

**APPLICATION OF FUZZY NETWORK GRAPH
IN HOSPITAL**

A PROJECT REPORT SUBMITTED TO V.H.N. SENTHIKUMARA
NADAR COLLEGE (AUTONOMOUS), VIRUDHUNAGAR.
IN THE PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE
AWARD OF THE DEGREE OF
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IN
MATHEMATICS

by

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Register Number: 19AUMA003, 19AUMA008, 19AUMA013



UNDER THE SUPERVISION OF

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V. H. N. SENTHIKUMARA NADAR COLLEGE, VIRUDHUNAGAR

(An Autonomous Institution Affiliated with Madurai Kamaraj University)

[Re-accredited with 'A' Grade by NAAC]

MAY – 2022

CERTIFICATE

This is to certify that the project report entitled “**APPLICATION OF FUZZY NETWORK GRAPH IN HOSPITAL**” submitted to Virudhunagar Hindu Nadars’ Senthikumara Nadar College (Autonomous), Virudhunagar 626 001, affiliated to Madurai Kamaraj University is a bonafide work carried out by **R. BIRUNDHA, M. HEMALATHA, V. KIRTHIKA (Register Number : 19AUMA003, 19AUMA008, 19AUMA013)** under the supervision of **Mrs. G. PETCHIAMMAL, M.Sc., M.Phil., ASSISTANT PROFESSOR**, Department of Mathematics, V. H. N. Senthikumara Nadar College (Autonomous), Virudhunagar – 626 001, during the academic year 2021 – 2022.

Place:

Supervisor

Date:

Head of the Department

DECLARATION

We hereby declare that this project entitled “**APPLICATION OF FUZZY NETWORK GRAPH IN HOSPITAL**” is a record of the work done by us at the Department of Mathematics, V. H. N. Senthikumara Nadar College (Autonomous), Virudhunagar – 626 001 under the guidance of **Mrs. G. PETCHIAMMAL, M.Sc., M.Phil., ASSISTANT PROFESSOR**, Department of Mathematics, V. H. N. Senthikumara Nadar College (Autonomous), Virudhunagar – 626 001, for the partial fulfilment of the requirement for the award of the degree of **Bachelor of Science in Mathematics**.

Place:

Date:

(R. BIRUNDHA, M. HEMALATHA, V. KIRTHIKA)

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(R. BIRUNDHA, M. HEMALATHA, V. KIRTHIKA)

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CHAPTER – 1

Introduction

1.1 Network:

Network analysis has originated as a branch of Sociology and Mathematics which provides formal models and methods for the systematic study of social structures and it has an especially long tradition in Sociology, Social psychology and Anthropology. But concepts of network analysis capture the common properties of all networks and its methods are applicable to the analysis of networks in general. Network analysis is carried out in area such as project planning, complex systems, electrical circuits, social networks, transportation systems, communication networks, epidemiology, bioinformatics, hypertext systems, text analysis, organization theory, event analysis, bibliometrics, genealogical research and others.

1.2 Graph theory:

Graph theory is a very important tool to represent many real world problems. Nowadays graphs do not represent all the systems properly due to the uncertainty or haziness of the parameters of systems. Crisp graphs and fuzzy graph both are structurally similar. But when there is an uncertainty on vertices and / or edges the fuzzy graph has a separate importance. Applications of fuzzy graph include data mining, image segmentation, clustering, image capturing, networking, communication, planning, scheduling, etc.

1.3 Fractals:

The word Fractal was coined by Mandelbrot in his fundamental essay from the Latin fractus meaning broken, to describe the objects that were too irregular to fit into a traditional geometrical setting. Many fractals have some degree of self-similarity – they are made up parts that resemble the whole in some ways. Sometimes, the resemblance may be weaker than strict geometrical similarity for example the similarity may be approximate or statistical. Sierpinski Triangle, Cantor Set and Von Koch Curve are the examples of fractals. He defined a fractal to be a set with Hausdorff dimension strictly greater than its topological dimensions.

Fractal has some form of self-similarity, perhaps approximate or statistical. The main tool of fractal geometry is dimension in its many forms. Fractals are used to describe the roughness of surfaces. A rough surface is characterized by a combination of two different fractals.

1.4 Diabetes:

Diabetes care is complex. The side effects of Diabetes, medications and the uncertainties of adherence to treatment, decision – making becomes more difficult. By adding hypertension, dyslipidemia and obesity, many or all of which are found in most people with Diabetes, and the treatment options become even more complex.

