How to Implement Quantum Galton Board?

1) Simulating a single peg

An integral part of a Galton board is the peg, which the board consists of. The single peg will decide which way the ball will follow, either right or left. For classical peg, the probability of going right is 50% and probability of going left is 50% as well. If we have these probabilities of each peg, then the overall distribution of the balls will follow normal distribution. To achieve different types of distributions, we need to introduce some bias to probabilities. Since, we deal with quantum Galton board (QGB), we must implement it using quantum circuits.

Simulation of a single peg in QGB will require 3 working qubits and one ancilla qubit. By default, Hadamard gate is applied to ancilla qubits to create a superposition and X gate is applied to a middle qubit. As for working gates, we will require 2 SWAP gates and one CNOT gate.

2) Creating a board

The board consists of layers of pegs, which follows the shape of a triangle. The first layer consists of a single peg, the second layer consists of 2 pegs, and so on. Layers with multiple pegs in a row will be connected with an additional CNOT gate. As a result, 4 gates on average are needed to implement a single peg.

3) Implementing different distributions

As we already discussed, Hadamard gate is applied to ancilla qubits at the beginning to introduce superposition. Because of this Hadamard gate, the distribution of balls will follow normal distribution. For achieving different distribution, e.g. exponential, all we need to do is replace the Hadamard gate with Rx gate and choose an appropriate angle of rotation.

Following all these steps, we arrive at the working Quantum Galton Board that simulates different distributions. The only thing that is left is to choose number of layers. Depending on the choice of number of layers, the number of qubits, number of gates, and overall complexity of the board will change accordingly.