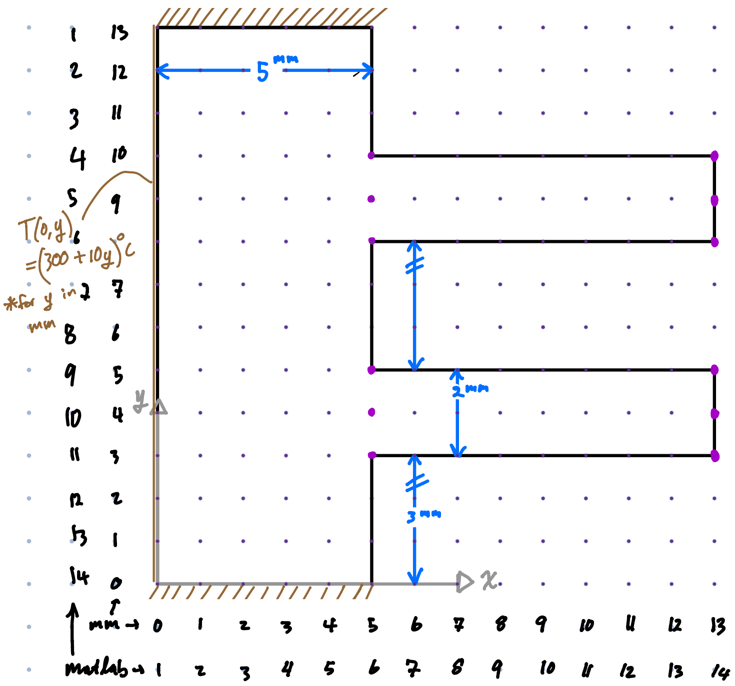
LAB 3 DRAFT

→ Show numbering of nodes
→ And groups of nodes with the same equation

$h = h_0\left\{1 + \frac{x-5}{8\text{ mm}}\right\}$

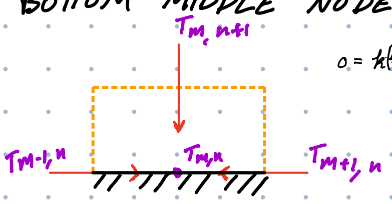
$T_\infty = 20\,^\circ C$

$\Delta x = \Delta y$



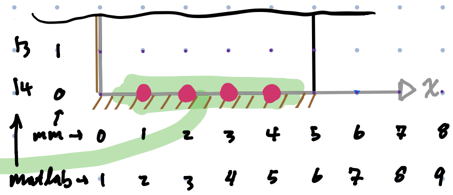
48 + 84 Nodes ? Check

BOTTOM MIDDLE NODES

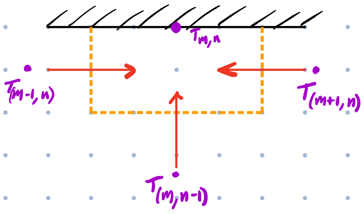


$$0 = k\left(\frac{\Delta x}{2}\right)\ell\left(\frac{T_{m-1,n} - T_{m,n}}{\Delta x}\right) + k\left(\frac{\Delta x}{2}\right)\ell\left(\frac{T_{m+1,n} - T_{m,n}}{\Delta x}\right) + k\Delta x\ell\left(\frac{T_{m,n+1} - T_{m,n}}{\Delta x}\right)$$

$$\boxed{T_{m,n} = \frac{\left(\frac{1}{2}\right)T_{m-1,n} + \left(\frac{1}{2}\right)T_{m+1,n} + T_{m,n+1}}{3}}$$

