

Дисциплина: «Математические методы теории систем»
Лабораторная работа № 1
на тему: «Модели структуры систем с использованием теории графов.
Топологическая декомпозиция»

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ЦЕЛЬ РАБОТЫ

Построение наглядной формальной модели, отображающей процесс взаимодействия между элементами или подсистемами, составляющими систему, а также их взаимодействие с внешней средой.

ЗАДАНИЕ

Задача 1. Дано описание системы на рис. 1.

1. Выполнить матричное и множественное описание графа топологии системы;
2. Выполнить топологическую декомпозицию системы (рис.1);
3. Разработать алгоритм решения задачи топологической декомпозиции на одном из языков программирования. Привести результаты работы программы

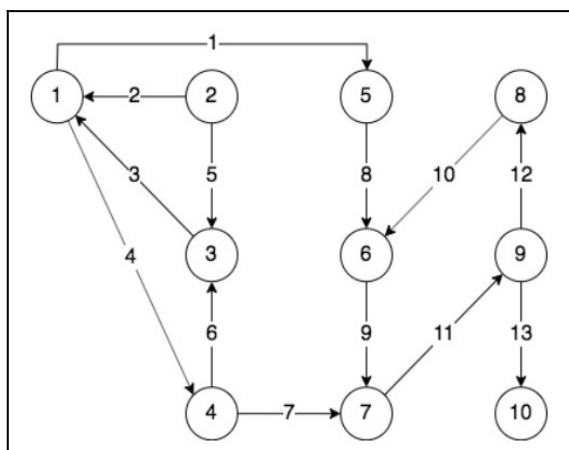


Рис. 1. Описание системы

Задача 2. Выбранный вариант системы приведён на рис. 2.

Выполнить топологическую декомпозицию одной системы из предложенных ниже вариантов, используя разработанную программу. Привести результаты работы программы.

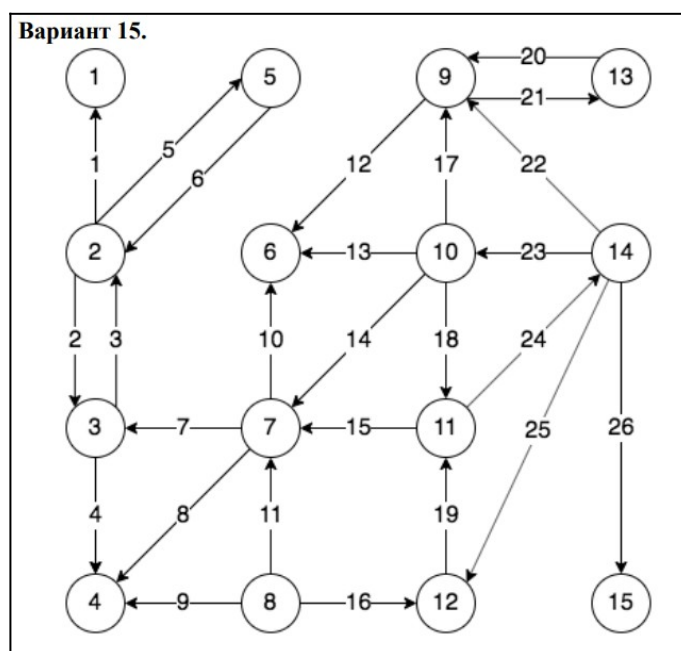


Рис. 2. Описание системы. Вариант 15

ВЫПОЛНЕНИЕ РАБОТЫ

Задание 1

1. Матричное представление

Матрицы смежности и инцидентности приведены в табл. 1 и 2 соответственно.

