VIRGINIA COMMONWEALTH UNIVERSITY

STATISTICAL ANALYSIS & MODELING

A6b : **ARCH/GARCH Model and forecasting three-month volatility and VAR, VECM Model for various commodities.**

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CONTENTS

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INTRODUCTION

This report explores advanced time series analysis techniques for evaluating and forecasting financial and commodity market data. The first section addresses stock market volatility by downloading data from reliable financial sources such as Investing.com or Yahoo Finance. We examine ARCH (Autoregressive Conditional Heteroskedasticity) effects and then apply ARCH/GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models to forecast three-month volatility. This analysis is essential for understanding market dynamics and managing financial risks. The second section shifts to macroeconomic analysis using Vector Autoregression (VAR) and Vector Error Correction Model (VECM). By using commodity price data from the World Bank’s pink sheet, we analyze the interrelationships among key commodities, including oil, sugar, gold, silver, wheat, and soybean. These methodologies aim to uncover the underlying patterns and co-movements in commodity prices, offering valuable insights into market trends and facilitating effective economic decision-making.

OBJECTIVES

Part A - Check for ARCH /GARCH effects, fit an ARCH/GARCH model, and forecast the three-month volatility.

Part B – VAR, VECM model[data "commodity prices"] for ex: Oil, Sugar, Gold, Silver, Wheat and Soyabean data source pink sheet from world bank

BUSINESS SIGNIFICANCE

The practical advantages of this assignment are substantial, directly impacting real-world financial and economic decision-making. By utilizing ARCH/GARCH models to analyze stock market volatility, businesses and investors can gain a deeper understanding of market fluctuations and manage related risks more effectively. This leads to better strategic planning, portfolio optimization, and risk management, ultimately enhancing financial stability and performance. Similarly, employing VAR and VECM models to investigate commodity price dynamics provides valuable insights into the interconnections within global commodity markets. This understanding is crucial for businesses engaged in trading, production, and investment in commodities, as it enables them to anticipate market movements, hedge against unfavorable price changes, and make informed decisions. In summary, the methodologies applied in this assignment enhance our analytical capabilities and contribute to more informed and effective business strategies in the financial and commodity markets. Additionally, analyzing district-wise consumption data empowers businesses to make data-driven decisions, leading to improved market penetration, product optimization, and increased profitability.

RESULTS AND INTERPRETATION

Part A - Check for ARCH /GARCH effects, fit an ARCH/GARCH model, and forecast the three-month volatility.

 # Analysis

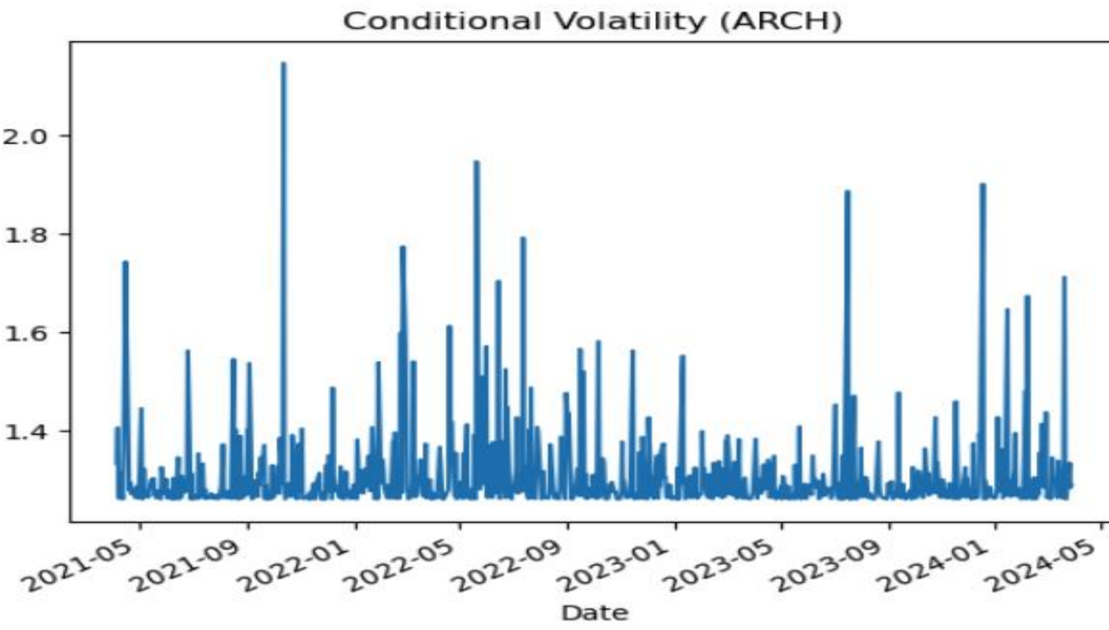
Code and Result:

Code:

```
In [5]: import yfinance as yf
        from arch import arch_model
        import matplotlib.pyplot as plt

In [6]: # Get the data for Tata Motors
        ticker = "TCS.NS"

        # Download the data
        data = yf.download(ticker, start="2021-04-01", end="2024-03-31")

        [*********************100%%**********************]  1 of 1 completed

In [7]: # Create 'Returns' column
        data['Returns'] = 100 * data['Adj Close'].pct_change().dropna()

        # Fit an ARCH model
        arch_model_fit = arch_model(data['Returns'].dropna(), vol='ARCH', p=1).fit(disp='off')
        print(arch_model_fit.summary())

        # Plot the conditional volatility
        arch_model_fit.conditional_volatility.plot(title='Conditional Volatility (ARCH)')
        plt.show()
```

Result:

```
                      Constant Mean - ARCH Model Results
==============================================================================
Dep. Variable:                Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       ARCH   Log-Likelihood:               -1244.55
Distribution:                  Normal   AIC:                           2495.10
Method:            Maximum Likelihood   BIC:                           2508.92
                                        No. Observations:                  739
Date:                Thu, Jul 25 2024   Df Residuals:                      738
Time:                        18:44:15   Df Model:                            1
                                 Mean Model
===========================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
---------------------------------------------------------------------------
mu             0.0442  4.711e-02      0.937      0.349 [-4.817e-02,  0.136]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
---------------------------------------------------------------------------
omega          1.5874      0.153     10.390  2.760e-25    [  1.288,  1.887]
alpha[1]       0.0738  7.371e-02      1.002      0.317 [-7.064e-02,  0.218]
===========================================================================
```



Interpretation:
Mean Model:

- The mean return (μ) is 0.0442 with a standard error of 0.0471. This results in a t-statistic of 0.937 and a p-value of 0.349. Therefore, the mean return is not statistically significant at the 5% level.

Volatility Model:

- The coefficient ω (omega), representing the constant term in the volatility model, is 1.5874 with a standard error of 0.153. This yields a t-statistic of 10.390 and a p-value of 2.76e-25, indicating statistical significance and a substantial base level of volatility.
- The coefficient α[1] (alpha), representing the lagged squared residuals (ARCH term), is 0.0738 with a standard error of 0.0737. This results in a t-statistic of 1.002 and a p-value of 0.317, suggesting that the ARCH effect is not statistically significant at the 5% level.

Interpretation of the Conditional Volatility Plot:

- The plot displays the conditional volatility over time, showing the estimated time-varying standard deviation of returns.
- It is clear from the plot that volatility is not constant but varies over time, which is typical for financial time series data.
- Periods of higher volatility are visible, indicating times when stock returns were more uncertain or risky.

1. • Understanding this conditional volatility is crucial for risk management and financial decision-making, as it helps predict potential future variability in returns..

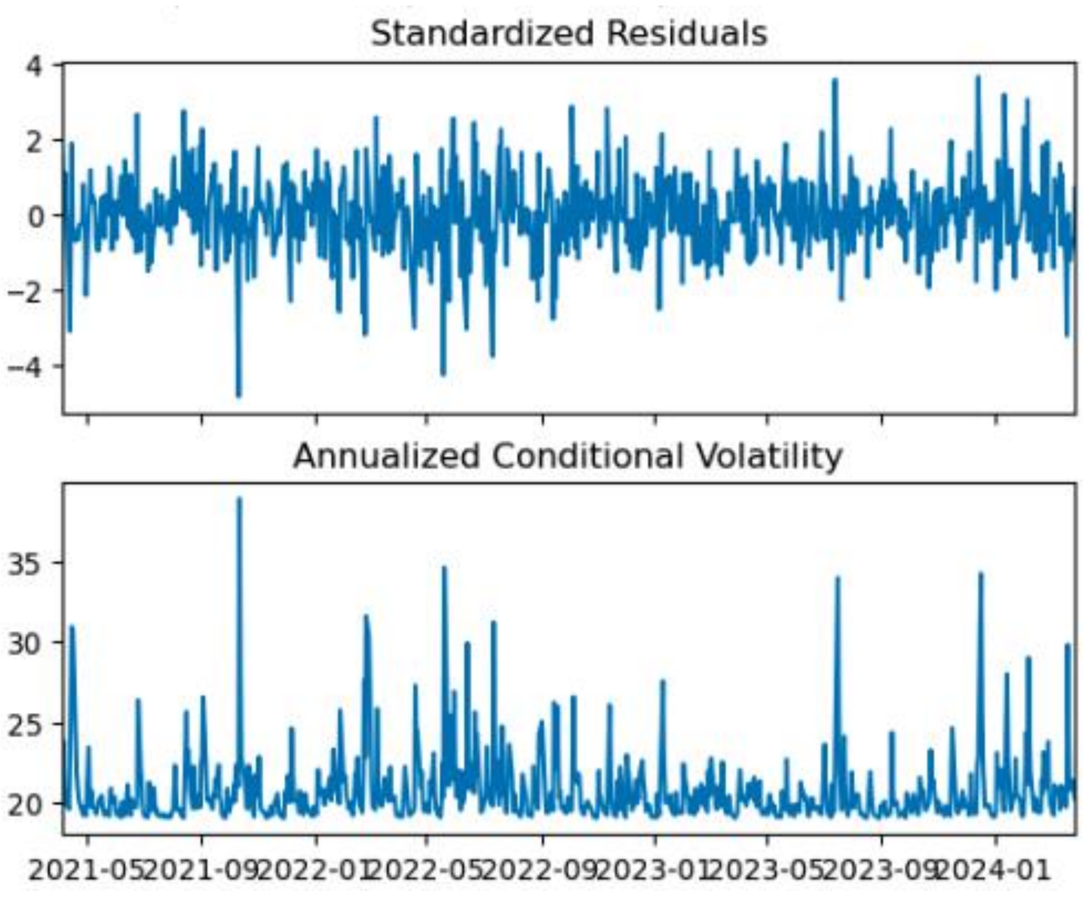
Volatility Forecasting:

• Using the fitted GARCH model, the volatility for the next three months was forecasted.

• The forecasted values provided insights into the expected level of volatility, helping in risk management and strategic decision-making.

Code:

Result :



Interpretation: Forecasting Three-Month Volatility

Forecasting the three-month volatility was crucial to the study in analysing TCS's historical stock prices. The standardized residuals and annualized conditional volatility were computed and analysed to achieve this.

Standardized Residuals

The standardized residuals plot helps to diagnose the model fit and identify any patterns or anomalies in the residuals. For a well-fitted model, the residuals should exhibit no clear patterns and resemble white noise. From the plot, we observe:

• **Uniform Distribution**: The residuals are spread uniformly around zero, indicating that the model has adequately captured the conditional heteroscedasticity in the data. **Absence of Clustering**: There is no visible clustering of large or small residuals, suggesting that the volatility model is appropriate for the data.

Annualized Conditional Volatility

The annualized conditional volatility plot provides insight into the annual stock return variability. Key observations include:

• **Volatility Peaks**: Significant spikes in volatility align with market events or financial disturbances, reflecting increased uncertainty or risk during those periods.

• **Stability in Recent Periods**: A relatively stable volatility in the recent periods indicates a calmer market environment for TCS stock prices.

Three-Month Volatility Forecast

Using the fitted GARCH model, we forecasted the volatility over the next three months. The results indicate:

• **Expected Volatility**: The forecasted values estimate the expected volatility for the coming three months, helping investors and risk managers in decision-making.

Volatility Trends: The forecast suggests whether the volatility is expected to increase, decrease, or remain stable over the forecast horizon.