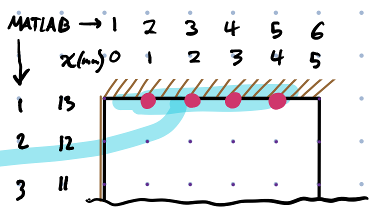


TOP MIDDLE NODES

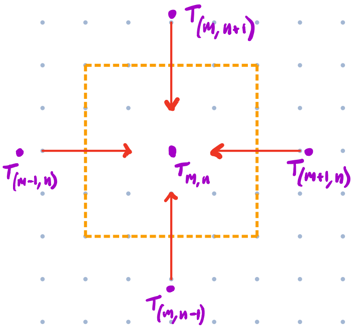


(Top is same except with $n-1$ instead of $n+1$)

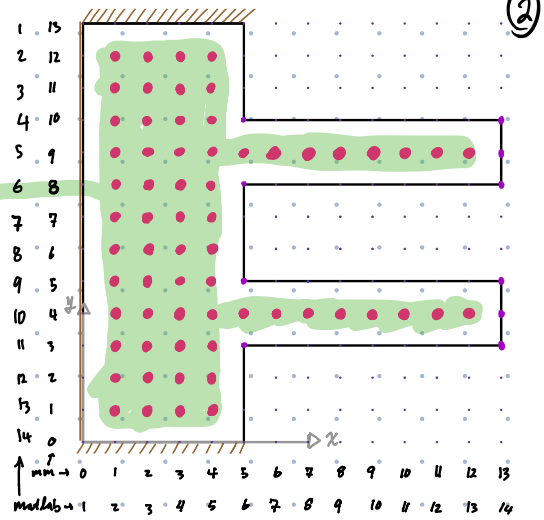
$$\boxed{T_{m,n} = \frac{\frac{1}{2}T_{(m-1,n)} + \frac{1}{2}T_{(m+1,n)} + T_{(m,n-1)}}{3}}$$



CENTRAL NODES $[(2,2)\to(5,13);\ (6,5)\to(13,5);\ (6,10)\to(13,10)]$

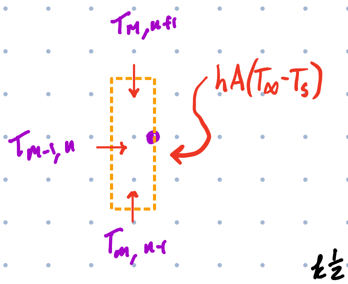


$$\boxed{T_{m,n} = \frac{T_{m-1,n}+T_{m,n-1}+T_{m+1,n}+T_{m,n+1}}{4}}$$



CONVECTION VERTICAL NODES

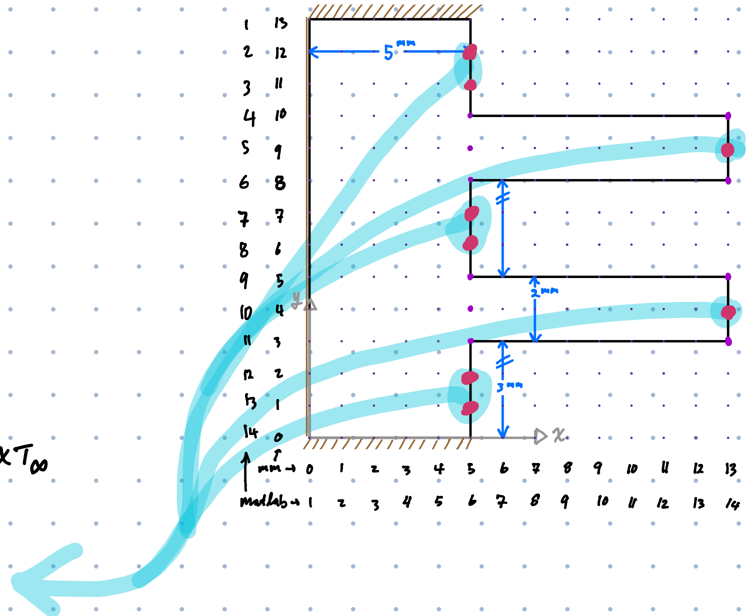
$(6,2); (6,3); (14,5); (6,7); (6,8); (14,10); (6,12); (6,13)$