Таблица 1. Матрица смежности

i\j	1	2	3	4	5	6	7	8	9	10
1	0	0	0	1	1	0	0	0	0	0
2	1	0	1	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	1	0	0	0
5	0	0	0	0	0	1	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	0	0	0	1	0
8	0	0	0	0	0	1	0	0	0	0
9	0	0	0	0	0	0	0	1	0	1
10	0	0	0	0	0	0	0	0	0	0

Таблица 2. Матрица инцидентностей

i\j	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	-1	-1	1	0	0	0	0	0	0	0	0	0
2	0	1	0	0	1	0	0	0	0	0	0	0	0
3	0	0	1	0	-1	-1	0	0	0	0	0	0	0
4	0	0	0	-1	0	1	1	0	0	0	0	0	0
5	-1	0	0	0	0	0	0	1	0	0	0	0	0
6	0	0	0	0	0	0	0	-1	1	-1	0	0	0
7	0	0	0	0	0	0	-1	0	-1	0	1	0	0
8	0	0	0	0	0	0	0	0	0	1	0	-1	0
9	0	0	0	0	0	0	0	0	0	0	-1	1	1
10	0	0	0	0	0	0	0	0	0	0	0	0	-1

2. Множественное представление

$G(1)=(4,5);$	$G^{-1}(1)=(2,3);$
$G(2)=(1,3);$	$G^{-1}(2)=(0);$
$G(3)=(1);$	$G^{-1}(3)=(2,4);$
$G(4)=(3,7);$	$G^{-1}(4)=(1);$
$G(5)=(6);$	$G^{-1}(5)=(1);$
$G(6)=(7);$	$G^{-1}(6)=(5,8);$
$G(7)=(9);$	$G^{-1}(7)=(6,4);$
$G(8)=(6);$	$G^{-1}(8)=(9);$
$G(9)=(8,10);$	$G^{-1}(9)=(7);$
$G(10)=(0);$	$G^{-1}(10)=(9);$

3. Топологическая декомпозиция системы

Достижимое множество:

$$R(i)=(i) \vee G(i) \vee \dots \vee G^{\lambda}(i) \vee \dots, \text{ где } \lambda - \text{длина пути графа};$$

Контрдостижимое множество:

$$R(i)=(i) \vee G(i)^{-1} \vee \dots \vee G^{\lambda}(i)^{-1} \vee \dots;$$

Сильно связный подграф:

$$V_n=R(i) \cap Q(i);$$

$$\begin{aligned} R(1) &= (1) \vee R(1)^1 \vee R(1)^2 \vee R(1)^3 \vee R(1)^4 = \\ &= (1) \vee (4,5)^1 \vee (3,7,6)^2 \vee (7,9)^3 \vee (8,10)^4 = \\ &= (1,4,5,3,7,6,8,9,10); \end{aligned}$$

$$\begin{aligned} Q(1) &= (1) \vee Q(1)^{-1} \vee Q(1)^{-2} = \\ &= (1) \vee (2,3)^{-1} \vee (2,4)^{-2} = \\ &= (1,2,3,4); \end{aligned}$$

$$V_1=R(1) \cap Q(1)=(1,3,4);$$

$$\begin{aligned} R(5) &= (5) \vee R(5)^1 \vee R(5)^2 \vee R(5)^3 \vee R(5)^4 = \\ &= (5) \vee (6)^1 \vee (7)^2 \vee (9)^3 \vee (8,10)^4 = \\ &= (5,6,7,8,9,10); \end{aligned}$$

$$Q(5)=(5);$$

$$V_2=R(5) \cap Q(5)=(5);$$

$$\begin{aligned} R(6) &= (6) \vee R(6)^1 \vee R(6)^2 \vee R(6)^3 = \\ &= (6) \vee (7)^1 \vee (9)^2 \vee (8,10)^3 = \\ &= (6,7,8,9,10); \end{aligned}$$

$$\begin{aligned} Q(6) &= (6) \vee Q(6)^{-1} \vee Q(6)^{-2} \vee Q(6)^{-3} = \\ &= (6) \vee (8)^{-1} \vee (9)^{-2} \vee Q(7)^{-3} = \\ &= (6,7,8,9); \end{aligned}$$

$$V_3=R(6) \cap Q(6)=(6,7,8,9);$$

$$R(10)=(10);$$

$$Q(10)=(10);$$

$$V_4=R(10) \cap Q(10)=(10);$$

И того имеем:

1. $G_1(V_1) = G_1(2);$
2. $G_2(V_2) = G_2(1,3,4);$
3. $G_3(V_3) = G_3(5);$
4. $G_4(V_4) = G_4(6,7,8,9);$
5. $G_5(V_5) = G_5(10).$

Результаты приведены на рис. 3, 4, 5.

