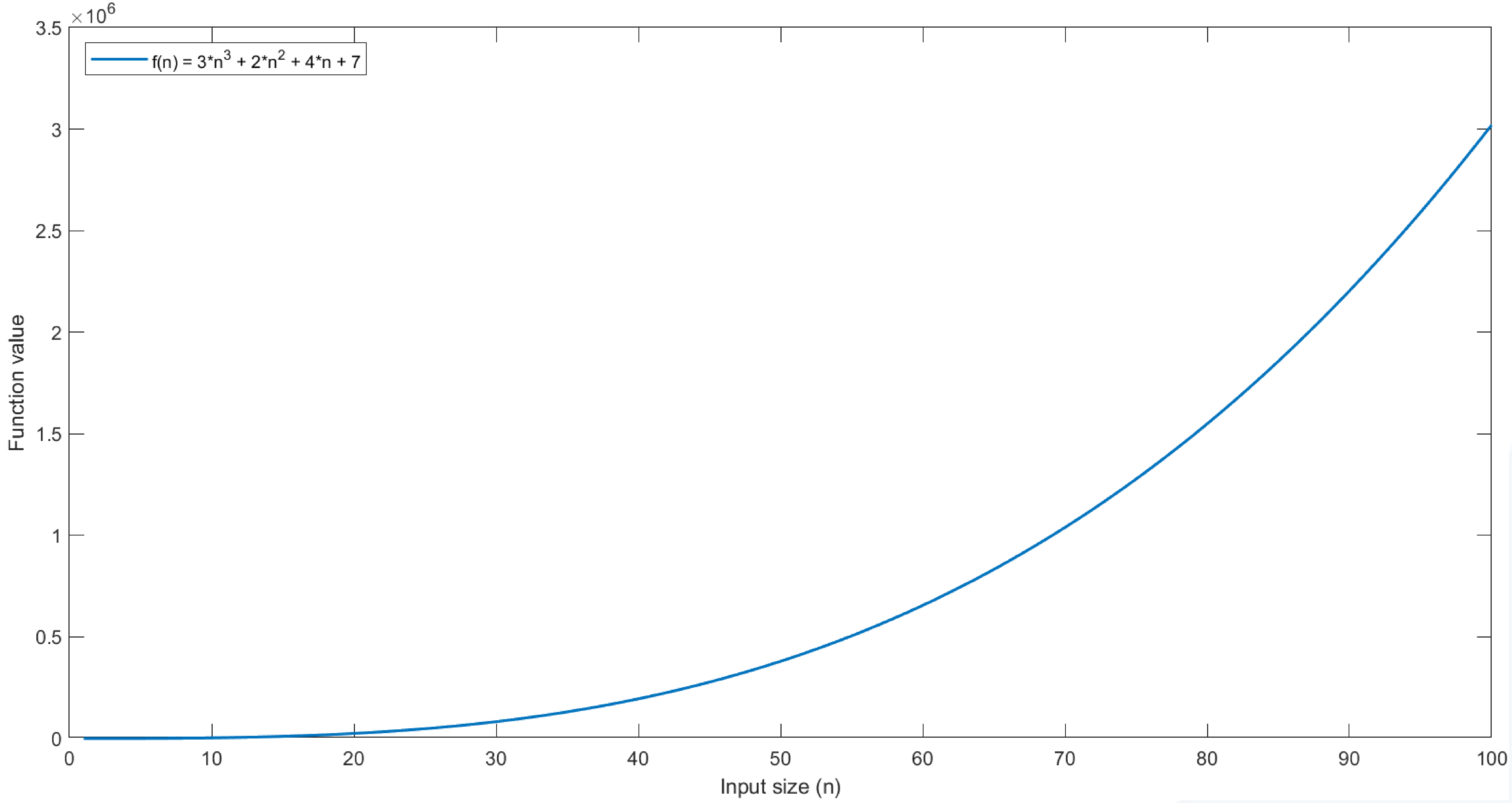
