

# Central Tendency and Variability

## Statistics

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# Grading System

## Course Evaluation

Term	Percentage
Attendance	15%
Mid-term	35%
Final	40%
In-class Project Presentation	10%
Penalty Exam I	100%
Penalty Exam II	100%
MakeUp	40%

### Exams

- ▶ Includes mid-term, final, makeup, excuse, upgrade, and penalty exams.

**Mid-term Exam:** Covers course content taught before the exam.

**Final Exam:** Covers all course content from the entire semester.

**Make-up Exam:** Equivalent to the final exam, replaces final exam results.

**Excuse Exam:** For students who missed mid-terms due to documented reasons.

**Upgrade Exam:** For students with final grades between 30.00 and 49.99, applicable to two courses per semester.

**Penalty Exam:** Extra exam for students who fail a course and need to retake it.

- ▶ Special provisions are available for graduating students, those with disabilities, and students needing to retake failed courses.

### Grading

- ▶ Based on semester activities, midterm, and final exams.
- ▶ Minimum 40 points from final exam required to pass a course.
- ▶ Minimum 50 points from all required to pass a course.

### Grade Announcements

- ▶ Grades are entered into the HELLO system and can be appealed within 48 hours.
- ▶ Students can object the grades via online tool in 48 hours after the grades are announced.

# Grade Calculation

	Percent	Hypothetical Points	Subtotal Points
<b>Attendance</b>	15%	100	15
<b>Midterm</b>	35%	60	21
<b>Final</b>	40%	85	34
<b>In-class</b>	10%	100	10
		<b>Total</b>	<b>80</b>

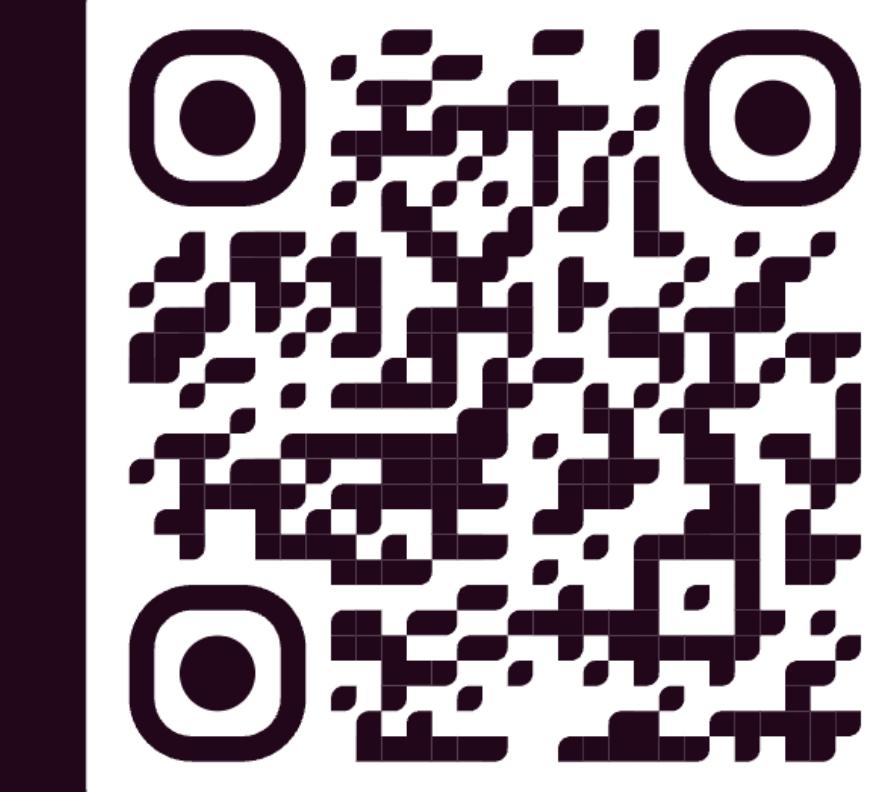
# <https://ibu.kirjakovski.mk>

## PROF. KIRJAKOVSKI IBU BLOG

### 2021

- |        |   |  |
|--------|---|--|
| Jun 12 | MAKE-UP EXAM: Computer Aided Education                  | <a href="#">Computer<br/>Aided<br/>Education<br/>(2020/2021)</a> |
| Jun 04 | IMPORTANT: Repeated Final Exam for Some Students        | <a href="#">Social<br/>Psychology<br/>(2020/2021)</a>            |
| May 24 | LECTURE 11: Organizational Theory, Dynamics, and Change | <a href="#">Organizational<br/>Psychology<br/>(2020/2021)</a>    |
| May 20 | COURSE EVALUATION: Organizational Psychology            | <a href="#">Organizational<br/>Psychology<br/>(2020/2021)</a>    |
| May 20 | FINAL EXAM: Computer Aided Education                    | <a href="#">Computer<br/>Aided<br/>Education<br/>(2020/2021)</a> |

SCAN ME



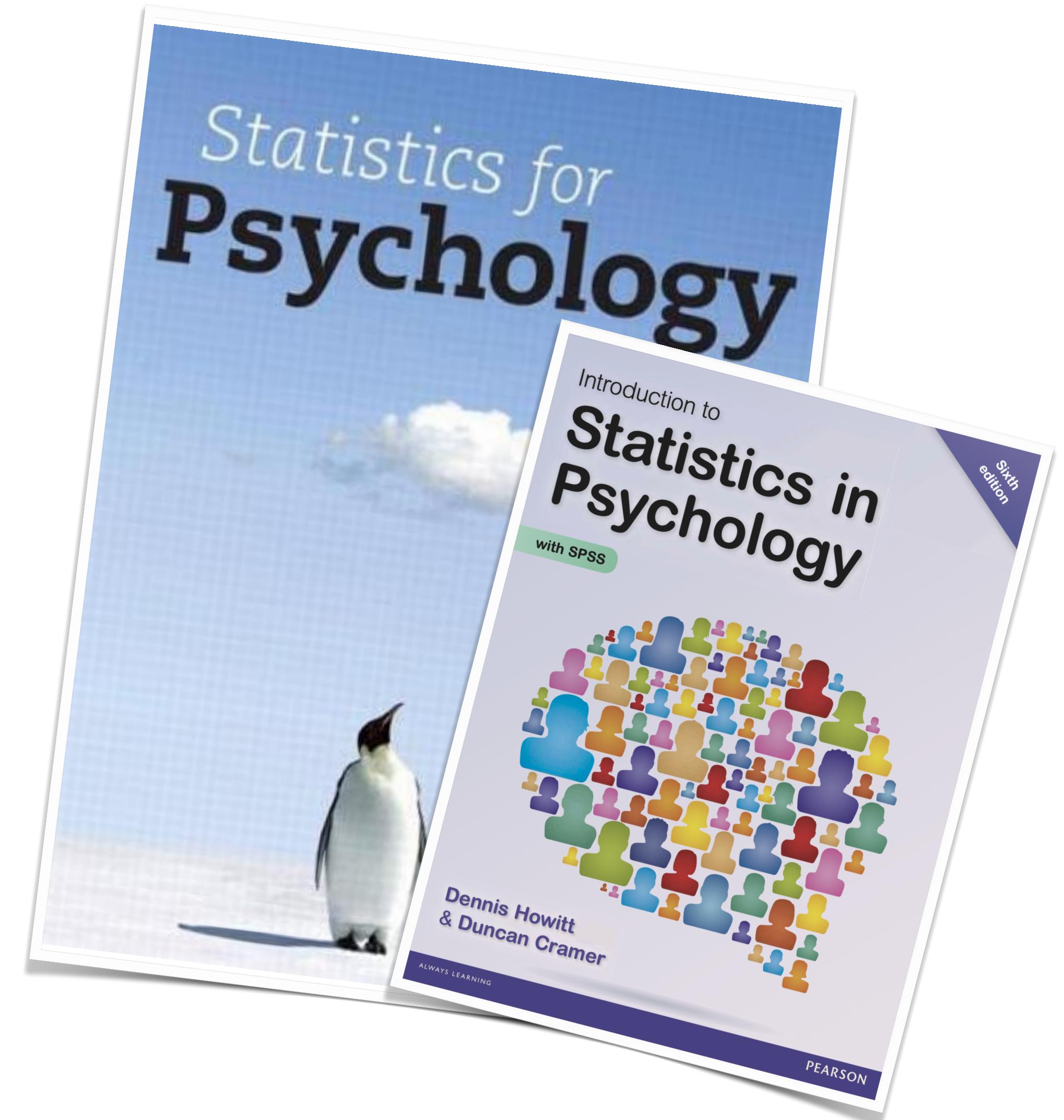
DO NOT RE-UPLOAD  
THE CONTENT!

