Question	Answer	Marks	Guidance
4	Substitute for y (or x) in first equation and simplify	*M1	All terms to one side and brackets expanded.
	Obtain $10x^2 + 3kx - 40$ $[=0]$ (or $10y^2 + 11ky + k^2 - 360[=0]$)	A1	
	Attempt $b^2 - 4ac$ for 3-term quadratic involving k	DM1	Not in quadratic formula unless $b^2 - 4ac$ is isolated.
	Obtain $9k^2 + 1600$ (or $81k^2 + 14400$)	A1	
	$9k^2 + 1600 > 0$	A1 FT	FT for $ak^2 + b > 0$ with $a, b > 0$.
		5	

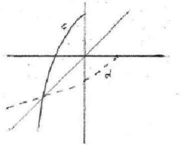
Question	Answer	Marks	Guidance
8(a)	Use $\tan^2\beta = \dfrac{\sin^2\beta}{\cos^2\beta}$	**B1**	E.g. $\tan^2\beta = \dfrac{\sin^2\beta}{\cos^2\beta}$ and then replaces $\sin^2\beta$ with a^2 or $\cos^2\beta$ with $1 - a^2$.
	$\cos\beta = -\sqrt{1-a^2}$	**B1**	
	Obtain $\dfrac{a^2}{1-a^2} + 3a\sqrt{1-a^2}$	**B1**	
		3	

Question	Answer	Marks	Guidance
8(b)	Use correct identity to obtain 3-term quadratic equation in $\sin\theta$	***M1**	
	Obtain $\sin^2\theta + 4\sin\theta + 1[=0]$	**A1**	
	Attempt to solve quadratic	**DM1**	At least as far as $\frac{-4 \pm \sqrt{12}}{2}$. $-15.5°$ implies attempt at solving quadratic.
	Obtain 195.5	**A1**	
	Obtain 344.5	**A1FT**	Following first answer; and no others for $0° < \theta < 360°$ but must be in 4th quadrant. **SC B1** for 3.41^c and 6.01^c.
		5	

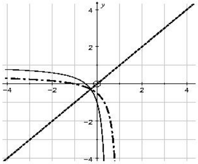
Question	Answer	Marks	Guidance
9(a)	Differentiate to obtain $5+12x-9x^2$	**B1**	
	Attempt to find two critical values by solving quadratic equation or inequality	**M1**	
	Obtain values $-\frac{1}{3}$ and $\frac{5}{3}$	**A1**	**SC B1** if no method for solving the quadratic.
	Conclude $x<-\frac{1}{3},\ x>\frac{5}{3}$	**A1FT**	**SC B1** if no method for solving the quadratic.
		4	

Question	Answer	Marks	Guidance
9(b)	Equate first derivative to 9 and simplify to 3 term quadratic	***M1**	
	Obtain $x=\frac{2}{3}$	**A1**	**SC B1** for solving $5+12x-9x^2=9$ without simplifying to a 3-term quadratic.
	Use x-value and corresponding y-value to determine value of k	**DM1**	
	Obtain $k=\frac{28}{9}$	**A1**	**SC B1** for $k=\frac{28}{9}$ from solving $5+12x-9x^2=9$ without simplifying to a 3-term quadratic.
		4	

Question	Answer	Marks	Guidance
11(a)	Obtain $b=2$ and $c=\frac{3}{2}$	**B1**	
	Obtain $\frac{15}{2}-2\left(x-\frac{3}{2}\right)^2$	**B1**	
	State range is $y \leqslant \frac{15}{2}$ or $f(x) \leqslant \frac{15}{2}$ with $\leqslant$ given or clearly implied (not $<$)	**B1 FT**	Following *their* value of a.
		3	
11(b)	State that reflection is in x-axis	**B1**	Accept transformations in any order.
	State or imply that translation is by $\begin{pmatrix} -\frac{3}{2} \\ \frac{15}{2} \end{pmatrix}$ or equivalent	**B1 FT**	Following *their* values of a and c in part **(a)**. Accept transformations in any order.
		2	