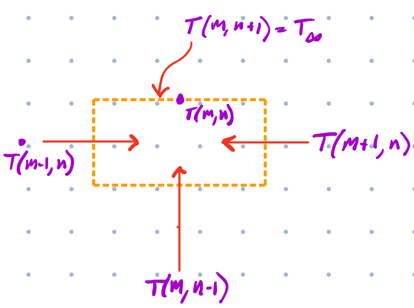
$$0 = \cancel{l}k\Delta x\left(\frac{T_{m-1,n}-T_{m,n}}{\Delta x}\right) + \cancel{l}k\frac{\Delta x}{2}\left(\frac{T_{m,n-1}-T_{m,n}}{\cancel{\Delta x}}\right) + \cancel{l}k\frac{\cancel{\Delta x}}{2}\left(\frac{T_{m,n+1}-T_{m,n}}{\cancel{\Delta x}}\right) + \cancel{l}h\Delta x(T_\infty - T_{m,n})$$

$$k\tfrac{1}{2}T_{m,n} + k\tfrac{1}{2}T_{m,n} + kT_{m,n} + h\Delta x T_{m,n} = kT_{m-1,n} + \frac{k}{2}T_{m,n-1} + \frac{k}{2}T_{m,n+1} + h\Delta x T_\infty$$

$$\boxed{T_{m,n} = \frac{k\left(T_{m-1,n} + \frac{1}{2}T_{m,n-1} + \frac{1}{2}T_{m,n+1} + \frac{h\Delta x}{k}T_\infty\right)}{(2k + h\Delta x)}}$$



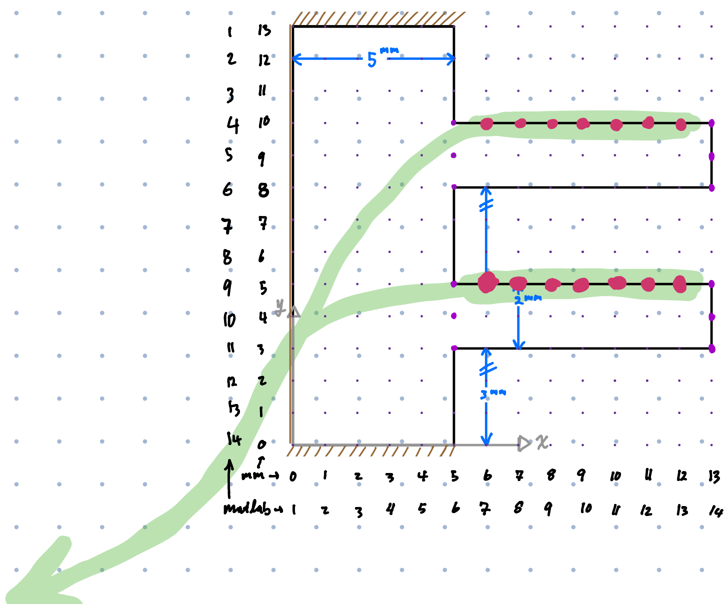
UPPER SURFACES OF FIN NODES $(7,4)\to(13,4)$ and $(7,9)\to(13,9)$



$$0 = \cancel{l}k\frac{\Delta x}{2}\big(T(m-1,n) - T(m,n)\big) + \cancel{l}k\Delta x\big(T(m,n-1) - T(m,n)\big) + \cancel{l}k\frac{\Delta x}{2}\big(T(m+1,n) - T(m,n)\big) + h\Delta x\cancel{l}\big(T_\infty - T(m,n)\big)$$