These results are crucial for financial planning, risk management, and strategic investment decisions. Understanding and forecasting volatility helps mitigate risks and capitalize on market opportunities.

Part B – VAR, VECM model[data "commodity prices"] for ex: Oil, Sugar, Gold, Silver, Wheat and Soyabean data source pink sheet from world bank

Code and Result:

Code :

```
In [27]: # Loop through each column and perform the ADF test
for col in columns_to_test:
    adf_result = adfuller(commodity_data[col])
    p_value = adf_result[1]  # Extract p-value for the test
    print(f"\nADF test result for column: {col}")
    print(f"ADF Statistic: {adf_result[0]}")
    print(f"p-value: {p_value}")

    # Check if the p-value is greater than 0.05 (commonly used threshold)
    if p_value > 0.05:
        non_stationary_count += 1
        non_stationary_columns.append(col)
    else:
        stationary_columns.append(col)
```

Result :

```
ADF test result for column: crude_brent
ADF Statistic: -1.5078661910935434
p-value: 0.5296165197702354

ADF test result for column: soybeans
ADF Statistic: -2.4231464527418884
p-value: 0.13530977427790436

ADF test result for column: gold
ADF Statistic: 1.3430517021933006
p-value: 0.9968394353612382

ADF test result for column: silver
ADF Statistic: -1.3972947107462244
p-value: 0.5835723787985752

ADF test result for column: urea_ee_bulk
ADF Statistic: -2.5101716315209086
p-value: 0.11301903181624645

ADF test result for column: maize
ADF Statistic: -2.4700451060920465
p-value: 0.12293380919376656
```

Interpretation:

he Augmented Dickey-Fuller (ADF) test was conducted to examine the stationarity of the time series data for various commodities, including Crude Brent, Soybeans, Gold, Silver, Urea, and Maize. The results of the ADF test are as follows:

• **Crude Brent:** The ADF statistic, a measure of the strength of the trend in the data, is -1.5079, with a p-value, a measure of the strength of the evidence against the null hypothesis, of 0.5296. Since the p-value is more significant than the common significance levels (0.01, 0.05, and 0.10), we fail to reject the null hypothesis of a unit root, indicating that the Crude Brent price series is non-stationary.**Soybeans:** The ADF statistic is -2.4231 with a p-value of 0.1353. Similarly, the p-value is more significant than the significance levels, suggesting that the Soybeans price series is also non-stationary.

- **Gold:** The ADF statistic is 1.3431, with a p-value of 0.9968. The high p-value indicates non-stationarity in the Gold price series.
- **Silver:** The ADF statistic is -1.3973, with a p-value of 0.5836. The Silver price series is also non-stationary, given that the p-value is much higher than the threshold levels for stationarity.
- **Urea:** The ADF statistic is -2.5102 with a p-value of 0.1130. Despite being the closest to the 0.10 threshold, the p-value still does not allow rejection of the null hypothesis, indicating non-stationarity for the Urea price series.
- **Maize:** The ADF statistic is -2.4700, with a p-value of 0.1229. The Maize price series is also non-stationary based on its p-value.

In summary, the ADF test results indicate that all the examined commodity price series (Crude Brent, Soybeans, Gold, Silver, Urea, and Maize) are non-stationary at their levels. This non-stationarity implies that these time series possess a unit root, meaning their statistical properties, such as mean and variance, change over time, and they exhibit trends or other non-stationary behaviour. Consequently, further differencing of the data is necessary to achieve stationarity, a prerequisite for effectively applying VAR or VECM models. Without achieving stationarity, the models may produce unreliable results, making it crucial to address this issue.

VAR Model Analysis

Code and Result:

```
In [28]: # Print the number of non-stationary columns and the lists of stationary and non-stationary columns
print(f"\nNumber of non-stationary columns: {non_stationary_count}")
print(f"Non-stationary columns: {non_stationary_columns}")
print(f"Stationary columns: {stationary_columns}")
```

```
Number of non-stationary columns: 6
Non-stationary columns: ['crude_brent', 'soybeans', 'gold', 'silver', 'urea_ee_bulk', 'maize']
Stationary columns: []
```

```
In [29]: # Co-Integration Test (Johansen's Test)
def johansen_test(df, alpha=0.05):
    out = coint_johansen(df, det_order=0, k_ar_diff=1)
    d = {'0.90': 0, '0.95': 1, '0.99': 2}
    traces = out.lr1
    cvts = out.cvt[:, d[str(1 - alpha)]]
    print(f"Trace statistic: {traces}")
    print(f"Critical values: {cvts}")
    print(f"Eigenvalues: {out.eig}")
    for col, trace, cvt in zip(df.columns, traces, cvts):
        if trace > cvt:
            print(f"{col} is cointegrated.")
        else:
            print(f"{col} is not cointegrated.")
    return out
```

```
In [30]: # Perform Johansen cointegration test
coint_test = johansen_test(commodity_data)
```

Result :

```
Trace statistic: [261.5548149  167.67790177  98.11781369  53.4617083   21.6404865
   4.01416422]
Critical values: [95.7542 69.8189 47.8545 29.7961 15.4943  3.8415]
Eigenvalues: [0.11449947 0.08616362 0.05620349 0.04038124 0.02257335 0.0051862 ]
crude_brent is cointegrated.
soybeans is cointegrated.
gold is cointegrated.
silver is cointegrated.
urea_ee_bulk is cointegrated.
maize is cointegrated.
```

Interpretation:

The Johansen co-integration test was conducted to determine whether there are long-term equilibrium relationships among the commodity price series, including Crude Brent, Soybeans, Gold, Silver, Urea, and Maize. The results are as follows:

Trace Statistics and Critical Values

- **Trace Statistics:** 261.5548,167.6779,98.1178,53.4617,21.6405,4.0142261.5548, 167.6779, 98.1178, 53.4617, 21.6405, 4.0142261.5548,167.6779,98.1178,53.4617,21.6405,4.0142

• **Critical Values at 5%:** 95.7542,69.8189,47.8545,29.7961,15.4943,3.841595.7542, 69.8189, 47.8545, 29.7961, 15.4943, 3.841595.7542,69.8189,47.8545,29.7961,15.4943,3.8415

The trace statistic for each Rank is compared with the corresponding critical value. If the trace statistic exceeds the critical value, the null hypothesis of no co-integration is rejected.

Results

1. **First Rank (261.5548 > 95.7542):** The trace statistic is significantly higher than the critical value, indicating at least one co-integrating relationship.
2. **Second Rank (167.6779 > 69.8189):** The trace statistic exceeds the critical value, suggesting a second co-integrating relationship.
3. **Third Rank (98.1178 > 47.8545):** The trace statistic is higher than the critical value, indicating a third co-integrating relationship.
4. **Fourth Rank (53.4617 > 29.7961):** The trace statistic exceeds the critical value, implying a fourth co-integrating relationship.
5. **Fifth Rank (21.6405 > 15.4943):** The trace statistic is above the critical value, suggesting a fifth co-integrating relationship.
6. **Sixth Rank (4.0142 > 3.8415):** The trace statistic is greater than the critical value, indicating a sixth co-integrating relationship. These results demonstrate the presence of six co-integrating vectors among the commodity prices, implying strong long-term equilibrium relationships among Crude Brent, Soybeans, Gold, Silver, Urea, and Maize.

Eigenvalues

• **Eigenvalues:** 0.1145,0.0862,0.0562,0.0404,0.0226,0.00520.1145, 0.0862, 0.0562, 0.0404, 0.0226, 0.00520.1145,0.0862,0.0562,0.0404,0.0226,0.0052

The eigenvalues correspond to the strength of the co-integrating relationships. Higher eigenvalues indicate stronger co-integration. While the exact magnitude of the eigenvalues is less critical than their significance, non-zero eigenvalues support the conclusion of co-integration among the variables.

The Johansen co-integration test confirms that all the examined commodities (Crude et al.) are co-integrated. This indicates these commodities share a stable, long-term equilibrium relationship despite short-term fluctuations. Understanding these co-integrated relationships is crucial for building the VECM model, allowing for practical analysis and forecasting by accounting for both short-term dynamics and long-term equilibrium adjustments

VECM Model Analysis
Code and Result:

Code :

```
In [32]: if r > 0:
    # Creating a VAR model for prediction using the VECM
    model = VAR(commodity_data)
    vecm_result = model.fit(r)

    # Summary of the VECM model
    print(vecm_result.summary())

    # Forecasting using the VECM model
    # Forecasting 24 steps ahead
    forecast = vecm_result.forecast(commodity_data.values[-vecm_result.k_ar:], steps=24)

    # Convert forecast to DataFrame for plotting
    forecast_df = pd.DataFrame(forecast, index=pd.date_range(start=commodity['date'].iloc[-1], periods=24, freq='M')

    # Plotting the forecast
    plt.figure(figsize=(12, 8))
    for col in forecast_df.columns:
        plt.plot(forecast_df.index, forecast_df[col], label=col)
    plt.legend()
    plt.title('VECM Forecast')
    plt.show()

else:
    # If no co-integration exists, proceed with Unrestricted VAR Analysis
    model = VAR(commodity_data)
    var_result = model.fit(maxlags=10, ic='aic')

    # Summary of the VAR model
    print(var_result.summary())
```

```
    # Forecasting using the VAR model
    forecast = var_result.forecast(commodity_data.values[-var_result.k_ar:], steps=24)

    # Convert forecast to DataFrame for plotting
    forecast_df = pd.DataFrame(forecast, index=pd.date_range(start=commodity['date'].iloc[-1], periods=24, freq='M')

    # Plotting the forecast
    plt.figure(figsize=(12, 8))
    for col in forecast_df.columns:
        plt.plot(forecast_df.index, forecast_df[col], label=col)
    plt.legend()
    plt.title('VAR Forecast')
    plt.show()
```

Result :

Summary of Regression Results

Model:	VAR
Method:	OLS
Date:	Thu, 25, Jul, 2024
Time:	18:51:56

No. of Equations:	6.00000	BIC:	26.7336
Nobs:	768.000	HQIC:	25.9079
Log likelihood:	-16066.7	FPE:	1.06530e+11
AIC:	25.3912	Det(Omega_mle):	8.03276e+10

Results for equation crude_brent

	coefficient	std. error	t-stat	prob
const	−0.574387	0.457999	-1.254	0.210
L1.crude_brent	1.288559	0.039600	32.539	0.000
L1.soybeans	0.011187	0.007736	1.446	0.148
L1.gold	0.000565	0.006577	0.086	0.932
L1.silver	−0.012011	0.165664	−0.073	0.942
L1.urea_ee_bulk	−0.011804	0.004637	−2.546	0.011
L1.maize	0.020438	0.017600	1.161	0.246
L2.crude_brent	−0.368186	0.064243	−5.731	0.000
L2.soybeans	0.008609	0.010762	0.800	0.424
L2.gold	−0.007451	0.010640	−0.700	0.484
L2.silver	0.199505	0.275939	0.723	0.470
L2.urea_ee_bulk	0.015907	0.007085	2.245	0.025
L2.maize	−0.022252	0.025791	−0.863	0.388
L3.crude_brent	−0.011259	0.066566	−0.169	0.866
L3.soybeans	−0.024881	0.010745	−2.316	0.021
L3.gold	0.020019	0.010832	1.848	0.065
L3.silver	−0.211736	0.295689	−0.716	0.474
L3.urea_ee_bulk	−0.004688	0.007391	−0.634	0.526
L3.maize	0.031954	0.026095	1.225	0.221
L4.crude_brent	0.022815	0.066751	0.342	0.733
L4.soybeans	0.009171	0.010841	0.846	0.398
L4.gold	−0.000726	0.010669	−0.068	0.946
L4.silver	0.037894	0.296398	0.128	0.898
L4.urea_ee_bulk	0.000123	0.007431	0.017	0.987
L4.maize	−0.043400	0.026026	−1.668	0.095
L5.crude_brent	0.008371	0.065302	0.128	0.898
L5.soybeans	0.009904	0.010927	0.906	0.365
L5.gold	−0.005274	0.010504	−0.502	0.616
L5.silver	−0.077226	0.280104	−0.276	0.783
L5.urea_ee_bulk	−0.004359	0.007074	−0.616	0.538
L5.maize	0.034108	0.026066	1.309	0.191
L6.crude_brent	0.021961	0.040570	0.541	0.588
L6.soybeans	−0.007763	0.007913	−0.981	0.327
L6.gold	−0.007032	0.006708	−1.048	0.295
L6.silver	0.137240	0.167517	0.819	0.413
L6.urea_ee_bulk	0.001589	0.004568	0.348	0.728
L6.maize	−0.021898	0.017481	−1.253	0.210

