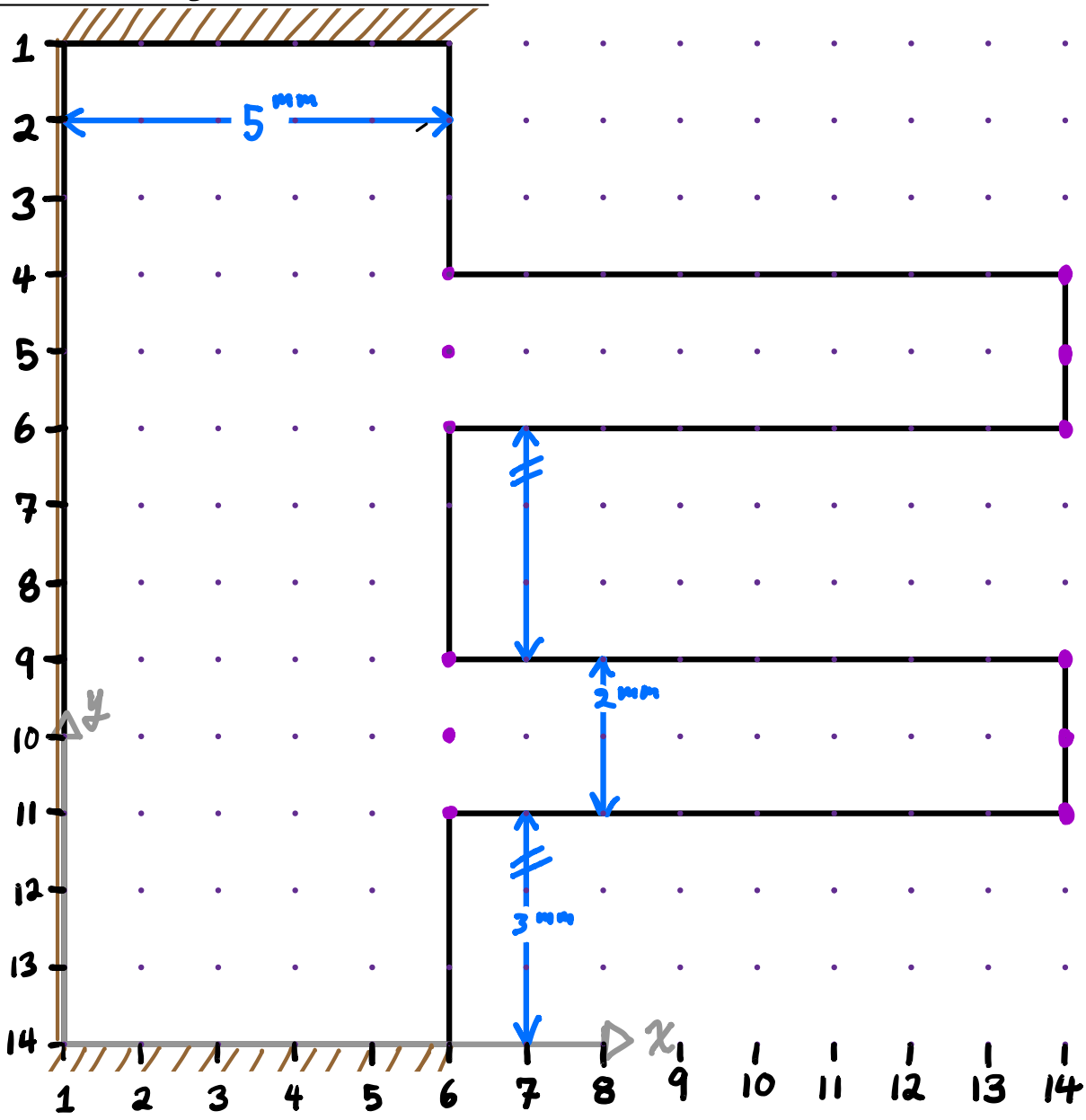
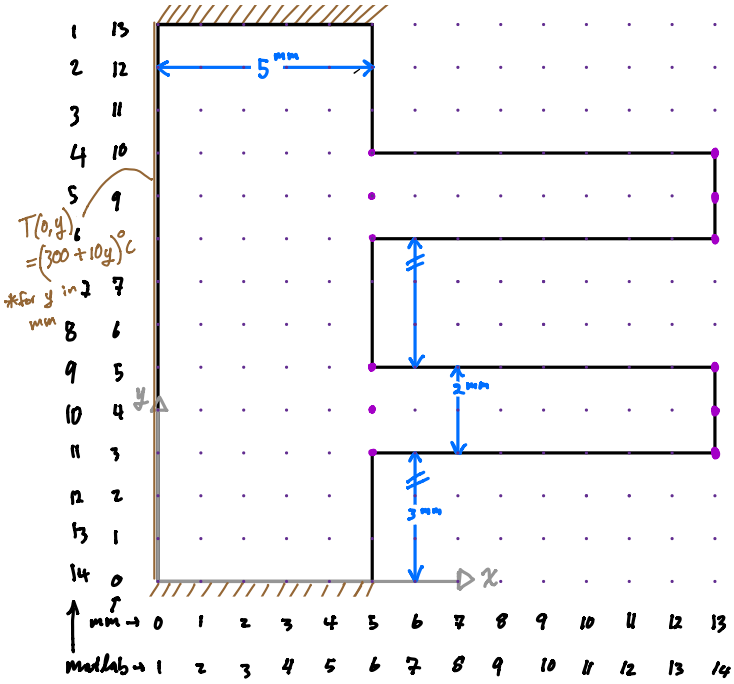
NODAL NUMBERING INDICES