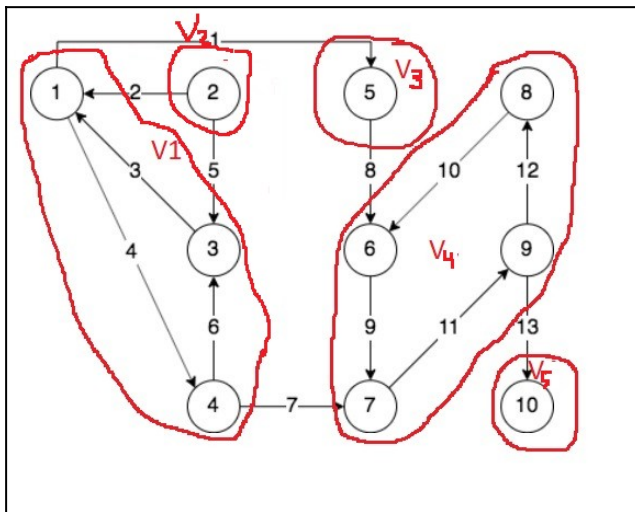


Рис. 3. Декомпозиция по блокам

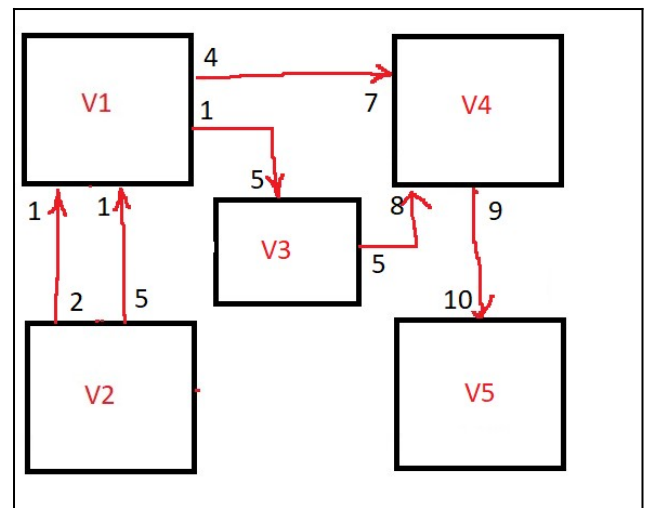


Рис. 4. Вид сильно связанных подграфов

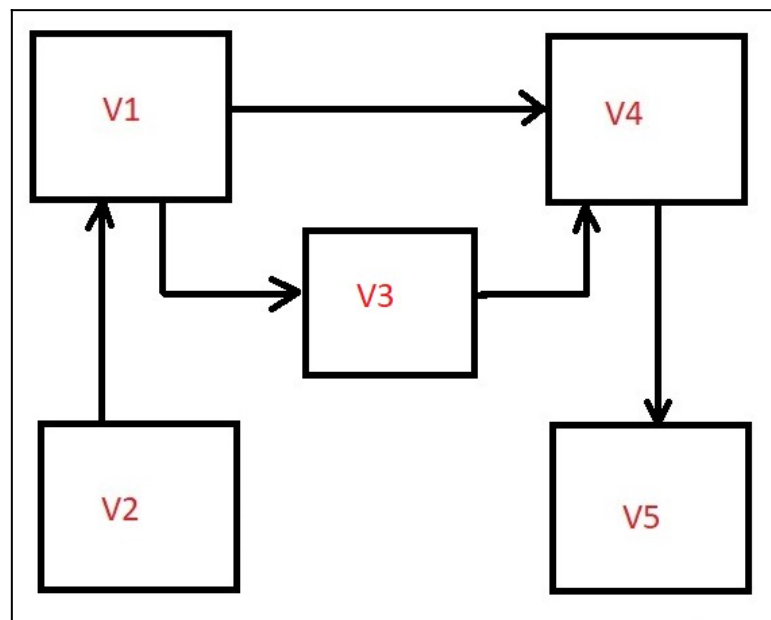


Рис. 5. Результат декомпозиции исходного графа

Задание 2

1. Матричное представление

Таблица 3. Матрица смежности

i\j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0
8	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0
9	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
10	0	0	0	0	0	1	1	0	1	0	1	0	0	0	0
11	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
12	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	1	1	0	1	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Таблица 4. Матрица инцидентий

i\j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	-1	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	-1	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	-1	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	-1	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	1	1	0	1	-1	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	-1	1	-1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	-1	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	-1	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0	-1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1

