4 Show that the curve with equation $x^2 - 3xy - 40 = 0$ and the line with equation $3x + y + k = 0$ meet for all values of the constant k. [5]

8 **(a)** It is given that β is an angle between $90°$ and $180°$ such that $\sin\beta = a$.

Express $\tan^2\beta - 3\sin\beta\cos\beta$ in terms of a. [3]

(b) Solve the equation $\sin^2\theta + 2\cos^2\theta = 4\sin\theta + 3$ for $0° < \theta < 360°$. [5]

9 The equation of a curve is $y = 4 + 5x + 6x^2 - 3x^3$.

(a) Find the set of values of x for which y decreases as x increases. [4]

(b) It is given that $y = 9x + k$ is a tangent to the curve.

Find the value of the constant k. [4]

11 The function f is defined by $f(x) = 3 + 6x - 2x^2$ for $x \in \mathbb{R}$.

(a) Express $f(x)$ in the form $a - b(x - c)^2$, where a, b and c are constants, and state the range of f. [3]

(b) The graph of $y = f(x)$ is transformed to the graph of $y = h(x)$ by a reflection in one of the axes followed by a translation. It is given that the graph of $y = h(x)$ has a minimum point at the origin.

Give details of the reflection and translation involved. [2]

The function g is defined by $g(x) = 3 + 6x - 2x^2$ for $x \leqslant 0$.

(c) Sketch the graph of $y = g(x)$ and explain why g is a one-one function. You are **not** required to find the coordinates of any intersections with the axes. [2]

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(d) Sketch the graph of $y = g^{-1}(x)$ on your diagram in **(c)**, and find an expression for $g^{-1}(x)$. You should label the two graphs in your diagram appropriately and show any relevant mirror line. [4]

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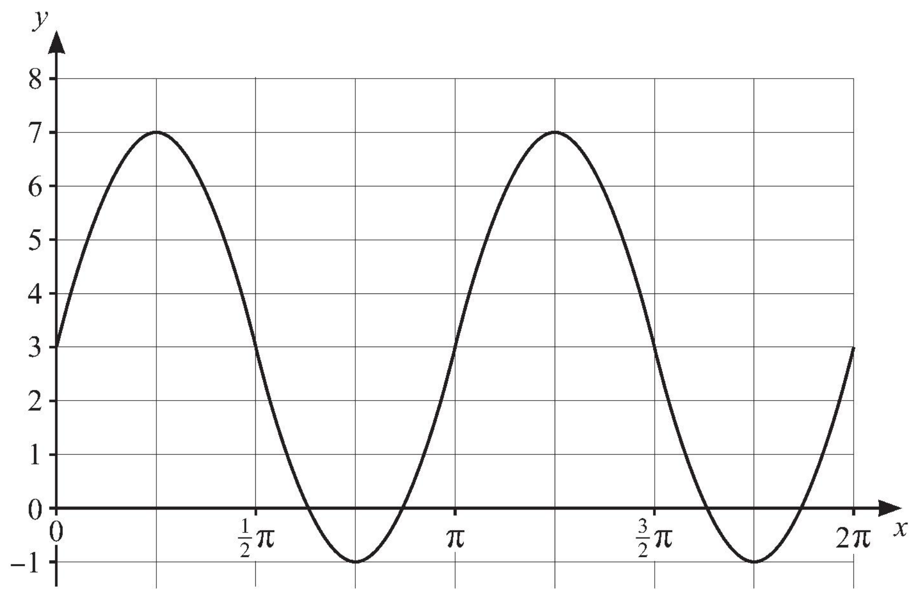

The diagram shows the curve with equation $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$, where a, b and c are positive constants.

(a) State the values of a, b and c. [3]

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(b) For these values of a, b and c, determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations:

(i) $a\sin(bx) + c = 7 - x$ [1]

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(ii) $a\sin(bx) + c = 2\pi(x - 1)$. [1]

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5 The function f is defined by $f(x) = \dfrac{2x+1}{2x-1}$ for $x < \dfrac{1}{2}$.