SEE BELOW FOR NODAL EQUATIONS & LOCATIONS

→ Show numbering of nodes
→ And groups of nodes with the same equation



$h = h_0\{1 + \frac{x-5}{8}\}$

$T_\infty = 20°C$

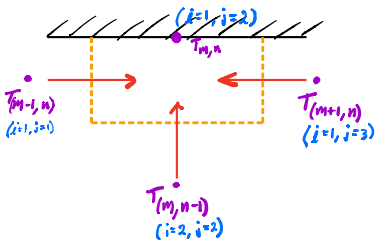
$\Delta x = \Delta y$

$h = h_0\{1 + \frac{x - 5\,mm}{8\,mm}\}$

$\frac{W}{m^2 \cdot K} = \frac{W}{m^2 \cdot K}\{1 + \frac{mm}{mm}\}$ ✓

Array order in Matlab: (j, i)

TOP MIDDLE NODES (2,1); (3,1); (4,1); (5,1)

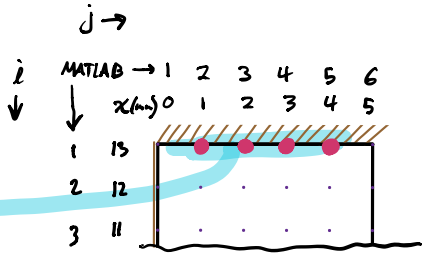


(TOP is same except with n−1 instead of n+1)

(A) $$T_{m,n} = \frac{\frac{1}{2}T_{(m-1,n)} + \frac{1}{2}T_{(m+1,n)} + T_{(m,n-1)}}{2}$$

$$\left(\frac{1}{2}\right)\left[\frac{1}{2}T_{(i,j-1)} + \frac{1}{2}T_{(i,j+1)} + T_{(i+1,j)}\right]$$

EQUATION USING MATLAB INDICES



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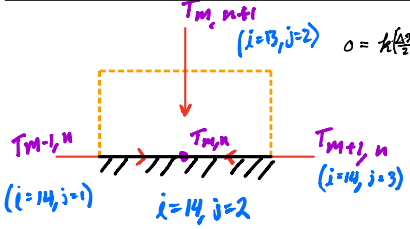
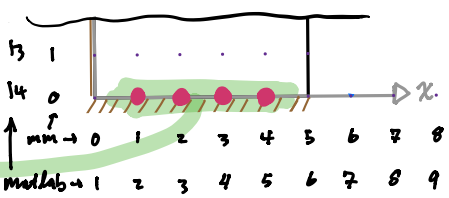
BOTTOM MIDDLE NODES (2,14); (3,14); (4,14); (5,14)



$$0 = k\left(\frac{\Delta x}{2}\right)l\left(\frac{T_{m-1,n} - T_{m,n}}{\Delta x}\right) + k\left(\frac{\Delta x}{2}\right)l\left(\frac{T_{m+1,n} - T_{m,n}}{\Delta x}\right) + k\,\Delta x\,l\left(\frac{T_{m,n+1} - T_{m,n}}{\Delta x}\right)$$

(B) $$T_{m,n} = \frac{\left(\frac{1}{2}\right)T_{m-1,n} + \left(\frac{1}{2}\right)T_{m+1,n} + T_{m,n+1}}{2}$$

$$T_{m,n} = \left(\frac{1}{2}\right)\left[0.5 * T_{(i,j-1)} + 0.5 * T_{(i,j+1)} + T_{(i-1,j)}\right]$$