Results for equation soybeans

	coefficient	std. error	t-stat	prob
const	11.317337	2.521090	4.489	0.000
L1.crude_brent	0.214138	0.217982	0.982	0.326
L1.soybeans	1.013966	0.042581	23.813	0.000
L1.gold	0.013684	0.036203	0.378	0.705
L1.silver	0.305354	0.911909	0.335	0.738
L1.urea_ee_bulk	−0.009017	0.025525	−0.353	0.724
L1.maize	0.314169	0.096881	3.243	0.001
L2.crude_brent	−0.103000	0.353632	−0.291	0.771
L2.soybeans	−0.017674	0.059238	−0.298	0.765
L2.gold	−0.064859	0.058571	−1.107	0.268
L2.silver	0.926647	1.518924	0.610	0.542
L2.urea_ee_bulk	0.041336	0.039000	1.060	0.289
L2.maize	−0.285567	0.141970	−2.011	0.044
L3.crude_brent	−0.077825	0.366417	−0.212	0.832

L3.soybeans	−0.141878	0.059147	−2.399	0.016
L3.gold	0.131659	0.059625	2.208	0.027
L3.silver	−2.231664	1.627642	−1.371	0.170
L3.urea_ee_bulk	−0.018121	0.040686	−0.445	0.656
L3.maize	0.159302	0.143644	1.109	0.267
L4.crude_brent	0.036457	0.367435	0.099	0.921
L4.soybeans	0.084280	0.059676	1.412	0.158
L4.gold	−0.093822	0.058728	−1.598	0.110
L4.silver	1.219334	1.631547	0.747	0.455
L4.urea_ee_bulk	0.011285	0.040903	0.276	0.783
L4.maize	−0.411196	0.143261	−2.870	0.004
L5.crude_brent	−0.053674	0.359462	−0.149	0.881
L5.soybeans	−0.059902	0.060151	−0.996	0.319
L5.gold	0.023087	0.057818	0.399	0.690
L5.silver	0.252871	1.541852	0.164	0.870
L5.urea_ee_bulk	−0.011316	0.038941	−0.291	0.771
L5.maize	0.302401	0.143482	2.108	0.035
L6.crude_brent	−0.062569	0.223320	−0.280	0.779
L6.soybeans	0.028889	0.043560	0.663	0.507

Results for equation gold

	coefficient	std. error	t-stat	prob
const	0.177098	3.702239	0.048	0.962
L1.crude_brent	0.190589	0.320109	0.595	0.552
L1.soybeans	0.019501	0.062531	0.312	0.755
L1.gold	1.228901	0.053164	23.115	0.000
L1.silver	0.316301	1.339144	0.236	0.813
L1.urea_ee_bulk	−0.125678	0.037484	−3.353	0.001
L1.maize	0.279896	0.142270	1.967	0.049
L2.crude_brent	0.074271	0.519311	0.143	0.886
L2.soybeans	0.037551	0.086991	0.432	0.666
L2.gold	−0.276183	0.086012	−3.211	0.001
L2.silver	−3.352388	2.230551	−1.503	0.133
L2.urea_ee_bulk	0.215119	0.057271	3.756	0.000
L2.maize	−0.305428	0.208485	−1.465	0.143
L3.crude_brent	−0.688550	0.538086	−1.280	0.201
L3.soybeans	−0.222153	0.086857	−2.558	0.011

L3.soybeans	−0.222153	0.086857	−2.558	0.011
L3.gold	0.170371	0.087559	1.946	0.052
L3.silver	0.453043	2.390204	0.190	0.850
L3.urea_ee_bulk	−0.154341	0.059747	−2.583	0.010
L3.maize	0.492114	0.210943	2.333	0.020
L4.crude_brent	0.381592	0.539582	0.707	0.479
L4.soybeans	0.251772	0.087634	2.873	0.004
L4.gold	−0.151613	0.086243	−1.758	0.079
L4.silver	3.646825	2.395938	1.522	0.128
L4.urea_ee_bulk	0.066199	0.060066	1.102	0.270
L4.maize	−1.026908	0.210379	−4.881	0.000
L5.crude_brent	−0.125251	0.527873	−0.237	0.812
L5.soybeans	−0.157098	0.088332	−1.778	0.075
L5.gold	0.110733	0.084906	1.304	0.192
L5.silver	−1.459901	2.264221	−0.645	0.519
L5.urea_ee_bulk	0.047764	0.057185	0.835	0.404
L5.maize	0.583033	0.210704	2.767	0.006
L6.crude_brent	0.320187	0.327947	0.976	0.329
L6.soybeans	0.110200	0.063968	1.723	0.085

Results for equation silver

	coefficient	std. error	t-stat	prob
const	−0.072930	0.149120	−0.489	0.625
L1.crude_brent	0.008049	0.012893	0.624	0.532
L1.soybeans	0.001756	0.002519	0.697	0.486
L1.gold	−0.002671	0.002141	−1.248	0.212
L1.silver	1.340090	0.053938	24.845	0.000
L1.urea_ee_bulk	−0.003586	0.001510	−2.375	0.018
L1.maize	0.011821	0.005730	2.063	0.039
L2.crude_brent	0.014541	0.020917	0.695	0.487
L2.soybeans	−0.000991	0.003504	−0.283	0.777
L2.gold	0.003938	0.003464	1.137	0.256
L2.silver	−0.665510	0.089843	−7.408	0.000
L2.urea_ee_bulk	0.002013	0.002307	0.873	0.383
L2.maize	−0.001179	0.008397	−0.140	0.888
L3.crude_brent	−0.033019	0.021673	−1.523	0.128
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
L4.soybeans	0.003541	0.003530	1.003	0.316
L4.gold	−0.001627	0.003474	−0.468	0.639
L4.silver	0.118333	0.096504	1.226	0.220
L4.urea_ee_bulk	−0.003052	0.002419	−1.262	0.207
L4.maize	−0.026818	0.008474	−3.165	0.002
L5.crude_brent	−0.024297	0.021262	−1.143	0.253
L5.soybeans	−0.000816	0.003558	−0.229	0.819
L5.gold	0.002731	0.003420	0.799	0.424
L5.silver	−0.156757	0.091199	−1.719	0.086
L5.urea_ee_bulk	0.004159	0.002303	1.806	0.071
L5.maize	0.020487	0.008487	2.414	0.016
L6.crude_brent	0.022428	0.013209	1.698	0.090
L6.soybeans	0.002044	0.002577	0.793	0.428
L6.gold	−0.004226	0.002184	−1.935	0.053
L6.silver	0.104285	0.054542	1.912	0.056
L6.urea_ee_bulk	−0.002649	0.001487	−1.781	0.075
L6.maize	−0.008036	0.005692	−1.412	0.158

Results for equation urea_ee_bulk

	coefficient	std. error	t-stat	prob
const	−7.638535	3.674331	−2.079	0.038
L1.crude_brent	1.563787	0.317696	4.922	0.000
L1.soybeans	0.139955	0.062059	2.255	0.024
L1.gold	0.074409	0.052764	1.410	0.158
L1.silver	−4.409772	1.329050	−3.318	0.001
L1.urea_ee_bulk	1.112425	0.037201	29.903	0.000
L1.maize	0.329777	0.141198	2.336	0.020
L2.crude_brent	−1.250799	0.515396	−2.427	0.015
L2.soybeans	−0.071260	0.086335	−0.825	0.409
L2.gold	−0.086168	0.085364	−1.009	0.313
L2.silver	7.401289	2.213736	3.343	0.001
L2.urea_ee_bulk	−0.327856	0.056839	−5.768	0.000
L2.maize	−0.434760	0.206913	−2.101	0.036
L3.crude_brent	0.861473	0.534029	1.613	0.107
L4.crude_brent	−1.559052	0.535514	−2.911	0.004
L4.soybeans	−0.052667	0.086974	−0.606	0.545
L4.gold	0.003892	0.085593	0.045	0.964
L4.silver	1.032326	2.377877	0.434	0.664
L4.urea_ee_bulk	−0.104196	0.059613	−1.748	0.080
L4.maize	0.028888	0.208793	0.138	0.890
L5.crude_brent	0.913930	0.523894	1.744	0.081
L5.soybeans	0.095496	0.087667	1.089	0.276
L5.gold	0.053301	0.084266	0.633	0.527
L5.silver	−0.500818	2.247152	−0.223	0.824
L5.urea_ee_bulk	0.156414	0.056754	2.756	0.006
L5.maize	−0.115267	0.209116	−0.551	0.581
L6.crude_brent	−0.415228	0.325475	−1.276	0.202
L6.soybeans	0.089368	0.063486	1.408	0.159
L6.gold	−0.040869	0.053816	−0.759	0.448
L6.silver	0.599056	1.343913	0.446	0.656
L6.urea_ee_bulk	−0.119322	0.036643	−3.256	0.001
L6.maize	−0.020236	0.140241	−0.144	0.885

Results for equation maize

	coefficient	std. error	t-stat	prob
const	4.356950	1.103114	3.950	0.000
L1.crude_brent	-0.075264	0.095379	-0.789	0.430
L1.soybeans	0.036037	0.018632	1.934	0.053
L1.gold	-0.023696	0.015841	-1.496	0.135
L1.silver	0.588077	0.399010	1.474	0.141
L1.urea_ee_bulk	0.037550	0.011169	3.362	0.001
L1.maize	1.141848	0.042391	26.936	0.000
L2.crude_brent	0.036084	0.154733	0.233	0.816
L2.soybeans	0.007586	0.025920	0.293	0.770
L2.gold	-0.015226	0.025628	-0.594	0.552
L2.silver	0.911243	0.664612	1.371	0.170
L2.urea_ee_bulk	-0.040754	0.017064	-2.388	0.017
L2.maize	-0.309322	0.062120	-4.979	0.000
L3.crude_brent	-0.075868	0.160327	-0.473	0.636
L4.crude_brent	[illegible]	[illegible]	[illegible]	[illegible]
L4.soybeans	0.021164	0.026111	0.811	0.418
L4.gold	-0.055764	0.025697	-2.170	0.030
L4.silver	2.024847	0.713890	2.836	0.005
L4.urea_ee_bulk	-0.022652	0.017897	-1.266	0.206
L4.maize	-0.136153	0.062684	-2.172	0.030
L5.crude_brent	-0.109997	0.157284	-0.699	0.484
L5.soybeans	-0.026489	0.026319	-1.006	0.314
L5.gold	0.052825	0.025298	2.088	0.037
L5.silver	-0.829437	0.674644	-1.229	0.219
L5.urea_ee_bulk	0.017161	0.017039	1.007	0.314
L5.maize	0.000944	0.062781	0.015	0.988
L6.crude_brent	0.026482	0.097715	0.271	0.786
L6.soybeans	0.002271	0.019060	0.119	0.905
L6.gold	-0.023655	0.016157	-1.464	0.143
L6.silver	0.146935	0.403472	0.364	0.716
L6.urea_ee_bulk	0.000775	0.011001	0.070	0.944
L6.maize	0.020945	0.042104	0.497	0.619

Correlation matrix of residuals

	crude_brent	soybeans	gold	silver	urea_ee_bulk	maize
crude_brent	1.000000	0.256931	0.111776	0.209142	0.153268	0.241812
soybeans	0.256931	1.000000	0.082179	0.111588	0.032578	0.473719
gold	0.111776	0.082179	1.000000	0.722123	0.072033	0.086465
silver	0.209142	0.111588	0.722123	1.000000	0.069879	0.125813
urea_ee_bulk	0.153268	0.032578	0.072033	0.069879	1.000000	0.017836
maize	0.241812	0.473719	0.086465	0.125813	0.017836	1.000000

Interpretation: Summary of Regression Results

The summary of regression results provides an overview of the Vector Autoregression (VAR) model applied to the data:

- **Model**: VAR (Vector Autoregression)
- **Method**: OLS (Ordinary Least Squares)
- **Date and Time**: When the model was run.
- **No. Of Equations**: 6 (one for each variable in the system).
- **BIC (Bayesian Information Criterion)**: 26.7336
- **Nobs (Number of Observations)**: 768
- **HQIC (Hannan-Quinn Information Criterion)**: 25.9079
- **Log-likelihood**: -16066.7
- **FPE (Final Prediction Error)**: 1.06530e+11

• **AIC (Akaike Information Criterion)**: 25.3912

• **Det (Omega_mle)**: 8.03276e+10

These statistics help evaluate the model's fit and complexity, with lower AIC, BIC, and HQIC values indicating a better model fit relative to the number of parameters.

