Workshop 1: Complexity Notation

Workshop 1 will take place in week 2. You should prepare solutions, but you don't have to hand them in and they won't get marked.

Exercise 1 *Complexity Notation*

Solve Exercise 10 and 11 in the book of Mehlhorn/Sanders (Chapter 2.1).

Exercise 2 *Complexity Notation*

Is it true that if $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$, then $h(n) = \Theta(f(n))$?

Exercise 3 *Complexity Notation*

Is it true that if $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $h(n) = \Omega(f(n))$?

Exercise 4 *Complexity Notation*

Is it true that a $\Theta(n^2)$ algorithm always takes longer to run than a $\Theta(logn)$ algorithm?

Exercise 5 *Complexity Notation*

For each pair of functions given below, point out the asymptotic relationships that apply: $f = O(g)$, $f = \Theta(g)$, $f = \Omega(g)$.

- $f(n) = \sqrt{n}$ and $g(n) = log(n)$ — $f(n) > g(n) \longrightarrow f = \Omega(g)$ ✓
- $f(n) = 1$ and $g(n) = 2$ — $f = \Theta(g)$ ✓
- $f(n) = 1000 \cdot 2^n$ and $g(n) = 3^n$ — $f = O(g)$ ✓
- $f(n) = 4^{n+4}$ and $g(n) = 2^{2n+2}$ — $f = \Omega(g)$ × Θ
- $f(n) = 5nlog(n)$ and $g(n) = nlog(5n)$ — $f = O(g)$ × Θ
- $f(n) = n!$ and $g(n) = (n+1)!$ — $f = \Theta(g)$ × O

Exercise 6 *Complexity Notation*

Prove that $n^k = o(c^n)$ for any integer k and any $c > 1$.

Exercise 1

Ex.10:

	Ans
a) $n^2 + 10^6 n \in O(n^2)$	✓
b) $n\log(n) \in O(n)$	✗
c) $n\log(n) \in \Omega(n)$	✓
d) $\log(n) \in o(n)$	✓

Ex.11:

> **Lemma 6**
>
> 1. $c \cdot f(n) = \Theta(f(n))$ for $\forall c > 0$.
> 2. $f(n) + g(n) = \Omega(f(n))$.
> 3. $f(n) + g(n) = O(f(n))$ if $g(n) = O(f(n))$.
> 4. $O(f(n)) \cdot O(g(n)) = O(f(n) \cdot g(n))$.

Proof: 1) For $n \geq 1$, $c \cdot f(n) \geqslant f(n)$

For $n \leq 1$, $c \cdot f(n) \leqslant f(n)$

Since, $n = 1$ satisfies upper & lower bound, $\Theta(f(n)) = c \cdot f(n)$

2) $\lim\limits_{n \to \infty} \frac{f(n)}{g(n)}$

> **L'Hopital's Rule:**
>
> 1) Take derivative until get out of '$\frac{0}{0}$' or '$\frac{\infty}{\infty}$'
>
> 2) Make sure that after subbing in $n \to \infty$ that the ratio is no longer '$\frac{\infty}{\infty}$' or '$\frac{0}{0}$'

Exercise 4

$$\lim_{n \to \infty} \frac{n^2}{\log(n)} \longrightarrow \frac{2n}{\frac{1}{n}} \longrightarrow 2n^2$$