قال تعالي " وما اوتيتم من العلم الا قليلا "

ملخصات

ايزى شوم

في

الرياضيات

👋 افضل وسيلة موجزة

👋 key=Tone

Some give up

Some still try

تحويلات لا بلاس

Laplace transforms

القانون

$$L[f(t)] = \int_0^{\infty} f(t)e^{-st}\ dt$$

Find Laplace transform where f(t) =1

$$L[\ 1\ ] = \int_0^{\infty} 1\ \times e^{-st}\ dt$$

$$= \int_0^{\infty} e^{-st}\ dt$$

$$= \frac{e^{-st}}{-s}\Bigg|_0^{\infty}$$

$$= \frac{e^{\infty}}{-s} - \frac{e^{0}}{-s}$$

$$= 0 + \frac{1}{s}$$

$$= \frac{1}{s}$$

<u>Prove</u> $\mathbf{L[c] = \frac{c}{s}}$

$$L[f(t)] = \int_0^{\infty} f(t)e^{-st}\, dt$$

$$L[c] = \int_0^{\infty} c.e^{-st}\, dt$$

$$= c\int_0^{\infty} e^{-st}\, dt$$

$$= c\left[\frac{e^{-st}}{-s}\right]_0^{\infty}$$

$$= c\left[\frac{e^{\infty}}{-s} - \frac{e^{0}}{-s}\right]$$

$$= c\left[0 - \frac{1}{-s}\right]$$

$$= c\left[\frac{1}{s}\right]$$

$$= \frac{c}{s}$$

Prove $L[e^{at}] = \frac{1}{s-a}$

$$L[f(t)] = \int_0^\infty f(t)e^{-st}\ dt$$

$$L[e^{at}] = \int_0^\infty e^{at}.e^{-st}\ dt$$

$$= \int_0^\infty e^{-(s-a)t}\ dt$$

$$= \frac{e^{-(s-a)t}}{-(s-a)}\Bigg|_0^\infty$$

$$= \frac{e^{\infty}}{-(s-a)} - \frac{e^{0}}{-(s-a)}$$

$$= 0 + \frac{1}{s-a}$$

$$= \frac{1}{s-a}$$

Prove $L[e^{-at}] = \frac{1}{s+a}$

$$L[f(t)] = \int_0^\infty f(t)e^{-st}\,dt$$

$$L[e^{-at}] = \int_0^\infty e^{-at}\,.\,e^{-st}\,dt$$

$$= \int_0^\infty e^{-(s+a)t}\,dt$$

$$= \frac{e^{-(s+a)t}}{-(s+a)}\Bigg|_0^\infty$$

$$= \frac{e^{\infty}}{-(s+a)} - \frac{e^{0}}{-(s+a)}$$

$$= 0 + \frac{1}{s+a}$$

$$= \frac{1}{s+a}$$

$$L[\cos at] = \frac{s}{s^2 + a^2}$$

$$L[\cosh at] = \frac{s}{s^2 - a^2}$$

$$L[\sin at\ ] = \frac{a}{s^2 + a^2}$$

$$L[\sinh at] = \frac{a}{s^2 - a^2}$$

$L[c] = \frac{c}{s}$	$L[e^{at}] = \frac{1}{s-a}$
$L[e^{-at}] = \frac{1}{s+a}$	$L[\cos at] = \frac{s}{s^2+a^2}$
$L[\cosh at] = \frac{s}{s^2-a^2}$	$L[\sin at] = \frac{a}{s^2+a^2}$
$L[\sinh at] = \frac{a}{s^2-a^2}$	

امثلة على القوانين السابقة

$$f(t) = 4cos(4t) - sin(4t) + 2cos(10t)$$

Find Laplace transform to f(t)

$$L[f(t)] = 4L[cos4t] - L[sin4t] + 2L[cos10t]$$

$$= 4\frac{s}{s^2+(4)^2} - \frac{4}{s^2+(4)^2} + 2\frac{s}{s^2+(10)^2}$$

$$= \frac{4s}{s^2+16} - \frac{4}{s^2+16} + \frac{2s}{s^2+100}$$

$$f(t) = 3sinh(2t) + 3sin(2t)$$

Find Laplace transform to f(t)

$$\mathrm{L}[\mathrm{f}(\mathrm{t}) = 3\mathrm{L}[\sinh 2\mathrm{t}] + 3\mathrm{L}[\sin 2\mathrm{t}]$$

$$= 3\frac{2}{s^2-(2)^2} + 3\frac{2}{s^2+(2)^2}$$

$$= \frac{6}{s^2-4} + \frac{6}{s^2+4}$$

لاحظ جيدا

$$L[e^{wt}\cos at]=\frac{s-w}{(s-w)^2+a^2}$$

$$L[e^{wt}\, sinat]=\frac{a}{(s-w)^2+a^2}$$

مثال

$$F(t)=e^{3t}+cos(6t)-e^{3t}cos(6t)$$

Find Laplace transform to f(t)

$$L[f(t)]=L\left[e^{3t}\right]+L[cos\ 6t]-L[e^{3t}\ cos\ 6t]$$

$$=\frac{1}{s-3}+\frac{s}{s^2+(6)^2}-\frac{(s-3)}{(s-3)^2+(6)^2}$$

$$=\frac{1}{s-3}+\frac{s}{s^2+36}+\frac{(s-3)}{(s-3)^2+36}$$

Prove $\mathbf{L[t^n] = \frac{n!}{s^{n+1}}}$

$$L[f(t)] = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^\infty t^n\, e^{-st}\, dt \ldots\ldots\ldots (1)$$

$$Let\ x = st \ldots\ldots.. (2)$$

$$dx = sdt$$

$$dt = \frac{dx}{s} \ldots\ldots (3)$$

From(2), (3) in (1)

$$= \int_0^\infty t^n\, e^{-x} \frac{dx}{s}$$

$$= \frac{1}{s} \int_0^\infty t^n\, e^{-x}\, dx$$

U	dv
t^n	e^{-x}
nt^{n-1}	$-e^{-x}$

$$= \frac{1}{s} = \frac{1}{s}\left[-e^{-x} t^n + n \int_0^\infty t^{n-1}\ e^{-x}\, dx\right]$$

$$= \frac{1}{s}\left[\cancel{-e^{-x} t^{n}}_{0} + n\int_0^\infty t^{n-1}\, e^{-x}\, dx\right]$$

$$= \frac{1}{s} \times n\int_0^\infty t^{n-1}\, e^{-x}\, dx$$

$$\frac{n}{s}\int_0^\infty t^{n-1}\, e^{-x}\, dx --(4)$$

$$let\ I = \int_0^\infty t^{n-1}\, e^{-x}\, dx$$

U	dv
t^{n-1}	e^{-x}
$(n-1)t^{n-2}$	$-e^{-x}$

$$I = \cancel{-e^{-x} t^{n-1}}_{0} - \int_0^\infty -e^{-x}(n-1)t^{n-2}\, dx$$

$$= (n-1)\int_0^\infty t^{n-2}\, e^{-x} dx --(5)$$

<u>From 5 in 4</u>

$$\mathrm{L[t^n]} = \frac{\mathrm{n}}{\mathrm{s}}(\mathrm{n}-1)\int_0^\infty \mathrm{t^{n-2}\, e^{-x}\, dx}$$

$$= \frac{n}{s}(n-1)(n-2)\ldots\ldots..3 \times 2 \times L[1]$$

$$= \frac{n!}{s^n} \times L[1]$$

$$= \frac{n!}{s^n} \times \frac{1}{s}$$

$$= \frac{n!}{s^{n+`1}}$$

مثال

$$f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$$

Find Laplace transform to f(t)

$$L[f(t) = 6L\left[e^{-5t}\right] + L[e^{3t}] + 5L\left[t^3\right] - L[9]$$

$$= 6\frac{1}{s+5} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - \frac{9}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$

