CS 74/174, Winter 2017

Prerequisite Quiz

Name: Class:

The goal of this quiz is to help you review the basic concepts in linear algebra and calculus that are essential for learning machine learning. It will not be counted into your final grades, nor decide your enrollment. However, these are the basic backgrounds that you **must** know in order to follow and understand this class. So if you have any problem, please review your math notes or ask TAs after the class. We will also go over some of the questions in the class.

1. What is the solution to this matrix product: $\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

2. Assume we have two vectors $\vec{x} = [1; 2]$ and $\vec{y} = [-1; 2]$. Please calculate the dot product $\langle \vec{x},\ \vec{y} \rangle$ (also called the inner product) between $\vec{x}$ and $\vec{y}$:

3. The dot product $\langle \vec{x},\ \vec{y} \rangle$ of two vectors $\vec{x}$ and $\vec{y}$ in $\mathcal{R}^3$ is

   [a] a vector that bisects $\vec{x}$ and $\vec{y}$

   [b] a vector that is mutually orthogonal to $\vec{x}$ and $\vec{y}$

   [c] the matrix $\vec{x}\vec{y}^\top$

   [d] a number that is proportional to the vector norm of $\vec{x} - \vec{y}$

   [e] a number that is proportional to the cosine of the angle between $\vec{x}$ and $\vec{y}$

   [f] none of the above

4. Assume we have a set of numbers $[-1,\ 1, 0, 0]$. Please calculate its empirical mean and variance:

5. Calculate the covariance and correlation between the following two sets of numbers $[-1,\ 1, 0, 0]$ and $[1, 1, 1, 1]$.

6. (i) Let A be a matrix and assume $B = A^{-1}$ is the inverse of A. Let I be the identity matrix of the same size as A. Find the correct statements in the following (there may be more than one answers):

   [a] We have $AB = I$ but $BA = I$ may not be true.

   [b] We always have $AB = BA = I$.

   [c] For any matrix, there exists an inverse matrix.

   [d] For any square matrix (matrices that have the same numbers of rows and columns), there exists an inverse matrix.

   [e] If we have $A\vec{x} = \vec{b}$ and $BA = I$, then $\vec{x} = B\vec{b}$.

   (ii) If [d] is not correct, please write a 2×2 matrix that has no inverse matrix (that is, not invertible).

7. Consider the following 2×2 matrix: $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$. Which of the following is the inverse of this matrix?

   [a] $\begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix}$

   [b] $\begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$

   [c] $\begin{pmatrix} 1 & -1 \\ -2 & -1 \end{pmatrix}$

   [d] $\begin{pmatrix} -1 & 1 \\ -2 & -1 \end{pmatrix}$

   [e] This matrix is not invertible.

   [f] none of the above.

8. Consider the following 2×2 matrix: $\begin{pmatrix} 12 & 0 \\ -12 & -2 \end{pmatrix}$. Which of the following is an eigenvector of this matrix, and if it exists what is its eigenvalue?

   [a] $\begin{pmatrix} 1 & 1 \end{pmatrix}^\top$

   [b] $\begin{pmatrix} 0 & 1 \end{pmatrix}^\top$

   [c] $\begin{pmatrix} 1 & 0 \end{pmatrix}^\top$

   [d] $\begin{pmatrix} 1 & -1 \end{pmatrix}^\top$

   [e] undefined

   [f] none of the above

9. What is the maximum rank of a $m \times n$ matrix where $m \leq n$? What is the minimum rank?

10. What is the derivative of $f(x) = ax + b$?

11. What is the derivative of $f(x) = \log(\exp(x) + 1)$? Hint: the derivative of $\log(x)$ is $1/x$ and the derivative of $\exp(x)$ is $\exp(x)$.

12. (a) Consider $f(\vec{x}) = 3x_1 + 2x_2 + 1$, where $\vec{x} = [x_1; x_2]$ is a column vector. So function $f(\vec{x})$ maps a 2×1 vector $\vec{x}$ to a number. The derivative of $f(\vec{x})$ is a 2×1vector of the same size as $\vec{x}$, defined as

$$\nabla f(\vec{x}) = \begin{bmatrix} \frac{\partial f(\vec{x})}{\partial x_1} \\ \frac{\partial f(\vec{x})}{\partial x_2} \end{bmatrix},$$

where $\frac{\partial f(\vec{x})}{x_1}$ is the partial derivative of $f(\vec{x})$ w.r.t. x_1. Please calculate $\nabla f(\vec{x})$.

(b) Please verify that $f(\vec{y}) = f(\vec{x}) \; + \; \langle \nabla f(\vec{x}), \; \vec{y} - \vec{x} \rangle$.

13. Following the above definition, calculate the derivative of the following function $f(\vec{x}) = (3x_1 + 2x_2 - 1)^2$ where $\vec{x} = [x_1; x_2]$.

14. Following the above definition, calculate the derivative of function $f(\vec{x}) = \log(e^{2x_1+x_2} + 1)$ where $\vec{x} = [x_1; x_2]$. Hint: the derivative of $\log(x)$ is $1/x$ and the derivative of $\exp(x)$ is $\exp(x)$.

15. (Optional) How machine learning may be useful for you?

What is your target grade in this class? Which part of the course do you think may be difficult for you?

Write any other comments you may have.