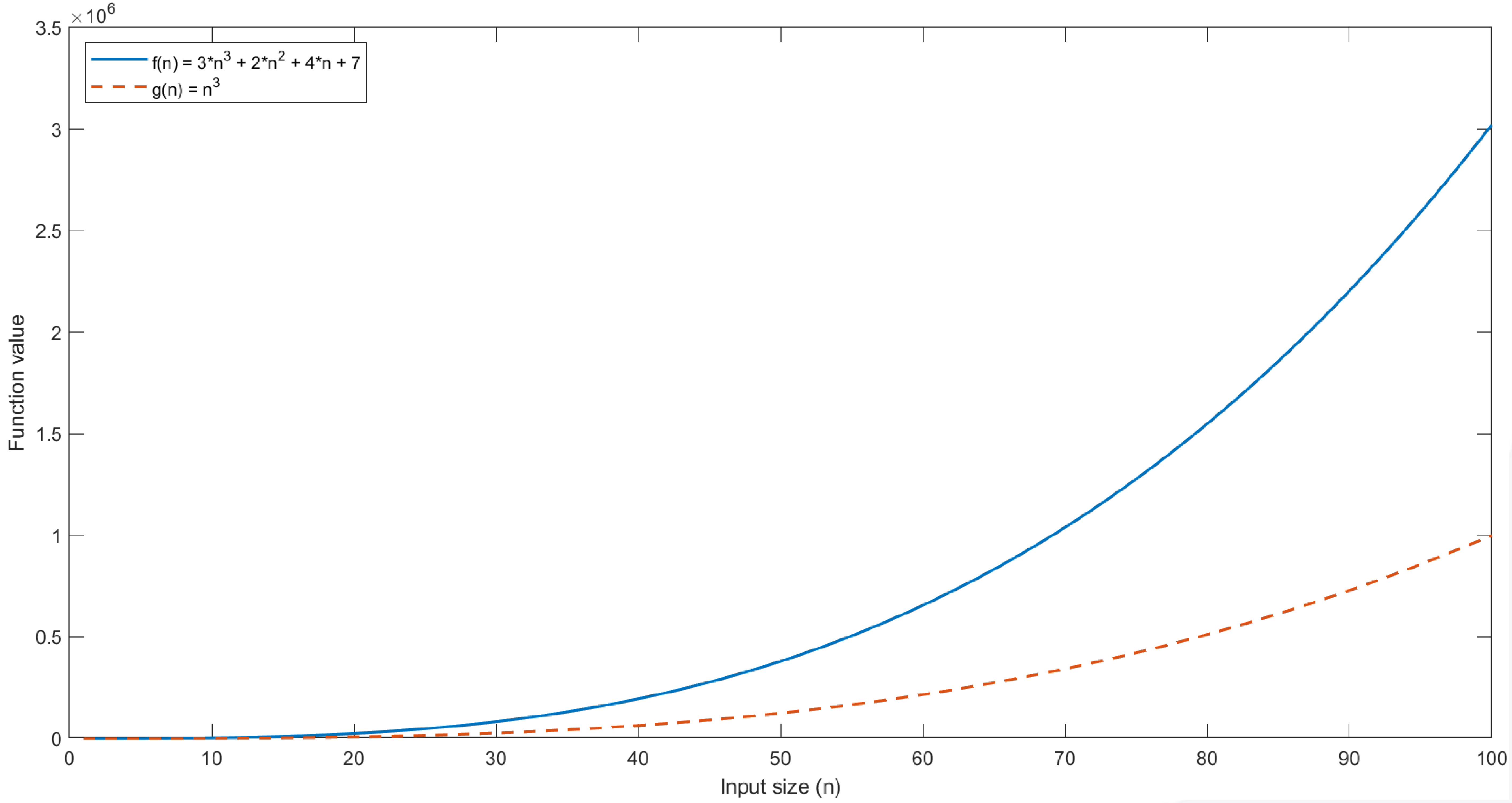
