

Question	Answer	Marks	Guidance
8(a)	$\left(x - \left(-\frac{1}{2}p\right)\right)^2 + (y - (-1))^2$ OE	B1*	Allow $a = -\frac{1}{2}p$ and $b = -1$, or centre is $\left(-\frac{1}{2}p, -1\right)$.
	$\left(x - \left(-\frac{1}{2}p\right)\right)^2 + (y - (-1))^2 = -q + 1 + \left(-\frac{1}{2}P\right)^2$ OE	DB1	
		2	
8(b)(i)	[Gradient of tangent =] $-\frac{1}{2}$	B1	OE SOI
	[Gradient of normal =] 2	M1	Use of $m_1m_2 = -1$ with <i>their</i> numeric tangent gradient.
	$\frac{y-3}{x-4} = 2$ [$y = 2x - 5$]	A1	OE ISW Allow $y = 2x + c$, $3 = 2 \times 4 + c \Rightarrow c = -5$.
		3	

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8(b)(ii)	Method 1 for the first two marks:		
	$-1-3=2\left(-\frac{1}{2}p-4\right)$ or $-1=-p-5$	M1*	Using <i>their</i> stated centre or $\left(\frac{\pm p}{2}, \pm 1\right)$ in <i>their</i> equation of the normal.
	$p=-4$	A1	
	Method 2 for the first two marks:		
	$-1=2x-5 \Rightarrow x=2 \Rightarrow -\frac{1}{2}p=2$	M1*	Using their normal equation and <i>their</i> stated centre or $\left(\frac{\pm p}{2}, \pm 1\right)$.
	$p=-4$	A1	
	Method 3 for the first two marks:		
	$2x+2y\frac{dy}{dx}+p+2\frac{dy}{dx}=0 \quad \left[\Rightarrow p=-8-8\frac{dy}{dx}\right]$	M1*	
	$\left[\frac{dy}{dx}=-\frac{1}{2}\Rightarrow\right] p=-4$	A1	

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8(b)(ii)	Method 1 for the last 3 marks:		
	$r^2 = (4-2)^2 + (3-(-1))^2 [=20]$	M1*	Using (4, 3) and <i>their</i> centre or $\left(\frac{\pm \text{their } p}{2}, \pm 1\right)$ to find r^2 or r .
	$-q+1+\frac{1}{4}p^2=20$	DM1	OE Using <i>their</i> expression for r^2 (from (a)) equated to <i>their</i> 20.
	$q=-15$	A1	
	Method 2 for the last 3 marks:		
	$r = \frac{ 2-2-10 }{\sqrt{5}} \left[= \frac{10}{\sqrt{5}} \right]$	M1*	Using (2, -1) and $x+2y-10=0$ (distance from a point to a line).
	$-q+1+\frac{1}{4}p^2 = \left(\frac{10}{\sqrt{5}}\right)^2$	DM1	OE Using <i>their</i> expression for r^2 equated to <i>their</i> $\left(\frac{10}{\sqrt{5}}\right)^2$.
	$q=-15$	A1	
	Method 3 for the last 3 marks:		
	$4^2+3^2+4p+6+q=0 \Rightarrow 4p+q+31=0$ OR $\left(4-\left(-\frac{1}{2}p\right)\right)^2 + (3-(-1))^2 = -q+1+\left(-\frac{1}{2}p\right)^2$	M1*	Substituting (4,3) into their circle equation.
	$4(-4)+q+31=0$	DM1	Substituting <i>their</i> $p=-4$.
	$q=-15$	A1	

Question	Answer	Marks	Guidance
8(b)(ii)	Alternative Method for Question 8(b)(ii)		
	$4^2 + 3^2 + 4p + 6 + q = 0$ $x^2 + (2x - 5)^2 + px + 2(2x - 5) + q = 0$ with $x = 4$ $x^2 + \left(\frac{10 - x}{2}\right)^2 + px + 2\left(\frac{10 - x}{2}\right) + q = 0$ with $x = 4$ $\left(\frac{y + 5}{2}\right)^2 + y^2 + p\left(\frac{y + 5}{2}\right) + 2y + q = 0$ with $y = 3$ $(10 - 2y)^2 + y^2 + p(10 - 2y) + 2y + q = 0$ with $y = 3$ $\{\text{Each of these} \Rightarrow 4p + q + 31 = 0\}$	M1*	Substituting $(4, 3)$ into <i>their</i> circle equation, or replacing y with $2x - 5$ from the normal equation, or replacing y with $\frac{10 - x}{2}$ from the tangent equation, or replacing x with $\frac{y + 5}{2}$ from the normal equation, or replacing x with $10 - 2y$ from the tangent equation, and using either $x = 4$ or $y = 3$ to form an equation in p and q .
	$\frac{5}{4}x^2 + (p - 6)x + 35 + q = 0 \Rightarrow (p - 6)^2 - 4 \times \frac{5}{4} \times (35 + q) = 0$ OR $5y^2 - y(38 + 2p) + 100 + 10p + q = 0 \Rightarrow (38 + 2p)^2 - 4 \times 5 \times (100 + 10p + q) = 0$ $\{\text{Each of these} \Rightarrow p^2 - 12p - 139 - 5q = 0\}$	M1*	Solving the tangent and circle equations simultaneously to form a quadratic equation in either x or y . Then using $b^2 - 4ac = 0$ on their quadratic to form an equation in p and q .
	Solving the equations simultaneously to find p or q	DM1	
	$p = -4$	A1	
	$q = -15$	A1	
		5	