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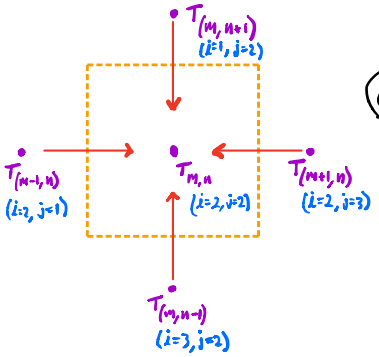
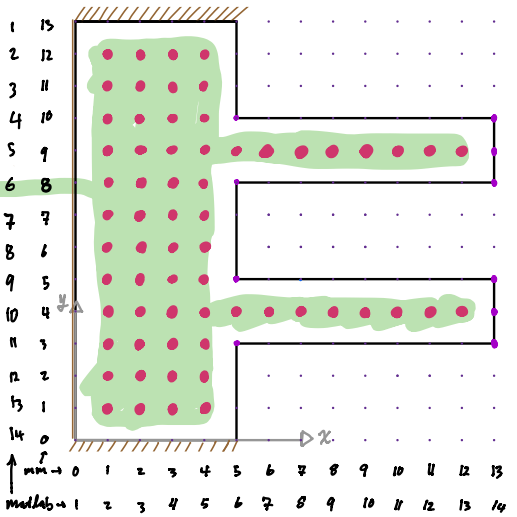
CENTRAL NODES [(2,2) → (5,13); (6,5) → (13, 5); (6,10) → (13,10)]



(C)

$$T_{m,n} = \frac{T_{m-1,n} + T_{m,n-1} + T_{m+1,n} + T_{m,n+1}}{4}$$

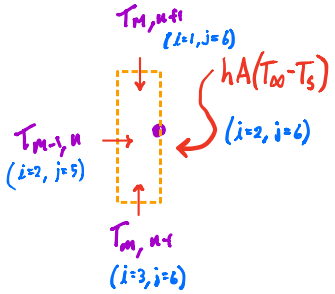
$$T_{m,n} = \left(\frac{1}{4}\right)\left[T_{(i,j-1)} + T_{(i+1,j)} + T_{(i,j+1)} + T_{(i-1,j)}\right]$$

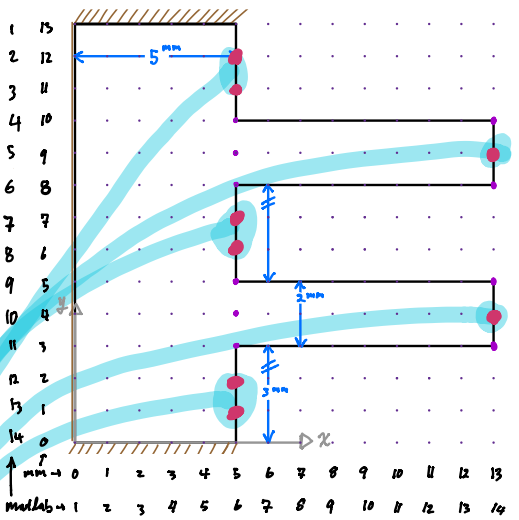


CONVECTION VERTICAL NODES (6,2); (6,3); (14,5); (6,7); (6,8); (14,10); (6,12); (6,13)

$$0 = \cancel{k}\Delta x\left(\frac{T_{m-1,n} - T_{m,n}}{\Delta x}\right) + \cancel{k}\frac{\Delta x}{2}\left(\frac{T_{m,n-1} - T_{m,n}}{\Delta x}\right) + \cancel{k}\frac{\Delta x}{2}\left(\frac{T_{m,n+1} - T_{m,n}}{\Delta x}\right) + \cancel{h}\,h\Delta x\,(T_\infty - T_{m,n})$$

$$\tfrac{1}{2}T_{m,n} + \tfrac{1}{2}T_{m,n} + T_{m,n} + \frac{h\Delta x}{k}T_{m,n} = T_{m-1,n} + \tfrac{1}{2}T_{m,n-1} + \tfrac{1}{2}T_{m,n+1} + \frac{h\Delta x}{k}T_\infty$$



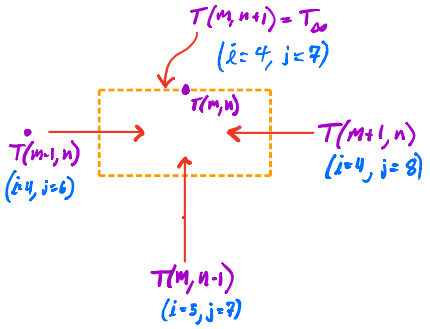


(D)

$$T_{m,n} = \frac{k\left(T_{m-1,n} + \frac{1}{2}T_{m,n-1} + \frac{1}{2}T_{m,n+1} + \frac{h\Delta x}{k}T_\infty\right)}{(2k + h\Delta x)}$$