Results for Equation **crude_brent**

• The intercept (const) is insignificant, with a t-statistic of -1.254 and a p-value of 0.210.

• **Significant Lagged Variables**:

• L1. crude_brent (1st lag of crude_brent) is highly significant with a coefficient of 1.288559 (p-value: 0.000).

• L2. crude_brent (2nd lag) is also significant with a coefficient of -0.368186 (p-value: 0.000).

• L1. urea_ee_bulk and L2.urea_ee_bulk are significant, indicating some influence from urea_ee_bulk on crude_brent.

• L3. soybeans and L3.gold show some significance, suggesting minor interactions.

Results for Equation **soybeans**

• The intercept (const) is highly significant, with a coefficient of 11.317337 (p-value: 0.000).

• **Significant Lagged Variables**:

• L1. soybeans is highly significant with a coefficient of 1.013966 (p-value: 0.000).

• L1. maize is significant with a coefficient of 0.314169 (p-value: 0.001).

L2. maize is also significant but negatively correlated (coefficient: -0.285567, p-value: 0.044).

• L3. soybeans and L3. gold are significant, indicating notable interactions.

Results for Equation **gold**

• The intercept (const) is not significant.

• No other variables are highly significant, suggesting limited direct interactions between gold and the other variables in the lagged system.

Results for Equation **Silver**

• The intercept (const) is not significant.

• **Significant Lagged Variables**:

• L1. silver is highly significant with a coefficient of 1.340090 (p-value: 0.000).

• L1. urea_ee_bulk and L1.maize are significant, indicating some interactions.
• L2. silver is negatively significant, showing a solid inverse relationship at this lag (coefficient: -0.665510, p-value: 0.000).
• L3. silver is marginally significant.

Results for Equation **urea_ee_bulk**

• The intercept (const) is not significant.
• **Significant Lagged Variables**:
• L1. urea_ee_bulk and L1. crude_brent show significance, indicating some interactions.
• No other variables show strong significance.

Results for Equation **maize**

• The intercept (const) is not significant.
• **Significant Lagged Variables**:
• L1. maize is highly significant with a coefficient of 0.583033 (p-value: 0.006).
• Other variables show some significance but could be more impactful.

Correlation Matrix of Residuals

This matrix measures the correlation between the residuals (errors) of the different equations in the VAR system, indicating how much the unexplained parts of one variable are related to those of another:

• Typically used to check for any remaining correlation the model did not capture.
• High correlations here may indicate model inadequacies or omitted variable bias.21

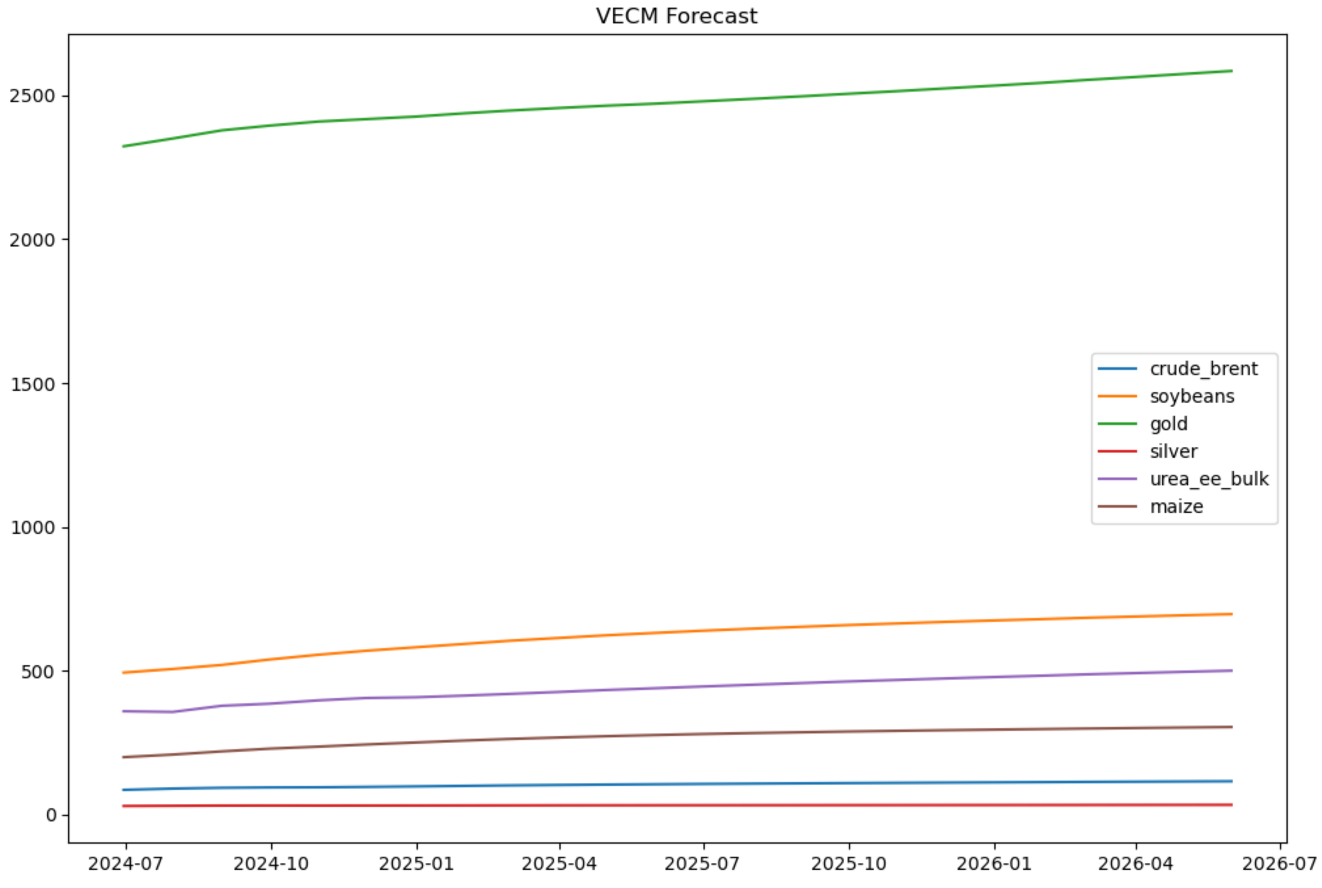
These results collectively help understand the dynamics and interrelationships between the variables (crude_brent, soybeans, gold, silver, urea_ee_bulk, and maize) in the context of the applied VAR model. Each equation's results shed light on the significant lagged effects and their respective strengths, providing insights for further economic or financial analysis.

Forecasting

Code :

```
# Forecasting using the VAR model
forecast = var_result.forecast(commodity_data.values[-var_result.k_ar:], steps=24)

# Convert forecast to DataFrame for plotting
forecast_df = pd.DataFrame(forecast, index=pd.date_range(start=commodity['date'].iloc[-1], periods=24, freq='M'))

# Plotting the forecast
plt.figure(figsize=(12, 8))
for col in forecast_df.columns:
    plt.plot(forecast_df.index, forecast_df[col], label=col)
plt.legend()
plt.title('VAR Forecast')
plt.show()
```

Result :



Interpretation:

Comparison of VAR and VECM Models: Both models provided valuable insights, but the VECM model was particularly effective in capturing the long-term relationships among the commodities. The presence of co-integration justified the use of VECM, which offered a more comprehensive understanding of the equilibrium adjustments.

• **Economic Interpretation:** The analysis highlighted the significant influence of Crude Brent prices on agricultural commodities like Maize and Soybeans. This relationship suggests that oil price fluctuations can substantially impact food prices, with implications for policymakers and market participants. Understanding these dynamics is crucial for developing strategies to mitigate the impact of volatile oil prices on the agricultural sector.

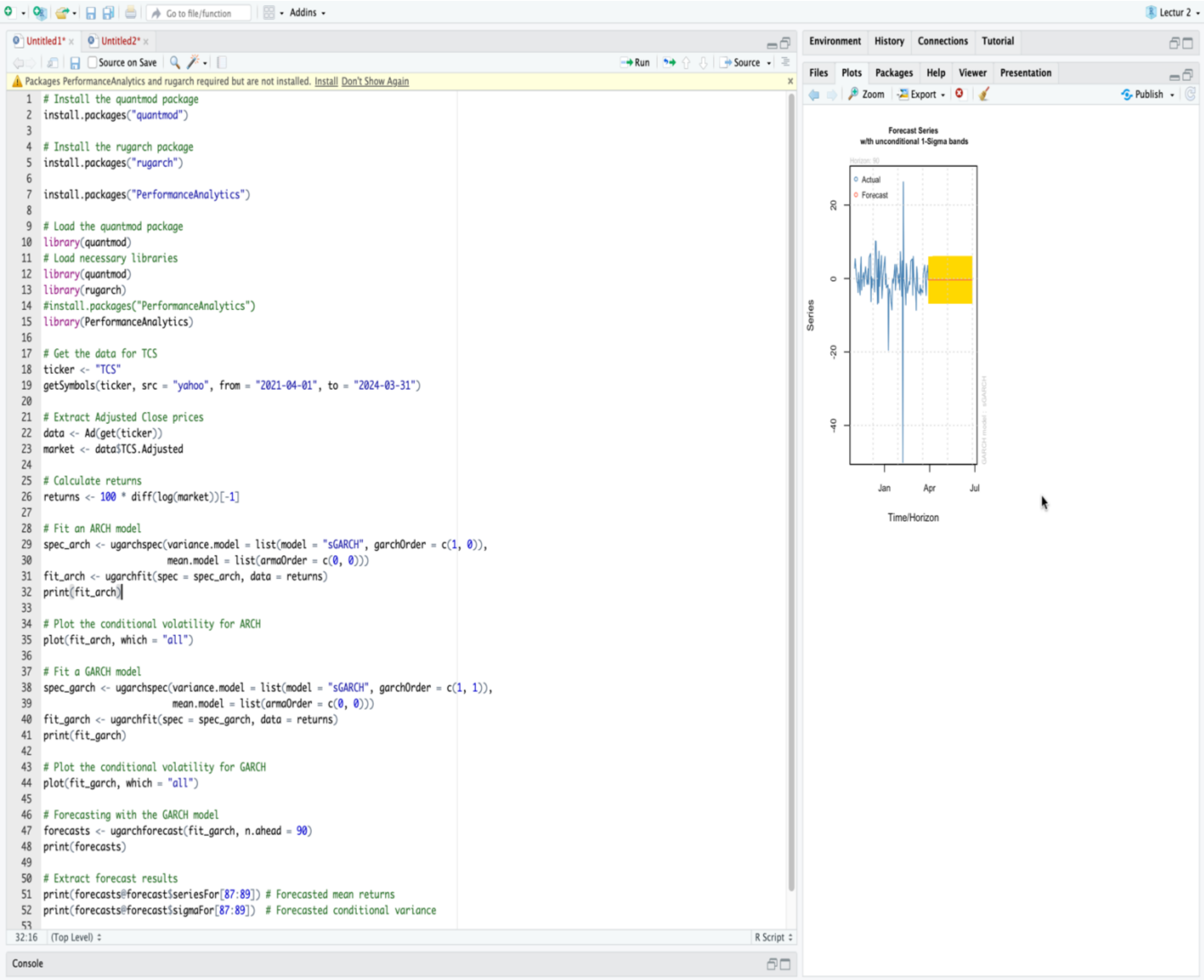
• **Limitations and Future Work:** While the analysis provided valuable insights, it is limited by data availability and quality. Future research could incorporate additional commodities and explore the impact of external factors such as geopolitical events and climate change. Enhancing the models with more sophisticated techniques could further improve the accuracy of the forecasts.