We are explained fuzzy relations between symptoms and diseases of Diabetic patients. Some basic definitions are discussed and methods of fuzzy network graph between diabetic patients, symptoms and related diseases are also discussed.

CHAPTER – 2

APPLICATION OF FUZZY NETWORK GRAPH IN HOSPITAL

2.1 Important definitions with examples:

Definition 2.1 (Fuzzy set):

A Fuzzy set on a set X is characterized by a mapping $m: X \rightarrow [0,1]$ which is called the Membership function. A fuzzy set is denoted by $A = (X, m)$

Example: 2.1

Let $X = \{G_1, G_2, G_3, G_4, G_5\}$ be a set of girls.

Let A be a fuzzy set of beautiful girls.

Now, $m: [0,1]$ is defined by,

$$m(G_1) = 0.7 ; m(G_2) = 0.9; m(G_3) = 0.4; m(G_4) = 0.5; m(G_5) = 0.8$$

$$* A = \{X, m\}$$

Example: 2.2

Let $X = \{G_1, G_2, G_3, G_4\}$ be a set of all girls in the class.

Let A be a fuzzy set of intelligent girls $m: X \rightarrow [0,1]$

$$\text{Fuzzy set } A = \{(G_1, 0.5), (G_2, 0.7), (G_3, 0.2), (G_4, 1)\}$$

Example: 2.3

Let $V = \{A, B, C, D, E\}$ set of some people.

Let A be a fuzzy set of smart people.

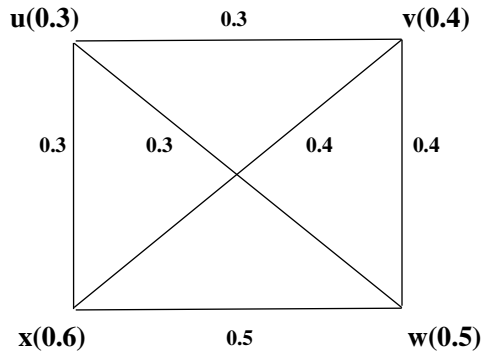
Now, $m: V \rightarrow [0,1]$ is defined by,

$$m(A) = 0.7 ; m(B) = 0.9 ; m(C) = 0.2 ; m(D) = 0.5; m(E) = 0.5$$

$$* A = \{V, m\}$$

Definition: 2.2 (Fuzzy graph):

A fuzzy graph $G = (V, \sigma, \mu)$ is an algebraic structure of non empty set of V together with a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ such that for all $x, y \in V$, $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ and μ is a symmetric fuzzy relation on σ .

Examples: 2.4

Here,

$\sigma(u) = 0.3$; $\sigma(v) = 0.4$; $\sigma(w) = 0.5$; $\sigma(x) = 0.6$; $\mu(u, v) = 0.3$; $\mu(v, w) = 0.4$; $\mu(w, x) = 0.5$; $\mu(x, u) = 0.3$; $\mu(u, w) = 0.3$ and $\mu(x, v) = 0.4$

Now, we check the condition:

$$\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$$

$$(i) \quad \mu(u, v) \leq \sigma(u) \wedge \sigma(v)$$

$$0.3 \leq 0.3 \wedge 0.4$$

$$0.3 = 0.3$$

$$(ii) \quad \mu(v, w) \leq \sigma(v) \wedge \sigma(w)$$

$$0.4 \leq 0.4 \wedge 0.5$$

$$0.4 = 0.4$$

$$(iii) \quad \mu(w, x) \leq \sigma(w) \wedge \sigma(x)$$

$$0.5 \leq 0.5 \wedge 0.6$$

$$0.5 = 0.5$$

$$(iv) \quad \mu(x, u) \leq \sigma(x) \wedge \sigma(u)$$

$$0.3 \leq 0.6 \wedge 0.3$$

$$0.3 = 0.3$$

$$(v) \quad \mu(u,w) \leq \sigma(u) \wedge \sigma(w)$$

$$0.3 \leq 0.3 \wedge 0.5$$

$$0.3 = 0.3$$

$$(vi) \quad \mu(v,w) \leq \sigma(v) \wedge \sigma(w)$$

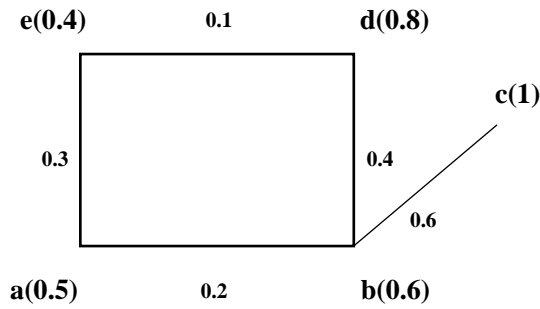
$$0.4 \leq 0.4 \wedge 0.5$$

$$0.4 = 0.4$$

$$\therefore \mu(x,y) \leq \sigma(x) \wedge \sigma(y) \quad \forall x,y \in V$$

♣The given graph is a fuzzy graph and also μ is a symmetric fuzzy relation on σ .

Example: 2.5



Here,

$\sigma(a) = 0.5$; $\sigma(b) = 0.6$; $\sigma(c) = 1$; $\sigma(d) = 0.8$; $\sigma(e) = 0.4$; $\mu(a,b) = 0.2$; $\mu(b,c) = 0.6$;
 $\mu(b,d) = 0.4$; $\mu(d,e) = 0.1$; $\mu(e,a) = 0.3$

Now, we check the condition:

$$\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$$

$$(i) \quad \mu(a,b) \leq \sigma(a) \wedge \sigma(b)$$

$$0.2 \leq 0.5 \wedge 0.6$$

$$0.2 < 0.5$$

$$(ii) \quad \mu(b,c) \leq \sigma(b) \wedge \sigma(c)$$

$$0.6 \leq 0.6 \wedge 1$$

$$0.6 = 0.6$$

$$(iii) \quad \mu(b,d) \leq \sigma(b) \wedge \sigma(d)$$

$$0.4 \leq 0.6 \wedge 0.8$$

$$0.4 < 0.6$$

$$(iv) \quad \mu(d,e) \leq \sigma(d) \wedge \sigma(e)$$

$$0.1 \leq 0.8 \wedge 0.4$$

$$0.1 < 0.4$$

$$(v) \quad \mu(e,a) \leq \sigma(e) \wedge \sigma(a)$$

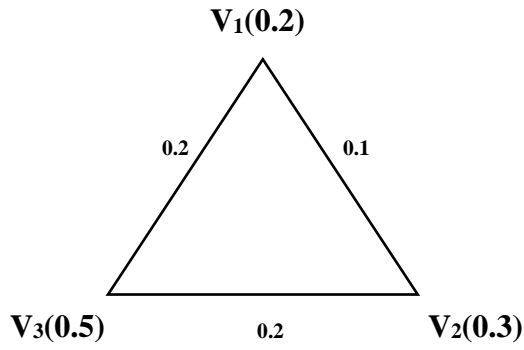
$$0.3 \leq 0.4 \wedge 0.5$$

$$0.3 < 0.4$$

$$\therefore \mu(x,y) \leq \sigma(x) \wedge \sigma(y) \quad \forall x,y \in V$$

∗ The given graph is a fuzzy graph.

Example: 2.6



Here,

$$\sigma(v_1) = 0.2, \sigma(v_2) = 0.3, \sigma(v_3) = 0.5, \mu(V_1, V_2) = 0.1, \mu(V_2, V_3) = 0.2, \mu(V_1, V_3) = 0.2$$

Now, we check the condition:

$$\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$$

$$(i) \quad \mu(V_1, V_2) \leq \sigma(V_1) \wedge \sigma(V_2)$$

$$0.1 \leq 0.2 \wedge 0.3$$

$$0.1 < 0.2$$

$$(ii) \quad \mu(V_2, V_3) \leq \sigma(V_2) \wedge \sigma(V_3)$$

$$0.2 \leq 0.3 \wedge 0.5$$

$$0.2 < 0.3$$

$$(iii) \quad \mu(V_3, V_1) \leq \sigma(V_3) \wedge \sigma(v_1)$$

$$0.2 \leq 0.5 \wedge 0.2$$

$$0.2 = 0.2$$

$$\mu(x,y) \leq \sigma(x) \wedge \sigma(y) \quad \forall x,y \in V$$

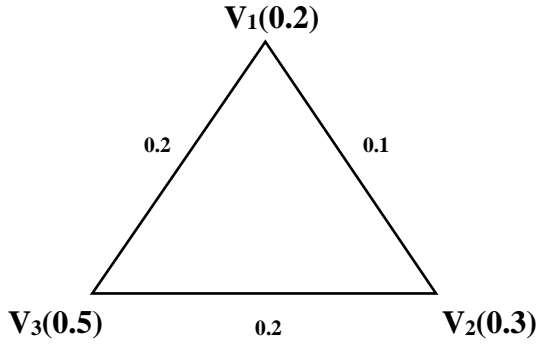
* The given graph is fuzzy graph.