Question	Answer	Marks	Guidance
11(c)	Sketch the correct graph appearing in second and third quadrants only	**B1**	
	State that each y-value is associated with a single x-value or equivalent	**B1**	Accept passes the horizontal line test. Ignore passes the vertical line test.
		2	
11(d)	Sketch the correct graph with suitable labelling to distinguish the two curves	**B1**	Appearing in third and fourth quadrants only.
	Draw the line $y = x$	**B1**	See above; no need to label the line.
	Attempt correct process for finding the inverse function	**M1**	Allowing use of $\pm$ and y so far.
	Obtain $\frac{3}{2} - \sqrt{\frac{15}{4} - \frac{1}{2}x}$ or equivalent	**A1**	Must involve x at the conclusion.
		4	

Question	Answer	Marks	Guidance
1(a)	$a=4$	B1	Allow $4\sin(2x)+3$ if values of a, b and c are not stated.
	$b=2$	B1	
	$c=3$	B1	
		3	
1(b)(i)	5	B1	Ignore attempts at finding solutions.
		1	
1(b)(ii)	1	B1	Ignore attempts at finding solutions.
		1	

Question	Answer	Marks	Guidance
5(a)(i)	$\left[\mathrm{f}(-1)=\right]\ \frac{1}{3}$	**B1**	Condone 0.333.
		1	
5(a)(ii)		**B1**	For showing the correct mirror line.
		B1	For correct shape: the curves should intersect in the first square in the third quadrant. To the left of the point of intersection, the reflection is below the original and crosses the *x*-axis. To the right of the point of intersection, the reflection is to the right the original.
		2	

Question	Answer	Marks	Guidance
5(a)(iii)	$\frac{2x+1}{2x-1}=y \Rightarrow 2x+1=y(2x-1)$	M1*	Equating y to the given function and clearing of fractions. x and y may be interchanged at this stage.
	$2xy-2x=y+1$	DM1	Condone ± errors during simplification.
	$\frac{x+1}{2(x-1)}, \frac{-x-1}{2-2x}$	A1	Allow ‘ f^{-1} ’or ‘$y =$’ but NOT ‘$x =$’, nor fractions within fractions.
	[Domain of f^{-1} is] $x<1$	B1	Accept $-\infty < x < 1$ or $(-\infty, 1)$, condone $[-\infty, 1)$.
	Alternative Method for Question 5(a)(iii)		
	$y=1+\frac{2}{2x-1} \Rightarrow y-1=\frac{2}{2x-1}$	M1*	Equating y to the given function after division by $2x-1$. Isolating the term in x. x and y may be interchanged at this stage.
	$2x=\frac{2}{y-1}+1$	DM1	Condone ± errors during simplification.
	$\frac{1}{x-1}+\frac{1}{2}$	A1	OE Allow ‘ f^{-1} ’or ‘$y =$’ but NOT ‘$x =$’, nor fractions within fractions.
	[Domain of f^{-1} is] $x<1$	B1	Accept $-\infty < x < 1$ or $(-\infty, 1)$, condone $[-\infty, 1)$.
		4	

Question	Answer	Marks	Guidance
5(b)	$gf\left(\frac{1}{4}\right) = -7$	**B1**	
	$\frac{2x+1}{2x-1} = -7$	**M1**	Equating $\frac{2x+1}{2x-1}$ to *their* $gf\left(\frac{1}{4}\right)$.
	$[x =]\ \frac{3}{8}$	**A1**	OE
	Alternative solution for Question 5(b)		
	$gf\left(\frac{1}{4}\right) = -7$	**B1**	
	$x = f^{-1}(-7)$	**M1**	$x = f^{-1}\left(their\ gf\left(\frac{1}{4}\right)\right)$
	$[x =]\ \frac{3}{8}$	**A1**	OE
		3	

Question	Answer	Marks	Guidance
7(a)	$-2\left((x \pm p)^2 \pm q\right)$ or $-2(x \pm p)^2 \pm q$	**M1***	$p \neq 0$.
	$-2\left((x-2)^2 \pm q\right)$ or $-2(x-2)^2 \pm q$	**DM1**	
	$-2(x-2)^2 + 19$ and (2, 19)	**A1**	Accept $x = 2, y = 19$ or 2, 19.
		3	