قانون اخر

$$L[t^n f(t)] = \frac{(-1)^n d^n}{ds^n} \text{L[f(t)]}$$

مثال

Find Laplace transform

(1) $\mathbf{f(t) = t \cosh(3t)}$

$$\mathbf{L[t \cosh 3t] = (-1)^1 \frac{d}{ds}\ \ l[\cosh 3t]}$$

$$= (-1) \frac{d}{ds} \frac{s}{s^2 - 9}$$

$$= (-1) \frac{\left(s^2 - 9\right) \times 1 - s \times 2s}{(s^2 - 9)^2}$$

$$= (-1) \frac{s^2 - 9 - 2s^2}{(s^2 - 9)^2}$$

$$= (-1) \frac{-s^2 - 9}{(s^2 - 9)^2}$$

$$= \frac{s^2 + 9}{(s^2 - 9)^2}$$

(2) $f(t) = t^2 \sin(2t)$

$$L\left[t^2 \sin(2t)\right] = (-1)^2 \frac{d^2}{ds^2} L[\sin 2t]$$

$$= \frac{d^2}{ds^2} \frac{2}{s^2+4}$$

$$= \frac{d}{ds} [\frac{d}{ds} \frac{2}{s^2+4}]$$

$$= \frac{d}{ds}\left[\frac{\left(s^2+4\right) \times 0 - 2 \times 2s}{(s^2+4)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{-4s}{(s^2+4)^2}\right]$$

$$= \frac{\left(s^2+4\right)^2 \times -4 - (-4s) \times 2(s^2+4) \times 2s}{(s^2+4)^4}$$

$$= \frac{-4\left(s^2+4\right)^2 + 16s^2(s^2+4)}{(s^2+4)^4}$$

$$= \frac{\cancel{(s^2+4)}\left[-4(s^2+4)+16s^2\right]}{\cancel{(s^2+4)}^4}$$

$$= \frac{-4s^2 - 16 + 16s^2}{(s^2+4)^3}$$

$$= \frac{12s^2 - 16}{(s^2+4)^3}$$

جميع قوانين لابلاس

$L[c] = \frac{c}{s}$	$L[e^{at}] = \frac{1}{s-a}$
$L[e^{-at}] = \frac{1}{s+a}$	$L[\cos at] = \frac{s}{s^2+a^2}$
$L[\cosh at] = \frac{s}{s^2-a^2}$	$L[\sin at] = \frac{a}{s^2+a^2}$
$L[\sinh at] = \frac{a}{s^2-a^2}$	$L[t^n] = \frac{n!}{s^{n+1}}$
$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^2} L[f(t)]$	
$L[e^{wt} \cos at] = \frac{(s-w)}{(s-w)^2+a^2}$	
$L[e^{wt} \sin at] = \frac{a}{(s-w)^2+a^2}$	

قوانين لابلاس العكسي

$L^{-1}\left[\frac{c}{s}\right]=c$	$L^{-1}\left[\frac{1}{s-a}\right]=e^{at}$
$L^{-1}\left[\frac{1}{s+a}\right]=e^{-at}$	$L^{-1}\left[\frac{s}{s^2+a^2}\right]=\cos at$
$L^{-1}\left[\frac{s}{s^2-a^2}\right]=\cosh at$	$L^{-1}\left[\frac{a}{s^2+a^2}\right]=\sin at$
$L^{-1}\left[\frac{a}{s^2-a^2}\right]=\sinh at$	$L^{-1}\left[\frac{1}{s^{n+1}}\right]=\frac{n}{n!}$
$L^{-1}\left[\frac{(s-w)}{(s-w)^2+a^2}\right]=e^{wt}\cos at$	
$L^{-1}\left[\frac{(s-w)}{(s-w)^2-a^2}\right]=e^{wt}\cosh at$	
$L^{-1}\left[\frac{a}{(s-w)^2+a^2}\right]=e^{wt}\sin at$	
$L^{-1}[\frac{a}{(s-w)^2-a^2}=e^{wt}\sinh at$	

Find the inverse transform

$$(a)\ \mathrm{F(s)} = \frac{6}{s} - \frac{1}{s-8} + \frac{4}{s-3}$$

$$\mathrm{L}^{-1}[f(s)] = 6L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s-8}\right] + 4L^{-1}\left[\frac{1}{s-3}\right]$$

$$= 6 - e^{8t} + 4e^{3t}$$

$$(b)\ \mathrm{F(s)} = \frac{19}{s+2} - \frac{1}{3s-5} + \frac{7}{s^5}$$

$$F(s) = \frac{19}{s+2} - \frac{\frac{1}{3}}{\frac{3s}{3} - \frac{5}{3}} + \frac{7}{s^5}$$

$$\mathrm{L}^{-1}[\mathrm{f(s)}] = 19\mathrm{L}^{-1}\left[\frac{1}{s+2}\right] - \frac{1}{3}\mathrm{L}^{-1}\left[\frac{1}{s-\frac{5}{3}}\right] + 7\mathrm{L}^{-1}\left[\frac{1}{s^5}\right]$$

$$= 19e^{-2t} - \frac{1}{3}e^{\frac{5}{3}t} + \frac{4}{4!}$$

$$(c) \quad F(s) = \frac{6s}{s^2+25} + \frac{3}{s^2+25}$$

$$L^{-1}[f(s)] = 6L^{-1}\left[\frac{s}{s^2+5^2}\right] + \frac{3}{5}L^{-1}\left[\frac{5}{s^2+5^2}\right]$$

$$= 6\cos 5t + \frac{3}{5}\sin 5t$$

$$(d) \quad F(s) = \frac{8}{3s^2+12} + \frac{3}{s^2-49}$$

$$L^{-1}[f(s)] = \frac{8}{6}L^{-1}\left[\frac{2}{s^2+2^2}\right] + \frac{3}{7}L^{-1}\left[\frac{7}{s^2-7^2}\right]$$

$$= \frac{8}{6}\sin 2t + \frac{3}{7}\sinh 7t$$

مثال اخر

Find the inverse transform

(a) $F(s)=\frac{6s-5}{s^2+7}$

$$F(s)=\frac{6s}{s^2+7}-\frac{5}{s^2+7}$$

$$L^{-1}[f(s)]=6L^{-1}\left[\frac{s}{s^2+\left(\sqrt{7}\right)^2}\right]-\frac{5}{\sqrt{7}}L^{-1}\left[\frac{\sqrt{7}}{s^2+\left(\sqrt{7}\right)^2}\right]$$

$$=6\cos\sqrt{7}\,t-\frac{5}{\sqrt{7}}\sin\sqrt{7}\,t$$

قاعده هامة

اذا كان المقام لا يحلل ، نقسم الحد الثاني علي 2 ، تقوم بتربيع ناتج القسمة وتضيفه وتطرحه

$$s^2 + 8s + 21$$

نجد انه لا يحلل ،

نقوم بقسمة الحد الثاني علي 2

$$\frac{8}{2} = 4$$

نقوم بتربيع ناتج القسمة

$$(4)^2 = 16$$

نقوم بأضافة وطرح 16

$$s^2 + 8s + 21 + 16 - 16$$

نقوم باخذ الجزء الموجب

$$(s^2 + 8s + 16) + 21 - 16$$

$$(s + 4)^2 + 5$$

مثال

Find the inverse transform

$$(b) \qquad F(s)=\frac{1-3s}{s^2+8s+21}$$

$$F(s)=\frac{1-3s}{(s^2+8s+16)+21-16}=\frac{1-3s}{(s+4)^2+5}$$

$$=\frac{1}{(s+4)^2+5}-\frac{3s}{(s+4)^2+5}$$

$$=\frac{1}{(s+4)^2+5}-\frac{3(s+4-4)}{(s+4)^2+5}$$

$$=\frac{1}{(s+4)^2+5}-\frac{3(s+4)+12}{(s+4)^2+5}$$

$$=\frac{1}{(s+4)^2+5}-\frac{3(s+4)}{(s+4)^2+5}+\frac{12}{(s+4)^2+5}$$

$$L^{-1}[f(s)]=\frac{1}{\sqrt{5}}L^{-1}\left[\frac{\sqrt{5}}{(s+4)^2+\left(\sqrt{5}\right)^2}\right]-3L^{-1}\left[\frac{(s+4)}{(s+4)^2+\left(\sqrt{5}\right)^2}\right]+\frac{12}{\sqrt{5}}L^{-1}\left[\frac{\sqrt{5}}{(s+4)^2+\left(\sqrt{5}\right)^2}\right]$$

$$=\frac{1}{\sqrt{5}}\,e^{-4t}\sin\sqrt{5}\,t-3e^{-4t}\cos\sqrt{5}\,t+\frac{12}{\sqrt{5}}e^{-4t}\sin\sqrt{5}\,t$$

$$=-3e^{-4t}\cos\sqrt{5}t+\frac{13}{\sqrt{5}}e^{-4t}\sin\sqrt{5}\,t$$

$$(c)\quad F(s)=\frac{3s-2}{2s^2-6s-2}$$

$$f(s)=\frac{\frac{3}{2}s-\frac{2}{2}}{\frac{2s^2}{2}-\frac{6s}{2}-\frac{2}{2}}=\frac{\frac{3}{2}s-1}{s^2-3s-1}$$

$$=\frac{\frac{3}{2}s-1}{s^2-3s-1+\frac{9}{4}-\frac{9}{4}}$$

$$=\frac{\frac{3}{2}s-1}{(s^2-3s+\frac{9}{4})-1-\frac{9}{4}}$$

$$=\frac{\frac{3}{2}s-1}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}}$$

$$=\frac{\frac{3}{2}s}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}}-\frac{1}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}}$$