Definition: 2.3 (Strong fuzzy graph):

Let $G = (V, \sigma, \mu)$ is a fuzzy graph. G is a strong fuzzy graph if,

$$\frac{1}{2} \{ \sigma(x) \wedge \sigma(y) \} \leq \mu(x,y) \quad \forall x,y \in V$$

Examples: 2.7



This is a fuzzy graph.

Now, we check the condition:

$$\frac{1}{2} \{ \sigma(x) \wedge \sigma(y) \} \leq \mu(x,y)$$

$$(i) \quad \frac{1}{2} \{ \sigma(V_1) \wedge \sigma(V_2) \} \leq \mu(V_1, V_2)$$

$$\frac{1}{2} \{ 0.2 \wedge 0.3 \} \leq 0.1$$

$$\frac{1}{2} (0.2) \leq 0.1$$

$$0.1 = 0.1$$

$$(ii) \quad \frac{1}{2} \{ \sigma(V_2) \wedge \sigma(V_3) \} \leq \mu(V_2, V_3)$$

$$\frac{1}{2} \{ 0.3 \wedge 0.5 \} \leq 0.2$$

$$\frac{1}{2} (0.3) \leq 0.2$$

$$0.15 < 0.2$$

$$(iii) \quad \frac{1}{2} \{ \sigma(V_1) \wedge \sigma(V_3) \} \leq \mu(V_1, V_3)$$

$$\frac{1}{2} \{ 0.2 \wedge 0.5 \} \leq 0.2$$

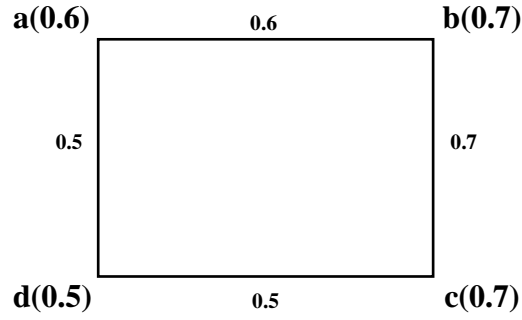
$$\frac{1}{2} (0.2) \leq 0.2$$

$$0.1 < 0.2$$

This graph satisfies the condition: $\frac{1}{2} \{\sigma(x) \wedge \sigma(y)\} \leq \mu(x,y) \forall x,y \in V$

Therefore, the given graph is a strong fuzzy graph.

Example: 2.8



It is a fuzzy graph.

Now, we check the condition:

$$\frac{1}{2} \{\sigma(x) \wedge \sigma(y)\} \leq \mu(x,y)$$

$$(i) \quad \frac{1}{2} \{\sigma(a) \wedge \sigma(b)\} \leq \mu(a,b)$$

$$\frac{1}{2} \{0.6 \wedge 0.7\} \leq 0.6$$

$$(0.6/2) \leq 0.6$$

$$0.3 < 0.6$$

$$(ii) \quad \frac{1}{2} \{\sigma(b) \wedge \sigma(c)\} \leq \mu(b,c)$$

$$\frac{1}{2} \{0.7 \wedge 0.7\} \leq 0.7$$

$$(0.7/2) \leq 0.7$$

$$0.35 < 0.7$$

$$(iii) \quad \frac{1}{2} \{\sigma(c) \wedge \sigma(d)\} \leq \mu(c,d)$$

$$\frac{1}{2} \{0.7 \wedge 0.5\} \leq 0.5$$

$$(0.5/2) \leq 0.5$$

$$0.25 < 0.5$$

$$(iv) \quad \frac{1}{2} \{ \sigma(d) \wedge \sigma(a) \} \leq \mu(d,a)$$

$$\frac{1}{2} \{ 0.5 \wedge 0.6 \} \leq 0.5$$

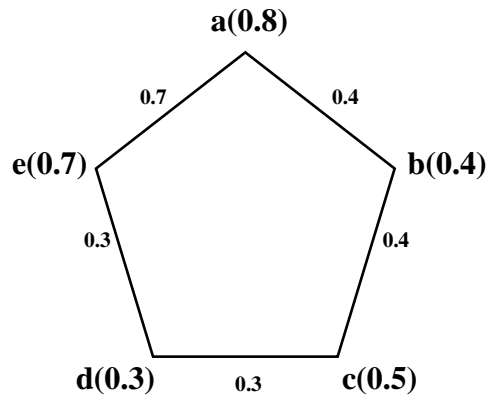
$$(0.5/2) \leq 0.5$$

$$0.25 < 0.5$$

This graph satisfies the condition: $\frac{1}{2} \{ \sigma(x) \wedge \sigma(y) \} \leq \mu(x,y) \quad \forall x,y \in V$

Therefore, the given graph is a strong fuzzy graph.

Example:2.9



It is a fuzzy graph.

Now, We check the condition: $\frac{1}{2} \{ \sigma(x) \wedge \sigma(y) \} \leq \mu(x,y)$

$$(i) \quad \frac{1}{2} \{ \sigma(a) \wedge \sigma(b) \} \leq \mu(a,b)$$

$$\frac{1}{2} \{ 0.8 \wedge 0.4 \} \leq 0.4$$

$$(0.4/2) \leq 0.4$$

$$0.2 < 0.4$$

$$(ii) \quad \frac{1}{2} \{ \sigma(b) \wedge \sigma(c) \} \leq \mu(b,c)$$

$$\frac{1}{2} \{ 0.4 \wedge 0.5 \} \leq 0.4$$

$$(0.4/2) \leq 0.4$$

$$0.2 < 0.4$$

$$(iii) \quad \frac{1}{2} \{ \sigma(c) \wedge \sigma(d) \} \leq \mu(c,d)$$

$$\frac{1}{2} \{ 0.5 \wedge 0.3 \} \leq 0.3$$

$$(0.3/2) \leq 0.3$$

$$0.15 < 0.3$$

$$(iv) \quad \frac{1}{2} \{ \sigma(d) \wedge \sigma(e) \} \leq \mu(d,e)$$

$$\frac{1}{2} \{ 0.3 \wedge 0.7 \} \leq 0.3$$

$$(0.3/2) \leq 0.3$$

$$0.15 < 0.3$$

$$(v) \quad \frac{1}{2} \{ \sigma(e) \wedge \sigma(a) \} \leq \mu(e,a)$$

$$\frac{1}{2} \{ 0.7 \wedge 0.8 \} \leq 0.7$$

$$(0.7/2) \leq 0.7$$

$$0.35 < 0.7$$

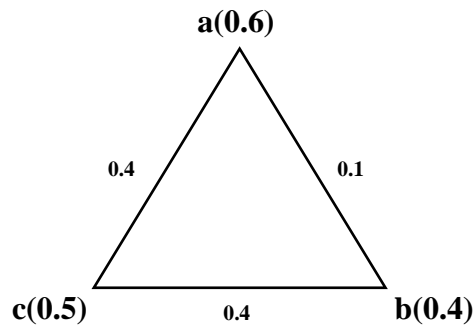
This graph satisfies the condition: $\frac{1}{2} \{ \sigma(x) \wedge \sigma(y) \} \leq \mu(x,y) \quad \forall x,y \in V$

Therefore, the given graph is a strong fuzzy graph.

Definition: 2.4 (Weak Fuzzy graph):

Let $G = (V, \sigma, \mu)$ is a fuzzy graph. G is a weak fuzzy graph if, $\frac{1}{2} \{ \sigma(x) \wedge \sigma(y) \} > \mu(x,y)$ for atleast one $x,y \in V$.

Example: 2.10



This is a fuzzy graph.

Now, we check the condition:

$$\frac{1}{2} \{ \sigma(x) \wedge \sigma(y) \} > \mu(x,y)$$

$$(i) \quad \frac{1}{2} \{ \sigma(a) \wedge \sigma(b) \} > \mu(a,b)$$

$$\frac{1}{2} \{ 0.6 \wedge 0.4 \} > 0.1$$

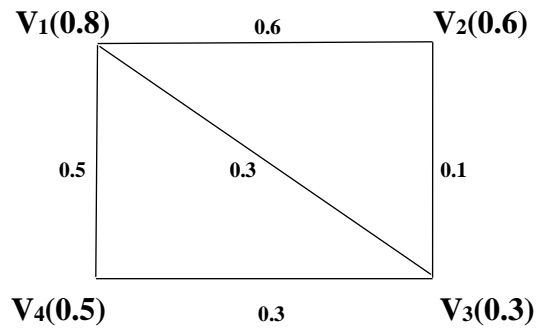
$$(0.4/2) > 0.1$$

$$0.2 > 0.1$$

The condition is satisfied.