Вывод программного кода приведен в приложениях А и Б.

ЗАКЛЮЧЕНИЕ

Программный код: https://github.com/Kirpo97/MMTS_labs/tree/main/lab_1.

ПРИЛОЖЕНИЕ А. Задание 1

Топологическая декомпозиция системы. Вычисление достижимого множества

```
R_lambda( 1 ) = [{4, 5}, {1, 3}, {1}, {3, 7}, {6}, {7}, {9}, {6}, {8, 10}, set()]
R_lambda( 2 ) = [{3, 6, 7}, {1, 4, 5}, {4, 5}, {1, 9}, {7}, {9}, {8, 10}, {7}, {6}, set()]
R_lambda( 3 ) = [{1, 9, 7}, {3, 4, 5, 6, 7}, {3, 6, 7}, {8, 10, 4, 5}, {9}, {8, 10}, {6}, {9}, {7}, set()]
R_lambda( 4 ) = [{4, 5, 8, 9, 10}, {1, 3, 6, 7, 9}, {1, 9, 7}, {3, 6, 7}, {8, 10}, {6}, {7}, {8, 10}, {9}, set()]
R_lambda( 5 ) = [{3, 6, 7, 8, 10}, {1, 4, 5, 7, 8, 9, 10}, {4, 5, 8, 9, 10}, {1, 9, 7}, {6}, {7}, {9}, {6}, {8, 10}, set()]

R( 1 ) = {1, 3, 4, 5, 6, 7, 8, 9, 10}
R( 2 ) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
R( 3 ) = {1, 3, 4, 5, 6, 7, 8, 9, 10}
R( 4 ) = {1, 3, 4, 5, 6, 7, 8, 9, 10}
R( 5 ) = {5, 6, 7, 8, 9, 10}
R( 6 ) = {6, 7, 8, 9, 10}
R( 7 ) = {6, 7, 8, 9, 10}
R( 8 ) = {6, 7, 8, 9, 10}
R( 9 ) = {6, 7, 8, 9, 10}
R( 10 ) = {10}
```