(a) (i) State the value of $f(-1)$. [1]

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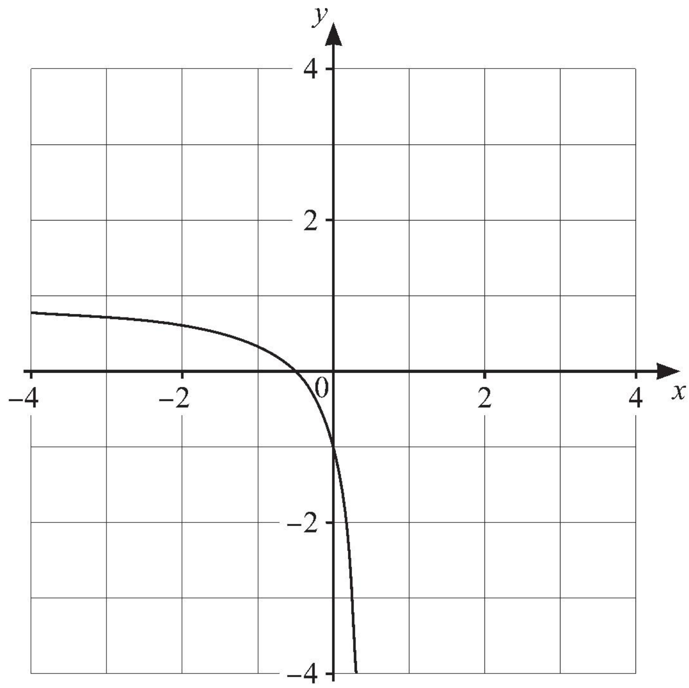
(ii)



The diagram shows the graph of $y = f(x)$. Sketch the graph of $y = f^{-1}(x)$ on this diagram. Show any relevant mirror line. [2]

(iii) Find an expression for $f^{-1}(x)$ and state the domain of the function f^{-1}. [4]

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The function g is defined by $g(x) = 3x + 2$ for $x \in \mathbb{R}$.

(b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$. [3]

7 (a) By expressing $-2x^2 + 8x + 11$ in the form $-a(x-b)^2 + c$, where a, b and c are positive integers, find the coordinates of the vertex of the graph with equation $y = -2x^2 + 8x + 11$. [3]

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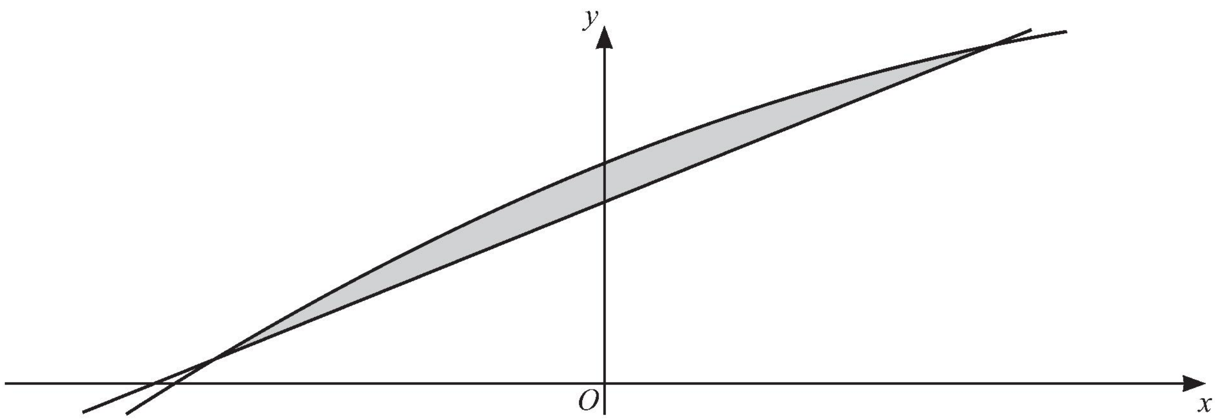
(b)



The diagram shows part of the curve with equation $y = -2x^2 + 8x + 11$ and the line with equation $y = 8x + 9$.