* The given graph is weak fuzzy graph.

Example: 2.11



It is a fuzzy graph.

Now, we check the condition:

$$\frac{1}{2} \{ \sigma(x) \wedge \sigma(y) \} > \mu(x,y)$$

$$(i) \quad \frac{1}{2} \{ \sigma(v_1) \wedge \sigma(v_2) \} > \mu(v_1,v_2)$$

$$\frac{1}{2} \{ 0.8 \wedge 0.6 \} > 0.6$$

$$(0.6/2) > 0.6$$

$$0.3 < 0.6$$

$$(ii) \quad \frac{1}{2} \{ \sigma(v_2) \wedge \sigma(v_3) \} > \mu(v_2,v_3)$$

$$\frac{1}{2} \{ 0.6 \wedge 0.3 \} > 0.1$$

$$(0.3/2) > 0.1$$

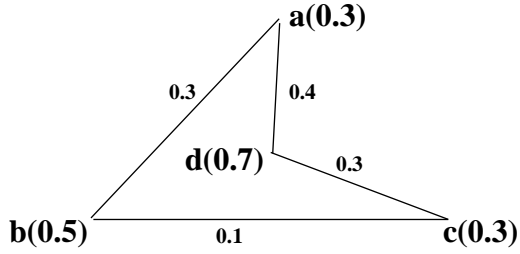
$$0.15 > 0.1$$

This implies, $\frac{1}{2} \{\sigma(x) \wedge \sigma(y)\} > \mu(x,y)$ for atleast one $x,y \in V$.

Therefore, the condition is satisfied.

So, the given graph is a weak fuzzy graph.

Example: 2.12



It is a fuzzy graph.

Now, we check the condition:

$$\frac{1}{2} \{\sigma(x) \wedge \sigma(y)\} > \mu(x,y)$$

$$(i) \quad \frac{1}{2} \{\sigma(a) \wedge \sigma(b)\} > \mu(a,b)$$

$$\frac{1}{2} \{0.3 \wedge 0.5\} > 0.3$$

$$(0.3/2) > 0.3$$

$$0.15 < 0.3$$

$$(ii) \quad \frac{1}{2} \{\sigma(b) \wedge \sigma(c)\} > \mu(b,c)$$

$$\frac{1}{2} \{0.5 \wedge 0.3\} > 0.1$$

$$(0.3/2) > 0.1$$

$$0.15 > 0.1$$

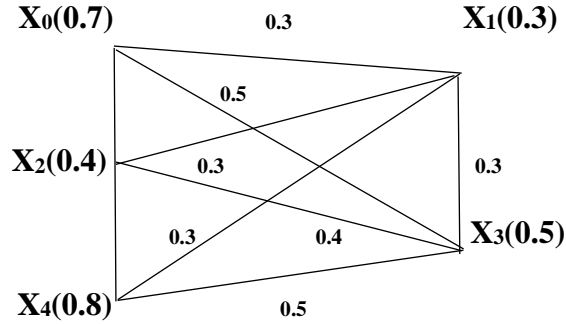
This implies, $\frac{1}{2} \{\sigma(x) \wedge \sigma(y)\} > \mu(x,y)$ for atleast one $x,y \in V$.

Therefore, the condition is satisfied.

So, the given graph is weak fuzzy graph.

Definition: 2.5 (Path in a fuzzy graph):

A path in fuzzy graph is a sequence of vertices $X_0, X_1, X_2, \dots, X_n$ such that $\mu(x_{i-1}, x_i) > 0$ and $i = 1, 2, 3, \dots, n$. The path is said to have length “n”.