Топологическая декомпозиция системы. Вычисление контрдостижимого множества

```
Q_lambda( 1 ) = [{2, 3}, set(), {2, 4}, {1}, {1}, {8, 5}, {4, 6}, {9}, {7}, {9}]
Q_lambda( 2 ) = [{2, 4}, set(), {1}, {2, 3}, {2, 3}, {1, 9}, {8, 1, 5}, {7}, {4, 6}, {7}]
Q_lambda( 3 ) = [{1}, set(), {2, 3}, {2, 4}, {2, 4}, {2, 3, 7}, {1, 2, 3, 9}, {4, 6}, {8, 1, 5}, {4, 6}]
Q_lambda( 4 ) = [{2, 3}, set(), {2, 4}, {1}, {1}, {2, 4, 6}, {2, 3, 4, 7}, {8, 1, 5}, {1, 2, 3, 9}, {8, 1, 5}]
Q_lambda( 5 ) = [{2, 4}, set(), {1}, {2, 3}, {2, 3}, {8, 1, 5}, {1, 2, 4, 6}, {1, 2, 3, 9}, {2, 3, 4, 7}, {1, 2, 3, 9}]

Q( 1 ) = {1, 2, 3, 4}
Q( 2 ) = {2}
Q( 3 ) = {1, 2, 3, 4}
Q( 4 ) = {1, 2, 3, 4}
Q( 5 ) = {1, 2, 3, 4, 5}
Q( 6 ) = {1, 2, 3, 4, 5, 6, 7, 8, 9}
Q( 7 ) = {1, 2, 3, 4, 5, 6, 7, 8, 9}
Q( 8 ) = {1, 2, 3, 4, 5, 6, 7, 8, 9}
Q( 9 ) = {1, 2, 3, 4, 5, 6, 7, 8, 9}
Q( 10 ) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

Декомпозиция системы

```
... V 1 = {2}
    V 2 = {1, 3, 4}
    V 3 = {5}
    V 4 = {8, 9, 6, 7}
    V 5 = {10}
```

Результат: Исходный граф можно разделить на 5 подграфов

ПРИЛОЖЕНИЕ Б. Задание 2

Топологическая декомпозиция системы. Вычисление достижимого множества

```

R_Lambda ( 1 ) = [set(), {1, 3, 5}, {2, 4}, set(), {2}, set(), {3, 4, 6}, {4, 12, 7}, {13, 6}, {9, 11, 6, 7}, {14, 7}, {11}, {9}, {9, 10, 12, 15}, set()]
R_Lambda ( 2 ) = [set(), {2, 4}, {1, 3, 5}, set(), {1, 3, 5}, set(), {2, 4}, {11, 3, 4, 6}, {9}, {3, 4, 6, 7, 13, 14}, {3, 4, 6, 9, 10, 12, 15}, {14, 7}, {13, 6}, {6, 7, 9, 11, 13}, set()]
R_Lambda ( 3 ) = [set(), {1, 3, 5}, {2, 4}, set(), {2, 4}, set(), {1, 3, 5}, {2, 4, 14, 7}, {13, 6}, {2, 3, 4, 6, 9, 10, 12, 15}, {2, 4, 6, 7, 9, 11, 13}, {3, 4, 6, 9, 10, 12, 15}, {9}, {3, 4, 6, 7, 9, 13, 14}, set()]
R_Lambda ( 4 ) = [set(), {2, 4}, {1, 3, 5}, set(), {1, 3, 5}, set(), {2, 4}, {1, 3, 4, 5, 6, 9, 10, 12, 15}, {9}, {1, 2, 3, 4, 5, 6, 7, 9, 11, 13}, {1, 3, 4, 5, 6, 7, 9, 13, 14}, {2, 4, 6, 7, 9, 11, 13}, {13, 6}, {2, 3, 4, 6, 9, 10, 12, 13, 15}, set()]