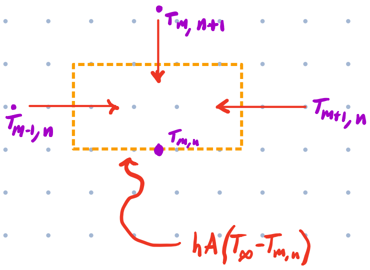
$$(2k+h)T(m,n) = \frac{k}{2}T(m-1,n) + kT(m,n-1) + \frac{k}{2}T(m+1,n) + hT_\infty$$

$$\boxed{T(m,n) = \frac{(k/2)T(m-1,n) + kT(m,n-1) + (k/2)T(m+1,n) + hT_\infty}{2k + h}}$$

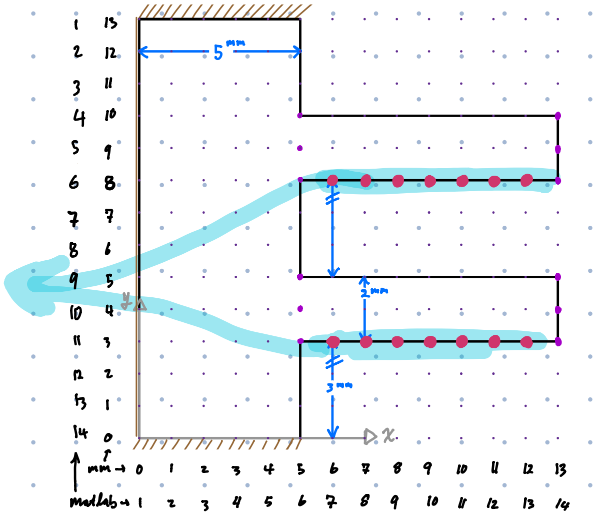


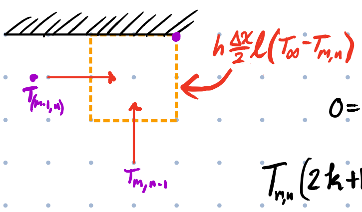
LOWER SURFACE OF FIN NODES (Same as above, but n+1 replaces n-1)

$(7,6)\rightarrow(13,6)$ and $(7,11)\rightarrow(13,11)$



$$\boxed{T(m,n) = \frac{(k/2)T(m-1,n) + kT(m,n+1) + (k/2)T(m+1,n) + hT_\infty}{2k+h}}$$

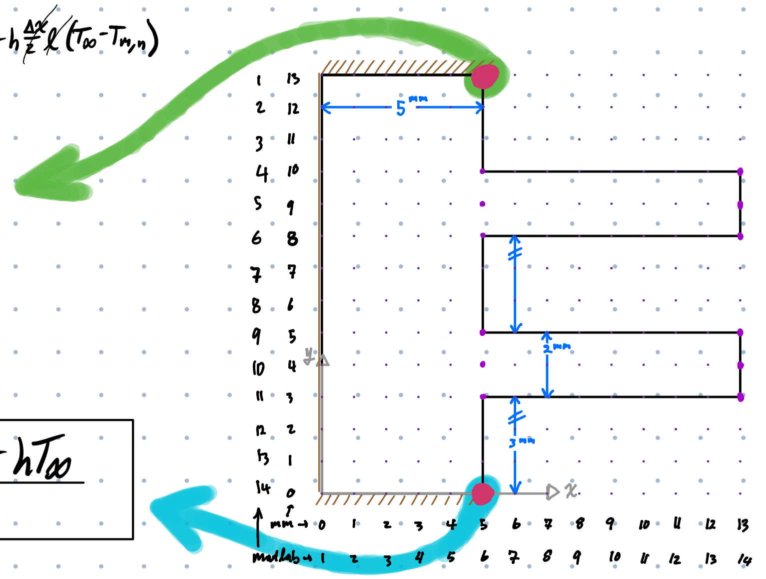


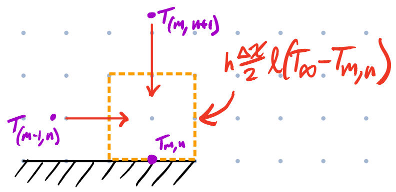


$$0 = \cancel{\ell}\cancel{\tfrac{\Delta x}{2}}k\left[T_{(m-1,n)} - T_{m,n}\right] + k\cancel{\ell}\cancel{\tfrac{\Delta x}{2}}\left[T_{m,n-1} - T_{m,n}\right] + h\cancel{\tfrac{\Delta x}{2}}\cancel{\ell}(T_\infty - T_{m,n})$$