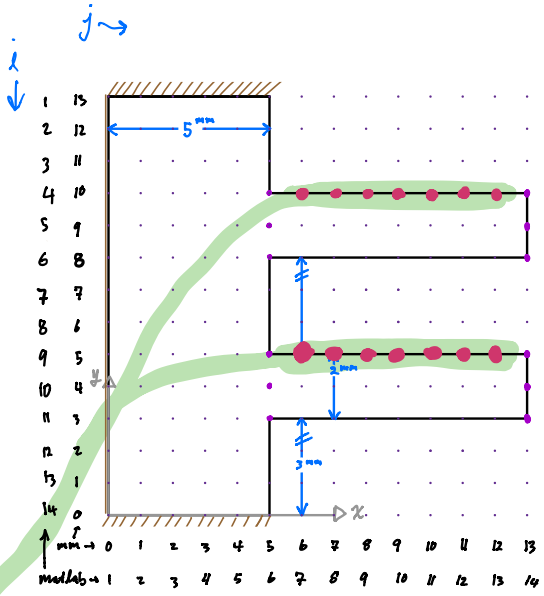
$$T_{m,n} = \left(\frac{k}{2k + h\Delta x}\right)\left[T_{(i,j-1)} + 0.5\,T_{(i+1,j)} + 0.5\,T_{(i-1,j)} + \frac{h\Delta x}{k}T_\infty\right]$$

UPPER SURFACES OF FIN NODES (7,4) → (13,4) and (7,9) → (13,9)



$$0 = \ell k \frac{\Delta x}{2}\left(\frac{T(m-1,n)-T(m,n)}{\Delta x}\right) + \ell k \Delta x\left(\frac{T(m,n-1)-T(m,n)}{\Delta x}\right)$$

$$+ \ell k \frac{\Delta x}{2}\left(\frac{T(m+1,n)-T(m,n)}{\Delta x}\right) + h\Delta x \ell (T_\infty - T(m,n))$$



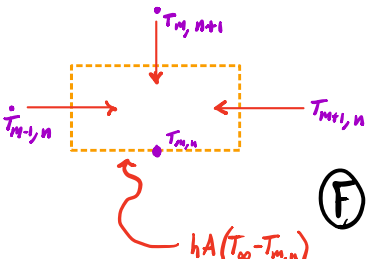
$$(2k + h\Delta x)T(m,n) = \frac{k}{2}T(m-1,n) + kT(m,n-1) + \frac{k}{2}T(m+1,n) + h\Delta x T_\infty$$

(E)
$$T(m,n) = \frac{(k/2)T(m-1,n) + kT(m,n-1) + (k/2)T(m+1,n) + h\Delta x T_\infty}{2k + \Delta x h}$$

$$T_{(m,n)} = \left(\frac{1}{2k+\Delta x h}\right)\left[\left(\frac{k}{2}\right)T_{(i,j-1)} + kT_{(i+1,j)} + \left(\frac{k}{2}\right)T_{(i,j+1)} + h\Delta x T_\infty\right]$$

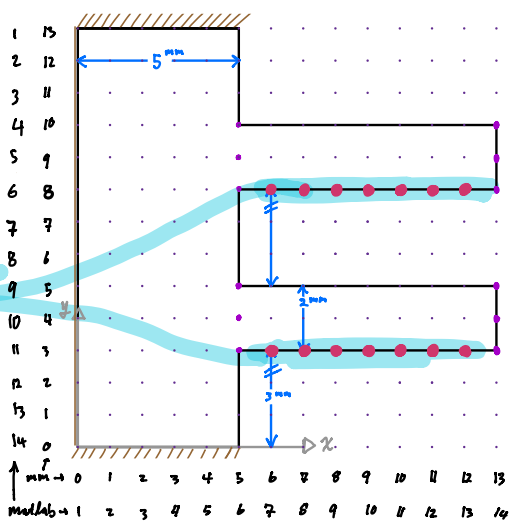
LOWER SURFACE OF FIN NODES (Same as above, but n+1 replaces n-1)

(7,6) → (13,6) and (7,11) → (13,11)