R_Lambda ( 5 ) = [set(), {1, 3, 5}, {2, 4}, set(), {1, 3, 5}, {2, 4, 6, 7, 9, 11, 13}, {13, 6}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 14}, {2, 3, 4, 6, 9, 10, 12, 13, 15}, {1, 3, 4, 5, 6, 7, 9, 11, 13}, set()]
R_Lambda ( 6 ) = [set(), {2, 4}, {1, 3, 5}, set(), {1, 3, 5}, set(), {2, 4}, {1, 3, 4, 5, 6, 7, 9, 13, 14}, {9}, {1, 2, 3, 4, 5, 6, 9, 10, 12, 13, 15}, {1, 2, 3, 4, 5, 6, 7, 9, 11, 13}, {2, 3, 4, 5, 6, 7, 9, 13, 14}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 15}, set()]
R_Lambda ( 7 ) = [set(), {1, 3, 5}, {2, 4}, set(), {2, 4}, set(), {1, 3, 5}, {2, 3, 4, 6, 9, 10, 12, 13, 15}, {13, 6}, {1, 2, 3, 4, 5, 6, 7, 9, 11, 13}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 14}, {1, 2, 3, 4, 5, 6, 7, 9, 11, 13}, {1, 2, 3, 4, 5, 6, 9, 10, 12, 13, 15}, set()]
R_Lambda ( 8 ) = [set(), {2, 4}, {1, 3, 5}, set(), {1, 3, 5}, set(), {2, 4}, {1, 2, 3, 4, 5, 6, 7, 9, 11, 13}, {9}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 14}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 15}, {1, 2, 3, 4, 5, 6, 7, 9, 11, 13}, set()]
R_Lambda ( 9 ) = [set(), {1, 3, 5}, {2, 4}, set(), {2, 4}, set(), {1, 3, 5}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 14}, {13, 6}, {1, 2, 3, 4, 5, 6, 9, 10, 12, 13, 15}, {1, 2, 3, 4, 5, 6, 7, 9, 11, 13}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 14}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 15}, set()]
R_Lambda ( 10 ) = [set(), {2, 4}, {1, 3, 5}, set(), {1, 3, 5}, set(), {2, 4}, {1, 2, 3, 4, 5, 6, 9, 10, 12, 13, 15}, {9}, {1, 2, 3, 4, 5, 6, 7, 9, 11, 13}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 14}, {1, 2, 3, 4, 5, 6, 7, 9, 13, 15}, set()]
R ( 1 ) = {1}
R ( 2 ) = {1, 2, 3, 4, 5}
R ( 3 ) = {1, 2, 3, 4, 5}
R ( 4 ) = {4}
R ( 5 ) = {1, 2, 3, 4, 5}
R ( 6 ) = {6}
R ( 7 ) = {1, 2, 3, 4, 5, 6, 7}
R ( 8 ) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
R ( 9 ) = {6, 9, 13}
R ( 10 ) = {1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15}
R ( 11 ) = {1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15}
R ( 12 ) = {1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15}
R ( 13 ) = {9, 13, 6}
R ( 14 ) = {1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15}
R ( 15 ) = {15}