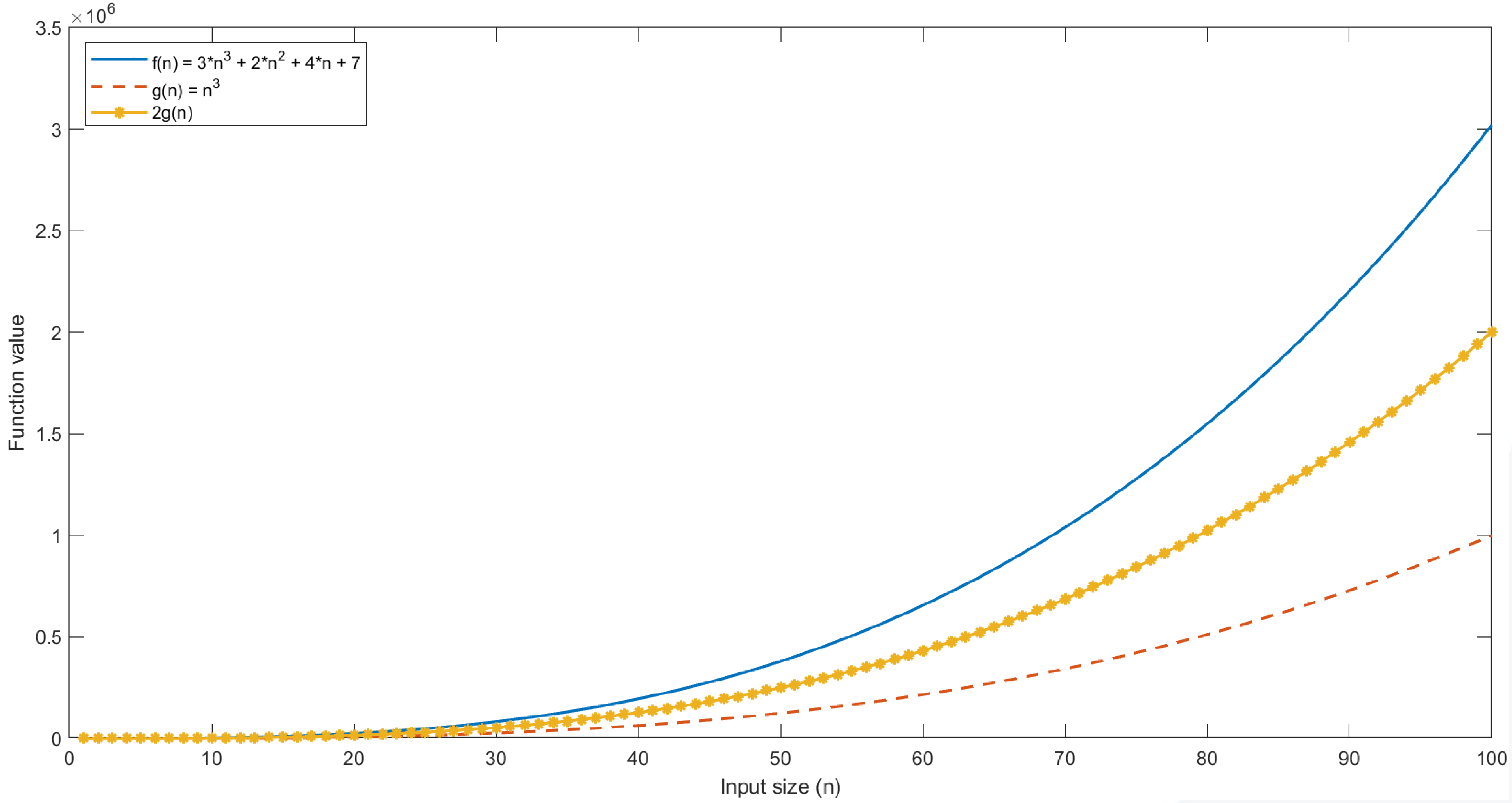
Visual inspection Ω



