Problem Set 2

Byron Ralph Boter Nhemuel Robles Giovanni Ross Sucalit

2024-02-25

Problem Set 2

1. Use the bisection method with a hand calculator or computer to find the real root of $x^3 - x - 3 = 0$. Use an error tolerance of $\epsilon = 0.0001$. Graph the function $f(x) = x^3 - x - 3$ and label the root.

2. The function $f(x) = -3x^3 + 2e^{\frac{x^2}{2}} - 1$ has values of zero near $x = -0.5$ and $x = 0.5$.

a. What is the derivative of f?

b. If you begin Newton's method at $x = 0$, which root is reached? How many iterations to achieve an error less than 10^{-5}?

c. Begin Newton's method at another starting point to get the other zero.

3. Use the function from no.2 and find the root using the secant method where $x_0 = 0$ and $x_1 = 1$. Use an error tolerance of $\epsilon = 0.001$.

4. Consider the system

$$10.2x + 2.4y - 4.5z = 14.067,$$

$$-2.3x - 7.7y + 11.1z = -0.996,$$

$$-5.5x - 3.2x + 0.9z = -12.645.$$

a. Present the augmented matrix of the system.

b. Solve the system using $A_x = LU_x = L_y = b$ and round the final answer to 4 decimal digits.

c. Find the residual vector if the correct solution is $x = 1.4531001$, $y = -1.5891949$, $z = -0.2748947$.

5. Compute the Frobenius norm, maximum column sum, and maximum row sum of the matrix:

$$\begin{pmatrix} 10.2 & 2.4 & 4.5 \\ -2.3 & 7.7 & 11.1 \\ -5.5 & -3.2 & 0.9 \end{pmatrix}$$

6. Solve the system of equations given in no. 4, starting with the initial vector of $[0, 0, 0]$:

a. Solve using the Jacobi method with 2-digit precision.

b. Solve using Gauss-Seidel method with 2-digit precision.

c. Solve for e if the true solution is $x = (1.5, 0.33, 0.45)^T$.