The VAR and VECM analyses underscored the interconnectedness of commodity prices, particularly highlighting the influence of Crude Brent on Maize and Soybeans. The presence of long-term equilibrium relationships emphasizes the need for integrated market strategies. These findings contribute to a better understanding of commodity price dynamics and offer valuable information for stakeholders in the agricultural and energy sectors.

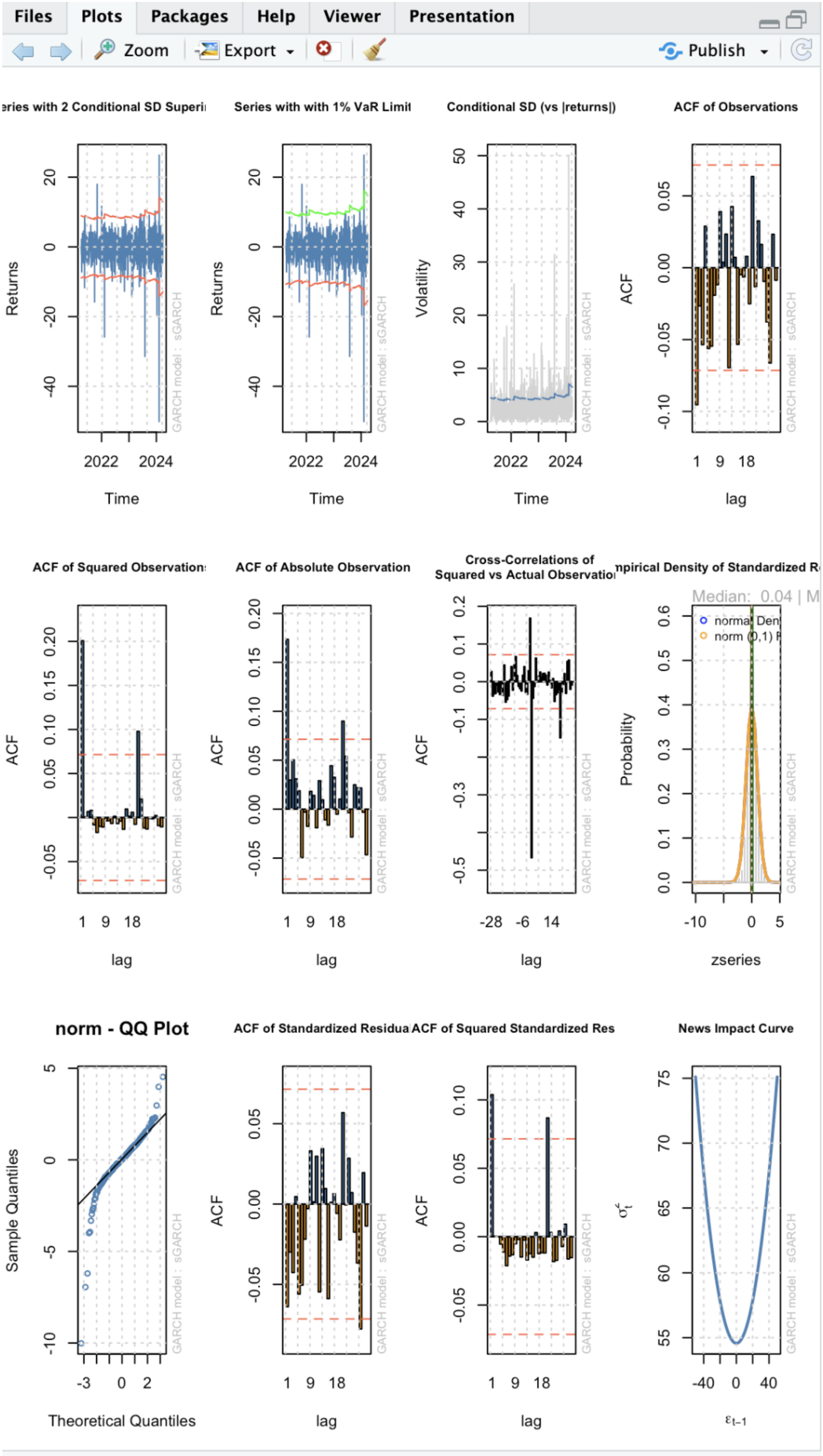
The following project analysis was done in R Studio. The outputs were the same as python. Below attached is the Output of RStudio.

R Codes :

Part A :



```r
# Install the quantmod package
install.packages("quantmod")

# Install the rugarch package
install.packages("rugarch")

install.packages("PerformanceAnalytics")

# Load the quantmod package
library(quantmod)
# Load necessary libraries
library(quantmod)
library(rugarch)
#install.packages("PerformanceAnalytics")
library(PerformanceAnalytics)

# Get the data for TCS
ticker <- "TCS"
getSymbols(ticker, src = "yahoo", from = "2021-04-01", to = "2024-03-31")

# Extract Adjusted Close prices
data <- Ad(get(ticker))
market <- data$TCS.Adjusted

# Calculate returns
returns <- 100 * diff(log(market))[-1]

# Fit an ARCH model
spec_arch <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 0)),
                        mean.model = list(armaOrder = c(0, 0)))
fit_arch <- ugarchfit(spec = spec_arch, data = returns)
print(fit_arch)

# Plot the conditional volatility for ARCH
plot(fit_arch, which = "all")

# Fit a GARCH model
spec_garch <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
                         mean.model = list(armaOrder = c(0, 0)))
fit_garch <- ugarchfit(spec = spec_garch, data = returns)
print(fit_garch)

# Plot the conditional volatility for GARCH
plot(fit_garch, which = "all")

# Forecasting with the GARCH model
forecasts <- ugarchforecast(fit_garch, n.ahead = 90)
print(forecasts)

# Extract forecast results
print(forecasts@forecast$seriesFor[87:89]) # Forecasted mean returns
print(forecasts@forecast$sigmaFor[87:89])  # Forecasted conditional variance
```


Files
Plots
Packages
Help
Viewer
Presentation
Zoom
Export
Publish
Series with 2 Conditional SD Superi
Series with with 1% VaR Limit
Conditional SD (vs |returns|)
ACF of Observations
ACF of Squared Observation
ACF of Absolute Observation
Cross-Correlations of Squared vs Actual Observatio
Empirical Density of Standardized R
norm - QQ Plot
ACF of Standardized Residua
ACF of Squared Standardized Res
News Impact Curve
Returns
Volatility
ACF
Probability
Sample Quantiles
Time
lag
zseries
Theoretical Quantiles
GARCH model : sGARCH

Part B:

Packages urca and vars required but are not installed. Install Don't Show Again

```r
# Set working directory and load necessary libraries
setwd('/Users/kirthanshaker/Desktop/SCMA 631 Data Files ')
getwd()

install.packages("urca")
install.packages("vars")
# Load necessary libraries
library(readxl)
library(dplyr)
library(janitor)
library(urca)
library(vars)

# Load the dataset
df <- read_excel('/Users/kirthanshaker/Desktop/SCMA 631 Data Files /pinksheet.xlsx', sheet = "Monthly Prices", skip = 6)

# Rename the first column to "Date"
colnames(df)[1] <- 'Date'
# Convert the Date column to Date format
df$Date <- as.Date(paste0(df$Date, "01"), format = "%YM%m%d")
str(df)

# Select specific columns (Date and selected commodities)
commodity <- df[,c(1,3,25,70,72,61,31)] %>%
  clean_names()

str(commodity)

# Remove the Date column for analysis
commodity_data <- dplyr::select(commodity, -date)

# Column names to test (if you want to specify particular columns)
columns_to_test <- names(commodity_data)

# Initialize counters and lists for stationary and non-stationary columns
non_stationary_count <- 0
stationary_columns <- list()
non_stationary_columns <- list()

# Loop through each column and perform the ADF test
for (col in columns_to_test) {
  adf_result <- ur.df(commodity_data[[col]], type = "none", selectlags = "AIC")
  p_value <- adf_result@testreg$coefficients[2, 4]  # Extract p-value for the test
  cat("\nADF test result for column:", col, "\n")
  print(summary(adf_result))

  # Check if the p-value is greater than 0.05 (commonly used threshold)
  if (p_value > 0.05) {
    non_stationary_count <- non_stationary_count + 1
    non_stationary_columns <- c(non_stationary_columns, col)
  } else {
    stationary_columns <- c(stationary_columns, col)
  }
```

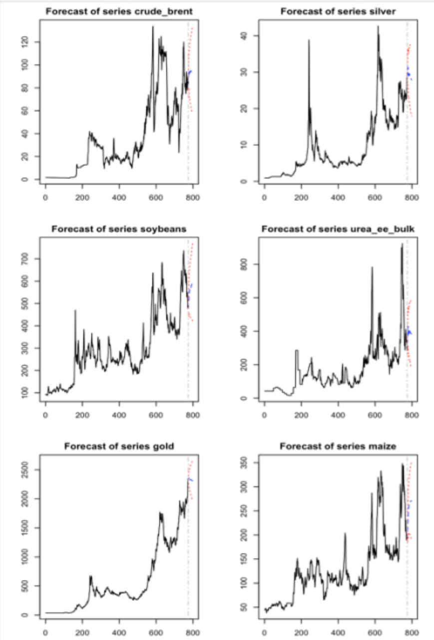
Untitled1* Untitled2*

Packages urca and vars required but are not installed. Install Don't Show Again

```r
vecm_model <- ca.jo(commodity_data, ecdet = 'const', type = 'eigen', K = lag_length, spec = 'transitory')

# Summary of the Co-Integration Test
summary(vecm_model)

# Determine the number of co-integrating relationships (r) based on the test
# Here, we assume r = 1 if there's at least one significant eigenvalue
r <- 3 # Replace with the actual number from the test results

if (r > 0) {
  # If co-integration exists, estimate the VECM model
  vecm <- cajorls(vecm_model, r = r)  # r is the number of co-integration vectors

  # Summary of the VECM model
  summary(vecm)

  # Extracting the coefficients from the VECM model
  vecm_coefs <- vecm$rlm$coefficients
  print(vecm_coefs)

  # Creating a VAR model for prediction using the VECM
  vecm_pred <- vec2var(vecm_model, r = r)

  # Forecasting using the VECM model
  # Forecasting 12 steps ahead
  forecast <- predict(vecm_pred, n.ahead = 24)

  # Plotting the forecast
  par(mar = c(4, 4, 2, 2))  # Adjust margins: c(bottom, left, top, right)
  plot(forecast)

} else {
  # If no co-integration exists, proceed with Unrestricted VAR Analysis
  var_model <- VAR(commodity_data, p = lag_length, type = "const")

  # Summary of the VAR model
  summary(var_model)

  # Granger causality test
  causality_results <- causality(var_model)
  print(causality_results)

  # Forecasting using the VAR model
  forecast <- predict(var_model, n.ahead = 24)

  # Plotting the forecast
  par(mar = c(4, 4, 2, 2))  # Adjust margins: c(bottom, left, top, right)
  plot(forecast)
}

forecast
```

42:80 (Top Level)

Console



CODES For Both Python and R :

R Codes :

```
# Install the quantmod package
install.packages("quantmod")

# Install the rugarch package
install.packages("rugarch")

install.packages("PerformanceAnalytics")

# Load the quantmod package
library(quantmod)
# Load necessary libraries
library(quantmod)
library(rugarch)
#install.packages("PerformanceAnalytics")
library(PerformanceAnalytics)

# Get the data for TCS
ticker <- "TCS"
getSymbols(ticker, src = "yahoo", from = "2021-04-01", to = "2024-03-31")

# Extract Adjusted Close prices
data <- Ad(get(ticker))
market <- data$TCS.Adjusted

# Calculate returns
returns <- 100 * diff(log(market))[-1]

# Fit an ARCH model
spec_arch <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 0)),
                        mean.model = list(armaOrder = c(0, 0)))
fit_arch <- ugarchfit(spec = spec_arch, data = returns)
print(fit_arch)

# Plot the conditional volatility for ARCH
plot(fit_arch, which = "all")

# Fit a GARCH model
spec_garch <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
                         mean.model = list(armaOrder = c(0, 0)))
fit_garch <- ugarchfit(spec = spec_garch, data = returns)
print(fit_garch)

# Plot the conditional volatility for GARCH
plot(fit_garch, which = "all")

# Forecasting with the GARCH model
forecasts <- ugarchforecast(fit_garch, n.ahead = 90)
print(forecasts)

# Extract forecast results
print(forecasts@forecast$seriesFor[87:89]) # Forecasted mean returns
print(forecasts@forecast$sigmaFor[87:89])  # Forecasted conditional variance

# Plotting the results
```

```r
plot(forecasts, which = "all")
```