Visual inspection Ω



Visual inspection Ω



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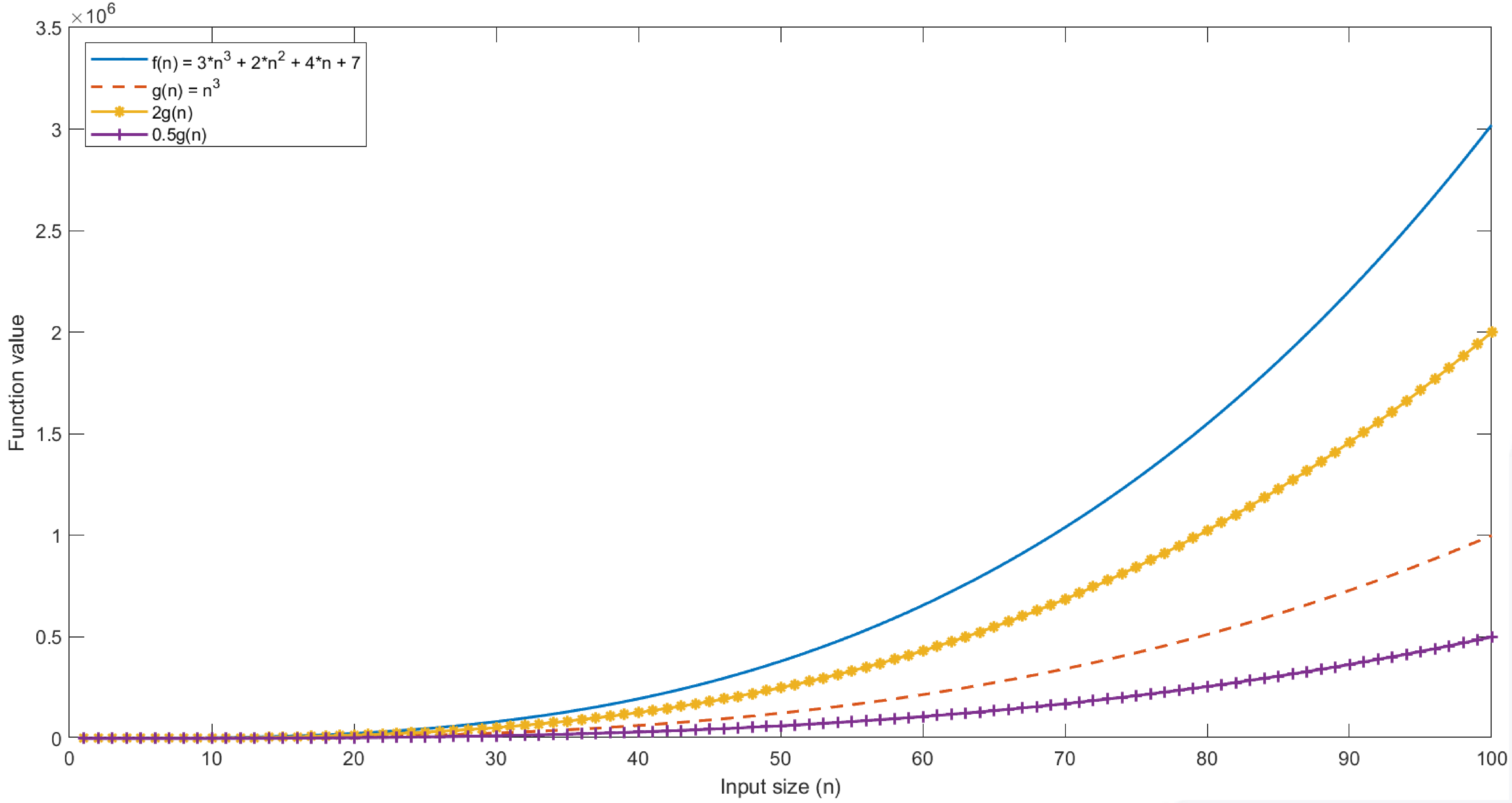