$$=\frac{\frac{3}{2}(s-\frac{3}{2}+\frac{3}{2})}{\left(s-\frac{3}{2}\right)^2-\left(\frac{\sqrt{13}}{2}\right)^2}-\frac{2}{\sqrt{13}}\frac{\frac{\sqrt{13}}{2}}{\left(s-\frac{3}{2}\right)^2-\left(\frac{\sqrt{13}}{2}\right)^2}$$

$$= \frac{\frac{3}{2}\left(s - /2\,\frac{3}{2}\right) + \frac{9}{4}}{\left(s - \frac{3}{2}\right)^2 - \left(\frac{\sqrt{13}}{2}\right)^2} - \frac{2}{\sqrt{13}} \frac{\frac{\sqrt{13}}{2}}{(s-3)^2 - \left(\frac{\sqrt{13}}{2}\right)^2}$$

$$= \frac{\frac{3}{2}\left(s - \frac{3}{2}\right)}{\left(s - \frac{3}{2}\right)^2 - \left(\frac{\sqrt{13}}{2}\right)^2} + \frac{9}{4} \times \frac{2}{\sqrt{13}} \frac{\frac{\sqrt{13}}{2}}{\left(s - \frac{3}{2}\right)^2 - \left(\frac{\sqrt{13}}{2}\right)^2} - \frac{2}{\sqrt{13}} \frac{\frac{\sqrt{13}}{2}}{\left(s - \frac{3}{2}\right)^2 - \left(\frac{\sqrt{13}}{2}\right)^2}$$

$$L^{-1}[f(s)] = \frac{3}{2} e^{\frac{3}{2}t} \cosh \frac{\sqrt{13}}{2}\, t + \frac{9}{2\sqrt{13}}\, e^{\frac{3}{2}t} \sinh \frac{\sqrt{13}}{2}\, t - \frac{2}{\sqrt{13}}\, e^{\frac{3}{2}t} \sinh \frac{\sqrt{13}}{2}\, t$$

$$= \frac{3}{2}\, e^{\frac{3}{2}t} \cosh \frac{\sqrt{13}}{2}\, t + \frac{5\sqrt{13}}{26}\, e^{\frac{3}{2}t} \sinh \frac{\sqrt{13}}{2}\, t$$

هام جدا :- اذا كان المقام يحلل نستخدم قوانين الكسور الجزئية لجعلها في صورة بسط ومقام ثم ايجاد التحويل العكسي

مثال

Find the inverse transform

(a) $\mathbf{F(s)=\frac{s+7}{s^2-3s-10}}$

$$\mathbf{F(s)=\frac{s+7}{(s\ +\ 2)(s\ -\ 5)}=\frac{A}{s+2}+\frac{B}{s-5}---(1)}$$

$$\boldsymbol{s+7=A(s-5)+B(s+2)}$$

When s=5

12=7B

$$\boldsymbol{B=\frac{12}{7}--(2)}$$

When s=-2

5=-7A

$$\boldsymbol{A=-\frac{5}{7}--(3)}$$

From 2,3 in 1

$$\mathbf{F(s)}=\boldsymbol{\frac{-\frac{5}{7}}{s+2}+\frac{\frac{12}{7}}{s-5}}$$

$$\mathbf{L^{-1}[f(s)] = -\frac{5}{7}L^{-1}\left[\frac{1}{s+2}\right] + \frac{12}{7}L^{-1}[\frac{1}{s-5}]}$$

$$= -\frac{5}{7}e^{-2t} + \frac{12}{7}\ e^{5t}$$

(b) $\mathbf{F(s) = \frac{2-5s}{(s-6)(s^2+11)}}$

$$\frac{2-5s}{(s-6)(s^2+11)} = \frac{A}{(s-6)} + \frac{Bs+c}{(s^2+11)} --(1)$$

$$2-5s = A(s^2+11) + (Bs+c)(s-6)$$

$$2-5s = As^2 + 11A + Bs^2 - 6Bs + cs - 6c$$

$$A + B = 0 --(2)$$

$$-6B + c = -5 \ , \ \ c = -5 + 6B --(3)$$

$$11A - 6c = 2 --(4)$$

From 3 in 4

$$11A - 6(-5 + 6B) = 2$$

$$11A + 30 - 36B = 2$$

$$11A - 36B = -28 --(5)$$

From 2 in 5

$$11A + 36A = -28$$

$$47A = -28$$

$$A = -\frac{28}{47} --(6)$$

<u>From 6 in 2</u>

$$\mathbf{B = \frac{28}{47} - -(7)}$$

<u>From 6 in 4</u>

$$\mathbf{c = -\frac{67}{47} - -(8)}$$

<u>From 6, 7, 8 in 1</u>

$$\mathbf{F(s) = \frac{-\frac{28}{47}}{s-6} + \frac{\frac{28}{47}s - \frac{67}{47}}{s^2+11}}$$

(c) $F(s)=\frac{25}{s^3(s^2+4s+5)}$

$$\frac{25}{s^3(s^2+4s+5)}=\frac{A}{s}+\frac{B}{s^2}+\frac{c}{s^3}+\frac{DS+E}{s^2+4s+5}--(1)$$

$$25=As^2(s^2+4s+5)+Bs(s^2+4s+5)+c(s^2+4s+5)+(Ds+E)(s^3)$$

$$25=As^4+4As^3+5As^2+Bs^3+4Bs^2+5Bs+cs^2+4cs+5c+Ds^4+Es^3$$

$$A+D=0--(2)$$

$$4A+B+E=0—(3)$$

$$5A+4B+c=0--(4)$$

$$5B+4c=0--(5)$$

$$5c=25\ ,,c=5--(6)$$

<u>From 6 in 5</u>

$$B=-4\ \ldots..(7)$$

<u>From 6, 7 in 4</u>

$$A=\frac{11}{5}\ \ldots..(8)$$

<u>From 7,8 in 3</u>

$$E=-\frac{24}{5}\ \ldots.(9)$$

From 8 in 2

$$\mathrm{D}=-\frac{11}{5}\ldots\ldots(10)$$

From 6,7,8,9,10 in 1

$$\frac{25}{s^3(s^2+4s+5)}=\frac{\frac{11}{5}}{s}+\frac{-4}{s^2}+\frac{5}{s^3}+\frac{-\frac{11}{5}s+\frac{-24}{5}}{s^2+4s+5}$$

$$=\frac{\frac{11}{5}}{s}-\frac{4}{s^2}+\frac{5}{s^3}+\frac{1}{5}[\frac{-11s-24}{s^2+4s+5}]$$

$$=\frac{\frac{11}{5}}{s}-\frac{4}{s^2}+\frac{5}{s^3}+\frac{1}{5}\left[\frac{-11s-24}{s^2+4s+4-4+5}\right]$$

$$=\frac{\frac{11}{5}}{s}-\frac{4}{s^2}+\frac{5}{s^3}+\frac{1}{5}\left[\frac{-11s-24}{(s^2+4s+4)+1}\right]$$

$$=\frac{\frac{11}{5}}{s}-\frac{4}{s^2}+\frac{5}{s^3}+\frac{1}{5}\left[\frac{-11(s+2-2)-24}{(s+2)^2+1}\right]$$

$$=\frac{\frac{11}{5}}{s}-\frac{4}{s^2}+\frac{5}{s^3}+\frac{1}{5}\left[\frac{-11(s+2-2)}{(s+2)^2+1}+\frac{-24}{(s+2)^2+1}\right]$$

$$=\frac{\frac{11}{5}}{s}-\frac{4}{s^2}+\frac{5}{s^3}+\frac{1}{5}\left[\frac{-11(s+2)+22}{(s+2)^2+1}+\frac{-24}{(s+s)^2+1}\right]$$

$$=\frac{\frac{11}{5}}{s}-\frac{4}{s^2}+\frac{5}{s^3}+\frac{1}{5}\left[\frac{-11(s+2)}{(s+2)^2+1}+\frac{22}{(s+2)^2+1}+\frac{-24}{(s+2)^2+1}\right]$$

$$L^{-1}[f(s)]=\frac{11}{5}-4t+\frac{5t^2}{2}+\frac{1}{5}[-11e^{-2t}\cos t-2e^{-2t}\sin t]$$