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Активация Windows

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Q.Lambda ( 1 ) = [{2}, {3, 5}, {2, 7}, {8, 3, 7}, {2}, {9, 10, 7}, {8, 10}, set(), {10, 13, 14}, {14}, {10, 12}, {8, 14}, {9}, {11}, {14}]
Q.Lambda ( 2 ) = [{3, 5}, {2, 7}, {6, 10, 3, 5}, {8, 2, 10, 7}, {3, 5}, {8, 10, 13, 14}, {14}, set(), {9, 11, 14}, {11}, {8, 14}, {11}, {10, 13, 14}, {10, 12}, {11}]
Q.Lambda ( 3 ) = [{2, 7}, {3, 5, 8, 10}, {2, 14, 7}, {3, 5, 8, 10, 14}, {2, 7}, {9, 11, 14}, {11}, set(), {10, 11, 12, 13, 14}, {10, 12}, {11}, {10, 12}, {9, 11, 14}, {8, 14}, {10, 12}]
Q.Lambda ( 4 ) = [{8, 10, 3, 5}, {2, 7, 14}, {3, 5, 8, 10, 11}, {2, 11, 14, 7}, {8, 10, 3, 5}, {10, 11, 12, 13, 14}, {10, 12}, set(), {8, 9, 10, 11, 12, 14}, {8, 14}, {10, 12}, {8, 14}, {10, 11, 12, 13, 14}, {11}, {8, 14}]
Q.Lambda ( 5 ) = [{2, 14, 7}, {3, 5, 8, 10, 11}, {2, 7, 10, 12, 14}, {3, 5, 8, 10, 11, 12}, {2, 14, 7}, {8, 9, 10, 11, 12, 14}, {8, 14}, set(), {8, 10, 11, 12, 13, 14}, {11}, {8, 9, 10, 11, 12, 14}, {10, 12}, {11}]
Q.Lambda ( 6 ) = [{3, 5, 8, 10, 11}, {2, 7, 10, 12, 14}, {3, 5, 8, 10, 11, 14}, {2, 7, 8, 10, 12, 14}, {3, 5, 8, 10, 11}, {8, 10, 11, 12, 13, 14}, {11}, set(), {8, 9, 10, 11, 12, 14}, {10, 12}, {11}, {10, 12}, {8, 10, 11, 12, 13, 14}, {8, 14}, {10, 12}]
Q.Lambda ( 7 ) = [{2, 7, 10, 12, 14}, {3, 5, 8, 10, 11, 14}, {2, 7, 10, 11, 12, 14}, {3, 5, 8, 10, 11, 14}, {2, 7, 10, 12, 14}, {8, 9, 10, 11, 12, 14}, {10, 12}, set(), {8, 10, 11, 12, 13, 14}, {8, 14}, {10, 12}, {8, 14}, {10, 11, 12, 13, 14}, {11}, {8, 14}]
Q.Lambda ( 8 ) = [{3, 5, 8, 10, 11, 14}, {2, 7, 10, 11, 12, 14}, {3, 5, 8, 10, 11, 12, 14}, {2, 7, 10, 11, 12, 14}, {3, 5, 8, 10, 11, 14}, {8, 10, 11, 12, 13, 14}, {10, 12}, set(), {8, 9, 10, 11, 12, 14}, {11}, {8, 14}]
Q.Lambda ( 9 ) = [{2, 7, 10, 11, 12, 14}, {3, 5, 8, 10, 11, 12, 14}, {2, 7, 8, 10, 11, 12, 14}, {3, 5, 8, 10, 11, 12, 14}, {2, 7, 10, 11, 12, 14}, {8, 9, 10, 11, 12, 14}, {10, 12}, set(), {8, 10, 11, 12, 13, 14}, {10, 12}, {11}, {10, 12}, {8, 9, 10, 11, 12, 14}, {10, 12}]
Q.Lambda ( 10 ) = [{3, 5, 8, 10, 11, 12, 14}, {2, 7, 8, 10, 11, 12, 14}, {3, 5, 8, 10, 11, 12, 14}, {2, 7, 8, 10, 11, 12, 14}, {3, 5, 8, 10, 11, 12, 14}, {8, 10, 11, 12, 13, 14}, {10, 12}, set(), {8, 9, 10, 11, 12, 14}, {10, 12}, {11}, {8, 14}]
Q ( 1 ) = {1, 2, 3, 5, 7, 8, 10, 14}
Q ( 2 ) = {2, 3, 5, 7, 8, 10, 11, 14}
Q ( 3 ) = {2, 3, 5, 7, 8, 10, 11, 12, 14}
Q ( 4 ) = {2, 3, 4, 5, 7, 8, 10, 11, 12, 14}
Q ( 5 ) = {2, 3, 5, 7, 8, 10, 14}
Q ( 6 ) = {6, 7, 8, 9, 10, 11, 12, 13, 14}
Q ( 7 ) = {7, 8, 10, 11, 12, 14}
Q ( 8 ) = {8}
Q ( 9 ) = {8, 9, 10, 11, 12, 13, 14}
Q ( 10 ) = {8, 10, 11, 12, 14}
Q ( 11 ) = {8, 10, 11, 12, 14}
Q ( 12 ) = {8, 10, 11, 12, 14}
Q ( 13 ) = {8, 9, 10, 11, 12, 13, 14}
Q ( 14 ) = {8, 10, 11, 12, 14}
Q ( 15 ) = {8, 10, 11, 12, 14, 15}

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__РЕЗУЛЬТАТ__

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'{15}',  
'{1}',  
'{2, 3, 5}',  
'{4}',  
'{6}',  
'{7}',  
'{8}',  
'{9, 13}'}
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