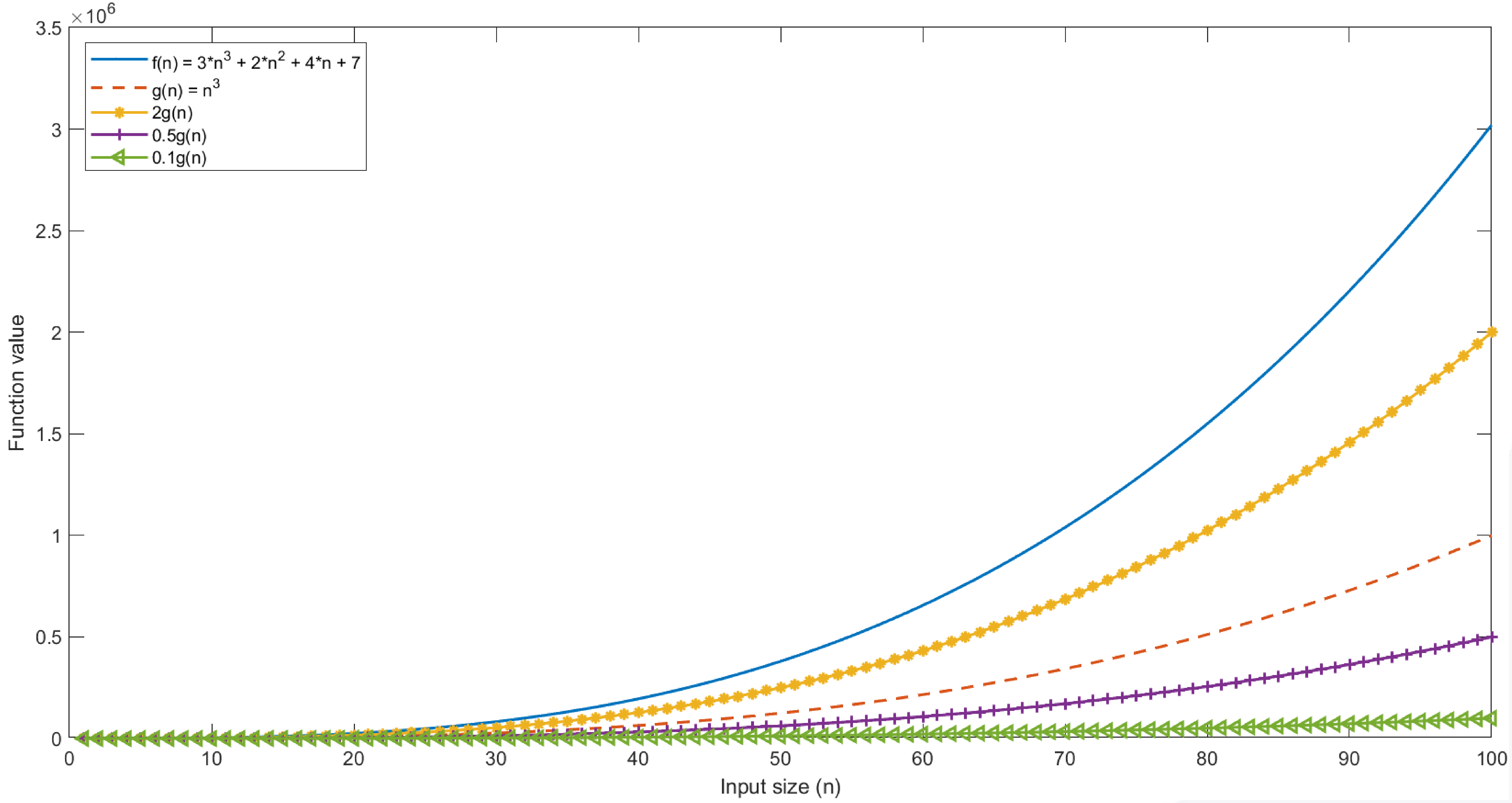
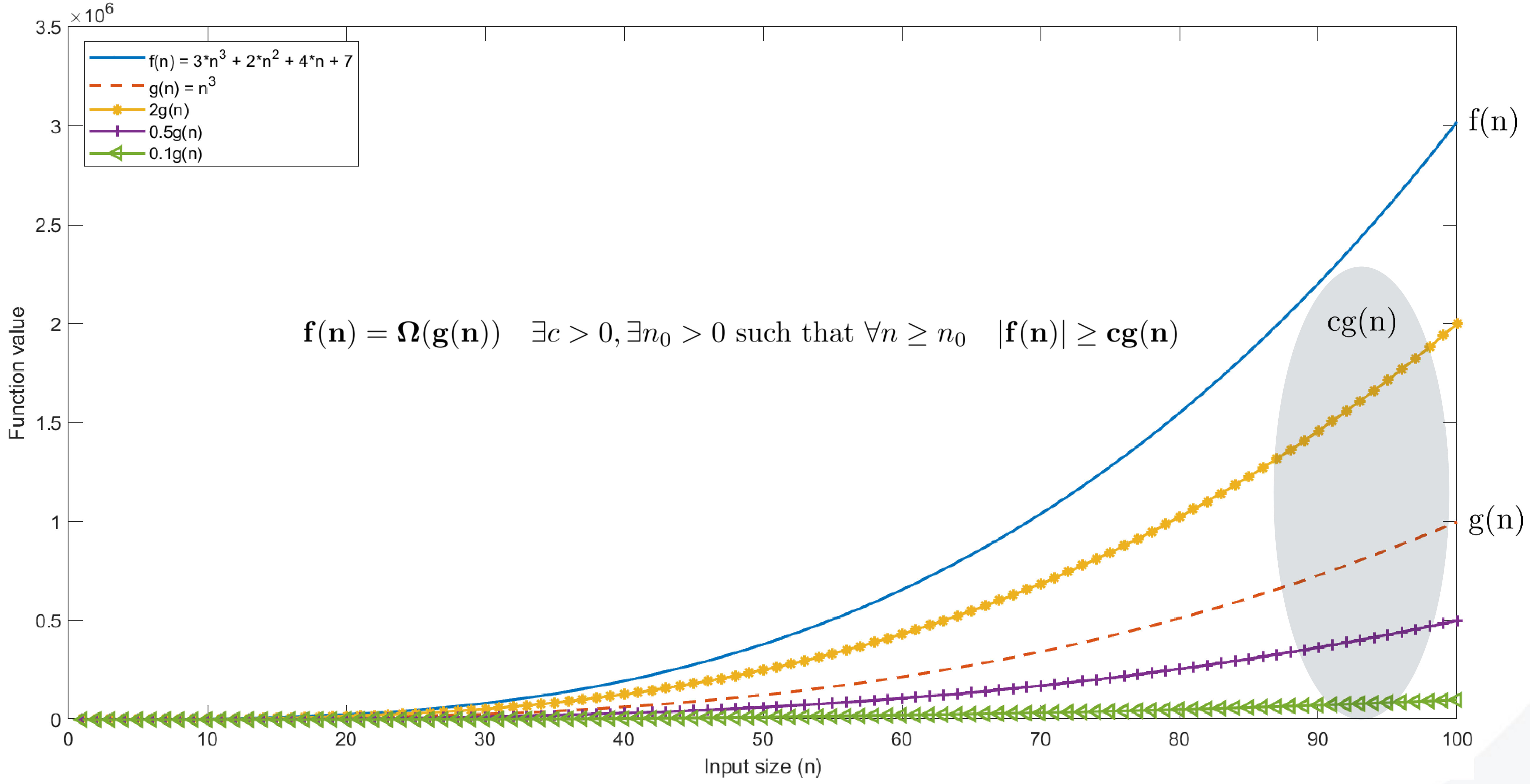
Visual inspection Ω



