Taylor's Theorem :-

Let $f:[a,b]\rightarrow R,\ f,f^{'},f^{''},.......,f^{(n-1)}$ be continuous on $[a,b]$ and suppose $f^{(n)}$exits on $(a,b)$. Then there exits $c\in(a,b)$ such that

$f\left(b\right)=f\left(a\right)+f^{'}\left(a\right)\left(b-a\right)+\frac{f^{''}(a)}{2!}\left(b-a\right)^{2}+.....+\frac{f^{(n-1)}(a)}{(n-1)!}\left(b-a\right)^{(n-1)}$
$\frac{f^{(n)}(c)}{n!}\left(b-a\right)^{n}$

Proof :- Define

$F\left(x\right)=f\left(b\right)-f\left(x\right)-f^{'}\left(x\right)\left(b-x\right)-\frac{f^{''}(x)}{2!}\left(b-x\right)^{2}..-\frac{f^{(n-1)}(x)}{(n-1)!}\left(b-x\right)^{(n-1)}$

We will show that $F\left(a\right)=\frac{(b-a)^{n}}{n!}f^{(n)}\left(c\right)$ for some $c\in(a,b)$, which will prove the theorem.

Note that

$$F^{'}\left(x\right)=-\frac{f^{(n)}\left(x\right)}{(n-1)!}\left(b-x\right)^{(n-1)}\cdots\cdots\cdots Eq\left(1.\right)$$

Define $g\left(x\right)=F\left(x\right)-\left(\frac{b-x}{b-a}\right)^{n}F\left(a\right)$. It is easy to check that $g\left(a\right)=g\left(b\right)=0$ and
hence by Rolle's theorem there exits some $c\in(a,b)$such that

$$g^{'}\left(c\right)=F^{'}\left(c\right)+\frac{n\left(b-c\right)^{(n-1)}}{\left(b-a\right)^{n}}F\left(a\right)=0\cdots\cdots\cdots Eq\left(2.\right)$$

From (1) and (2) we obtain that $\frac{f^{(n)}(c)}{(n-1)!}\left(b-c\right)^{(n-1)}=\frac{n(b-c)^{(n-1)}}{(b-a)^{n}}F\left(a\right)$.This implies that $F\left(a\right)=\frac{(b-a)^{n}}{n!}f^{(n)}\left(c\right)$. This proves the theorem.