# Academic Calendar

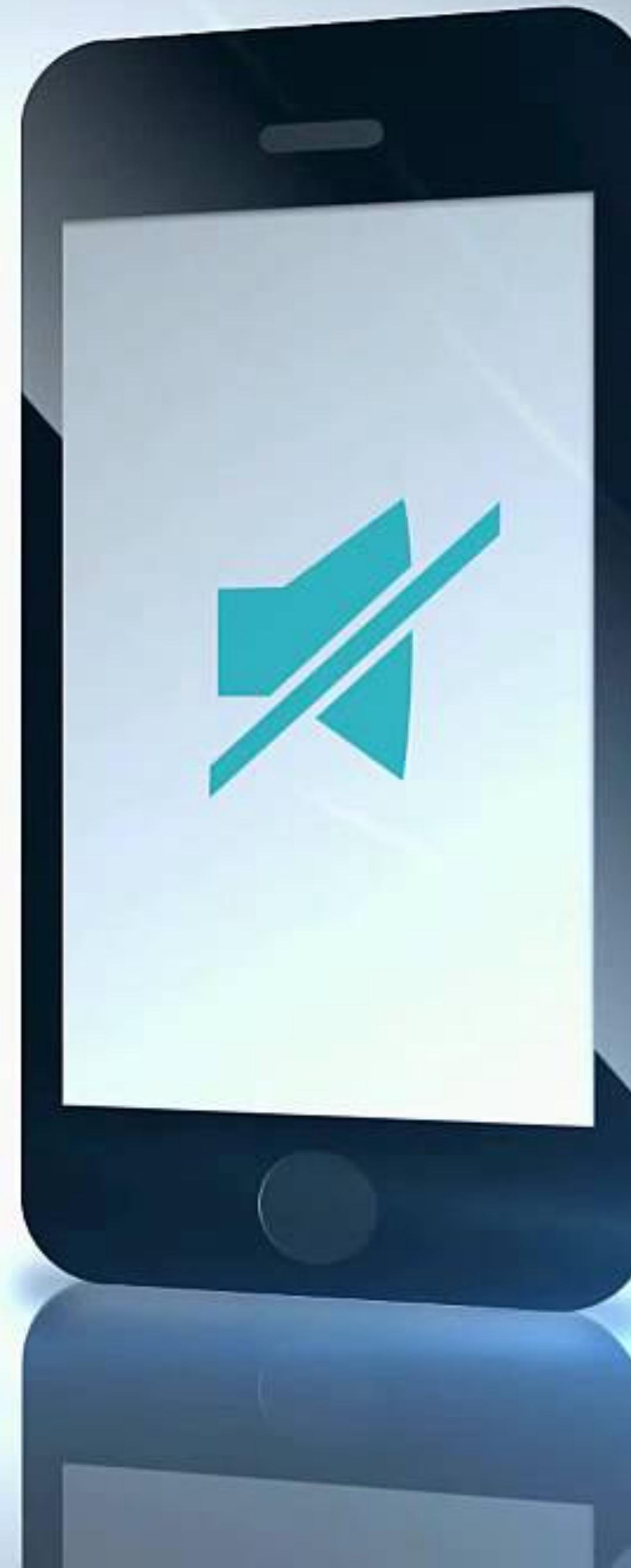
- 7 Oct 2025, Week 1  
**DATA, MEASUREMENT, TABLES, AND GRAPHS**
- 14 Oct 2025, Week 2  
**CENTRAL TENDENCY AND VARIABILITY**
- 21 Oct 2025, Week 3  
**BASICS OF INFERENTIAL STATISTICS**
- 28 Oct 2025, Week 4  
**INTRODUCTION TO HYPOTHESIS TESTING**
- 4 Nov 2025, Week 5  
**HYPOTHESIS TESTS WITH MEANS OF SAMPLES**
- **MIDTERM EXAMS (10–15 NOV 2025)**
- **EXCUSE EXAMS (24–28 NOV 2025)**
- 18 Nov 2025, Week 6  
**STATISTICAL SIGNIFICANCE: DECISION ERRORS, EFFECT SIZE, AND STATISTICAL POWER**
- 25 Nov 2025, Week 7  
**INTRODUCTION TO T TESTS: SINGLE SAMPLE AND DEPENDENT MEANS**
- 2 Dec 2025, Week 8  
**THE T TEST FOR INDEPENDENT MEANS**
- 9 Dec 2025, Week 9  
**INTRODUCTION TO THE ANALYSIS OF VARIANCE**
- 16 Dec 2025, Week 10  
**CORRELATION AND REGRESSION**
- 23 Dec 2025, Week 11  
**CHI-SQUARE TESTS**
- 30 Dec 2025, Week 12  
**NON-PARAMETRIC STATISTICS**
- **WINTER BREAK (31 DEC 2025 – 9 JAN 2026)**
- **FINAL EXAMS (12–17 JANUARY 2026)**
- **MAKEUP EXAMS (19–24 JAN 2026)**

# Textbook and software

- **Main textbook:**  
Aron, A., Coups, E. J., & Aron, E. (2013).  
*Statistics for psychology* (6th ed). Pearson.
- **Supplementary textbook:**  
Howitt, D., & Cramer, D. (2014).  
*Introduction to statistics in psychology:*  
With SPSS (6th Edition). Pearson.
- **Bring your laptops to class!**
- Excel, Jamovi, R, plain text editor  
(Windows: e.g., Notepad++, macOS: e.g.,  
Textmate)



**PLEASE  
SILENCE  
YOUR PHONE**



# Brief Math Review

## Addition and Subtraction

$$8 - 12 = ?$$

$$-8 + 12 = ?$$

## Multiplication and division

$$2 \times 3 \times 6 = ?$$

$$\frac{2 \times 8}{4} = ?$$

$$7 \times 3 + 6 = ?$$

$$(-2) \times (-3) = ?$$

$$\frac{-6}{-3} = ?$$

$$(-2) \times 3 = ?$$

$$(-2) \times 3 \times (-6) \times (-4) = ?$$

## Parentheses

$$2 \times (7 - 6 + 3) = ?$$

$$2 \times (7 - 6 + 3)^2 = ?$$

## Fractions

$$\frac{1}{5} = ?$$

$$3 \times \frac{6}{5} = ?$$

$$\frac{3}{5} \times \frac{6}{7} \times \frac{1}{2} = ?$$

$$\frac{1}{3} + \frac{4}{3} = ?$$

$$\frac{1}{6} + \frac{4}{3} = ?$$

$$\left(\frac{25}{25} \times \frac{8}{5}\right) + \left(\frac{13}{13} \times \frac{12}{25}\right) = ?$$