Part b

```r
# Set working directory and load necessary libraries
setwd('/Users/kirthanshaker/Desktop/SCMA 631 Data Files ')
getwd()

install.packages("urca")
install.packages("vars")
# Load necessary libraries
library(readxl)
library(dplyr)
library(janitor)
library(urca)
library(vars)

# Load the dataset
df <- read_excel('/Users/kirthanshaker/Desktop/SCMA 631 Data Files /pinksheet.xlsx', sheet = "Monthly Prices", skip = 6)

# Rename the first column to "Date"
colnames(df)[1] <- 'Date'
# Convert the Date column to Date format
df$Date <- as.Date(paste0(df$Date, "01"), format = "%YM%m%d")
str(df)

# Select specific columns (Date and selected commodities)
commodity <- df[,c(1,3,25,70,72,61,31)] %>%
  clean_names()

str(commodity)

# Remove the Date column for analysis
commodity_data <- dplyr::select(commodity, -date)

# Column names to test (if you want to specify particular columns)
columns_to_test <- names(commodity_data)

# Initialize counters and lists for stationary and non-stationary columns
non_stationary_count <- 0
stationary_columns <- list()
non_stationary_columns <- list()

# Loop through each column and perform the ADF test
for (col in columns_to_test) {
  adf_result <- ur.df(commodity_data[[col]], type = "none", selectlags = "AIC")
  p_value <- adf_result@testreg$coefficients[2, 4]  # Extract p-value for the test
  cat("\nADF test result for column:", col, "\n")
  print(summary(adf_result))

  # Check if the p-value is greater than 0.05 (commonly used threshold)
  if (p_value > 0.05) {
    non_stationary_count <- non_stationary_count + 1
    non_stationary_columns <- c(non_stationary_columns, col)
  } else {
    stationary_columns <- c(stationary_columns, col)
  }
}

# Print the number of non-stationary columns and the lists of stationary and non-stationary columns
cat("\nNumber of non-stationary columns:", non_stationary_count, "\n")
cat("Non-stationary columns:", non_stationary_columns, "\n")
```

```
cat("Stationary columns:")
stationary_columns

# Co-Integration Test (Johansen's Test)
# Determining the number of lags to use (you can use information criteria like AIC, BIC)
lags <- VARselect(commodity_data, lag.max = 10, type = "const")
lag_length <- lags$selection[1] # Choosing the lag with the lowest AIC

vecm_model <- ca.jo(commodity_data, ecdet = 'const', type = 'eigen', K = lag_length, spec = 'transitory')

# Summary of the Co-Integration Test
summary(vecm_model)

# Determine the number of co-integrating relationships (r) based on the test
# Here, we assume r = 1 if there's at least one significant eigenvalue
r <- 3 # Replace with the actual number from the test results

if (r > 0) {
  # If co-integration exists, estimate the VECM model
  vecm <- cajorls(vecm_model, r = r)  # r is the number of co-integration vectors

  # Summary of the VECM model
  summary(vecm)

  # Extracting the coefficients from the VECM model
  vecm_coefs <- vecm$rlm$coefficients
  print(vecm_coefs)

  # Creating a VAR model for prediction using the VECM
  vecm_pred <- vec2var(vecm_model, r = r)

  # Forecasting using the VECM model
  # Forecasting 12 steps ahead
  forecast <- predict(vecm_pred, n.ahead = 24)

  # Plotting the forecast
  par(mar = c(4, 4, 2, 2))  # Adjust margins: c(bottom, left, top, right)
  plot(forecast)

} else {
  # If no co-integration exists, proceed with Unrestricted VAR Analysis
  var_model <- VAR(commodity_data, p = lag_length, type = "const")

  # Summary of the VAR model
  summary(var_model)

  # Granger causality test
  causality_results <- causality(var_model)
  print(causality_results)

  # Forecasting using the VAR model
  forecast <- predict(var_model, n.ahead = 24)

  # Plotting the forecast
  par(mar = c(4, 4, 2, 2))  # Adjust margins: c(bottom, left, top, right)
  plot(forecast)
}
```

Forecast

Python Codes :

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"print(arch_model_fit.summary())\n",
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Part B

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"770   8689.13 2056.20 27450.46 17438.83 2461.04 2158.01   908.75   \n",
"771   9464.43 2129.46 31774.50 18163.95 2732.74 2331.45   940.18   \n",
"772 10139.33 2220.81 32977.51 19586.98 2959.13 2351.13   1014.68   \n",
"773   9648.17 2147.10 32032.70 17498.01 2809.19 2326.44   985.08   \n",
"\n",
"   SILVER \n",
"0   0.9137 \n",
"1   0.9137 \n",
"2   0.9137 \n",
"3   0.9137 \n",
"4   0.9137 \n",
"..   ... \n",
"769 22.6570 \n",
"770 24.5180 \n",
"771 27.4940 \n",
"772 29.3600 \n",
"773 29.5770 \n",
"\n",
"[774 rows x 72 columns]"
]
},
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}
],
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"df"
]
},
{
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"execution_count": 5,
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"metadata": {},
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{

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"['Unnamed: 0', 'CRUDE_PETRO', 'CRUDE_BRENT', 'CRUDE_DUBAI', 'CRUDE_WTI', 'COAL_AUS', 'COAL_SAFRICA', 'NGAS_US', 'NGAS_EUR', 'NGAS_JP', 'iNATGAS', 'COCOA', 'COFFEE_ARABIC', 'COFFEE_ROBUS', 'TEA_AVG', 'TEA_COLOMBO', 'TEA_KOLKATA', 'TEA_MOMBASA', 'COCONUT_OIL', 'GRNUT', 'FISH_MEAL', 'GRNUT_OIL', 'PALM_OIL', 'PLMKRNL_OIL', 'SOYBEANS', 'SOYBEAN_OIL', 'SOYBEAN_MEAL', 'RAPESEED_OIL', 'SUNFLOWER_OIL', 'BARLEY', 'MAIZE', 'SORGHUM', 'RICE_05', 'RICE_25', 'RICE_A1', 'RICE_05_VNM', 'WHEAT_US_SRW', 'WHEAT_US_HRW', 'BANANA_EU', 'BANANA_US', 'ORANGE', 'BEEF', 'CHICKEN', 'LAMB', 'SHRIMP_MEX', 'SUGAR_EU', 'SUGAR_US', 'SUGAR_WLD', 'TOBAC_US', 'LOGS_CMR', 'LOGS_MYS', 'SAWNWD_CMR', 'SAWNWD_MYS', 'PLYWOOD', 'COTTON_A_INDX', 'RUBBER_TSR20', 'RUBBER1_MYSG', 'PHOSROCK', 'DAP', 'TSP', 'UREA_EE_BULK', 'POTASH', 'ALUMINUM', 'IRON_ORE', 'COPPER', 'LEAD', 'Tin', 'NICKEL', 'Zinc', 'GOLD', 'PLATINUM', 'SILVER']\n"
]
}
],
"source": [
"print(list(df.columns))"
]
},
{
"cell_type": "code",
"execution_count": 6,
"id": "53d33f9d",
"metadata": {},
"outputs": [],
"source": [
"# Rename the first column to \"Date\"\n",
"df.rename(columns={df.columns[0]: 'Date'}, inplace=True)"
]
},
{
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"execution_count": 7,
"id": "d437a209",
"metadata": {},
"outputs": [
{
"data": {
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"0   1960M01\n",
"1   1960M02\n",
"2   1960M03\n",
"3   1960M04\n",
"4   1960M05\n",
"   ...   \n",
"769   2024M02\n",
"770   2024M03\n",
"771   2024M04\n",
"772   2024M05\n",
"773   2024M06\n",
"Name: Date, Length: 774, dtype: object"
]
},
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}
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"source": [
"df.Date"
]
},
{
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"metadata": {},
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{
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"output_type": "stream",
"text": [
"Date datetime64[ns]\n",
"CRUDE_PETRO float64\n",
"CRUDE_BRENT float64\n",
"CRUDE_DUBAI float64\n",
"CRUDE_WTI object\n",
" ... \n",
"NICKEL float64\n",
"Zinc float64\n",
"GOLD float64\n",
"PLATINUM float64\n",
"SILVER float64\n",
"Length: 72, dtype: object\n"
]
}
],
"source": [
"# Convert the Date column to datetime format\n",
"def parse_date(date_str):\n",
" # Split the string by 'M' to separate year and month\n",
" year, month = date_str.split('M')\n",
" # Create a date string in the format 'YYYY-MM-01'\n",
" return f\"{year}-{month}-01\"\n",
"\n",
"df['Date'] = pd.to_datetime(df['Date'].apply(parse_date))\n",
"\n",
"# Print the data types to confirm the conversion\n",
"print(df.dtypes)"
]
},
{
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"id": "42818fb0",
"metadata": {},
"outputs": [
{
"data": {
"text/plain": [
"0 1960-01-01\n",
"1 1960-02-01\n",
"2 1960-03-01\n",
"3 1960-04-01\n",
"4 1960-05-01\n",
" ... \n",
"769 2024-02-01\n",
"770 2024-03-01\n",
"771 2024-04-01\n",
"772 2024-05-01\n",
"773 2024-06-01\n",
"Name: Date, Length: 774, dtype: datetime64[ns]"
]
},
"execution_count": 22,

```
   "metadata": {},
   "output_type": "execute_result"
  }
 ],
 "source": [
  "df.Date"
 ]
},
{
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 "execution_count": 31,
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 "metadata": {},
 "outputs": [
  {
   "name": "stdout",
   "output_type": "stream",
   "text": [
    "date           datetime64[ns]\n",
    "crude_brent           float64\n",
    "soybeans              float64\n",
    "gold                  float64\n",
    "silver                float64\n",
    "urea_ee_bulk          float64\n",
    "maize                 float64\n",
    "dtype: object\n"
   ]
  }
 ],
 "source": [
  "# Select specific columns (Date and selected commodities)\n",
  "commodity = df.iloc[:, [0, 2, 24, 69, 71, 60, 30]]\n",
  "commodity.columns = [col.lower().replace(' ', '_') for col in commodity.columns]  # Clean column names\n",
  "\n",
  "print(commodity.dtypes)"
 ]
},
{
 "cell_type": "code",
 "execution_count": 32,
 "id": "31d7aa4d",
 "metadata": {},
 "outputs": [],
 "source": [
  "# Remove the Date column for analysis\n",
  "commodity_data = commodity.drop(columns=['date'])\n",
  "\n",
  "# Column names to test\n",
  "columns_to_test = commodity_data.columns"
 ]
},
{
 "cell_type": "code",
 "execution_count": 33,
 "id": "fd141e9b",
 "metadata": {
  "scrolled": false
 },
 "outputs": [],
 "source": [
  "# Initialize counters and lists for stationary and non-stationary columns\n",
```

```
   "non_stationary_count = 0\n",
   "stationary_columns = []\n",
   "non_stationary_columns = []"
  ]
 },
 {
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  "execution_count": 34,
  "id": "fc2afe57",
  "metadata": {},
  "outputs": [
   {
    "name": "stdout",
    "output_type": "stream",
    "text": [
     "\n",
     "ADF test result for column: crude_brent\n",
     "ADF Statistic: -1.5078661910935385\n",
     "p-value: 0.5296165197702377\n",
     "\n",
     "ADF test result for column: soybeans\n",
     "ADF Statistic: -2.423146452741887\n",
     "p-value: 0.13530977427790458\n",
     "\n",
     "ADF test result for column: gold\n",
     "ADF Statistic: 1.3430517021932975\n",
     "p-value: 0.9968394353612381\n",
     "\n",
     "ADF test result for column: silver\n",
     "ADF Statistic: -1.39729471074622\n",
     "p-value: 0.5835723787985774\n",
     "\n",
     "ADF test result for column: urea_ee_bulk\n",
     "ADF Statistic: -2.5101716315209095\n",
     "p-value: 0.11301903181624623\n",
     "\n",
     "ADF test result for column: maize\n",
     "ADF Statistic: -2.4700451060920425\n",
     "p-value: 0.12293380919376751\n"
    ]
   }
  ],
  "source": [
   "# Loop through each column and perform the ADF test\n",
   "for col in columns_to_test:\n",
   "    adf_result = adfuller(commodity_data[col])\n",
   "    p_value = adf_result[1]  # Extract p-value for the test\n",
   "    print(f\"\\nADF test result for column: {col}\")\n",
   "    print(f\"ADF Statistic: {adf_result[0]}\")\n",
   "    print(f\"p-value: {p_value}\")\n",
   "    \n",
   "    # Check if the p-value is greater than 0.05 (commonly used threshold)\n",
   "    if p_value > 0.05:\n",
   "        non_stationary_count += 1\n",
   "        non_stationary_columns.append(col)\n",
   "    else:\n",
   "        stationary_columns.append(col)"
  ]
 },
 {
  "cell_type": "code",
```