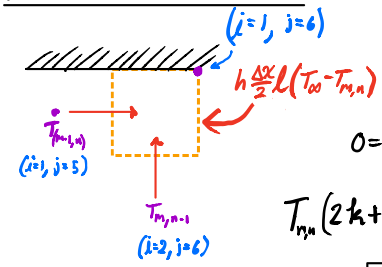


(F)
$$T(m,n) = \frac{(k/2)T(m-1,n) + kT(m,n+1) + (k/2)T(m+1,n) + h\Delta x T_\infty}{2k + \Delta x h}$$

$$T_{(m,n)} = \left(\frac{1}{2k+\Delta x h}\right)\left[\left(\frac{k}{2}\right)T_{(i,j-1)} + kT_{(i-1,j)} + \left(\frac{k}{2}\right)T_{(i,j+1)} + h\Delta x T_\infty\right]$$



TOP CORNER (6,1)

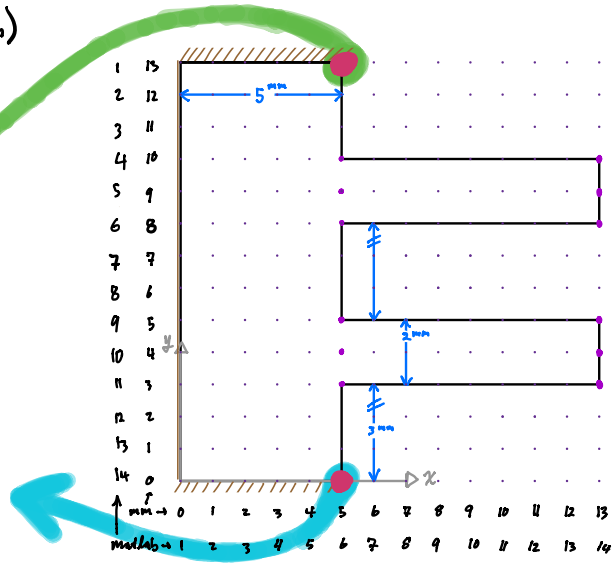


$$0 = \ell\frac{\Delta x}{2}k\left[\frac{T_{(m-1,n)} - T_{m,n}}{\Delta x}\right] + k\ell\frac{\Delta x}{2}\left[\frac{T_{m,n-1} - T_{m,n}}{\Delta x}\right] + h\frac{\Delta x}{2}\ell(T_\infty - T_{m,n})$$

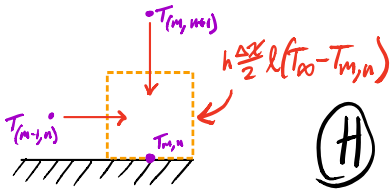
$$T_{m,n}(2k + h\Delta x) = kT_{(m-1,n)} + kT_{(m,n-1)} + h\Delta x T_\infty$$

(G) $$T_{m,n} = \frac{kT_{(m-1,n)} + kT_{(m,n-1)} + h\Delta x T_\infty}{2k + h\Delta x}$$

$$T_{m,n} = \left(\frac{1}{2k + \Delta x h}\right)\left[k\,T_{(i,j-1)} + kT_{(i+1,j)} + h\Delta x T_\infty\right]$$



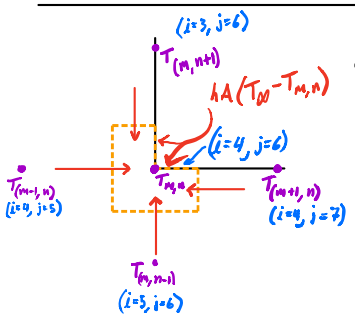
BOTTOM CORNER (6,14)



(H) $$T_{m,n} = \frac{kT_{(m-1,n)} + kT_{(m,n+1)} + h\Delta x T_\infty}{2k + h\Delta x}$$

$$T_{m,n} = \left(\frac{1}{2k + h\Delta x}\right)\left[k\,T_{(i,j-1)} + kT_{(i-1,j)} + h\Delta x T_\infty\right]$$

TOP LEFT FIN CORNERS (6,4) and (6,9)