Question	Answer	Marks	Guidance
7(b)	**Method 1**		
	$[x=]\pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Subtract and attempt to integrate	**M1***	
	$\left[\int\left(-2x^2+2\right)\mathrm{d}x\right]-\frac{2}{3}x^3+2x$	**B1***	Both terms correct.
	$\left(-\frac{2}{3}+2\right)-\left(\frac{2}{3}-2\right)$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve to their integrated expression.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW. Condone $\frac{-8}{3}\to\frac{8}{3}$. **SC B1** for mistaking triangle for trapezium leading to $\frac{11}{3}$, i.e. a total of 2/5.
	Method 2		
	$[x=]\pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Attempt to integrate and subtract	**M1***	The second integral can be replaced with what is clearly their area of a trapezium.
	$\left\{\frac{-2x^3}{3}+\frac{8}{2}x^2+11x\right\}[-]\left\{\frac{8}{2}x^2+9x\right\}$	**B1***	OE All terms correct. The second integral can be replaced by $\frac{1}{2}(1+17)\times 2$ OE.

Question	Answer	Marks	Guidance
7(b)	$\left\{\left(\frac{-2}{3}+4+11\right)-\left(\frac{2}{3}+4-11\right)\right\}$ [−] $\{(4+9)-(4-9)\}$	M1	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve, to *their* integrated expressions. If the trapezium has been used, the second integral can be replaced by *their* 18.
	$=\frac{8}{3},\ 2\frac{2}{3}$	DB1	AWRT 2.67 WWW. Condone $\frac{-8}{3}\to\frac{8}{3}$. **SC B1** for mistaking triangle for trapezium leading to $\frac{11}{3}$, i.e. a total of 2/5.
	Method 3		
	$[x=]\pm 1$	B1*	Both x co-ordinates for the points of intersection.
	Subtract and attempt to integrate	M1*	
	$-\frac{2}{3}(x-2)^3-\frac{8}{2}x^2+10x$	B1*	All terms correct.
	$\left(\frac{2}{3}-4+10\right)-(18-4-10)$	M1	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve, to *their* integrated expression.
	$=\frac{8}{3},\ 2\frac{2}{3}$	DB1	AWRT 2.67 WWW.

Question	Answer	Marks	Guidance
7(b)	**Method 4**		
	$[x=]\pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Attempt to integrate and subtract	**M1***	The second integral can be replaced with what is clearly *their* area of a trapezium.
	$\left\{-\frac{2}{3}(x-2)^3+19x\right\}$ [−] $\left\{\frac{8}{2}x^2+9x\right\}$	**B1***	All terms correct. The second integral can be replaced with $\frac{1}{2}(1+17)\times 2$ OE.
	$\left\{\left(\frac{2}{3}+19\right)-(18-19)\right\}$ [−] $\{(4+9)-(4-9)\}$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve, to *their* integrated expression. If the trapezium has been used the second integral can be replaced with *their* 18 OE.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW. Condone $\frac{-8}{3}\to\frac{8}{3}$. **SC B1** for mistaking triangle for trapezium leading to $\frac{11}{3}$, i.e. a total of 2/5.
		5	

