Workshop 3: Master Theorem and Amortized Analysis

General Instructions

Please attempt these questions before attending your workshop session. You do not need to submit your solutions beforehand.

Exercise 1 *Master theorem*

We refer to the simplified version of the Master theorem (p.16, slides on Karatsuba).

1) Let $T(1) = 1$ and $T(n) = n + \frac{T(n/2)}{2}$. Use the theorem to calculate the complexity of $T(n)$.

2) What if I change it to $T(1) = 1$ and $T(n) = \frac{T(n/2)}{2}$? Is the result still $\Theta(n)$?

Exercise 2 *Amortized Analysis*

Read the Chapters 3.2 and 3.3 in the book of Mehlhorn and Sanders.

1. Explain in your own words the general ideas and the approach of an amortized analysis.
2. Explain this type of analysis in detail for unbounded arrays.
3. Your manager asks you to change the initialization of α to $\alpha = 2$ in the unbounded array implementation (Section 3.2). He argues that it is wasteful to shrink an array only when three-fourths of it are unused. He proposes to shrink it when $n \leq w/2$. How are you going to convince him that this is a bad idea?
4. Suppose, for a real-time application, you need an unbounded array data structure with a worst-case constant execution time for all operations. Design such a data structure.