Example: 2.13

Path in the given graph is X_0, X_1, X_2, X_3, X_4

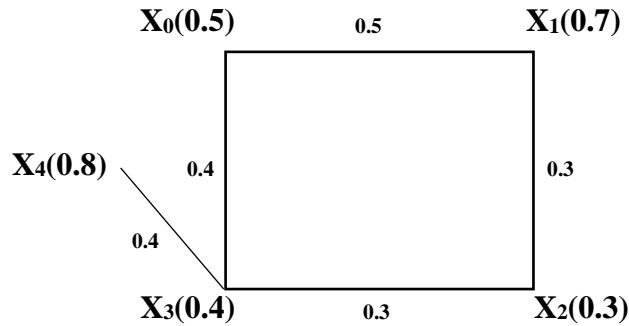
If $i = 1$; $\mu(x_0, x_1) = 0.3 > 0$

If $i = 2$; $\mu(x_1, x_2) = 0.3 > 0$

If $i = 3$; $\mu(x_2, x_3) = 0.4 > 0$

If $i = 4$; $\mu(x_3, x_4) = 0.5 > 0$

Also the path satisfies the condition: $\mu(x_{i-1}, x_i) > 0$

Example: 2.14

Path in the given graph is X_0, X_1, X_2, X_3, X_4

If $i = 1$; $\mu(x_0, x_1) = 0.5 > 0$

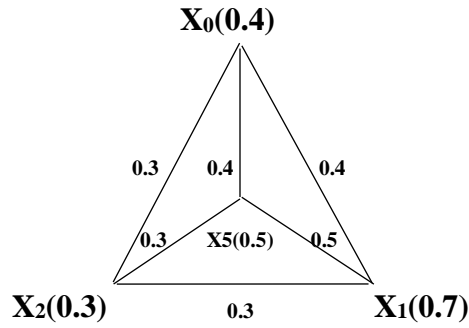
If $i = 2$; $\mu(x_1, x_2) = 0.3 > 0$

If $i = 3$; $\mu(x_2, x_3) = 0.3 > 0$

If $i = 4$; $\mu(x_3, x_4) = 0.4 > 0$

Also the path satisfies the condition: $\mu(x_{i-1}, x_i) > 0$

Example: 2.15



Path in the given graph is X_0, X_1, X_2, X_3

If $i = 1$; $\mu(x_0, x_1) = 0.4 > 0$

If $i = 2$; $\mu(x_1, x_2) = 0.3 > 0$

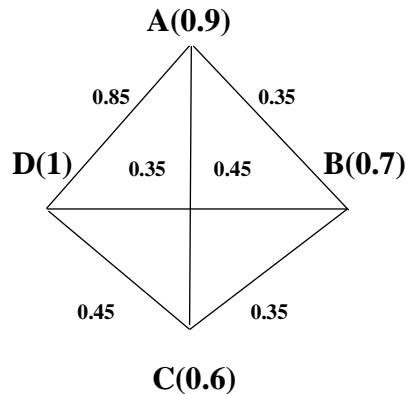
If $i = 3$; $\mu(x_2, x_3) = 0.3 > 0$

Also the path satisfies the condition: $\mu(x_{i-1}, x_i) > 0$

Definition: 2.6 (Connected graph):

Two nodes that are joined by a path are said to be connected. A component of a fuzzy graph is the fuzzy subgraph such that any two vertices are connected by path. So, a fuzzy graph is said to be connected fuzzy graph if it has one component.

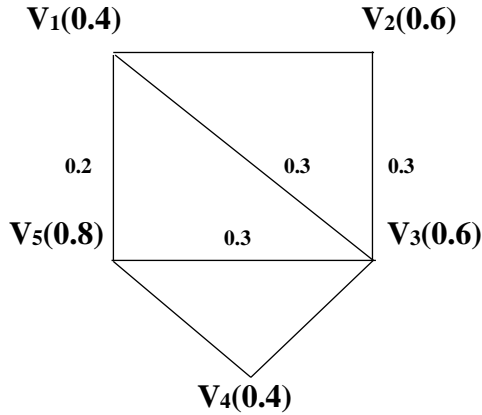
Example: 2.16



In this graph, two nodes are joined by a path and it has one component.

* The given graph is connected fuzzy graph.

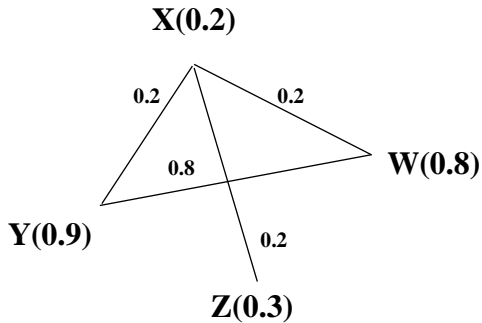
Example: 2.17



In this graph, two nodes are joined by a path and it has one component.