$$\frac{8}{1/3} = ?$$

## Algebraic Operations

$$3 + X = 8$$

$$2 \times X = 21$$

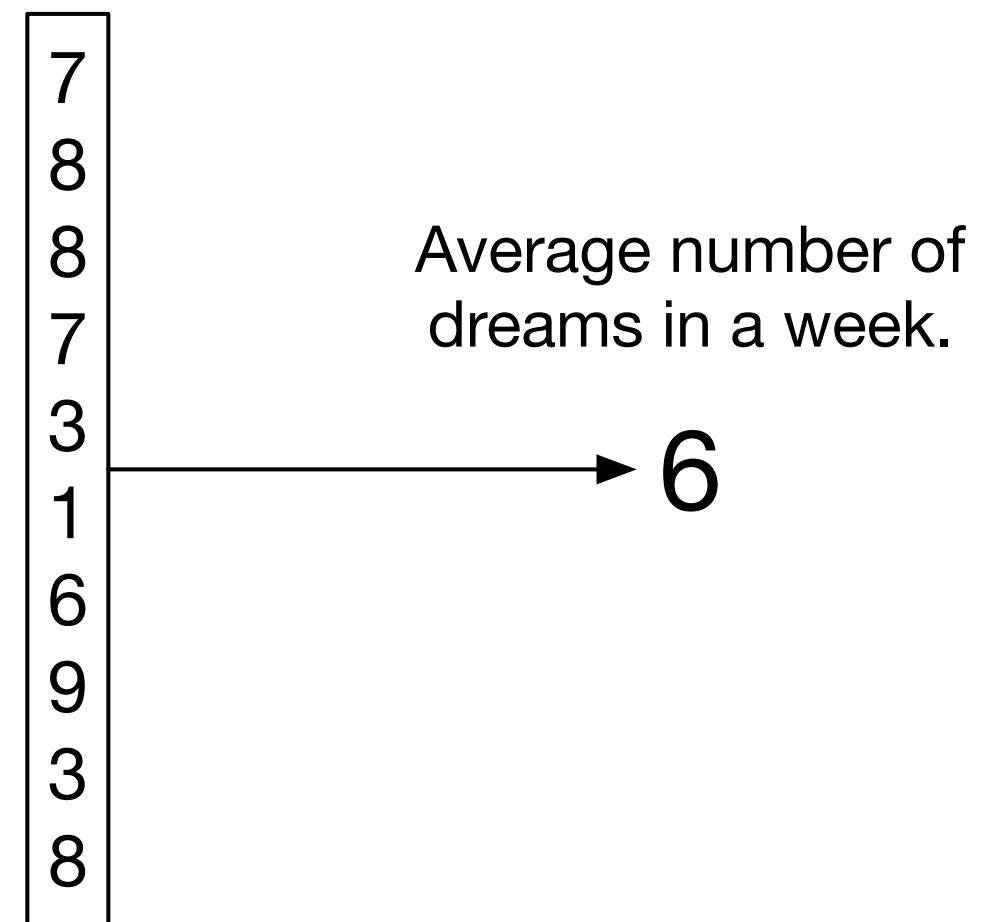
$$7.6 \times (X + 8) = \frac{14}{7} - 5$$

# Symbols

<b>Numerator</b>	The thing on the top	$X \leq Y$	$X$ less than or equal to $Y$
<b>Denominator</b>	The thing on the bottom	$X > Y$	$X$ greater than $Y$
$a/b$	$a$ = Numerator; $b$ = Denominator	$X \geq Y$	$X$ greater than or equal to $Y$
$+, -, \times, \div$ (or /)	Symbols for addition, subtraction, multiplication, and division; called <i>operators</i>	$X < Y < Z$	$X$ less than $Y$ less than $Z$ (i.e., $Y$ is between $X$ and $Z$ )
$X = Y$	$X$ equals $Y$	$X \pm Y$	$X$ plus or minus $Y$
$X \approx Y$ or $X \simeq Y$	$X$ approximately equal to $Y$	$ X $	Absolute value of $X$ —ignore the sign of $X$
$X \neq Y$	$X$ unequal to $Y$	$\frac{1}{X}$	The reciprocal of $X$
$X < Y$	$X$ less than $Y$ ( <i>Hint:</i> The smaller end points to the smaller number.)	$X^2$	$X$ squared
		$X^n$	$X$ raised to the $n$ th power
		$\sqrt{X} = X^{1/2}$	Square root of $X$

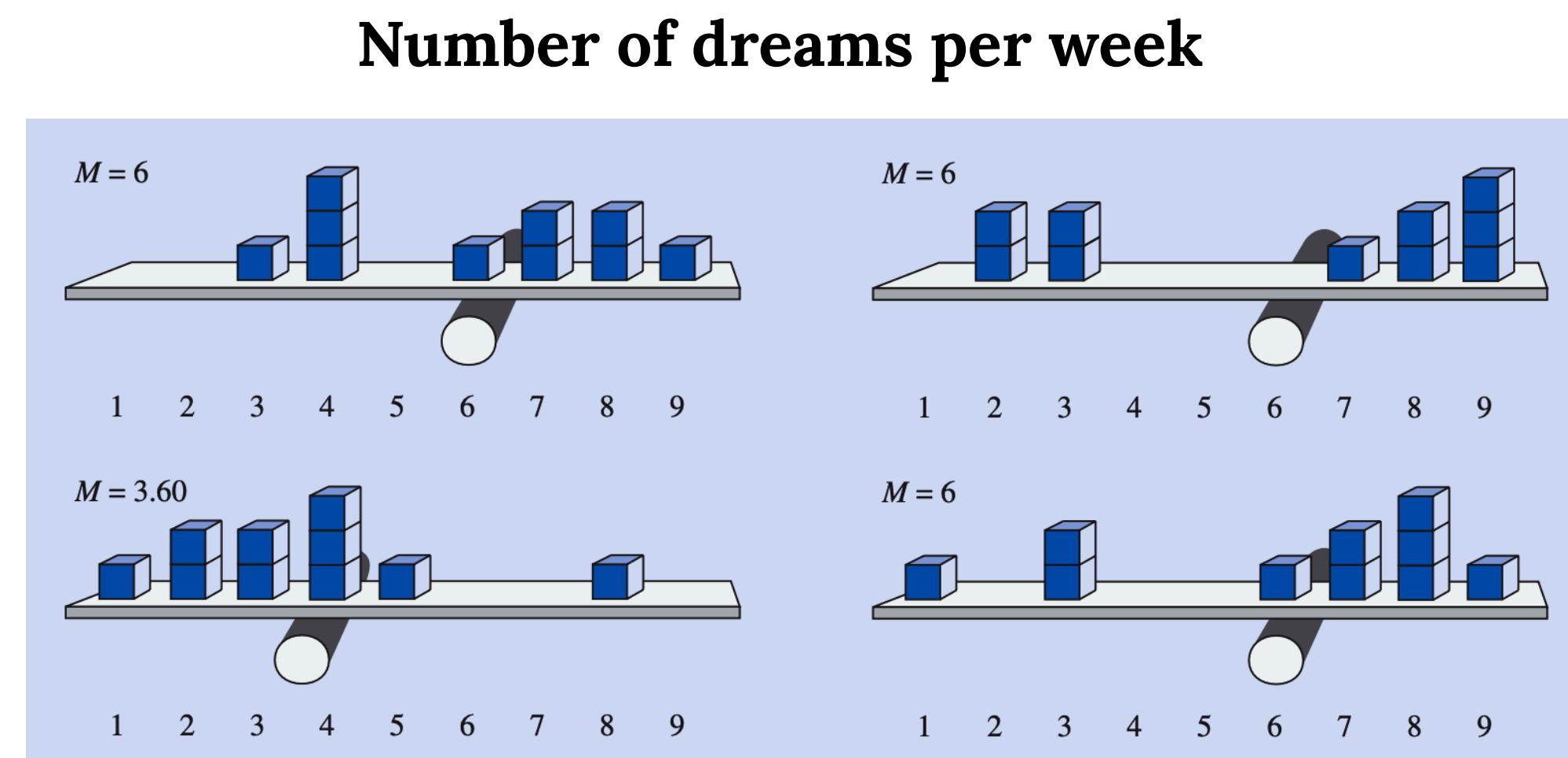
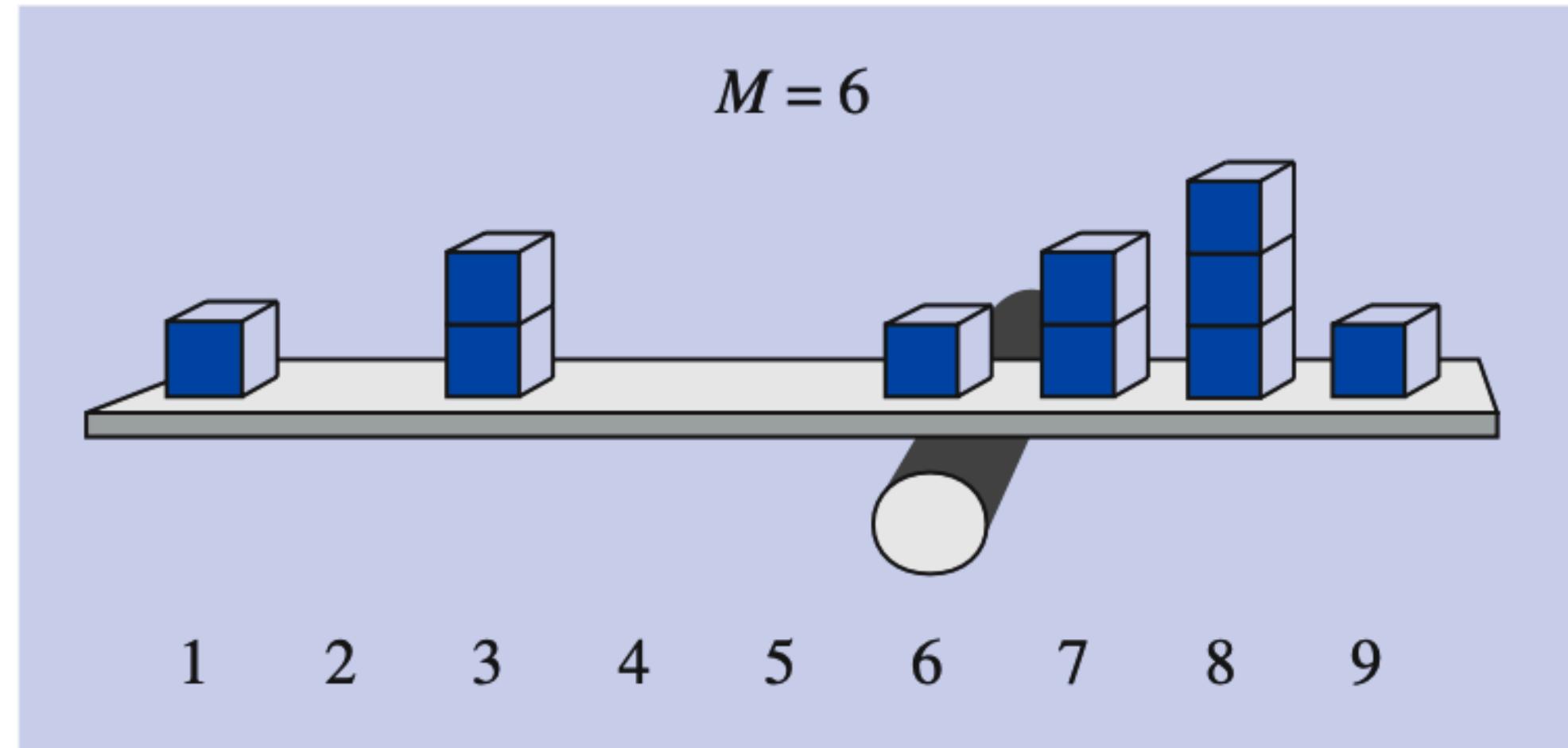
# Central Tendency