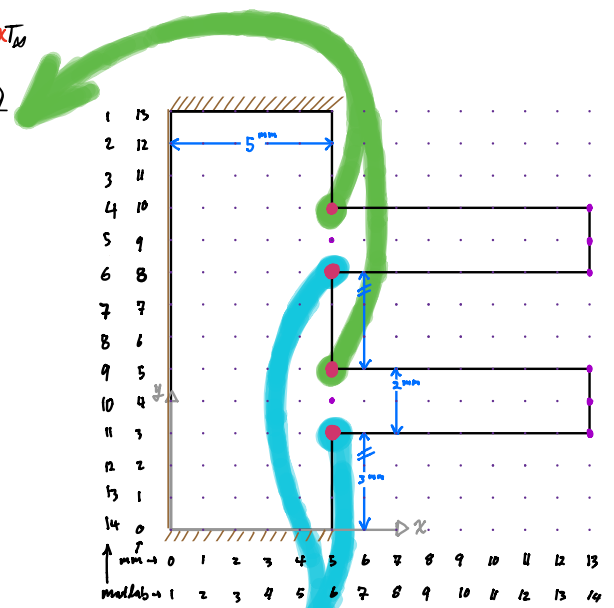
$$0 = \ell k\frac{\Delta x}{2}\left(\frac{T_{(m,n+1)} - T_{m,n}}{\Delta x}\right) + \ell k\Delta x\left(\frac{T_{(m-1,n)} - T_{m,n}}{\Delta x}\right) + \ell k\left(\frac{T_{(m,n-1)} - T_{m,n}}{\Delta x}\right) + \ell k\frac{\Delta x}{2}\left(\frac{T_{(m+1,n)} - T_{m,n}}{\Delta x}\right) + h\ell\Delta x(T_\infty - T_{m,n})$$

$$T_{m,n}\left\{\frac{k}{2} + k + k + \frac{k}{2} + h\Delta x\right\} = \frac{k}{2}T_{(m,n+1)} + kT_{(m-1,n)} + kT_{(m,n-1)} + \frac{k}{2}T_{(m+1,n)} + h\Delta x T_\infty$$

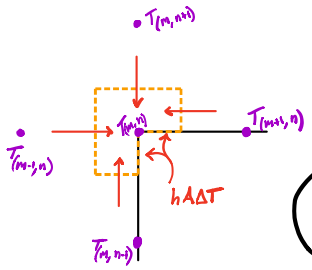
$$3k + h = k\left(3 + \frac{h}{k}\right) \qquad \frac{k\left(\frac{1}{2}T_{(m,n+1)} + T_{(m-1,n)} + T_{(m,n-1)} + \frac{1}{2}T_{(m+1,n)} + \frac{h\Delta x}{k}T_\infty\right)}{k\left(3 + \frac{h\Delta x}{k}\right)}$$

(I) $$T_{m,n} = \frac{\frac{1}{2}T_{(m,n+1)} + T_{(m-1,n)} + T_{(m,n-1)} + \frac{1}{2}T_{(m+1,n)} + \left(\frac{h\Delta x}{k}\right)T_\infty}{3 + \left(\frac{h\Delta x}{k}\right)}$$

$$T_{m,n} = \frac{1}{3 + (h\Delta x/k)}\left[0.5*T_{(i-1,j)} + T_{(i,j-1)} + T_{(i+1,j)} + 0.5*T_{(i,j+1)} + \left(\frac{h\Delta x}{k}\right)T_\infty\right]$$



BOTTOM LEFT FIN CORNERS (6,6) and (6,11)

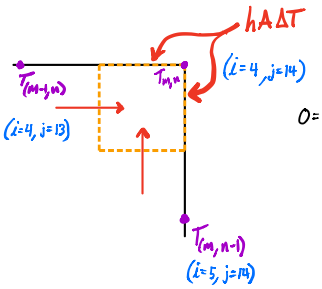


Same exact setup, but the $(\frac{1}{2})$ coefficient swaps from $T_{(m,n+1)}$ to $T_{(m,n-1)}$

(J) $$T_{m,n} = \frac{T_{(m,n+1)} + T_{(m-1,n)} + \left(\frac{1}{2}\right)T_{(m,n-1)} + \left(\frac{1}{2}\right)T_{(m+1,n)} + \left(\frac{h\Delta x}{k}\right)T_\infty}{3 + (h\Delta x/k)}$$

$$T_{m,n} = \frac{1}{3 + (h\Delta x/k)}\left[T_{(i-1,j)} + T_{(i,j-1)} + 0.5*T_{(i+1,j)} + 0.5*T_{(i,j+1)} + \left(\frac{h\Delta x}{k}\right)T_\infty\right]$$

TOP RIGHT FIN CORNERS (14, 4) and (14, 9)



