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Section : S5 , S6

1) (a) $y\cos(y)\dfrac{dy}{dx} = \dfrac{x^2}{\sqrt{1-x^2}}$ $\quad \therefore \int y\cos(y)\, dy = \int \dfrac{x^2}{\sqrt{1-x^2}}\, dx$

$\therefore$

D	I
y	$\cos y$
1	$\sin y$
0	$-\cos y$

$\therefore \int y\cos y\, dy = y\sin y + \cos y + C$

let $x = \sin\theta$

$\therefore dx = \cos\theta\, d\theta$

$$\therefore \int \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}} \cos\theta\, d\theta = \int \frac{\sin^2\theta}{\cos\theta}\cos\theta\, d\theta = \int \sin^2\theta\, d\theta$$

$$= \int \frac{1}{2} - \frac{1}{2}\cos 2\theta\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C = \frac{1}{2}\sin^{-1}x - \frac{1}{4}\sin(2\sin^{-1}x)$$

$$\therefore y\sin y + \cos y = \frac{1}{2}\sin^{-1}x - \frac{1}{4}\sin^{-1}(2\sin^{-2}x) + C$$

1) (b) $\int \frac{y-x}{x+y} \, d\cancel{x} = -\int \frac{x-y}{x+y} \, d\cancel{x}$ $\quad \therefore \int dy = -\int \frac{x-y}{x+y} \, dx$

$y = -\int \frac{x-y}{x+y} \, dx$ $\quad$ let $u = x+y$ $\quad \therefore dx = du$

$\therefore -\int \frac{x-y}{x+y} \, dx = -\int \frac{u-2y}{u} \, du = -\int 1 - \frac{2y}{u} \, du = -u + 2y \ln u$

$\therefore y = -x + y + 2y \ln|x+y|$ $\quad \therefore y = \frac{2y \ln|x+y| - x}{2} + c$

c) $3x^2 \cos(3y) \, dy = -2x \sin(3y) \, dx$

$\therefore \int \frac{\cos(3y)}{\sin(3y)} \, dy = -\int \frac{2x}{3x^2} \, dx$

$\therefore \frac{1}{3} \ln|\sin 3y| = -\frac{2}{3} \ln|x| + c$

2) $x y' + y = 3x^3 y^3$ $\quad \therefore y' + \frac{y}{x} = 3x^2 y^3$

$n = 3 \quad \therefore 1 - n = -2$ $\quad , z_{(x)} = y^{-2} \quad \therefore z'_{(x)} = -2y^{-3}$

$P(x) = \frac{1}{x}, \quad q(x) = 3x^2$ $\quad \therefore z'_{(x)} + z_{(x)} \cdot P_{(x)} \cdot (1-n) = (1-n) \cdot q_{(x)}$

$\therefore z' - \frac{2}{x} z = -6x^2 \quad , \quad M = e^{-2 \ln x} = x^{-2}$

$z_{(x)} = x^2 \left[\int -6x^2 \cdot x^{-2} \, dx \right] \quad , z_{(x)} = x^2(-6x + c) \quad , z_{(x)} = -6x^3 + x^2 c$

$y^{-2} = -6x^3 + x^2 c \quad , \quad y(1) = 1$

$\therefore -6(1)^3 + (1)^2 c = (1)^{-2} \quad \therefore c = 7$

$\therefore y^{-2} = -6x^3 + 7x^2 \quad \therefore y = \frac{1}{\sqrt{x^2(-6x+7)}}$

3) (a) $y'' - 3y' = 9e^{3x} + 10$, $y = e^{mx}$

$\therefore m^2 - 3m = 0$ $\therefore m = 0$, $m = 3$

$y_c(x) = c_1 + c_2 e^{3x}$

$y_p = Axe^{3x} + Bx$

$y' = Ae^{3x} + 3Axe^{3x} + B$ $\therefore y' = A(1+3x)e^{3x} + B$

$\therefore y'' = 3Ae^{3x} + 3Ae^{3x} + 9Axe^{3x}$ $\therefore y'' = 3A(2+3x)e^{3x}$

$\therefore 6A + 9Ax - 3A - 9Ax = 9$ $\therefore A = 3$

$3A(2+3x)e^{3x} - 3A(1+3x)e^{3x} - 3B = 9e^{3x} + 10$, $\therefore B = \frac{-10}{3}$

$\therefore y_p = 3xe^{3x} - \frac{10}{3}x$

$y_c(x) + y_p(x) = c_1 + c_2 e^{3x} + 3xe^{3x} - \frac{10}{3}x$

3) (c) $x^2 y'' + 3x y' + y = \sin(\ln x) + 5\ln x$

$e^t = x \qquad \therefore t = \ln x$

$x y' = \dot{y} \quad , x^2 y'' = (\ddot{y} - \dot{y}) \quad , \ddot{y} - \dot{y} + 3\dot{y} + 1 = \sin t + 5t$

$\ddot{y} + 2\dot{y} + 1 = 0 \quad , r^2 + 2r + 1 = 0 \quad , r_1 = -1 = r_2$

$y_c(t) = C_1 e^{-t} + C_2 t e^{-t}$

$y_p(t) = 5t + \sin t \qquad , \dot{y} = -A\sin t + B\cos t + C$

$\ddot{y} = -A\cos t - B\sin t$

$(-A\cos t - B\sin t) + 2(-A\sin t + B\cos t + C) + A\cos t + B\sin t + Ct + D$
$= \sin t + 5t$

$\therefore B = 0 , \ A = \frac{-1}{2} , C = 5 , D = 0$

$y_p = \frac{-1}{2}\cos t + 5t$

$y_g(x) = C_1 e^{-x} + C_2 \ln x \, e^{-x} - \frac{1}{2}\cos t \ln x + 5\ln x$

$\boxed{5}$

4) $x^2 y'' - 6y = 0$, $y(1) = 1$, $y'(1) = 0$

$x = e^t$ $\therefore t = \ln x$

$(\ddot{y} - \dot{y}) - 6 = 0$, $r_2 = 3$; $r_1 = -2$

$y(t) = c_1 e^{3t} + c_2 e^{-2t}$

$y_c(x) = c_1 x^3 + c_2 x^{-2}$

$y'_c(x) = 3c_1 x^2 - 2c_2 x^{-3}$

$y_c(1) = c_1 + c_2 = 1$

$y'_c(1) = 3c_1 - 2c_2 = 0$ $\therefore \frac{2}{3} c_2 = c_1$

$\therefore c_2 + \frac{2}{3} c_2 = \frac{4}{3} c_2$ $\therefore c_2 = \frac{3}{4}$, $c_1 = \frac{1}{2}$

$\therefore y_{(x)} = \frac{1}{2} x^3 + \frac{3}{4} x^{-2}$

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