- **Descriptive statistics:** Allows a researcher to quantify and describe the basic characteristics of a data set. Determines which advanced statistical tests are appropriate and provides information about measures of **central tendency** and measures of **spread and distribution**.
- **Representative value:** A simple way, with a single number, to describe a group of scores (there may be hundreds—or even thousands—of scores).
- **Central tendency:** Typical or most representative value of a group of scores.
- Three measures of central tendency: **mean**, **mode**, and **median**.
- Each measure of central tendency uses its own method to come up with a single number describing the middle of a group of scores.



# The Mean

- **Mean:** Arithmetic average of a group of scores.
- The arithmetic mean is calculated by adding all values in a data set, then dividing by the number of values in that data set.
- $M = \frac{\Sigma X}{N}$
- **M:** Mean (average)  
 **$\Sigma$ :** Sum of; add up all the scores following this symbol (“Sigma”)  
**X:** Scores in the distribution of the variable X  
**N:** Number of scores in a distribution
- Mathematically, the mean is the point where the total distance to scores above equals the total distance to scores below. It's an unbiased average, so large and small outliers affect the mean equally. It's useful for describing and inferring data, and it's used in many statistical calculations.
- **Population mean ( $\mu$ ):** Case where a data set includes values for every member of a population.
- **Sample mean ( $\bar{x}$  or  $M$ ):** Case where a data set only includes values for selected members of a population.



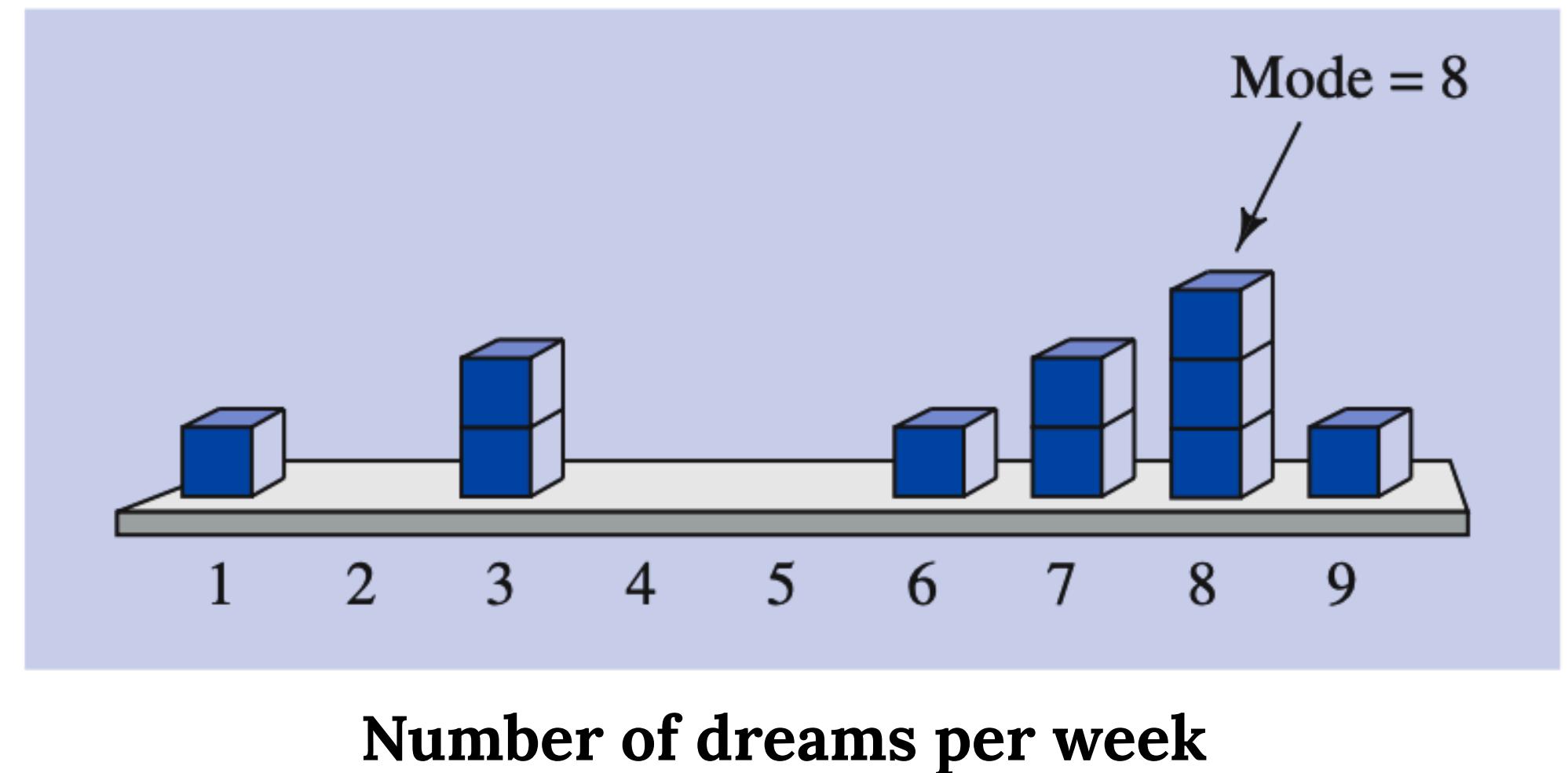
# Stress Scores Exercise – Mean

8, 7, 4, 10, 8, 6, 8, 9, 9, 7, 3, 7, 6, 5, 0, 9,  
10, 7, 7, 3, 6, 7, 5, 2, 1, 6, 7, 10, 8, 8