(d) $\mathbf{F(s) = \frac{86s-78}{(s+3)(s-4)(5s-1)}}$

$$\frac{86s-78}{(s+3)(s-4)(5s-1)} = \frac{A}{s+3} + \frac{B}{s-4} + \frac{c}{5s-1} - (1)$$

$$86s - 78 = A(s-4)(5s-1) + B(s+3)(5s-1) + C(s+3)(s-4)$$

<u>when $s = 4$</u>

$$B = 2 - -(2)$$

<u>**When s=-3**</u>

$$A = -3\text{-----}(3)$$

when $s = \frac{1}{5}$

$$c = 5 - -(4)$$

<u>**From 2,3,4 in 1**</u>

$$F(s) = -\frac{3}{s+3} + \frac{2}{s-4} + \frac{5}{5s-1}$$

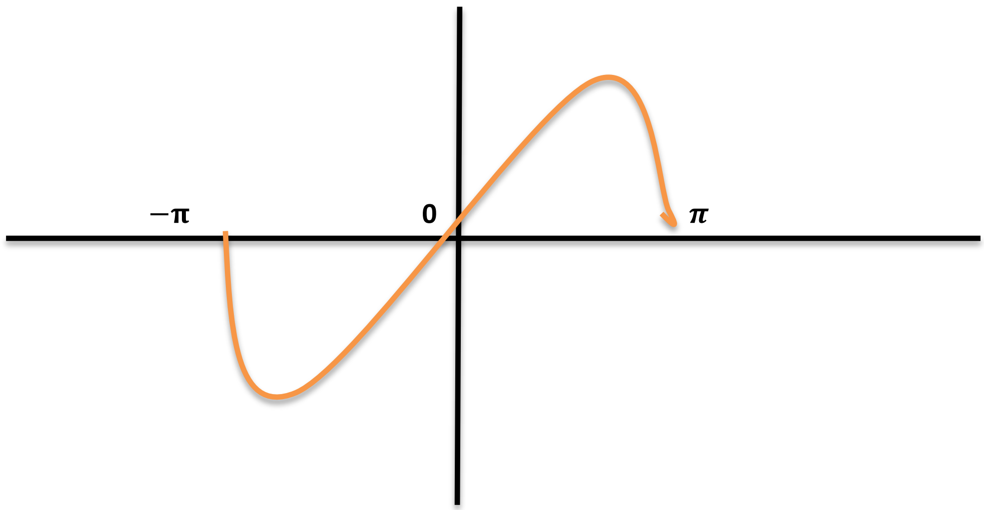
$$L^{-1}[f(s)] = -3e^{-3t} + 2e^{4t} + e^{\frac{1}{5}t}$$

Fourier Transforms

تحويلات فورير

اساسيات الفصل :- نتعرف على شكل دالتى جيب الزاوية وجيب تمام الزاوية

(1) Sin x



نجد ان المساحة فوق المنحني تساوي المسافة اسفل المنحني يبقا التكامل بصفر

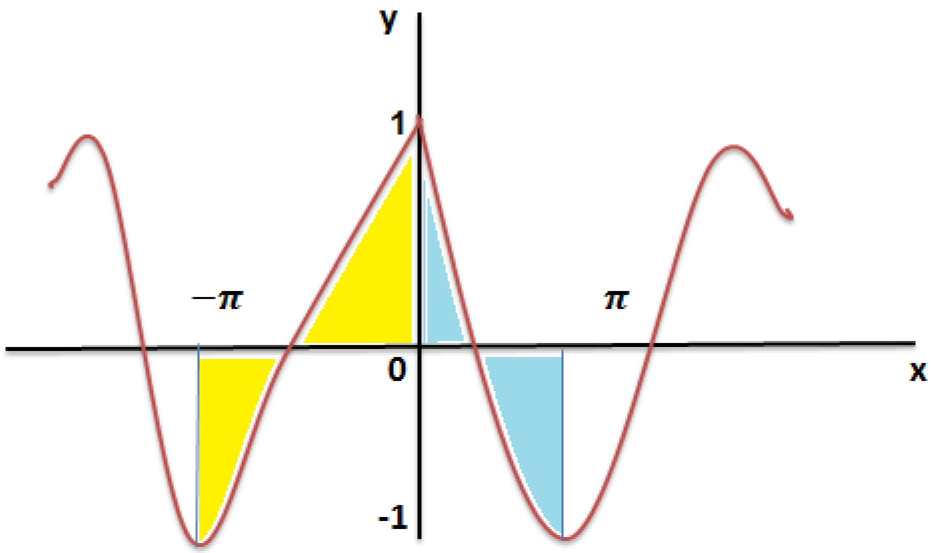
$$\int_{-\pi}^{\pi} \sin x \, dx = 0$$

من الشكل نلاحظ ان جيب الزاوية لها قيمة واحده وهيا صفر حيث

Sin 0=0

Sin 180=0

(2) $cos\ x$



نلاحظ ان جيب تمام الزاوية لها قيمتان 1و-1 حيث

$$cos\ 0 = 1$$

$$cos\ 180 = -1$$

كذلك هناك تماثل يبقا التكامل بصفر

$$\int_{-\pi}^{\pi} cos\ x = 0$$

ممكن نكتبها كد

$$cos\ nx = (-1)^n$$

ممكن تعميم القاعدة

$$\cos(n+1)x = (-1)^{n+1}$$

$$cos(n-1)x = (-1)^{n-1}$$

Prove $\int_{-\pi}^{\pi} sin\, x\, dx \ = 0$

$$\int_{-\pi}^{\pi} sin\, x\, dx = -cos\, x = [-cos(-\pi) + cos(\pi)] = 1 - 1 = 0$$

Prove $\int_{-\pi}^{\pi} cos\, x\, dx = 0$

$$\int_{-\pi}^{\pi} \cos x \, dx = \sin x = [\sin(-\pi) - \sin(\pi)] = 0 - 0 = 0$$

ثلاث قوانين هامة

$$\sin(mx)\cos(n\,x) = \frac{1}{2}[\sin(m+n)x + \sin(m-n)x]$$

$$\cos(mx)\,\cos(nx) = \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]$$

$$\sin(mx)\sin(nx) = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$$

Prove $\int_{-\pi}^{\pi} \sin nx\, dx = 0$

$$\int_{-\pi}^{\pi} \sin nx\, dx = -\frac{\cos nx}{n} = \left[\frac{-\cos n(-\pi)}{n} + \frac{\cos n(\pi)}{n}\right] = 0$$

prove $\int_{-\pi}^{\pi} \cos nx\, dx = 0$

$$\int_{-\pi}^{\pi} \cos nx\, dx = \frac{\sin nx}{n} = \left[\frac{\sin n(-\pi)}{n} - \frac{\sin n(\pi)}{n}\right] = 0$$

prove $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\, dx = 0$

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\, dx = \int_{-\pi}^{\pi} \frac{1}{2}[\sin(m+n)x + \sin(m-n)x]dx$$

$$= \frac{1}{2}\left[-\frac{\cos(m+n)x}{m+n} - \frac{\cos(m-n)x}{m-n}\right]$$

$$= \frac{1}{2}\left[-\cancel{\frac{\cos(m+n)(-\pi)}{m+n}} - \cancel{\frac{\cos(m-n)(-\pi)}{m-n}} + \cancel{\frac{\cos(m+n)(\pi)}{m+n}} + \cancel{\frac{\cos(m-n)(\pi)}{m-n}}\right]$$

=0

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prove $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx$

$$\int_{-\pi}^{\pi} cos(mx)cos(nx)dx = \int_{-\pi}^{\pi} \frac{1}{2}[cos(m+n)x + cos(m-n)x]dx$$

<u>If $m \neq n$</u>

$$= \frac{1}{2}\left[\frac{sin(m+n)x}{(m+n)} + \frac{sin(m-n)x}{m-n}\right]$$

$$= \frac{1}{2}[\frac{\sin(m+n)(-\pi)}{(m+n)} + \frac{\sin(m-n)(-\pi)}{m-n} - \frac{\sin(m+n)(\pi)}{(m+n)} - \frac{\sin(m+n)(\pi)}{m-n}]$$

=0

<u>If m=n</u>

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx = \int_{-\pi}^{\pi} \frac{1}{2}[\cos(2n)x + \cos(0)x]dx$$

$$= \frac{1}{2}[\int_{-\pi}^{\pi} cos(2n)x\, dx + \int_{-\pi}^{\pi} dx\,]$$

$$= \frac{1}{2}\left[\frac{sin(2n)x}{2n} + x\right]$$

$$= \frac{1}{2}\left[\frac{sin(2n)(\pi)}{2n} - \frac{sin(2n)(-\pi)}{2n} + \pi + \pi\right]$$

$$= \frac{1}{2}[0 - 0 + \pi + \pi]$$

$$= \frac{1}{2}[2\pi]$$

$$= \pi$$

Find $\int_{-\pi}^{\pi} sin(m)x\ sin(n)x\ dx$

<u>if n ≠ m</u>

$$\int_{-\pi}^{\pi} sin(m)x\ sin(n)x = \frac{1}{2}\int_{-\pi}^{\pi}[cos(m-n)x - cos(m+n)x]$$

$$= \frac{1}{2}\left[\frac{sin(m-n)x}{m-n} - \frac{sin(m+n)x}{m+n}\right]$$

$$= \frac{1}{2}\left[\frac{\sin(m-n)(-\pi)}{m-n} - \frac{\sin(m+n)(-\pi)}{m+n} - \frac{\sin(m-n)(\pi)}{m-n} + \frac{\sin(m+n)x}{m+n}\right]$$