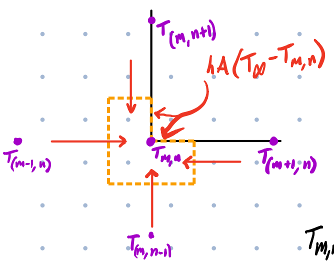
$$T_{m,n}(2k+h) = kT_{(m-1,n)} + kT_{(m,n-1)} + hT_\infty$$

$$\boxed{T_{m,n} = \frac{kT_{(m-1,n)} + kT_{(m,n-1)} + hT_\infty}{2k+h}}$$





$$\boxed{T_{m,n} = \frac{kT_{(m-1,n)} + kT_{(m,n+1)} + hT_\infty}{2k+h}}$$

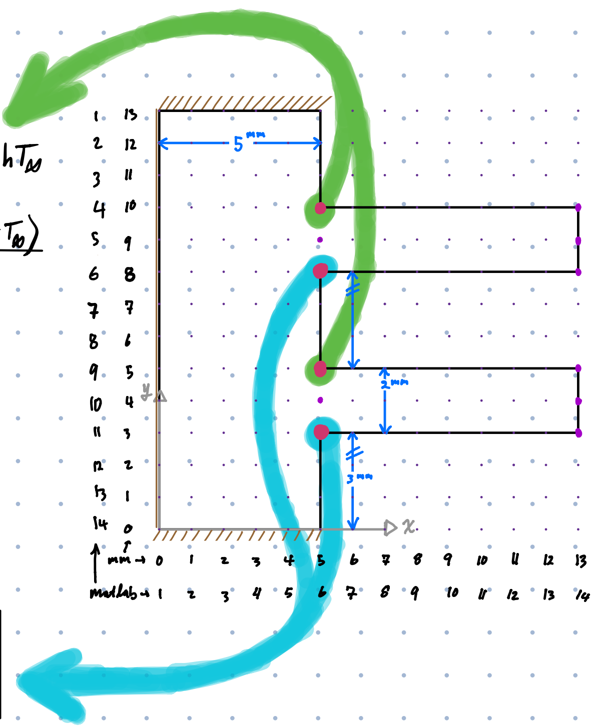


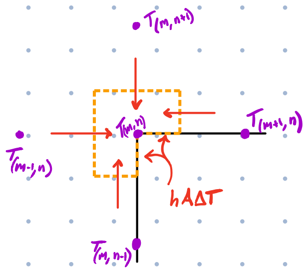
$$0 = \cancel{\ell}k\frac{\Delta x}{2}(T_{(m,n+1)} - T_{m,n}) + \cancel{\ell}k\cancel{\Delta x}(T_{(m-1,n)} - T_{m,n})$$
$$+ \cancel{\Delta x}\cancel{\ell}k(T_{(m,n-1)} - T_{m,n}) + \cancel{\ell}k\frac{\Delta x}{2}(T_{(m+1,n)} - T_{m,n})$$
$$+ h\cancel{\ell}\cancel{\Delta x}(T_\infty - T_{m,n})$$

$$T_{m,n}\left\{\frac{k}{2} + k + k + \frac{k}{2} + h\right\} = \frac{k}{2}T_{(m,n+1)} + kT_{(m-1,n)} + kT_{(m,n-1)} + \frac{k}{2}T_{(m+1,n)} + hT_\infty$$

$$3k + h = k\left(3 + \frac{h}{k}\right) \qquad \frac{\cancel{k}\left(\frac{1}{2}T_{(m,n+1)} + T_{(m-1,n)} + T_{(m,n-1)} + \frac{1}{2}T_{(m+1,n)} + \frac{h}{k}T_\infty\right)}{\cancel{k}(3 + h/k)}$$