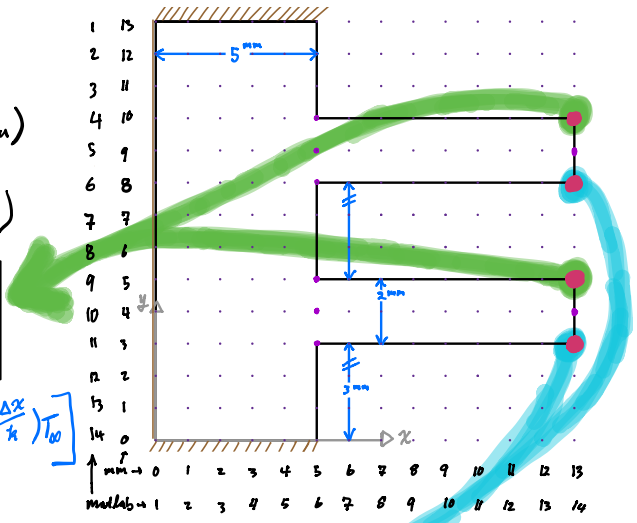
$$0 = k\frac{\Delta x}{2}\left(\frac{T_{(m-1,n)} - T_{m,n}}{\Delta x}\right) + k\frac{\Delta x}{2}\left(\frac{T_{(m,n-1)} - T_{m,n}}{\Delta x}\right) + h\Delta x\left(T_\infty - T_{m,n}\right)$$

$$T_{m,n}\cdot k\left(\frac{1}{2} + \frac{1}{2} + \frac{h\Delta x}{k}\right) = k\left(\frac{1}{2}\right)T_{(m-1,n)} + \left(\frac{1}{2}\right)T_{(m,n-1)} + \left(\frac{h\Delta x}{k}\right)T_\infty$$

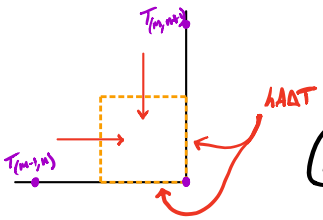
(K)

$$T_{m,n} = \frac{\left(\frac{1}{2}\right)T_{(m-1,n)} + \left(\frac{1}{2}\right)T_{(m,n-1)} + \left(\frac{h\Delta x}{k}\right)T_\infty}{1 + \left(\frac{h\Delta x}{k}\right)}$$

$$T_{m,n} = \left(\frac{1}{1+\left(\frac{h\Delta x}{k}\right)}\right)\left[0.5 * T_{(i,j-1)} + 0.5 * T_{(i+1,j)} + \left(\frac{h\Delta x}{k}\right)T_\infty\right]$$



BOTTOM RIGHT FIN CORNERS (14, 6) and (14, 11)

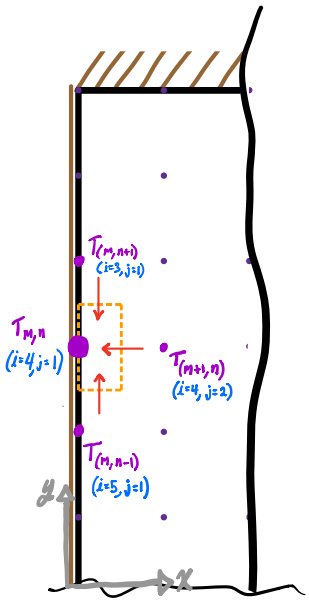


(L)

$$T_{m,n} = \frac{\left(\frac{1}{2}\right)T_{(m-1,n)} + \left(\frac{1}{2}\right)T_{(m,n+1)} + \left(\frac{h\Delta x}{k}\right)T_\infty}{1 + \left(\frac{h\Delta x}{k}\right)}$$

$$T_{m,n} = \left(\frac{1}{1+\left(\frac{h\Delta x}{k}\right)}\right)\left[0.5 * T_{(i,j-1)} + 0.5 * T_{(i-1,j)} + \left(\frac{h\Delta x}{k}\right)T_\infty\right]$$