=0

<u>If n=m</u>

$$\int_{-\pi}^{\pi} \sin(m)x \sin(n)x = \frac{1}{2}\int_{-\pi}^{\pi}[\cos(0)x - \cos(2n)x]\, dx$$

$$= \frac{1}{2}\left[x - \frac{sin(2n)x}{2n}\right]$$

$$= \frac{1}{2}\left[\pi - \frac{sin(2n)(\pi)}{2n} + \pi + \frac{sin(2n)(-\pi)}{2n}\right]$$

$$= \frac{1}{2}[\pi - 0 + \pi + 0]$$

$$\frac{1}{2} \times 2\pi = \pi$$

ملخص القوانين

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx$$ If $m \neq n --\rightarrow 0$ $m = n-\rightarrow \pi$	$$\int_{-\pi}^{\pi} \sin(m)x\,\sin(n)x\,dx$$ If $m \neq n-\rightarrow 0$ $m = n-\rightarrow \pi$
$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx = 0$$ $if\ n \neq m-\rightarrow 0$ $n = m-\rightarrow 0$	

قوانين اخرى

$$\int_{-\pi}^{\pi} \cos^2(nx)dx = \pi$$

$$\int_{-\pi}^{\pi} \sin^2 (nx)dx = \pi$$

<u>Fourier series</u>

$$F(x) = ao + \sum an \cos(nx) + bn \sin(nx)$$

<u>Where</u>

$$ao = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\ dx$$

$$an = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx$$

$$bn = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx$$

<u>خطوات حل المسائل</u>

! كتابة الصيغة العامة لفورير

! ايجاد الثلاث مجاهيل

$$ao , an , bn$$

! التعويض في الصيغة العامة لفورير

Prove that

$$ao = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$F(x) = ao + \sum an\cos(nx) + bn\sin(nx)$$

By integration

$$\int_{-\pi}^{\pi} f(x)dx = ao\int_{-\pi}^{\pi} dx + \sum an\underbrace{\int_{-\pi}^{\pi}\cos(nx)dx}_{0} + bn\underbrace{\int_{-\pi}^{\pi}\sin(nx)dx}_{0}$$

$$\int_{-\pi}^{\pi} f(x)dx = ao[x]$$

$$\int_{-\pi}^{\pi} f(x)dx = ao[\pi - (-\pi)]$$

$$\int_{-\pi}^{\pi} f(x)dx = 2\pi\, ao$$

$$ao = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

Prove that

$$an = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx$$

$$F(x) = ao + \sum an\cos(nx) + bn\sin(nx)$$

x cos(nx)

$$\mathrm{f(x)\ cos(nx) = aocos(nx)} + \sum \mathrm{an\ cos(nx)cos(nx) + bn\ sin(nx)cos(nx)}$$

by integration

$$\int_{-\pi}^{\pi} \mathrm{f(x)cos(nx)dx} = \mathrm{ao}\underbrace{\int_{-\pi}^{\pi} \mathrm{cos(nx)\ dx}}_{0} + \sum \mathrm{an}\underbrace{\int_{-\pi}^{\pi} \mathrm{cos(nx)cos(nx)}}_{\pi} + \mathrm{bn}\underbrace{\int_{-\pi}^{\pi} sin(nx)cos(nx)}_{0}$$

$$\int_{-\pi}^{\pi} f(x)\cos(nx)dx = \pi \times an$$

$$\mathrm{an} = \frac{1}{\pi}\int_{-\pi}^{\pi} \mathrm{f(x)cos(nx)dx}$$

Prove that

$$bn = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, sin(nx)\, dx$$

$$\mathrm{F(x) = ao + \sum an\ cos(nx) + bn\ sin(nx)}$$

$\times \mathrm{sin(nx)}$

$$\mathrm{f(x)\ sin(nx) = aosin(nx) + \sum an\ cos(nx)sin(nx) + bn\ sin(nx)sin(nx)}$$

by integration

$$\int_{-\pi}^{\pi} \mathrm{f(x)sin(nx)dx = ao} \int_{-\pi}^{\pi} \underbrace{\mathrm{sin(nx)}\ \mathrm{dx}}_{\mathbf{0}} + \sum \mathrm{an} \int_{-\pi}^{\pi} \underbrace{\mathrm{cos(nx)sin\ (nx)}dx}_{\mathbf{0}} + bn \int_{-\pi}^{\pi} \underbrace{sin(nx)sin(nx)dx}_{\boldsymbol{\pi}}$$

$$\int_{-\pi}^{\pi} \mathrm{f(x)\ sin(nx)dx = \pi \times bn}$$

$$\mathrm{bn = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\ sin(nx)dx}$$

Find the Fourier series of the function

$$F(x) = x \ , \quad -\pi \le x \le \pi$$

$$F(x) \sim ao + \sum an\cos(nx) + bn\sin(nx) --(1)$$

$$ao = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x\,dx$$

$$= \frac{1}{2\pi}[\frac{x^2}{2}]$$

$$= \frac{1}{4\pi}[(-\pi)^2 - (\pi)^2]$$

$$ao = 0 --- (2)$$

$$an = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos(nx)dx$$

$$= \frac{1}{\pi}\left[\frac{x\sin(nx)}{n} - \int_{-\pi}^{\pi}\frac{\sin(nx)}{n}\,dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi\sin(-n\pi)}{n} - \frac{\pi\sin(n\pi)}{n}\right]$$

$$= \frac{1}{\pi}\left[\cancel{\frac{\pi\sin(n\pi)}{n}} - \cancel{\frac{\pi\sin(n\pi)}{n}}\right]$$

$$an = 0 - - - (3)$$

$$\mathbf{bn} = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin(nx)dx$$

$$= \frac{1}{\pi}\left[-\frac{x\cos(nx)}{n} + \int_{-\pi}^{\pi}\frac{\cos(nx)}{n}\,dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi\cos(-n\pi)}{n} - \frac{\pi\cos(n\pi)}{n} + \cancel{\frac{\sin(-n\pi)}{n^2}} - \cancel{\frac{\sin(n\pi)}{n^2}}\right]$$

$$= \frac{1}{\pi}\left[\frac{-2\pi\cos(n\pi)}{n}\right]$$

$$= \frac{-2}{n}\cos(n\pi)$$

$$= (-1)\frac{2}{n}\,(-1)^n$$

$$= \frac{2}{n}(-1)^{n+1} - -(4)$$

لاحظ ان ن جيب التمام ليها قيمتين -1و1 فعلشان كدا حطيت قيمتها ب

$$(-1)^n$$

From 4, 3,2 in 1

$$\mathbf{F(x)\sim\sum\frac{2}{n}(-1)^{n+1}\sin(nx)}$$

$$F(x)\sim\left(2sinx-sin2x+2\frac{sin3x}{3}-\frac{sin4x}{2}\dots\dots.\right)$$

$$\sim2\left(sinx-\frac{sin2x}{2}+\frac{sin3x}{3}-\frac{sin4x}{4}\dots..\right)$$

Find the Fourier series

$$f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ \pi \ , & 0 \le x \le \pi \end{cases}$$

$$F(x) = ao + \sum an\cos(nx) + bn\sin(nx) - -(1)$$

$$ao = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} f(x)dx + \int_{0}^{\pi} f(x)dx\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} 0\ dx + \int_{0}^{\pi} \pi\ dx\right]$$

$$= \frac{1}{2\pi} \times \pi \int_{0}^{\pi} dx$$

$$= \frac{1}{2}[x]$$

$$= \frac{1}{2}[\pi - 0]$$

$$ao = \frac{\pi}{2} \dots \dots .. (2)$$

$$an = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\,(nx)dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos(nx)dx + \int_{0}^{\pi} f(x)\,\cos(nx)dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,\cos(nx)dx + \int_{0}^{\pi} \pi\,\cos(nx)dx\right]$$

$$= \frac{1}{\pi} \times \pi \int_{0}^{\pi} \cos\,(nx)dx$$

$$= \left[\frac{\sin(nx)}{n}\right]$$

$$an = 0 - - - (3)$$

$$\mathbf{bn} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\sin(nx)dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\, \sin(nx)dx + \int_{0}^{\pi} f(x)\, \sin(nx)dx \right]$$

$$= \frac{1}{\pi} \times \pi \int_0^{\pi} \sin(nx)dx$$

$$= -\frac{\cos(nx)}{n}$$

$$= -\frac{\cos(n\pi)}{n} + \frac{\cos(0)}{n}$$

$$= \frac{-(-1)^n}{n} + \frac{1}{n}$$

$$bn = \frac{1}{n}(1 - (-1)^n) \dots\dots\dots..(4)$$

From 4,3,2 in 1

$$\mathbf{F(x)} \sim \sum \frac{\mathbf{1}}{\mathbf{n}} (\mathbf{1} - (-\mathbf{1})^{\mathbf{n}}) \mathbf{sin(\ nx)}$$

$$\sim \frac{\pi}{2} + \left(2sinx + \frac{2sin3x}{3} + \frac{2sin5x}{5} + \frac{2sin7x}{7} \ldots \ldots \right)$$

$$\sim \frac{\pi}{2} + 2 \left(sinx + \frac{sin3x}{3} + \frac{sin5x}{5} + \frac{sin7x}{7} + \cdots \ldots . \right)$$