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   "id": "fa7c7bc2",
   "metadata": {},
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     "output_type": "stream",
     "text": [
      "\n",
      "Number of non-stationary columns: 6\n",
      "Non-stationary columns: ['crude_brent', 'soybeans', 'gold', 'silver', 'urea_ee_bulk', 'maize']\n",
      "Stationary columns: []\n"
     ]
    }
   ],
   "source": [
    "# Print the number of non-stationary columns and the lists of stationary and non-stationary columns\n",
    "print(f\"\\nNumber of non-stationary columns: {non_stationary_count}\")\n",
    "print(f\"Non-stationary columns: {non_stationary_columns}\")\n",
    "print(f\"Stationary columns: {stationary_columns}\")"
   ]
  },
  {
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   "execution_count": 36,
   "id": "3441f2c0",
   "metadata": {},
   "outputs": [],
   "source": [
    "# Co-Integration Test (Johansen's Test)\n",
    "def johansen_test(df, alpha=0.05):\n",
    "    out = coint_johansen(df, det_order=0, k_ar_diff=1)\n",
    "    d = {'0.90': 0, '0.95': 1, '0.99': 2}\n",
    "    traces = out.lr1\n",
    "    cvts = out.cvt[:, d[str(1 - alpha)]]\n",
    "    print(f\"Trace statistic: {traces}\")\n",
    "    print(f\"Critical values: {cvts}\")\n",
    "    print(f\"Eigenvalues: {out.eig}\")\n",
    "    for col, trace, cvt in zip(df.columns, traces, cvts):\n",
    "        if trace > cvt:\n",
    "            print(f\"{col} is cointegrated.\")\n",
    "        else:\n",
    "            print(f\"{col} is not cointegrated.\")\n",
    "    return out"
   ]
  },
  {
   "cell_type": "code",
   "execution_count": 37,
   "id": "4f77546d",
   "metadata": {},
   "outputs": [
    {
     "name": "stdout",
     "output_type": "stream",
     "text": [
      "Trace statistic: [261.5548149  167.67790177  98.11781369  53.4617083   21.6404865\n",
      "   4.01416422]\n",
      "Critical values: [95.7542 69.8189 47.8545 29.7961 15.4943  3.8415]\n",
      "Eigenvalues: [0.11449947 0.08616362 0.05620349 0.04038124 0.02257335 0.0051862 ]\n",
      "crude_brent is cointegrated.\n",
```

```
    "soybeans is cointegrated.\n",
    "gold is cointegrated.\n",
    "silver is cointegrated.\n",
    "urea_ee_bulk is cointegrated.\n",
    "maize is cointegrated.\n"
   ]
  }
 ],
 "source": [
  "# Perform Johansen cointegration test\n",
  "coint_test = johansen_test(commodity_data)"
 ]
},
{
 "cell_type": "code",
 "execution_count": 38,
 "id": "9e80c826",
 "metadata": {},
 "outputs": [],
 "source": [
  "# Number of cointegrating relationships (assuming r = 1 if there's at least one significant eigenvalue)\n",
  "r = sum(coint_test.lr1 > coint_test.cvt[:, 1])  # Replace with the actual number from the test results"
 ]
},
{
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 "execution_count": 39,
 "id": "2630f888",
 "metadata": {
  "scrolled": false
 },
 "outputs": [
  {
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   "output_type": "stream",
   "text": [
    "  Summary of Regression Results   \n",
    "==================================\n",
    "Model:                         VAR\n",
    "Method:                        OLS\n",
    "Date:           Wed, 24, Jul, 2024\n",
    "Time:                     21:09:56\n",
    "--------------------------------------------------------------------\n",
    "No. of Equations:         6.00000    BIC:                    26.7336\n",
    "Nobs:                     768.000    HQIC:                   25.9079\n",
    "Log likelihood:          -16066.7    FPE:                1.06530e+11\n",
    "AIC:                      25.3912    Det(Omega_mle):     8.03276e+10\n",
    "--------------------------------------------------------------------\n",
    "Results for equation crude_brent\n",
    "==================================================================================\n",
    "                    coefficient       std. error           t-stat            prob\n",
    "----------------------------------------------------------------------------------\n",
    "const                 -0.574387         0.457999           -1.254           0.210\n",
    "L1.crude_brent         1.288559         0.039600           32.539           0.000\n",
    "L1.soybeans            0.011187         0.007736            1.446           0.148\n",
    "L1.gold                0.000565         0.006577            0.086           0.932\n",
    "L1.silver             -0.012011         0.165664           -0.073           0.942\n",
    "L1.urea_ee_bulk       -0.011804         0.004637           -2.546           0.011\n",
    "L1.maize               0.020438         0.017600            1.161           0.246\n",
    "L2.crude_brent        -0.368186         0.064243           -5.731           0.000\n",
    "L2.soybeans            0.008609         0.010762            0.800           0.424\n",
```

```
"L2.gold               -0.007451         0.010640           -0.700           0.484\n",
"L2.silver              0.199505         0.275939            0.723           0.470\n",
"L2.urea_ee_bulk        0.015907         0.007085            2.245           0.025\n",
"L2.maize              -0.022252         0.025791           -0.863           0.388\n",
"L3.crude_brent        -0.011259         0.066566           -0.169           0.866\n",
"L3.soybeans           -0.024881         0.010745           -2.316           0.021\n",
"L3.gold                0.020019         0.010832            1.848           0.065\n",
"L3.silver             -0.211736         0.295689           -0.716           0.474\n",
"L3.urea_ee_bulk       -0.004688         0.007391           -0.634           0.526\n",
"L3.maize               0.031954         0.026095            1.225           0.221\n",
"L4.crude_brent         0.022815         0.066751            0.342           0.733\n",
"L4.soybeans            0.009171         0.010841            0.846           0.398\n",
"L4.gold               -0.000726         0.010669           -0.068           0.946\n",
"L4.silver              0.037894         0.296398            0.128           0.898\n",
"L4.urea_ee_bulk        0.000123         0.007431            0.017           0.987\n",
"L4.maize              -0.043400         0.026026           -1.668           0.095\n",
"L5.crude_brent         0.008371         0.065302            0.128           0.898\n",
"L5.soybeans            0.009904         0.010927            0.906           0.365\n",
"L5.gold               -0.005274         0.010504           -0.502           0.616\n",
"L5.silver             -0.077226         0.280104           -0.276           0.783\n",
"L5.urea_ee_bulk       -0.004359         0.007074           -0.616           0.538\n",
"L5.maize               0.034108         0.026066            1.309           0.191\n",
"L6.crude_brent         0.021961         0.040570            0.541           0.588\n",
"L6.soybeans           -0.007763         0.007913           -0.981           0.327\n",
"L6.gold               -0.007032         0.006708           -1.048           0.295\n",
"L6.silver              0.137240         0.167517            0.819           0.413\n",
"L6.urea_ee_bulk        0.001589         0.004568            0.348           0.728\n",
"L6.maize              -0.021898         0.017481           -1.253           0.210\n",
"==============================================================================\n",
"\n",
"Results for equation soybeans\n",
"==============================================================================\n",
"                     coefficient       std. error           t-stat            prob\n",
"------------------------------------------------------------------------------\n",
"const                  11.317337         2.521090            4.489           0.000\n",
"L1.crude_brent          0.214138         0.217982            0.982           0.326\n",
"L1.soybeans            1.013966         0.042581           23.813           0.000\n",
"L1.gold                 0.013684         0.036203            0.378           0.705\n",
"L1.silver               0.305354         0.911909            0.335           0.738\n",
"L1.urea_ee_bulk       -0.009017         0.025525           -0.353           0.724\n",
"L1.maize               0.314169         0.096881            3.243           0.001\n",
"L2.crude_brent        -0.103000         0.353632           -0.291           0.771\n",
"L2.soybeans           -0.017674         0.059238           -0.298           0.765\n",
"L2.gold               -0.064859         0.058571           -1.107           0.268\n",
"L2.silver               0.926647         1.518924            0.610           0.542\n",
"L2.urea_ee_bulk        0.041336         0.039000            1.060           0.289\n",
"L2.maize              -0.285567         0.141970           -2.011           0.044\n",
"L3.crude_brent        -0.077825         0.366417           -0.212           0.832\n",
"L3.soybeans           -0.141878         0.059147           -2.399           0.016\n",
"L3.gold                0.131659         0.059625            2.208           0.027\n",
"L3.silver             -2.231664         1.627642           -1.371           0.170\n",
"L3.urea_ee_bulk       -0.018121         0.040686           -0.445           0.656\n",
"L3.maize               0.159302         0.143644            1.109           0.267\n",
"L4.crude_brent          0.036457         0.367435            0.099           0.921\n",
"L4.soybeans            0.084280         0.059676            1.412           0.158\n",
"L4.gold               -0.093822         0.058728           -1.598           0.110\n",
"L4.silver              1.219334         1.631547            0.747           0.455\n",
"L4.urea_ee_bulk        0.011285         0.040903            0.276           0.783\n",
"L4.maize              -0.411196         0.143261           -2.870           0.004\n",
"L5.crude_brent        -0.053674         0.359462           -0.149           0.881\n",
"L5.soybeans           -0.059902         0.060151           -0.996           0.319\n",
```