LEFT WALL HEAT TRANSFER RATE



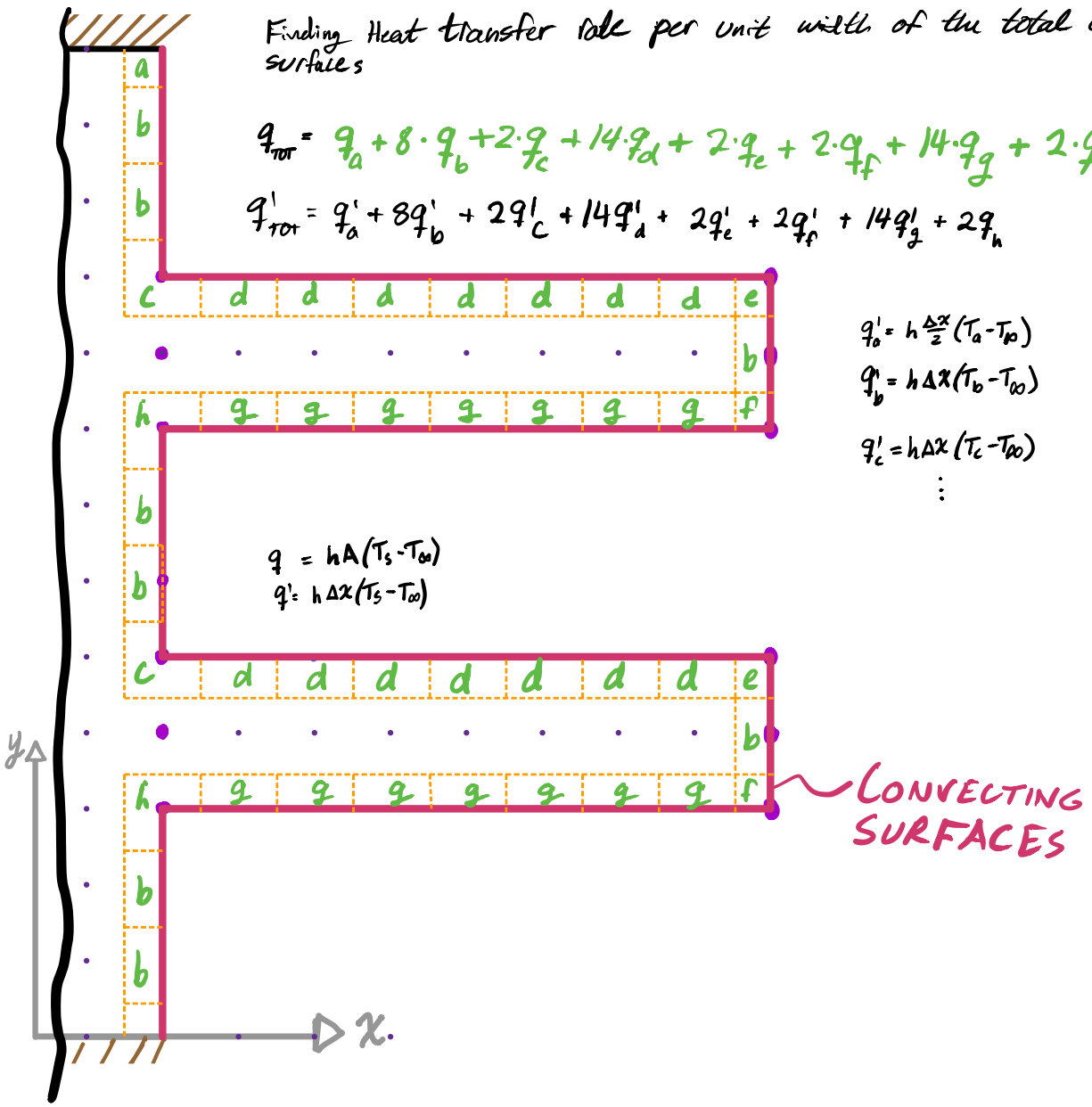
$$q' = \sum_{k=1}^{14} q'_k$$

$$q_{(n,n)} = k\cancel{l}\cancel{(\Delta x)} \cdot \left(\frac{T_{(m+1,n)} - T_{(m,n)}}{\cancel{\Delta x}}\right)$$

$$\boxed{q'_{(i,j)} = k\left(T_{(i,j+1)} - T_{(i,j)}\right)}$$

top & Bottom:

$$q' = .05 \cdot k\left(T_{(i,j+1)} - T_{(i,j)}\right)$$



Finding Heat transfer rate per unit width of the total connecting surfaces

$$q_{TOT} = q_a + 8 \cdot q_b + 2 \cdot q_c + 14 \cdot q_d + 2 \cdot q_e + 2 \cdot q_f + 14 \cdot q_g + 2 \cdot q_h$$

$$q'_{TOT} = q'_a + 8q'_b + 2q'_c + 14q'_d + 2q'_e + 2q'_f + 14q'_g + 2q_h$$

$$q'_a = h\frac{\Delta x}{2}(T_a - T_\infty)$$

$$q'_b = h\Delta x(T_b - T_\infty)$$

$$q'_c = h\Delta x(T_c - T_\infty)$$

$$\vdots$$

$$q = hA(T_s - T_\infty)$$

$$q' = h\Delta x(T_s - T_\infty)$$

CONVECTING SURFACES

(F.)(iii.)

20.9265 when $cc = 0.00000001$

0.1% of 20.9265 $= \frac{0.1}{100} \cdot 20.9265 = 0.0209265$

$20.9265 + 0.0209265 = 20.9474265$

To get this value for q', use guess & check.

For convection criteria 0.000176, $q' = 20.9474$ ✓

(iv.) $q' = 0.9252$ for Conv Criteria 0.000176