Find the area of the shaded region. [5]

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9 The equation of a curve is $y = \frac{1}{2}k^2x^2 - 2kx + 2$ and the equation of a line is $y = kx + p$, where k and p are constants with $0 < k < 1$.

(a) It is given that one of the points of intersection of the curve and the line has coordinates $\left(\frac{5}{2}, \frac{1}{2}\right)$.

Find the values of k and p, and find the coordinates of the other point of intersection. [7]

(b) It is given instead that the line and the curve do **not** intersect.

Find the set of possible values of p. [3]

2 Find the exact solution of the equation

$$\cos\frac{1}{6}\pi + \tan 2x + \frac{\sqrt{3}}{2} = 0 \text{ for } -\frac{1}{4}\pi < x < \frac{1}{4}\pi. \qquad [2]$$

4 Solve the equation $4\sin^4\theta + 12\sin^2\theta - 7 = 0$ for $0° \leqslant \theta \leqslant 360°$. [4]

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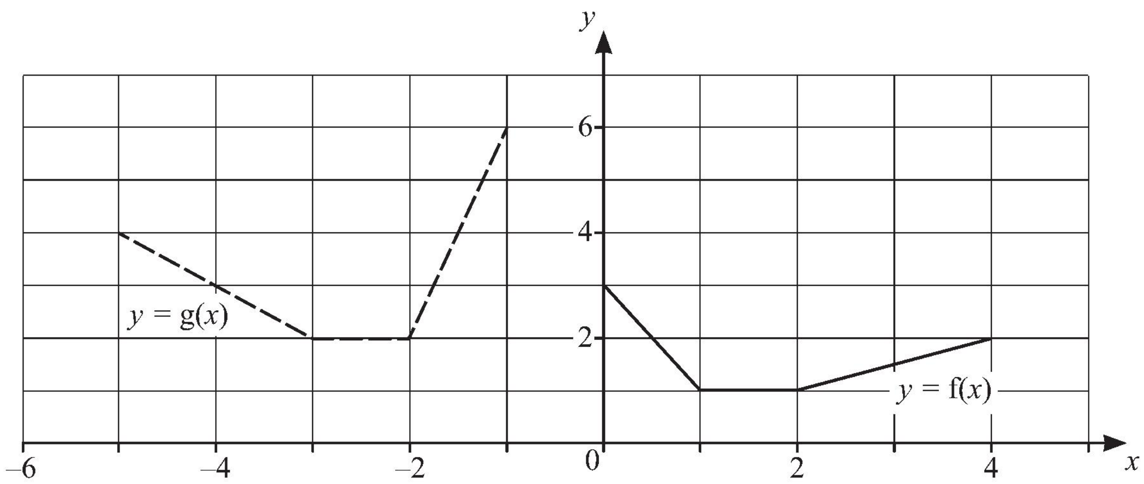

In the diagram, the graph with equation $y = \mathrm{f}(x)$ is shown with solid lines and the graph with equation $y = \mathrm{g}(x)$ is shown with broken lines.

(a) Describe fully a sequence of three transformations which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = \mathrm{g}(x)$. [6]

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(b) Find an expression for $\mathrm{g}(x)$ in the form $a\mathrm{f}(bx+c)$, where a, b and c are integers. [2]

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8 **(a)** Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. [2]

The function $f(x) = 3x^2 - 12x + 14$ is defined for $x \geqslant k$, where k is a constant.

(b) Find the least value of k for which the function f^{-1} exists. [1]

For the rest of this question, you should assume that k has the value found in part **(b)**.

(c) Find an expression for $f^{-1}(x)$. [3]

(d) Hence or otherwise solve the equation $ff(x) = 29$. [3]