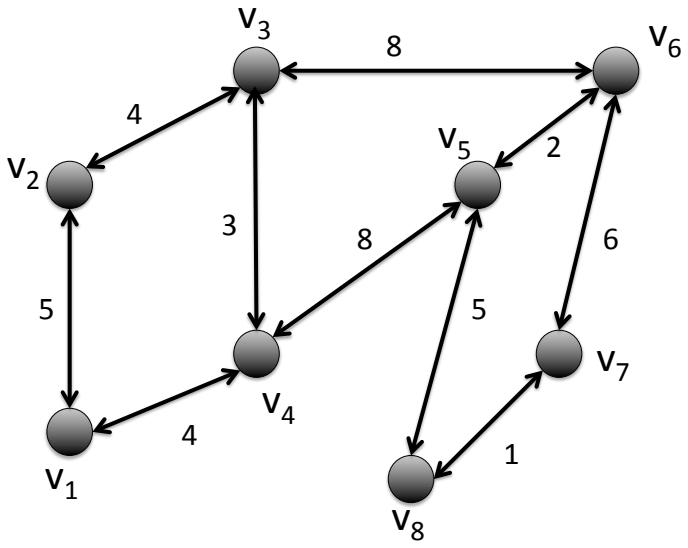
Minimum Spanning Trees, Euler Path, Undecidable Problems, and Turing Machines

Exercise 1 *Kruskal's Algorithm*

Compute a minimum spanning for the following graph using Kruskal's algorithm. Show the status of your partial minimum spanning tree after each edge insertion and indicate for each edge whether it is included in the minimum spanning tree.



Exercise 2 *Euler tours of directed graphs*

A Hamiltonian tour is a tour that visits every node of a strongly directed graph just once. Determining if a Hamiltonian tour exists is an *NP*-Complete problem.

In contrast an *Euler tour* of a strongly connected directed graph $G = (V, E)$ is a *cycle* that traverses each edge of G exactly once, although it may visit a node more than once. Answer the following questions:

- Show that G has an Euler tour if and only if the in-degree of every node v equals the out degree of v.
- Describe an $O(m)$ (where m is the number of edges in G) algorithm to find an Euler tour of G if such a tour exists (*Hint:* Merge edge-disjoint cycles).

Exercise 3 *We define the following program: Does program Q, given input y, print either "Hello world" or "Goodbye world" as its first output?*

Use reduction to prove that the above program is undecidable.

Exercise 4 *On* `https://turingmachine.io/`*, implement the Turing machine that tests whether the input is a UoA ID. That is, whether the input starts with a, and is then followed by 7 digits.*