Find the Fourier series

$$f(x)=\begin{cases}-\dfrac{\pi}{2}, & -\pi\le x<0\\ \dfrac{\pi}{2}, & 0\le x\le\pi\end{cases}$$

$$F(x)\sim ao+\sum an\cos(nx)+bn\sin(nx)--(1)$$

$$ao=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(x)dx$$

$$=\frac{1}{2\pi}\left[\int_{-\pi}^{0}f(x)dx+\int_{0}^{\pi}f(x)dx\right]$$

$$=\frac{1}{2\pi}\left[\int_{-\pi}^{0}-\frac{\pi}{2}dx+\int_{0}^{\pi}\frac{\pi}{2}\,dx\right]$$

$$=\frac{1}{2\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0}dx+\frac{\pi}{2}\int_{0}^{\pi}dx\right]$$

$$=\frac{1}{2\pi}\left[-\frac{\pi}{2}[x]+\frac{\pi}{2}[x]\ \right]$$

$$=\frac{1}{2\pi}\left[-\frac{\pi}{2}(0+\pi)+\frac{\pi}{2}(\pi-0)\right]=\frac{1}{2\pi}\left[-\cancel{\frac{\pi^2}{2}}+\cancel{\frac{\pi^2}{2}}\right]$$

$$ao=0---(2)$$

$$\mathbf{an} = \frac{1}{\pi}\int_{-\pi}^{\pi} \mathbf{f(x)\cos(nx)dx}$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} \mathbf{f(x)\cos(nx)\,dx} + \int_{0}^{\pi} \mathbf{f(x)\cos(nx)dx}\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\cos(nx)dx + \int_{0}^{\pi}\frac{\pi}{2}\cos(nx)\,dx\right]$$

$$= \frac{1}{\pi}\times\frac{-\pi}{2}\left[\frac{\sin(nx)}{n} - \frac{\sin(nx)}{n}\right]$$

$$an = 0 - - - (3)$$

$$\mathbf{bn} = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\sin(nx)\,dx + \int_{0}^{\pi}\frac{\pi}{2}\sin(nx)\,dx\right]$$

$$= \frac{1}{\pi} \times -\frac{\pi}{2}\left[\int_{-\pi}^{0} \sin(nx)dx - \int_{0}^{\pi} \sin(nx)dx\right]$$

$$= \frac{-1}{2}\left[-\frac{cos(nx)}{n} + \frac{cos(nx)}{n}\right]$$

$$= -\frac{1}{2}\left[-\frac{cos0}{n} + \frac{cos(n\pi)}{n} + \frac{cos(n\pi)}{n} - \frac{cos0}{n}\right]$$

$$= -\frac{1}{2}\left[-\frac{2}{n} + \frac{2cos(n\pi)}{n}\right]$$

$$= \frac{1}{n}\left(1 - cos(n\pi)\right)$$

$$bn = \frac{1}{n}(1 - (-1)^n) \ldots\ldots\ldots (4)$$

From 4,3,2 in 1

$$\mathbf{F(x)} \sim \sum \frac{\mathbf{1}}{\mathbf{n}} (\mathbf{1} - (-\mathbf{1})^{\mathbf{n}} \, \mathbf{sin(nx)}$$

$$\sim \mathbf{2}\left(\boldsymbol{sinx} + \frac{\boldsymbol{sin3x}}{\mathbf{3}} + \frac{\boldsymbol{sin5x}}{\mathbf{5}} + \cdots \ldots \ldots\right)$$

اذا كانت حدود التكامل محصورة في الفترة

[-L, L]

$$F(x) \sim ao + \sum an\cos\left(\frac{n\pi x}{L}\right) + bn\sin\left(\frac{n\pi x}{L}\right)$$

$$ao = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$an = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$bn = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{2}\right)dx$$

Find the Fourier series

$$f(x) = \begin{cases} 0 & , -2 \le x < 0 \\ x & , 0 \le x \le 2 \end{cases}$$

$$F(x) \sim ao + \sum an \cos\left(\frac{n\pi x}{L}\right) + bn \sin\left(\frac{n\pi x}{L}\right) - -(1)$$

$$ao = \frac{1}{2L}\int_{-L}^{L} f(x)dx = \frac{1}{2L}\left[\int_{-2}^{0} f(x)dx + \int_{0}^{2} f(x)dx\right]$$

$$= \frac{1}{4}\left[\int_{-2}^{0} 0\,dx + \int_{0}^{2} x\,dx\right]$$

$$= \frac{1}{4}\left[\frac{x^2}{2}\right]$$

$$= \frac{1}{8}\left[x^2\right]$$

$$= \frac{1}{8}[4 - 0]$$

$$ao = \frac{1}{2} - -(2)$$

$$\mathrm{an} = \frac{1}{L}\int_{-2}^{2} f(x)\cos\left(\frac{n\pi x}{L}\right)dx = \frac{1}{L}\left[\int_{-2}^{0} f(x)\cos\left(\frac{n\pi x}{L}\right)dx + \int_{0}^{2} f(x)\cos\left(\frac{n\pi x}{L}\right)dx\right]$$

$$= \frac{1}{2}\left[\int_{-2}^{0} 0\cos\left(\frac{n\pi x}{2}\right)dx + \int_{0}^{2} x\cos\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{1}{2}\int_{0}^{2} x\cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \frac{1}{2}\left[\frac{2x}{n\pi}\sin\left(\frac{n\pi x}{2}\right) - \int_{0}^{2}\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{1}{2}\left[\left[\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\right] + \left[\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right)\right]\right]$$

$$= \frac{1}{2}\left[\left[\frac{2}{n\pi}\sin(n\pi) - \frac{2}{n\pi}\sin(0)\right] + \left[\frac{4}{n^2\pi^2}\cos(n\pi) - \frac{4}{n^2\pi^2}\cos(0)\right]\right]$$

$$= \frac{1}{2}\times\frac{4}{n^2\pi^2}[\cos(n\pi) - 1]$$

$$an = \frac{2}{n^2\pi^2}[(-1)^n - 1] - - - (3)$$

$$bn = \frac{1}{L}\int_{-2}^{2} f(x)sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\left[\int_{-2}^{0} f(x)sin\left(\frac{n\pi x}{L}\right)dx + \int_{0}^{2} f(x)sin\left(\frac{n\pi x}{L}\right)dx\right]$$

$$= \frac{1}{2}\left[\int_{-2}^{0} 0\ sin\left(\frac{n\pi x}{2}\right)dx + \int_{0}^{2} x\ sin\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{1}{2}\left[\int_{0}^{2} x\ sin\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{1}{2}\left[-\frac{2x}{n\pi}cos\left(\frac{n\pi x}{2}\right) + \int_{0}^{2}\frac{2}{n\pi}cos\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{1}{2}\left[\left[-\frac{2x}{n\pi}cos\left(\frac{n\pi x}{2}\right)\right] + \left[\frac{4}{n^2\pi^2}sin\left(\frac{n\pi x}{2}\right)\right]\right]$$

$$= \frac{1}{2}\left[\left[-\frac{4}{n\pi}cos(n\pi) + 0\right] + \left[\frac{4}{n^2\pi^2}sin(n\pi) - \frac{4}{n^2\pi^2}sin(0)\right]\right]$$

$$= \frac{1}{2}\times\frac{4}{n\pi}[-cos(n\pi)]$$

$$= \frac{2}{n\pi}[(-1)\ (-1)^n]$$

$$bn = \frac{2}{n\pi}\left[(-1)^{n+1}\right] - - - (4)$$

From 2,3,4 in 1

$$\mathbf{F(x)} \sim \frac{\mathbf{1}}{\mathbf{2}} + \sum \frac{\mathbf{2}}{\mathbf{n^2\pi^2}}[(-\mathbf{1})^{\mathbf{n}} - \mathbf{1}]\mathbf{\cos}\left(\frac{\mathbf{n\pi x}}{\mathbf{2}}\right) + \frac{\mathbf{2}}{\mathbf{n\pi}}[(-\mathbf{1})^{\mathbf{n+1}}]\,\mathbf{\sin}\left(\frac{\mathbf{n\pi x}}{\mathbf{2}}\right)$$

