**Finite-Difference Heat Transfer Analysis Utilizing Gauss-Seidel Iteration in MATLAB**

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Wednesday Lab

ENGR 372

Lab Report 3

**A.) See appendix for hard copy of full program.**

**B.) Nodal Equations (See above attachments)**

**C.) Flowchart (See above attachments)**

**D.) Run Instructions**

1. Open MATLAB R2018b or later

2. Open file “lab3.m”

3. Click “Editor” tab in the upper toolbar, or click anywhere in the code to enter editor view.

4. Click the green arrow in the upper toolbar in that is labeled “Run”

5. Enter values of Thermal Conductivity [k], Convection Coefficient [h0], and Convergence Criteria (in Degrees Celsius) when prompted. After typing each number, click “ok” to continue.

6. Below in the “Command Window” notice the six fin base temperatures and six fin tip temperatures, as well as the number of iterations and heat transfer rates per unit width [q’].

7. If other nodal temperatures are desired, click “Yes” when prompted.

8. Enter the x-coordinate of the desired node, then click “OK”

9. Enter the y-coordinate of the desired node, then click “OK”

10. Nodal temperature will appear below in the “Command Window”

11. Repeat steps 7-10 as desired for other node temperature values

12. If another run is desired, select “No” when asked to see another node value. Then select “Yes” when asked if another run is desired.

13. Repeat Steps 5-12 as desired.

14. To close the program, simply click “No” for each option when prompted, and click the “X” in the top right corner of the window to close MATLAB.

**E.) Temperature Values of the Six Nodes along the Vertical Surfaces on the Tips of the Two Fins**

|  |  |
| --- | --- |
| **Convergence Criteria** | 0.0001 ºC |
| **Thermal Conductivity [k]** | 1 W/mK |
| **Convection Coefficient [h0]** | 1 W/m2K |
| **Top Corner of Upper Fin Tip** | 347.4547 ºC |
| **Center of Upper Fin Tip Temperature** | 347.7748 ºC |
| **Bottom Corner of Upper Fin Tip Temperature** | 347.4543 ºC |
| **Top Corner of Lower Fin Tip Temperature** | 318.9116 ºC |
| **Center of Lower Fin Tip Temperature** | 319.2039 ºC |
| **Bottom Corner of Lower Fin Tip Temperature** | 318.9113 ºC |

**F.) i.) For k = 1 W/mK and h0 = 1 W/m2K,**

|  |  |
| --- | --- |
| **Convergence Criteria** | **Number of Iterations** |
| 1 ºC | 38 |
| 0.1 ºC | 275 |
| 0.01 ºC | 626 |
| 0.001 ºC | 980 |
| 0.0001 ºC | 1337 |
| 0.00001 ºC | 1696 |

**ii.)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Thermal Conductivity [k] (W/m\*K)** | **Convection Coefficient [h0] (W/m2K)** | **Total Heat Transfer Rate per Unit Width Through Left Wall [q’] (W/m)** | **Convergence Criteria** |
| 10 | 1 | 22.2641 | 0.00000001 |
| 1 | 1 | 20.9265 | 0.00000001 |
| 0.1 | 1 | 13.5129 | 0.00000001 |
| 10 | 10 | 209.2647 | 0.00000001 |
| 1 | 10 | 135.1286 | 0.00000001 |
| 0.1 | 10 | 41.0692 | 0.00000001 |

**iii.)** Temperature convergence criteria required to get an accuracy of 0.1% for q’ when k=1 and h0=1. Work shown below.

Text, letter

Description automatically generated

The percent accuracy of q’ is different from the percent accuracy of the temperatures because as the temperatures get higher with different values of k and/or h0, the difference is higher, and has a greater effect on the value of q’. This is opposed to finding the value of temperatures using the convergence criteria, which is always a delta between two temperatures without a multiplicative factor. For any temperature differences using the same convergence criteria, the difference will be the same even if there are very high or very low temperatures.

**iv.)** Accurate value of heat transfer rate per unit width

|  |  |  |  |
| --- | --- | --- | --- |
| **Thermal Conductivity [k] (W/m\*K)** | **Convection Coefficient [h0] (W/m2K)** | **Total Heat Transfer Rate per Unit Width Through the Total Convecting Surfaces [q’] (W/m)** | **Convergence Criteria** |
| 10 | 1 | 22.2641 | 0.00000001 |
| 1 | 1 | 20.9265 | 0.00000001 |
| 0.1 | 1 | 13.5129 | 0.00000001 |
| 10 | 10 | 209.2646 | 0.00000001 |
| 1 | 10 | 135.1287 | 0.00000001 |
| 0.1 | 10 | 41.0692 | 0.00000001 |

With a very small convergence criteria, the values of the total heat transfer through the left wall per unit width and the total heat transfer through the convecting surfaces of the wall are nearly identical. This makes sense because if one were to draw a control volume around the entire object, q’IN must be equal to q’OUT. The fact that there are any discrepancies can be attributed to calculation errors due to rounding within the program.