**Make a frequency table, draw a histogram, and calculate the mean.**

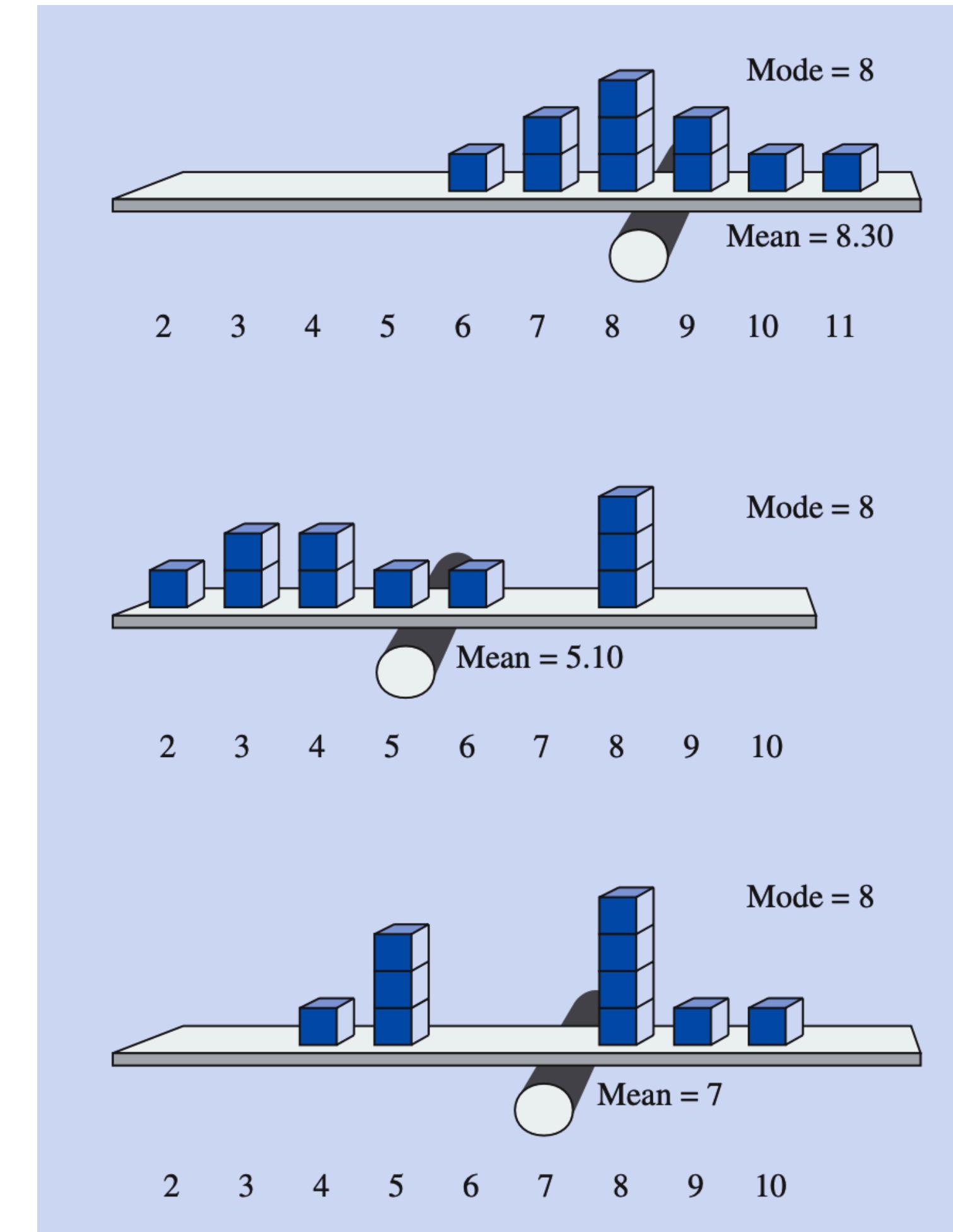
# The Mode

- **Mode:** Value with the greatest frequency in a distribution.
- In a perfectly symmetrical unimodal distribution, the mode is the same as the mean.
- The mode is the usual way of describing the central tendency for a nominal variable.



# The Mode

- You can change some of the scores in a distribution without affecting the mode—but this is not true of the mean, which is affected by any changes in the scores in the distribution.



# Stress Scores Exercise – Mode

8, 7, 4, 10, 8, 6, 8, 9, 9, 7, 3, 7, 6, 5, 0, 9,  
10, 7, 7, 3, 6, 7, 5, 2, 1, 6, 7, 10, 8, 8

**Find the mode.**

# The Median

- **Median:** Middle score when all the scores in a distribution are arranged from lowest to highest.

- $Mdn = \frac{N+1}{2}^{\text{th position}}$  for odd  $N$

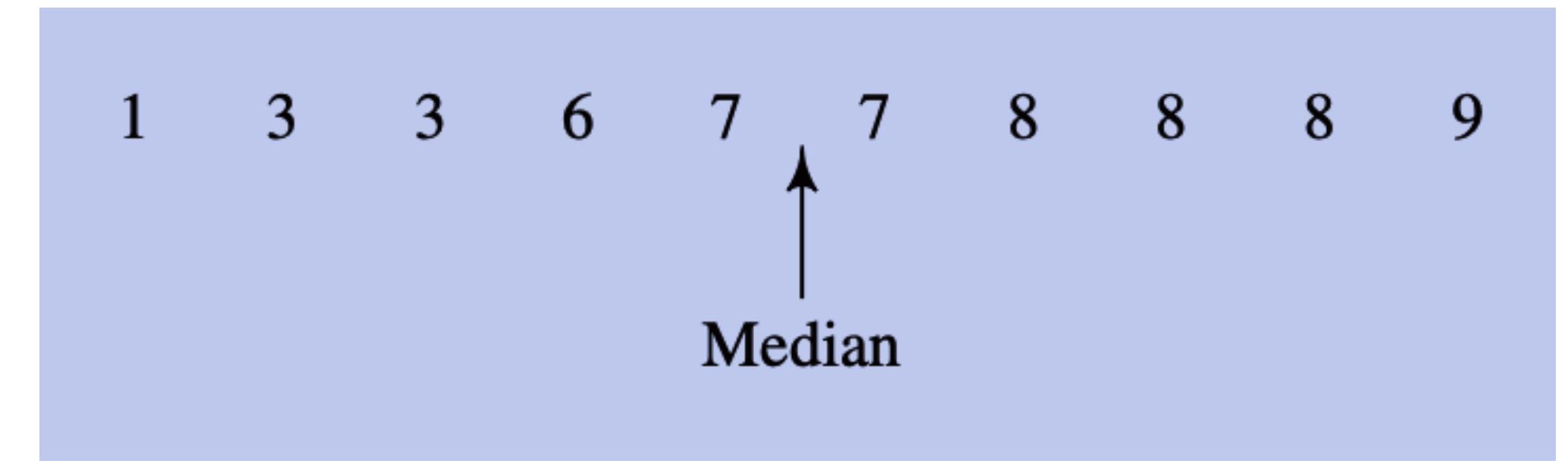
- $Mdn = \frac{N}{2}^{\text{th position}} \text{ and } \frac{N}{2} + 1^{\text{th position}}$  for even  $N$

- Calculation steps:

1. Line up all the scores from lowest to highest.

2. Figure how many scores there are to the middle score by adding 1 to the number of scores and dividing by 2.

3. Count up to the middle score or scores.



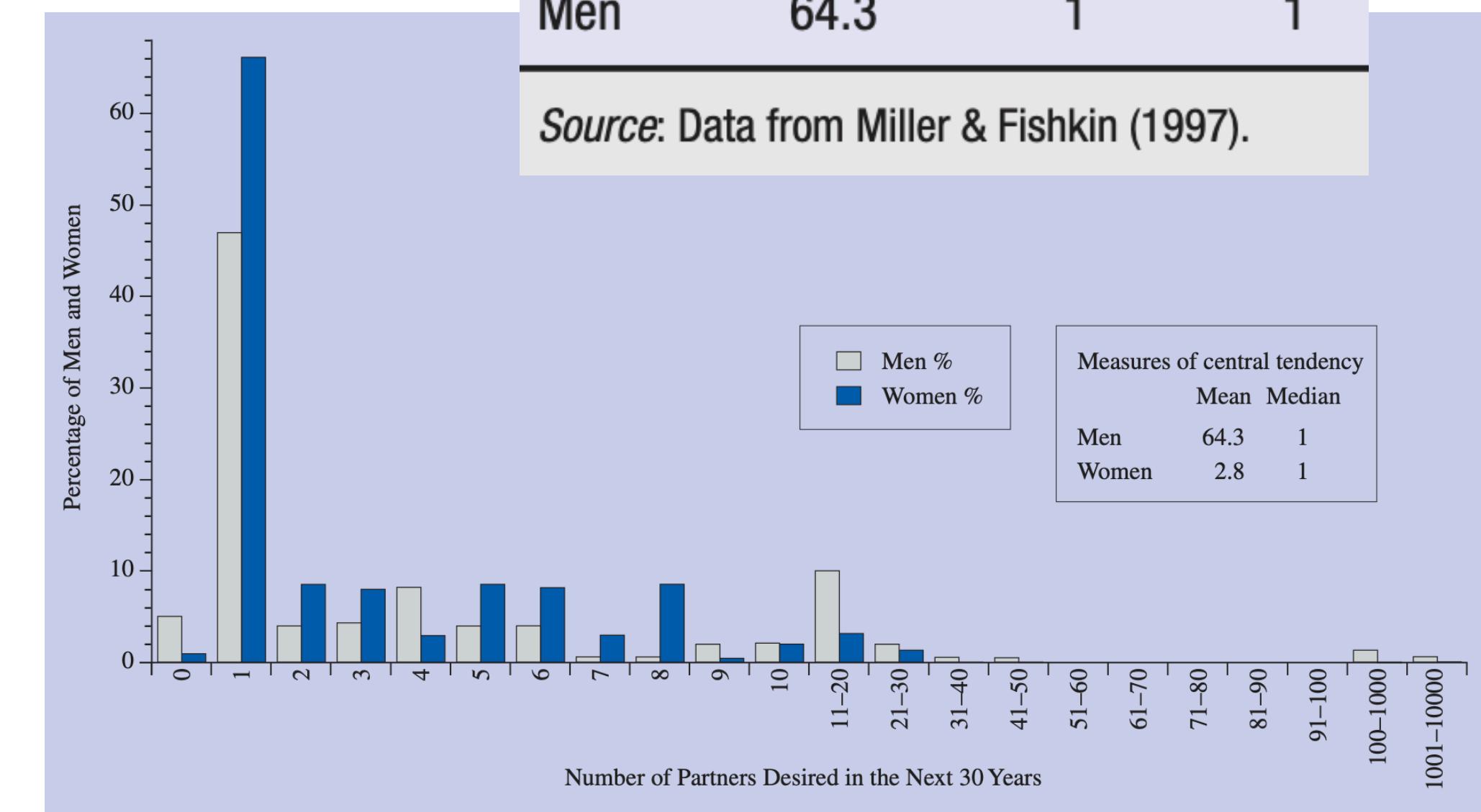
# Stress Scores Exercise – Median

8, 7, 4, 10, 8, 6, 8, 9, 9, 7, 3, 7, 6, 5, 0, 9,  
10, 7, 7, 3, 6, 7, 5, 2, 1, 6, 7, 10, 8, 8

**Find the median.**

# Outliers

- Sometimes, the median is better than the mean (or mode) as a representative value for a group of scores. This happens when a few extreme scores would strongly affect the mean but would not affect the median.
- **Outlier:** Score with an extreme value (very high or very low) in relation to the other scores in the distribution.

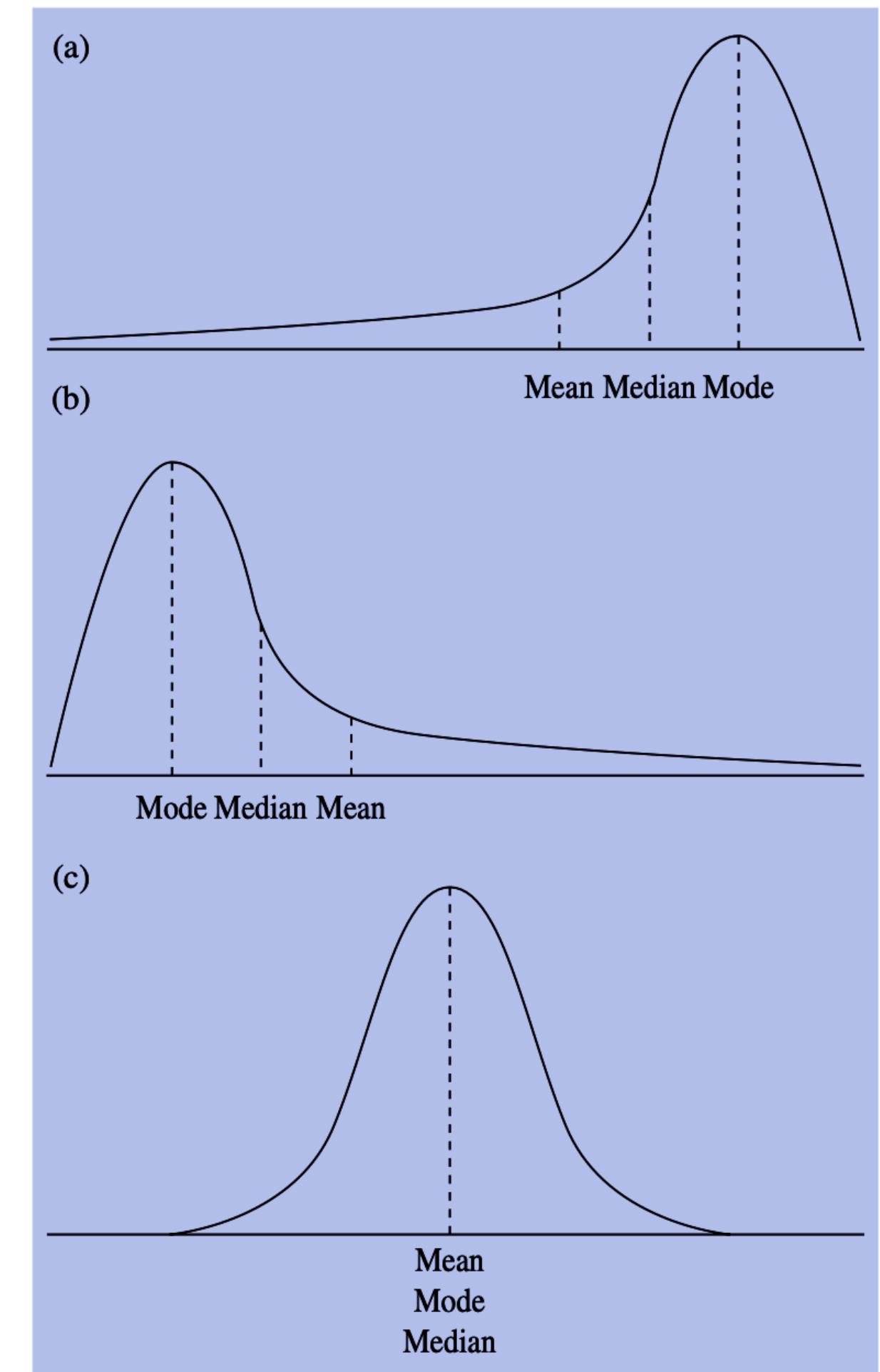


**Table 2-1** Responses of 106 Men and 160 Women to the Question, "How many partners would you ideally desire in the next 30 years?"