Find the Fourier cosine

$$\mathbf{ao} = \frac{1}{\pi}\int_{0}^{\pi} \mathbf{f(x)dx}\,,$$

$$an = \frac{2}{\pi}\int_{0}^{\pi} f(x)cos(nx)dx,$$

$$bn = 0$$

Find the Fourier sine

$$ao = 0,$$

$$an = 0,$$

$$\mathbf{bn} = \frac{2}{\pi}\int_{0}^{\pi} \mathbf{f(x)sin(nx)dx}$$

خلي بالك هنا فترة التكامل هتبقا

$$[0\,,\pi]$$

Find the Fourier Cosine series

$$f(x) = x \qquad\qquad for \qquad\qquad x \in [0, \pi]$$

$$f(x) \sim ao + \sum an\, cos(nx) + bn\, sin(nx) \,--(1)$$

It's cosine

$$\mathbf{bn = 0 \,--(2)}$$

$$ao = \frac{1}{\pi}\int_{0}^{\pi} f(x)dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} x\, dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]$$

$$= \frac{1}{2\pi}\left[\pi^2 - 0\right]$$

$$ao = \frac{\pi}{2} \,--(3)$$

$$\mathbf{an} = \frac{2}{\pi}\int_{0}^{\pi} \mathbf{f(x)\cos(nx)dx}$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x\cos(nx)dx$$

$$= \frac{2}{\pi}\left[\frac{x\sin(nx)}{n} - \int_{0}^{\pi}\frac{\sin(nx)}{n}\,dx\right]$$

$$= \frac{2}{\pi}\left[\left[\frac{x\sin(nx)}{n}\right] + \left[\frac{\cos(nx)}{n^2}\right]\right]$$

$$= \frac{2}{\pi}\left[[\pi\sin(n\pi) - 0] + \left[\frac{\cos(n\pi)}{n^2} - \frac{\cos(0)}{n^2}\right]\right]$$

$$= \frac{2}{\pi}\left[\frac{\cos(n\pi)}{n^2} - \frac{1}{n^2}\right]$$

$$an = \frac{2}{n^2\pi}[(-1)^n - 1] - - - -(4)$$

From 2,3,4 in 1

$$\mathbf{F(x)\sim\frac{\pi}{2}+\sum\frac{2}{n^2\pi}[(-1)^n-1]\cos(nx)}$$

$$\boldsymbol{\sim\frac{\pi}{2}+\frac{4}{\pi}\left(\cos x+\frac{\cos 3x}{9}+\frac{\cos 5x}{25}+\cdots\ldots..\right)}$$

Find the Fourier sine series

$$\mathbf{F(x) = 1 \quad for \quad x\epsilon[0, \pi]}$$

$$f(x) \sim ao + \sum an\cos(nx) + bn\sin(nx) - - - (1)$$

It's sine

$$\mathbf{ao = 0 - - - (2)}$$

$$\mathbf{an = 0 - - - (3)}$$

$$\mathbf{bn = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)dx}$$

$$= \frac{2}{\pi}\int_0^{\pi} \sin(nx)dx$$

$$= \frac{2}{\pi}[-\frac{\cos(nx)}{n}]$$

$$= -\frac{2}{n\pi}[\cos[nx]]$$

$$= -\frac{2}{n\pi}[\cos(n\pi) - \cos(0)]$$

$$= -\frac{2}{n\pi}[(-1)^n - 1]$$

$$\mathbf{bn} = \frac{\mathbf{2}}{\mathbf{n\pi}}[\mathbf{1} - (-\mathbf{1})^{\mathbf{n}}] - - - (\mathbf{4})$$

<u>From 2,3,4 in 1</u>

$$\mathbf{f}(\mathbf{x}) \sim \sum \frac{\mathbf{2}}{\mathbf{n\pi}}[\mathbf{1} - (-\mathbf{1})^{\mathbf{n}} \ \mathbf{sin}(\mathbf{nx})$$

$$\sim \frac{4}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots.\right)$$

Find the Fourier sine series

$$\mathbf{F(x) = \cos(x) \qquad for \qquad x\epsilon[0, \pi]}$$

$$\mathbf{F(x) \sim ao + \sum an\cos(nx) + bn\sin(nx) - -(1)}$$

<u>**It's sine**</u>

$$\mathbf{ao = 0 - -(2)}$$

$$\boldsymbol{an} = \mathbf{0 - -(3)}$$

$$\mathbf{bn = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)dx}$$

$$= \frac{2}{\pi}\int_0^{\pi} \cos(x)\sin(nx)dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{1}{2}[\sin(n+1)x + \sin(n-1)x]\,dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos(n+1)x}{n+1} - \frac{\cos(n-1)x}{n-1}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\cos(n+1)\pi}{n+1} - \frac{\cos(n-1)\pi}{n-1} + \frac{\cos(0)}{n+1} + \frac{\cos(0)}{n-1}\right]$$

$$=\frac{1}{\pi}\left[-\frac{(-1)^{n+1}}{n+1}-\frac{(-1)^{n-1}}{n-1}+\frac{1}{n+1}+\frac{1}{n-1}\right]$$

$$=\frac{1}{\pi}\left[\frac{-(n-1)(-1)^{n+1}-(n+1)(-1)^{n-1}}{n^2-1}+\frac{n-1+n+1}{n^2-1}\right]$$

$$=\frac{1}{\pi}\left[\frac{-(n-1)(-1)^{n+1}-(n+1)(-1)^{n-1}}{n^2-1}+\frac{2n}{n^2-1}\right]$$

$$=\frac{1}{\pi}\left[\frac{(n-1)(-1)^{n}+(n+1)(-1)^{n}}{n^2-1}+\frac{2n}{n^2-1}\right]$$

$$=\frac{1}{\pi}\left[\frac{(-1)^{n}(n-1+n+1)}{n^2-1}+\frac{2n}{n^2-1}\right]$$

$$=\frac{1}{\pi}\left[\frac{2n(-1)^{n}}{n^2-1}+\frac{2n}{n^2-1}\right]$$

$$= \frac{1}{\pi}\left[\frac{2n(-1)^n + 2n}{n^2 - 1}\right]$$

$$= \frac{2n}{\pi}\left[\frac{(-1)^n + 1}{n^2 - 1}\right]$$

$$bn = \frac{2n}{\pi}\left[\frac{1 + (-1)^n}{n^2 - 1}\right] - - - (4)$$

<u>From 2,3,4 in 1</u>

$$\cos(x) \sim \frac{8}{\pi}\sum \frac{n\sin(2nx)}{4n^2 - 1}$$

Ch3

$$\mathbf{F(x)=\int_{-\infty}^{\infty}F(k)e^{2\pi ikx}\ \ dk}$$ **.............(1)**

$$\boldsymbol{F(k)=\int_{-\infty}^{\infty}F(x)e^{-2\pi ikx}\ \ dx}$$ **..........(2)**

From 1,2

$$\boldsymbol{Fx[F(x)](k)=\int_{-\infty}^{\infty}F(x)e^{-2\pi ikx}\ dx}$$

Prove that $Fx[F^n(x)](k) = (2\pi ik)^n\, Fx[F(x)](k)$

Where $Fx[F(x)](k) = \int_{-\infty}^{\infty} F(x)e^{-2\pi ikx}\ dx$

$$Fx[F(x)](k) = \int_{-\infty}^{\infty} F(x)e^{-2\pi ikx}\ dx$$

By differentiation

$$Fx[F^{/}(x)](k) = \int_{-\infty}^{\infty} \underset{\underset{dv}{\downarrow}}{F^{/}(x)}\,\underset{\underset{u}{\downarrow}}{e^{-2\pi ikx}}\ dx$$

$$u = e^{-2\pi ikx} \qquad\qquad dv = F^{/}(x)$$

$$du = (-2\pi ik)e^{-2\pi ikx} \qquad\qquad v = F(x)$$

$$Fx\left[F^{/}(x)\right](k) = F(x)e^{-2\pi ikx} - \int_{-\infty}^{\infty} F(x)(-2\pi ik)e^{-2\pi ikx}\,dx$$

$$= \cancelto{0}{F(x)e^{-2\pi ikx}} + (2\pi ik)\int_{-\infty}^{\infty} F(x)e^{-2\pi ikx}\ dx$$

$$= (2\pi ik)\int_{-\infty}^{\infty} F(x)e^{-2\pi ikx}\,dx\text{.........(1)}$$

$$\mathbf{Fx}\left[\mathbf{F}^{//}\,(\mathbf{x})\right](\mathbf{k}) = (\mathbf{2\pi ik})\left[\mathbf{F}(\mathbf{x})\mathbf{e}^{-\mathbf{2\pi ikx}} - \int_{-\infty}^{\infty}\mathbf{F}(\mathbf{x})(-\mathbf{2\pi ik})\mathbf{e}^{-\mathbf{2\pi ikx}}\,\mathbf{dx}\right]$$

$$= (\mathbf{2\pi ik})\left[\cancelto{0}{\mathbf{F}(\mathbf{x})\mathbf{e}^{-\mathbf{2\pi ikx}}} + (\mathbf{2\pi ik})\int_{-\infty}^{\infty}\mathbf{F}(\mathbf{x})\mathbf{e}^{-\mathbf{2\pi ikx}}\,\mathbf{dx}\right]$$

$$= (\mathbf{2\pi ik})^2\int_{-\infty}^{\infty}\mathbf{F}(\mathbf{x})\mathbf{e}^{-\mathbf{2\pi ikx}}\,\mathbf{dx}\text{............}(2)$$