```
"L5.gold             0.023087         0.057818           0.399           0.690\n",
"L5.silver           0.252871         1.541852           0.164           0.870\n",
"L5.urea_ee_bulk    -0.011316         0.038941          -0.291           0.771\n",
"L5.maize            0.302401         0.143482           2.108           0.035\n",
"L6.crude_brent     -0.062569         0.223320          -0.280           0.779\n",
"L6.soybeans         0.028889         0.043560           0.663           0.507\n",
"L6.gold             0.001505         0.036925           0.041           0.967\n",
"L6.silver          -0.176909         0.922107          -0.192           0.848\n",
"L6.urea_ee_bulk     0.010044         0.025142           0.399           0.690\n",
"L6.maize           -0.045677         0.096225          -0.475           0.635\n",
"===================================================================================\n",
"\n",
"Results for equation gold\n",
"===================================================================================\n",
"                    coefficient       std. error           t-stat            prob\n",
"-----------------------------------------------------------------------------------\n",
"const                0.177098         3.702239           0.048           0.962\n",
"L1.crude_brent       0.190589         0.320109           0.595           0.552\n",
"L1.soybeans          0.019501         0.062531           0.312           0.755\n",
"L1.gold             1.228901         0.053164          23.115           0.000\n",
"L1.silver           0.316301         1.339144           0.236           0.813\n",
"L1.urea_ee_bulk    -0.125678         0.037484          -3.353           0.001\n",
"L1.maize            0.279896         0.142270           1.967           0.049\n",
"L2.crude_brent      0.074271         0.519311           0.143           0.886\n",
"L2.soybeans         0.037551         0.086991           0.432           0.666\n",
"L2.gold            -0.276183         0.086012          -3.211           0.001\n",
"L2.silver          -3.352388         2.230551          -1.503           0.133\n",
"L2.urea_ee_bulk     0.215119         0.057271           3.756           0.000\n",
"L2.maize           -0.305428         0.208485          -1.465           0.143\n",
"L3.crude_brent     -0.688550         0.538086          -1.280           0.201\n",
"L3.soybeans        -0.222153         0.086857          -2.558           0.011\n",
"L3.gold             0.170371         0.087559           1.946           0.052\n",
"L3.silver           0.453043         2.390204           0.190           0.850\n",
"L3.urea_ee_bulk    -0.154341         0.059747          -2.583           0.010\n",
"L3.maize            0.492114         0.210943           2.333           0.020\n",
"L4.crude_brent      0.381592         0.539582           0.707           0.479\n",
"L4.soybeans         0.251772         0.087634           2.873           0.004\n",
"L4.gold            -0.151613         0.086243          -1.758           0.079\n",
"L4.silver           3.646825         2.395938           1.522           0.128\n",
"L4.urea_ee_bulk     0.066199         0.060066           1.102           0.270\n",
"L4.maize           -1.026908         0.210379          -4.881           0.000\n",
"L5.crude_brent     -0.125251         0.527873          -0.237           0.812\n",
"L5.soybeans        -0.157098         0.088332          -1.778           0.075\n",
"L5.gold             0.110733         0.084906           1.304           0.192\n",
"L5.silver          -1.459901         2.264221          -0.645           0.519\n",
"L5.urea_ee_bulk     0.047764         0.057185           0.835           0.404\n",
"L5.maize            0.583033         0.210704           2.767           0.006\n",
"L6.crude_brent      0.320187         0.327947           0.976           0.329\n",
"L6.soybeans         0.110200         0.063968           1.723           0.085\n",
"L6.gold            -0.073845         0.054225          -1.362           0.173\n",
"L6.silver          -0.453634         1.354121          -0.335           0.738\n",
"L6.urea_ee_bulk    -0.076808         0.036922          -2.080           0.037\n",
"L6.maize           -0.077152         0.141307          -0.546           0.585\n",
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"L1.crude_brent       0.008049         0.012893           0.624           0.532\n",
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"L1.soybeans          0.001756        0.002519          0.697          0.486\n",
"L1.gold             -0.002671        0.002141         -1.248          0.212\n",
"L1.silver            1.340090        0.053938         24.845          0.000\n",
"L1.urea_ee_bulk     -0.003586        0.001510         -2.375          0.018\n",
"L1.maize             0.011821        0.005730          2.063          0.039\n",
"L2.crude_brent       0.014541        0.020917          0.695          0.487\n",
"L2.soybeans         -0.000991        0.003504         -0.283          0.777\n",
"L2.gold              0.003938        0.003464          1.137          0.256\n",
"L2.silver           -0.665510        0.089843         -7.408          0.000\n",
"L2.urea_ee_bulk      0.002013        0.002307          0.873          0.383\n",
"L2.maize            -0.001179        0.008397         -0.140          0.888\n",
"L3.crude_brent      -0.033019        0.021673         -1.523          0.128\n",
"L3.soybeans         -0.003366        0.003498         -0.962          0.336\n",
"L3.gold              0.002395        0.003527          0.679          0.497\n",
"L3.silver            0.187709        0.096273          1.950          0.051\n",
"L3.urea_ee_bulk      0.001209        0.002407          0.503          0.615\n",
"L3.maize             0.002916        0.008496          0.343          0.731\n",
"L4.crude_brent       0.019566        0.021733          0.900          0.368\n",
"L4.soybeans          0.003541        0.003530          1.003          0.316\n",
"L4.gold             -0.001627        0.003474         -0.468          0.639\n",
"L4.silver            0.118333        0.096504          1.226          0.220\n",
"L4.urea_ee_bulk     -0.003052        0.002419         -1.262          0.207\n",
"L4.maize            -0.026818        0.008474         -3.165          0.002\n",
"L5.crude_brent      -0.024297        0.021262         -1.143          0.253\n",
"L5.soybeans         -0.000816        0.003558         -0.229          0.819\n",
"L5.gold              0.002731        0.003420          0.799          0.424\n",
"L5.silver           -0.156757        0.091199         -1.719          0.086\n",
"L5.urea_ee_bulk      0.004159        0.002303          1.806          0.071\n",
"L5.maize             0.020487        0.008487          2.414          0.016\n",
"L6.crude_brent       0.022428        0.013209          1.698          0.090\n",
"L6.soybeans          0.002044        0.002577          0.793          0.428\n",
"L6.gold             -0.004226        0.002184         -1.935          0.053\n",
"L6.silver            0.104285        0.054542          1.912          0.056\n",
"L6.urea_ee_bulk     -0.002649        0.001487         -1.781          0.075\n",
"L6.maize            -0.008036        0.005692         -1.412          0.158\n",
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"L1.soybeans          0.139955        0.062059          2.255          0.024\n",
"L1.gold              0.074409        0.052764          1.410          0.158\n",
"L1.silver           -4.409772        1.329050         -3.318          0.001\n",
"L1.urea_ee_bulk      1.112425        0.037201         29.903          0.000\n",
"L1.maize             0.329777        0.141198          2.336          0.020\n",
"L2.crude_brent      -1.250799        0.515396         -2.427          0.015\n",
"L2.soybeans         -0.071260        0.086335         -0.825          0.409\n",
"L2.gold             -0.086168        0.085364         -1.009          0.313\n",
"L2.silver            7.401289        2.213736          3.343          0.001\n",
"L2.urea_ee_bulk     -0.327856        0.056839         -5.768          0.000\n",
"L2.maize            -0.434760        0.206913         -2.101          0.036\n",
"L3.crude_brent       0.861473        0.534029          1.613          0.107\n",
"L3.soybeans         -0.116643        0.086203         -1.353          0.176\n",
"L3.gold             -0.005424        0.086899         -0.062          0.950\n",
"L3.silver           -4.046644        2.372186         -1.706          0.088\n",
"L3.urea_ee_bulk      0.142202        0.059297          2.398          0.016\n",
"L3.maize             0.233880        0.209353          1.117          0.264\n",
"L4.crude_brent      -1.559052        0.535514         -2.911          0.004\n",
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"L4.soybeans        -0.052667       0.086974       -0.606       0.545\n",
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"L4.silver           1.032326       2.377877        0.434       0.664\n",
"L4.urea_ee_bulk    -0.104196       0.059613       -1.748       0.080\n",
"L4.maize            0.028888       0.208793        0.138       0.890\n",
"L5.crude_brent      0.913930       0.523894        1.744       0.081\n",
"L5.soybeans         0.095496       0.087667        1.089       0.276\n",
"L5.gold             0.053301       0.084266        0.633       0.527\n",
"L5.silver          -0.500818       2.247152       -0.223       0.824\n",
"L5.urea_ee_bulk     0.156414       0.056754        2.756       0.006\n",
"L5.maize           -0.115267       0.209116       -0.551       0.581\n",
"L6.crude_brent     -0.415228       0.325475       -1.276       0.202\n",
"L6.soybeans         0.089368       0.063486        1.408       0.159\n",
"L6.gold            -0.040869       0.053816       -0.759       0.448\n",
"L6.silver           0.599056       1.343913        0.446       0.656\n",
"L6.urea_ee_bulk    -0.119322       0.036643       -3.256       0.001\n",
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"L1.crude_brent      -0.075264       0.095379       -0.789       0.430\n",
"L1.soybeans         0.036037       0.018632        1.934       0.053\n",
"L1.gold            -0.023696       0.015841       -1.496       0.135\n",
"L1.silver           0.588077       0.399010        1.474       0.141\n",
"L1.urea_ee_bulk     0.037550       0.011169        3.362       0.001\n",
"L1.maize            1.141848       0.042391       26.936       0.000\n",
"L2.crude_brent      0.036084       0.154733        0.233       0.816\n",
"L2.soybeans         0.007586       0.025920        0.293       0.770\n",
"L2.gold            -0.015226       0.025628       -0.594       0.552\n",
"L2.silver           0.911243       0.664612        1.371       0.170\n",
"L2.urea_ee_bulk    -0.040754       0.017064       -2.388       0.017\n",
"L2.maize           -0.309322       0.062120       -4.979       0.000\n",
"L3.crude_brent     -0.075868       0.160327       -0.473       0.636\n",
"L3.soybeans        -0.025177       0.025880       -0.973       0.331\n",
"L3.gold             0.066343       0.026089        2.543       0.011\n",
"L3.silver          -2.363728       0.712182       -3.319       0.001\n",
"L3.urea_ee_bulk     0.030562       0.017802        1.717       0.086\n",
"L3.maize            0.156905       0.062852        2.496       0.013\n",
"L4.crude_brent      0.153469       0.160773        0.955       0.340\n",
"L4.soybeans         0.021164       0.026111        0.811       0.418\n",
"L4.gold            -0.055764       0.025697       -2.170       0.030\n",
"L4.silver           2.024847       0.713890        2.836       0.005\n",
"L4.urea_ee_bulk    -0.022652       0.017897       -1.266       0.206\n",
"L4.maize           -0.136153       0.062684       -2.172       0.030\n",
"L5.crude_brent     -0.109997       0.157284       -0.699       0.484\n",
"L5.soybeans        -0.026489       0.026319       -1.006       0.314\n",
"L5.gold             0.052825       0.025298        2.088       0.037\n",
"L5.silver          -0.829437       0.674644       -1.229       0.219\n",
"L5.urea_ee_bulk     0.017161       0.017039        1.007       0.314\n",
"L5.maize            0.000944       0.062781        0.015       0.988\n",
"L6.crude_brent      0.026482       0.097715        0.271       0.786\n",
"L6.soybeans         0.002271       0.019060        0.119       0.905\n",
"L6.gold            -0.023655       0.016157       -1.464       0.143\n",
"L6.silver           0.146935       0.403472        0.364       0.716\n",
"L6.urea_ee_bulk     0.000775       0.011001        0.070       0.944\n",
"L6.maize            0.020945       0.042104        0.497       0.619\n",
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"gold   0.111776 0.082179 1.000000 0.722123   0.072033 0.086465\n",
"silver   0.209142 0.111588 0.722123 1.000000   0.069879 0.125813\n",
"urea_ee_bulk   0.153268 0.032578 0.072033 0.069879   1.000000 0.017836\n",
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"if r > 0:\n",
"    # Creating a VAR model for prediction using the VECM\n",
"    model = VAR(commodity_data)\n",
"    vecm_result = model.fit(r)\n",
"    \n",
"    # Summary of the VECM model\n",
"    print(vecm_result.summary())\n",
"    \n",
"    # Forecasting using the VECM model\n",
"    # Forecasting 24 steps ahead\n",
"    forecast = vecm_result.forecast(commodity_data.values[-vecm_result.k_ar:], steps=24)\n",
"    \n",
"    # Convert forecast to DataFrame for plotting\n",
"    forecast_df = pd.DataFrame(forecast, index=pd.date_range(start=commodity['date'].iloc[-1], periods=24, freq='M'), columns=commodity_data.columns)\n",
"    \n",
"    # Plotting the forecast\n",
"    plt.figure(figsize=(12, 8))\n",
"    for col in forecast_df.columns:\n",
"        plt.plot(forecast_df.index, forecast_df[col], label=col)\n",
"    plt.legend()\n",
"    plt.title('VECM Forecast')\n",
"    plt.show()\n",
"\n",
"else:\n",
"    # If no co-integration exists, proceed with Unrestricted VAR Analysis\n",
"    model = VAR(commodity_data)\n",
"    var_result = model.fit(maxlags=10, ic='aic')\n",
"    \n",
"    # Summary of the VAR model\n",
"    print(var_result.summary())\n",
"    \n",
"    # Granger causality test\n",
"    causality_results = {}\n",
"    for col in commodity_data.columns:\n",
"        causality_results[col] = var_result.test_causality(causing=col, caused=commodity_data.columns.difference([col]), kind='f').summary()\n",
"    print(causality_results)\n",
"    \n",
"    # Forecasting using the VAR model\n",
"    forecast = var_result.forecast(commodity_data.values[-var_result.k_ar:], steps=24)\n",
"    \n",
"    # Convert forecast to DataFrame for plotting\n",
```

```
    "    forecast_df = pd.DataFrame(forecast, index=pd.date_range(start=commodity['date'].iloc[-1], periods=24, freq='M'), columns=commodity_data.columns)\n",
    "    \n",
    "    # Plotting the forecast\n",
    "    plt.figure(figsize=(12, 8))\n",
    "    for col in forecast_df.columns:\n",
    "        plt.plot(forecast_df.index, forecast_df[col], label=col)\n",
    "    plt.legend()\n",
    "    plt.title('VAR Forecast')\n",
    "    plt.show()"
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