	Mean	Median	Mode
Women	2.8	1	1
Men	64.3	1	1

# Comparing the Mean, Mode, and Median

- The median is most likely to be used when a few extreme scores would make the mean unrepresentative of the main body of scores.
- For a perfect normal curve, the mean, mode, and median are always the same value.
- In some situations, psychologists use the median as part of more complex statistical methods.
- The median is the usual way of describing the central tendency for a rank-order variable.

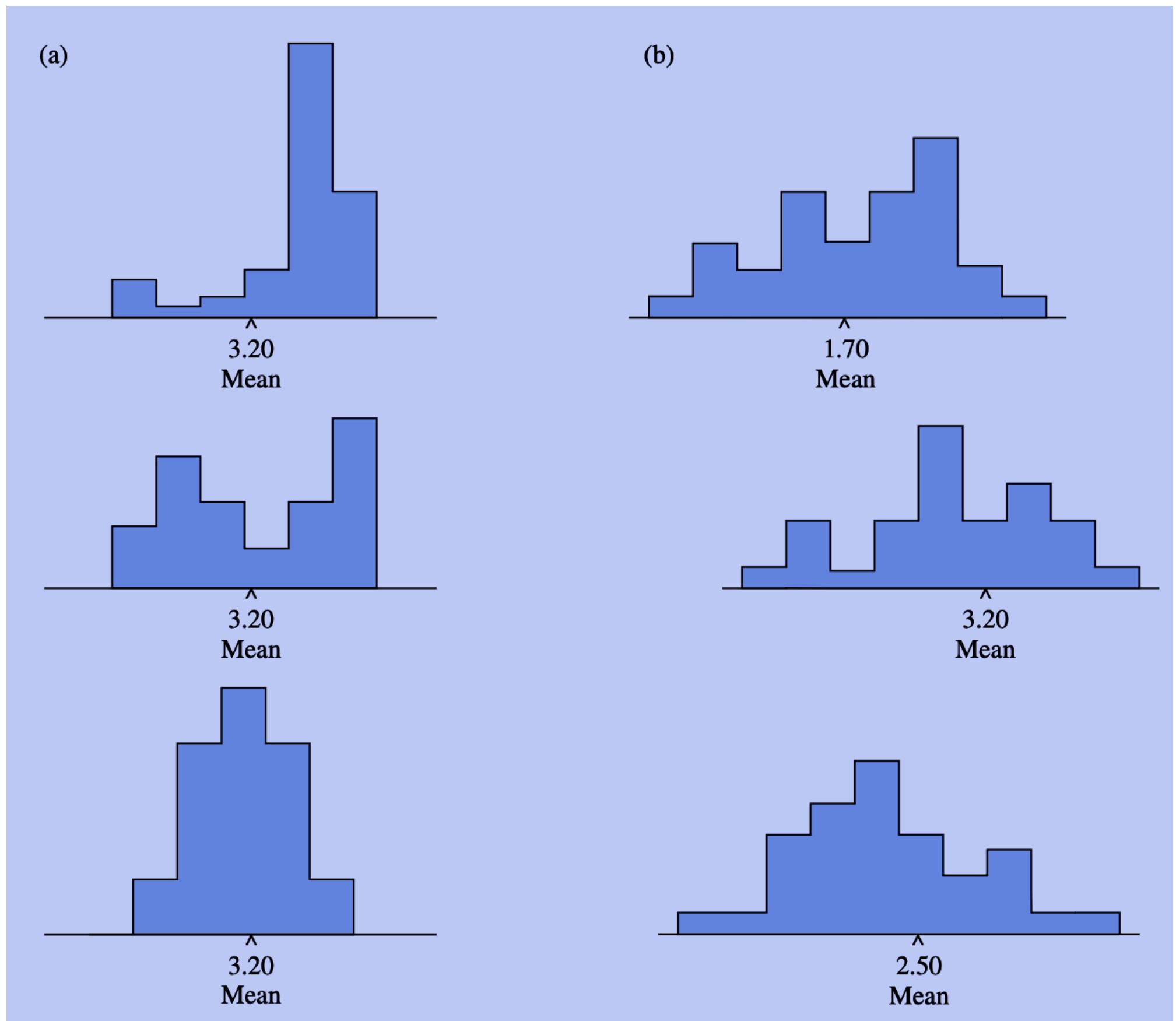


# Comparing the Mean, Mode, and Median

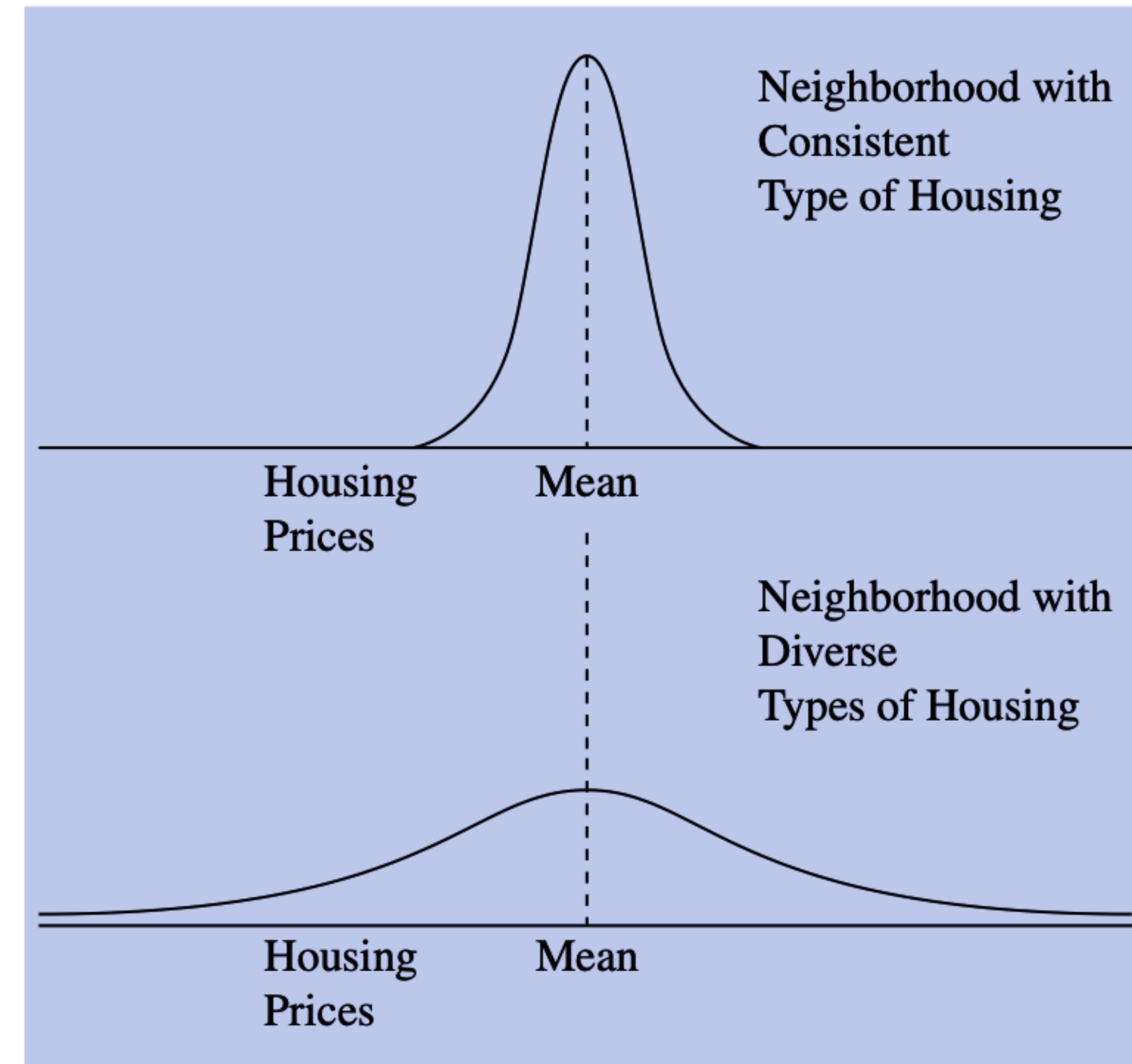
<b>Measure</b>	<b>Definition</b>	<b>When Used</b>
Mean	Sum of the scores divided by the number of scores	<ul style="list-style-type: none"><li>With equal-interval variables</li><li>Very commonly used in psychology research</li></ul>
Mode	Value with the greatest frequency in a distribution	<ul style="list-style-type: none"><li>With nominal variables</li><li>Rarely used in psychology research</li></ul>
Median	Middle score when all the scores in a distribution are arranged from lowest to highest	<ul style="list-style-type: none"><li>With rank-ordered variables</li><li>When a distribution has one or more outliers</li><li>Rarely used in psychology research</li></ul>

# The Concept of Variability

- Researchers also want to know how spread out the scores are in a distribution.
- You can think of the variability of a distribution as the amount of spread of the scores around the mean. In other words, how close or far from the mean are the scores in a distribution?
- Distributions with the same mean can have very different amounts of spread around the mean or can have the same amounts of spread but different means.
- Two measures of the variability of a group of scores: the **variance** and **standard deviation**.