Question	Answer	Marks	Guidance
9(a)	$\left[\frac{1}{2}k^2\times\frac{25}{4}-2k\times\frac{5}{2}+2=\frac{1}{2}\right.$ OR $\left.\frac{1}{2}k^2\times\frac{25}{4}-2k\times\frac{5}{2}+2=k\times\frac{5}{2}+\left(\frac{1}{2}-\frac{5}{2}k\right)\right]$ $25k^2-40k+12\ [=0]$	M1*	Using $\left(\frac{5}{2},\frac{1}{2}\right)$ in the curve equation or equating the line and the curve and then using $x=\frac{5}{2}$ and $p=\frac{1}{2}-\frac{5}{2}k$. Simplify to get a three-term quadratic in k. Condone errors in simplification.
	$k=\frac{2}{5}$	A1	OE Condone inclusion of $k=\frac{6}{5}$.
	$\frac{1}{2}=\left(\textit{their}\,\frac{2}{5}\right)\left(\frac{5}{2}\right)+p\ \Rightarrow p=$	DM1*	Using $\left(\frac{5}{2},\frac{1}{2}\right)$ and *their* k in an equation in p. Either the line (as shown) or $4p^2+12p+5=0$ are the most likely and solving for p.
	$p=-\frac{1}{2}$	A1	OE Condone inclusion of $p=-\frac{5}{2}$.
	$\frac{2}{25}x^2-\frac{6}{5}x+\frac{5}{2}\ [=0]\ \left[4x^2-60x+125\ [=0]\right]$	DM1	Equating the line and curve using *their* k and p and simplify to get a three-term quadratic [= 0].
	$\left(\frac{25}{2},\frac{9}{2}\right)$	A1 A1	OE Accept $x=\frac{25}{2}$, $y=\frac{9}{2}$.

Question	Answer	Marks	Guidance
9(a)	**Alternative Method for Question 9(a)**		
	$\left[\frac{1}{2}k^2 \times \frac{25}{4} - 2k \times \frac{5}{2} + 2 = k \times \frac{5}{2} + p\right]$ $4p^2 + 12p + 5\ [=0]$	**M1***	OE Using $\left(\frac{5}{2}, \frac{1}{2}\right)$ in the curve equation or equating the line and the curve and then using $x = \frac{5}{2}$ and $k = \frac{1}{5} - \frac{2}{5}p$. Simplify to get a three-term quadratic in p [= 0].
	$p = -\frac{1}{2}$ OE	**A1**	Condone inclusion of $p = -\frac{5}{2}$.
	$\frac{1}{2} = \left(\frac{5}{2}k\right) + \left(\mathit{their} - \frac{1}{2}\right) \Rightarrow k =$	**DM1***	Using $\left(\frac{5}{2}, \frac{1}{2}\right)$ and *their* p in the line equation and solving for k.
	$k = \frac{2}{5}$	**A1**	OE Condone inclusion of $k = \frac{6}{5}$.
	$\frac{2}{25}x^2 - \frac{6}{5}x + \frac{5}{2}[=0]\ \left[4x^2 - 60x + 125[=0]\right]$	**DM1**	Equating the line and curve using *their* k and p and simplify to get a three-term quadratic [= 0].
	$\left(\frac{25}{2}, \frac{9}{2}\right)$	**A1 A1**	OE Accept $x = \frac{25}{2}$, $y = \frac{9}{2}$.
		7	

Question	Answer	Marks	Guidance
9(b)	$\left[\frac{1}{2}k^2x^2 - 2kx + 2 = kx + p \quad \Rightarrow\right] \quad \frac{1}{2}k^2x^2 - 3kx + 2 - p$	**M1***	Equate the original equations of the curve and the line and collect like terms; k and p must still be present.
	$9k^2 - 4 \times \frac{1}{2}k^2(2-p)$	**DM1**	Use of $b^2 - 4ac$ for their quadratic in x to give an expression in k and p. This expression can come from *their* equation in **(a)**.
	$p < -\frac{5}{2}$	**A1**	
		3	

Question	Answer	Marks	Guidance
2	$\cos\left(\frac{\pi}{6}\right)+\tan 2x+\frac{\sqrt{3}}{2}=0 \;\Rightarrow\; \tan 2x=-\sqrt{3}$	**M1**	Making $\tan 2x$ the subject. $\tan 2x=0$ is M0. Accept decimals and one sign error.
	$\Rightarrow 2x=-\frac{\pi}{3} \;\Rightarrow x=-\frac{\pi}{6}$	**A1**	May come from non-exact working. Ignore answers outside the given range.
		2	