Visual inspection Ω



Visual inspection Ω



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Visual Inspection

$$f(n) = 3n^3 + 2n^2 + 5n + 7$$

$$1 < log(n) < \sqrt{n} < n < nlog(n) < n^2 < ... < n^3 < ... < 2^n < e^n < ... < n^n$$

Visual Inspection

$$f(n) = 3n^3 + 2n^2 + 5n + 7$$

$$1 < log(n) < \sqrt{n} < n < nlog(n) < n^2 < \ldots < \underbrace{n^3 < \ldots < 2^n < e^n < \ldots < n^n}_{\text{O}}$$

Visual Inspection

$$f(n) = 3n^3 + 2n^2 + 5n + 7$$

$$\underbrace{1 < log(n) < \sqrt{n} < n < nlog(n) < n^2 < ... < n^3}_{\Omega} \underbrace{< ... < 2^n < e^n < ... < n^n}_{O}$$

Visual Inspection

$$f(n) = 3n^3 + 2n^2 + 5n + 7$$

$$\underbrace{1 < log(n) < \sqrt{n} < n < nlog(n) < n^2 < \ldots < n^3}_{\Omega} \underbrace{< \ldots < 2^n < e^n < \ldots < n^n}_{\mathrm{O}}$$

While it is mathematically correct to say that $f(n) = O(2^n)$ or even $f(n) = O(n^n)$, it does not capture the growth rate in a useful way

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Visual Inspection

$$f(n) = 3n^3 + 2n^2 + 5n + 7$$

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While it is mathematically correct to say that $f(n) = O(2^n)$ or even $f(n) = O(n^n)$, it does not capture the growth rate in a useful way

In general, the most useful asymptotic notation is one that accurately captures the dominant term or terms of the function, e.g. the smallest upper bound or the largest lower bound.

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Example

$$f(n) = \sqrt{6n^3 + 7n^2 + 5n + 5}$$

$$\mathbf{f(n) = O(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq \infty$$

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Example

$$f(n) = n^3 \log_2 n$$

$$g(n) = 3n \log_8 n$$

$\mathbf{f(n) = O(g(n))}$ $\lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq \infty$

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Example

$$f(n) = 8^n \qquad g(n) = 4^n$$

$$\mathbf{f(n) = O(g(n))} \quad \lim_{n\to\infty} \frac{\mathbf{f(n)}}{\mathbf{g(n)}} \neq \infty$$

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