# Two Distributions With the Same Mean but Different Amounts of Spread



# The Variance

- **Variance:** Measure of how spread out a set of scores are; average of the squared deviations from the mean.
- Variance of a group of scores is one kind of number that tells you how spread out the scores are around the mean.
- **Deviation score:** Score minus the mean.
- **Squared deviation score:** Square of the difference between a score and the mean.
- **Sum of squared deviations:** Total of each score's squared difference from the mean.
- $$SD^2 = \frac{\sum(X - M)^2}{N}$$
- Calculation steps:
  1. Subtract the mean from each score.
  2. Square each of these deviation scores (this gives each score's squared deviation score).
  3. Add up the squared deviation scores (sum of squares).
  4. Divide the sum of squared deviations by the number of scores.
- However, the variance is rarely used as a descriptive statistic.

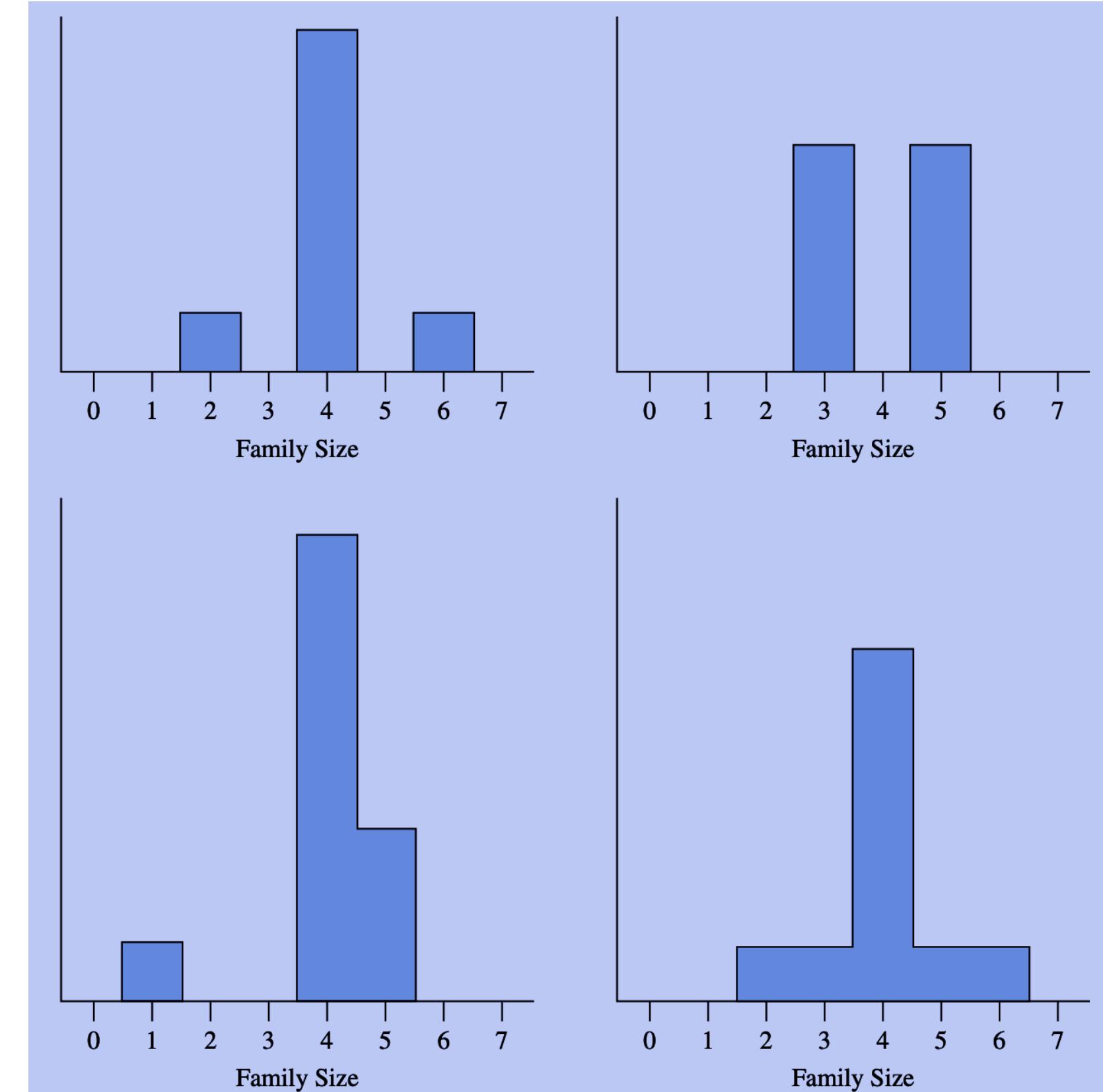
# The Standard Deviation

- The most widely used number to describe the spread of a group of scores.
- **Standard deviation:** Square root of the average of the squared deviations from the mean; the most common descriptive statistic for variation; approximately the average amount that scores in a distribution vary from the mean.

- Knowing the standard deviation gives you a general sense of the degree of spread.
- The standard deviation does not, however, perfectly describe the shape of the distribution.

- $SD = \sqrt{SD^2}$  or more precisely  $SD = \sqrt{\frac{\sum(X - M)^2}{N}}$

- $SD^2$  is the symbol of *variance*.  $SD$  is short for *standard deviation*. The standard deviation  $SD$  is usually reported instead of the variance  $SD^2$  because it is in the same units as the data, making it easier to interpret.



**Some possible distributions for family size in a country where the mean is 4 and the standard deviation is 1.**

# Examples of Figuring the Variance and Standard Deviation

**Table 2-3** Figuring the Variance and Standard Deviation in the Number of Dreams Example

Score (Number of Dreams)	-	Mean Score (Mean Number of Dreams)	=	Deviation Score	Squared Deviation Score
7		6		1	1
8		6		2	4
8		6		2	4
7		6		1	1
3		6		-3	9
1		6		-5	25
6		6		0	0
9		6		3	9
3		6		-3	9
8		6		2	4
				$\Sigma: 0$	$66$
Variance = $SD^2 = \frac{\sum(X - M)^2}{N} = \frac{SS}{N} = \frac{66}{10} = 6.60$					
Standard deviation = $SD = \sqrt{SD^2} = \sqrt{6.60} = 2.57$					

# Computational and Definitional Formulas

- **Computational formula:** Equation mathematically equivalent to the definitional formula.  
Easier to use for figuring by hand, it does not directly show the meaning of the procedure.
- **Definitional formula:** Equation for a statistical procedure directly showing the meaning of the procedure.
- $SD^2 = \frac{\sum(X - M)^2}{N}$  is transformed into  $SD^2 = \frac{\sum X^2 - [\sum X]^2/N}{N}$
- The researcher did not have to first find the deviation score for each score.
- **Bessel's correction:** The use of  $N - 1$  instead of  $n$  in the formula for the sample variance, and sample standard deviation, where  $N$  is the number of observations in a sample. This method corrects the bias in the estimation of the population variance, samples tend to have less variability than the populations from which they are drawn.

# Questions/Comments?