Question	Answer	Marks	Guidance
4	Let $x = \sin^2\theta$ $(2x+7)(2x-1)=0$ or $\left(2\sin^2\theta+7\right)\left(2\sin^2\theta-1\right)$	**M1**	Or equivalent method.
	$\Rightarrow \sin^2\theta = \frac{1}{2} \Rightarrow \sin\theta = [\pm]\frac{1}{\sqrt{2}}$	**M1**	Finding $\sin^2\theta$ and then $\sin\theta$ (may be implied).
	$\theta = 45°,\ 135°,\ 225°,\ 315°$	**A1 A1**	A1 for any two correct values. A1 for all correct and no others within the range. For answers in radians, A1 only for all 4 angles. If no (correct) working, then **SC B1** for all 4 solutions.
		4	

Question	Answer	Marks	Guidance
5(a)	Reflection [in] y-axis	**B1 B1**	B1 for reflection B1 mention of y-axis, OE. **SC B2** for stretch, SF –1, parallel to x-axis.
	Translation or shift $\begin{pmatrix}-1\\0\end{pmatrix}$	**B1***	B1 for 'translation' and a correct vector/description. Do not accept 'left'/'right'. If two translations then B0 and B0 for the order.
	Stretch, factor 2, parallel to y-axis	**B2,1,0**	B2 all correct OE. B1 any 2 parts correct. This can be at any point in the sequence.
	Correct order and three correctly named transformations only	**DB1**	If a fourth transformation is given this mark is not awarded and no marks are given for the two transformations of the same type, except where the reflection is described as a stretch. If any transformation is incorrectly named this cannot be given. If translation is not $\begin{pmatrix}-1\\0\end{pmatrix}$ or $\begin{pmatrix}1\\0\end{pmatrix}$ then DB0 is given.
	Alternative Solution for first 3 marks		
	Translation or shift $\begin{pmatrix}1\\0\end{pmatrix}$	**B1***	B1 for 'translation' and correct vector/description.
	Reflection [in] y-axis	**B1 B1**	B1 for 'reflection', B1 for 'in y-axis'.
	Alternative solutions		
	There are alternative solutions which can be marked in the same way e.g. the given stretch, translation $\begin{pmatrix}-4\\0\end{pmatrix}$, reflect in $x = -2.5$		
		6	

Question	Answer	Marks	Guidance
5(b)	$g(x)=2f(-x-1)$ or $a=2,\ b=-1,\ c=-1$	B1	First B1 for $a=2$ and no additional terms added to the function. $a=-2$ is B0.
		B1	Second B1 for $b=-1$ and $c=-1$.
		2	

Question	Answer	Marks	Guidance
8(a)	$3(x-2)^2+2$ or $a=-2,\ b=2$	**B1 B1**	
		2	
8(b)	2 or $k=2$ or $k\geq 2$	**B1FT**	FT on *their* a. Do not accept $x=2$ or $x\geqslant 2$.
		1	
8(c)	$3(x-2)^2+14-12=y \Rightarrow (x-2)^2=\dfrac{y-2}{3}$	**M1**	Using *their* completed square form.
	$x=[\pm]\sqrt{\dfrac{y-2}{3}}+2$	**DM1**	
	$\mathrm{f}^{-1}(x)=\sqrt{\dfrac{x-2}{3}}+2$	**A1**	OE, e.g. $y=\dfrac{\sqrt{3x-6}}{3}+2$.
		3	

Question	Answer	Marks	Guidance
8(d)	Finding $f^{-1}(29)$ [= 5]	M1	Or solving f(x) = 29 [using *their* completed square form, OE].
	Finding $f^{-1}($ *their* $5)$	M1	Or solving f(x) = *their* 5.
	$x=3$	A1	If using f(x) method, $x = 1$ must be discarded.
	Alternative solution for Question 8(d)		
	$3\left(3(x-2)^2+2\right)-2)^2+2=29$ using *their* completed square form	M1	Or $3\left(3x^2-12x+14\right)^2-12\left(3x^2-12x+14\right)+14=29$. Allow if the $'=29'$ appears later in the working.
	Solving as far as $9(x-2)^4=9$ or $x^2-4x+3=0$	DM1	OE Or $[27]\left(x^4-8x^3+24x^2-32x+15\right)=0$.
	$x=3$ only	A1	WWW Only dependent on the first M1.
		3	