<u>From 1,2</u>

$$\mathbf{Fx}[\mathbf{F}^{\mathbf{n}}(\mathbf{x})](\mathbf{k}) = (\mathbf{2\pi ik})^{\mathbf{n}}\int_{-\infty}^{\infty}\mathbf{F}(\mathbf{x})\,\mathbf{e}^{-\mathbf{2\pi ikx}}\mathbf{dx}$$

$$= (\mathbf{2\pi ik})^{\mathbf{n}}\,\boldsymbol{Fx}[\boldsymbol{F}(\boldsymbol{x})](\boldsymbol{k})$$

$$\mathrm{F(x)} = \frac{1}{2}[F(x) + F(x)]$$

$$= \frac{1}{2}[F(x) + F(x) + F(-x) - F(-x)]$$

$$= \frac{1}{2}\left[[\mathrm{F(x)} + \mathrm{F(-x)}] + [\mathrm{F(x)} - \mathrm{F(-x)}]\right]$$

$$= E + O$$

$$\mathrm{Fx[\cos(2\pi kox)F(x)](k)} = \int_{-\infty}^{\infty} \mathrm{\cos(2\pi kox)F(x)e^{-2\pi ikx}\,dx}$$

$$where\ \ cos(2\pi kox) = \frac{e^{2\pi ikox} + e^{-2\pi ikox}}{2}$$

$$\mathrm{Fx[\cos(2\pi kox)F(x)](k)} = \int_{-\infty}^{\infty} \frac{\mathrm{e^{2\pi ikox} + e^{-2\pi ikox}}}{2}\mathrm{F(x)e^{-2\pi ikx}dx}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} F(x)e^{-2\pi i(k-ko)x} + F(x)e^{-2\pi i(k+ko)x}$$

$$= \frac{1}{2}[F(k - ko) + F(k + ko)]$$

$$\mathbf{F(k)} = \int_{-\infty}^{\infty} F(x)e^{-2\pi ikx}\, dx$$

$$\mathbf{F^{/}(k)} = \frac{\mathbf{d}}{\mathbf{dk}} \int_{-\infty}^{\infty} F(x)e^{-2\pi ikx}\, dx$$

$$= (-2\pi i) \int_{-\infty}^{\infty} X\, e^{-2\pi ikx}\, dx$$

$$F^{n}(k) = (-2\pi i)^{n} \int_{-\infty}^{\infty} X^{n}\, e^{-2\pi ikx}\, dx \text{-------(1)}$$

$$u1 = \int_{-\infty}^{\infty} X\, e^{-2\pi ikx}\, dx$$

$$un = \int_{-\infty}^{\infty} X^{n}\, e^{-2\pi ikx}\, dx \ldots \ldots \ldots \ldots (2)$$

From 1,2

$$F^{n}(k) = (-2\pi i)^{n}\, un$$

Z- transform

We know Laplace transform to a function x(t)

$$\mathbf{L[x(t)] = \int_0^{\infty} x(t)e^{-st}\,dt}$$

If we want to compute Laplace transform by computer

$$\mathbf{z[x(nT)] = \sum_{n=0}^{\infty} x(nT)e^{-snt}}$$

$$= \sum_{n=0}^{\infty} x(nT)(e^{st})^{-n}$$

Where $z = e^{st}$

$$= \sum_{n=0}^{\infty} x(nT)\, z^{-n}$$

هتعرف القانون دا + انك عارف ان الفتره من صفر لمالانهاية

Find z Transform

$$x(nT) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$L[1] = \frac{1}{s} - -(1)$$

انت عارف ان الفترة من صفر لمالانهاية يبقا الجزء اللي تحت مش معانا

$$z[x(nT)] = \sum_{n=0}^{\infty} x(nT)\, z^{-n}$$

$$= x(0)z^{0} + x(T)z^{-1} + \cdots \ldots.$$

$$= 1 + z^{-1} + z^{-2} + \cdots \ldots..$$

المتسلسلة = الحد الاول / الحد الاول – الحد الثاني

$$= \frac{1}{1-z^{-1}}$$

$x\, T$

$$\mathrm{T.x(nT)} = \frac{\mathrm{T}}{1 - \mathrm{z}^{-1}}$$

Where $z = e^{st}$

$$T.x(nT) = \frac{T}{1 - e^{-st}}$$

ثالثا ايجاد النهاية= تفاضل البسط/ تفاضل المقام وبعدين عوض بصفر

$$\lim_{T\to 0} \frac{T}{1 - e^{-st}} = \frac{1}{se^{-st}} = \frac{1}{se^{0}} = \frac{1}{s} - -(2)$$

<u>From (1) ,(2)</u>

$$\boldsymbol{L[1] = Z(1)}$$

Z- transform

$$x(nT)=e^{-at}=e^{-ant}=(e^{-at})^{n}=K^{n}$$

$$L[e^{-at}]=\frac{1}{s+a}--(1)$$

$$z[x(nT)]=\sum_{n=0}^{\infty}x(nT)z^{-n}$$

$$=x(0)+x(T)z^{-1}+x(2T)z^{-2}+x(3T)z^{-3}+\cdots..$$

$$=1+kz^{-1}+k^{2}z^{-2}+k^{3}z^{-3}+\cdots\ldots\ldots$$

$$=1+\left(K^{-1}Z\right)^{-1}+\left(K^{-1}Z\right)^{-2}+\left(K^{-1}Z\right)^{-3}+\cdots\ldots$$

المتسلسلة = الحد الاول / الحد الاول – الحد الثاني

$$=\frac{1}{1-(K^{-1}Z)^{-1}}$$

x T

$$T.x(nT)=\frac{T}{1-(k^{-1}z)^{-1}}$$

Where $z=e^{st}\ ,\ k=e^{-at}$

$$T.x(nT)=\frac{T}{1-e^{-at}\,e^{-st}}=\frac{T}{1-e^{-(s+a)t}}$$

ثالثا ايجاد النهاية = تفاضل البسط / تفاضل المقام ثم عوض بصفر

$$\lim_{T\to 0}\frac{T}{1-e^{-(s-a)t}}=\frac{1}{(s-a)e^{-(s+a)t}}=\frac{1}{(s+a)\,e^{0}}=\frac{1}{s+a}--(2)$$

From(1),(2)

$$\mathbf{L}[\mathbf{e}^{-\mathbf{at}}] = \mathbf{z}(\mathbf{e}^{-\mathbf{at}})$$

طبعا انت عارف ان الاخت جيب التمام ليها قيمتين 1و-1

Find z- Transform

$$x(nT) = a^n \cos(\frac{n\pi}{2})$$

If n is odd n=2k+1

$$\cos\frac{(2k+1)\pi}{2} = \cos\left(k\pi + \frac{\pi}{2}\right) = 0$$

If n is even n=2k

$$\cos\left(\frac{2k\pi}{2}\right) = \cos(k\pi) = (-1)^n = \begin{cases} 1 & k \ \ even \\ -1 & k \ \ odd \end{cases}$$

$$z[x(nT)] = \sum_{n=0}^{\infty} a^{2k}\cos(k\pi)z^{-1}$$

$$= a^o + a^4z^{-4} + a^8z^{-8} + \cdots ....$$

$$-a^2z^{-2} - a^6z^{-6} - a^{10}z^{-10} \ldots \ldots$$

انا عوضت بالاعداد الزوجية ثم عوضت بالاعداد الفردية

كدا عندي متسلسلتين

$$= \frac{1}{1 - a^4z^{-4}} - \frac{a^2z^{-2}}{1 - a^4z^{-4}}$$

$$= \frac{\cancel{1-a^2z^{-2}}}{\cancel{(1-a^2z^{-2})}(1+a^2z^{-2})} = \frac{1}{(1+a^2z^{-2})}$$

Using symbolic toolbox

Sym a n z

Xn=a^n *cos(n*pi/2);

$$\mathbf{xz = ztrans(xn, n, z)};$$

Xz Enter

Xz=z^2 /a^2 +z^2

☺☺ يا فرحه يعقوب

بيوسف