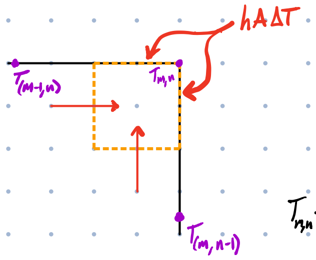
$$\boxed{T_{m,n} = \frac{\frac{1}{2}T_{(m,n+1)} + T_{(m-1,n)} + T_{(m,n-1)} + \frac{1}{2}T_{(m+1,n)} + \left(\frac{h}{k}\right)T_\infty}{3 + (h/k)}}$$





Same exact setup, but the $(\frac{1}{2})$ coefficient swaps from $T_{(m,n+1)}$ to $T_{(m,n-1)}$

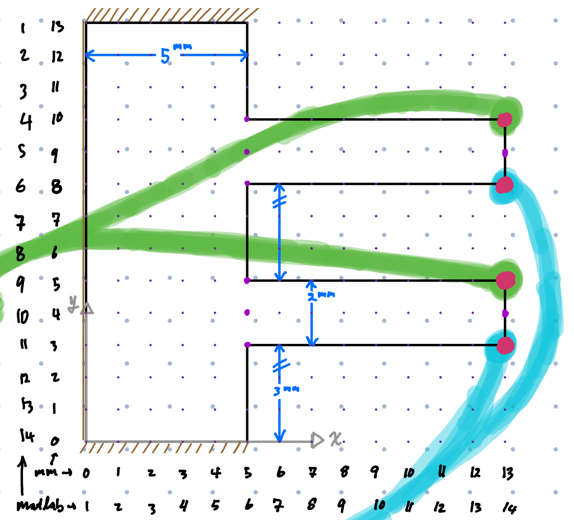
$$\boxed{T_{m,n} = \frac{T_{(m,n+1)} + T_{(m-1,n)} + (\frac{1}{2})T_{(m,n-1)} + (\frac{1}{2})T_{(m+1,n)} + \left(\frac{h}{k}\right)T_\infty}{3 + (h/k)}}$$

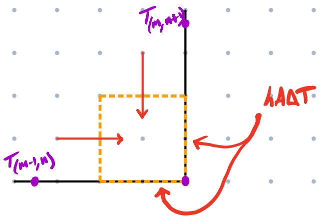


$$0 = k\frac{\cancel{\Delta x}}{2}\cancel{\ell}\left(T_{(m-1,n)} - T_{m,n}\right) + k\frac{\cancel{\Delta x}}{2}\cancel{\ell}\left(T_{(m,n-1)} - T_{m,n}\right) + h\cancel{\Delta x}\cancel{\ell}\left(T_\infty - T_{m,n}\right)$$

$$T_{m,n}\cdot k\left(\frac{1}{2}+\frac{1}{2}+\frac{h}{k}\right) = k\left(\left(\frac{1}{2}\right)T_{(m-1,n)} + \left(\frac{1}{2}\right)T_{(m,n-1)} + \left(\frac{h}{k}\right)T_\infty\right)$$

$$\boxed{T_{m,n} = \frac{\left(\frac{1}{2}\right)T_{(m-1,n)} + \left(\frac{1}{2}\right)T_{(m,n-1)} + \left(\frac{h}{k}\right)T_\infty}{1+\left(\frac{h}{k}\right)}}$$





$$\boxed{T_{m,n} = \frac{\left(\frac{1}{2}\right)T_{(m-1,n)} + \left(\frac{1}{2}\right)T_{(m,n+1)} + \left(\frac{h}{k}\right)T_\infty}{1+\left(\frac{h}{k}\right)}}$$