* The given graph is connected fuzzy graph.

Example: 2.18



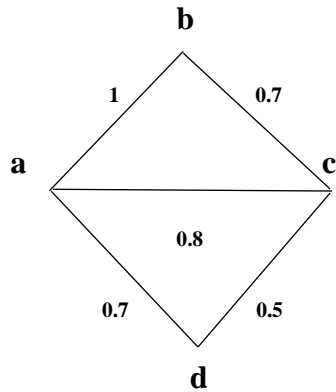
In this graph, two nodes are joined by a path and it has one component.

* The given graph is connected fuzzy graph.

Definition: 2.7 (Strength of the path):

The strength of a path is defined as $\min \{\mu(x_{i-1}, x_i) : i = 1, 2, 3, \dots, n\}$. In other words Strength of a path is the weight of the weakest arc of the path. The strength of the connectedness between two nodes X and Y is defined as the maximum of the strength of all paths between X and Y and it is denoted by $CONN_G(x, y)$.

Example: 2.19



Path in the given graph is **$P = a - b - c - d$** .

$$\mu(a,b) = 1, \mu(b,c) = 0.7, \mu(c,d) = 0.5, \mu(d,a) = 0.7, \mu(a,c) = 0.8.$$

$$\text{Strength of the path } P = \min \{ \mu(a,b), \mu(b,c), \mu(c,d) \}$$

$$= \min \{ 1, 0.7, 0.5 \}$$

$$= 0.5$$

$$\therefore \text{Strength of the path} = 0.5$$

Strength of the connectedness between ‘a’ and ‘c’:

There are three paths between ‘a’ and ‘c’

$$P_1 : a - b - c$$

$$P_2 : a - d - c$$

$$P_3 : a - c$$

$$\text{Strength of the path } P_1 = \min \{ \mu(a,b) ; \mu(b,c) \}$$

$$= \min \{ 1, 0.7 \}$$

$$P_1 = 0.7$$

$$\text{Strength of the path } P_2 = \min \{ \mu(a,d) ; \mu(d,c) \}$$

$$= \min \{ 0.7, 0.5 \}$$

$$P_2 = 0.5$$

Strength of the path $P_3 = \min \{\mu(a,c)\}$

$$P_3 = 0.8$$

Strength of the connectedness between 'a' and 'c' = $\max \{\text{strength of the paths } P_1, P_2, \text{ and } P_3\}$

$$= \max \{0.7, 0.5, 0.8\}$$

$$= 0.8$$

$$\text{ie) } \text{CONN}_G(a,c) = 0.8$$

Strength of the connectedness between 'a' and 'b':

There are three paths between 'a' and 'b'

$$P_1 : a - b$$

$$P_2 : a - c - b$$

$$P_3 : a - d - c - b$$

Strength of the path $P_1 = \min \{\mu(a,b)\}$

$$P_1 = 1$$

Strength of the path $P_2 = \min \{\mu(a,c) ; \mu(c,b)\}$

$$= \min \{0.8, 0.7\}$$

$$P_2 = 0.7$$

Strength of the path $P_3 = \min \{\mu(a,d) ; \mu(d,c) ; \mu(c,b)\}$

$$= \min \{0.7, 0.5, 0.7\}$$

$$P_3 = 0.5$$

Strength of the connectedness between 'a' and 'b' = $\max \{\text{strength of the paths } P_1, P_2 \text{ and } P_3\}$

$$= \max \{1, 0.7, 0.5\}$$

$$= 1$$

$$\text{ie) } \text{CONN}_G(a,b) = 1$$

Strength of the connectedness between ‘a’ and ‘d’:

There are three paths between ‘a’ and ‘d’

$$P_1 : a - b - c - d$$

$$P_2 : a - c - d$$

$$P_3 : a - d$$

$$\text{Strength of the path } P_1 = \min \{ \mu(a,b) ; \mu(b,c) ; \mu(c,d) \}$$

$$= \min \{ 1, 0.7, 0.5 \}$$

$$P_1 = 0.5$$

$$\text{Strength of the path } P_2 = \min \{ \mu(a,c) ; \mu(c,d) \}$$

$$= \min \{ 0.8, 0.5 \}$$

$$P_2 = 0.5$$

$$\text{Strength of the path } P_3 = \min \{ \mu(a,c) ; \mu(c,d) \}$$

$$= \min \{ 0.8, 0.5 \}$$

$$P_3 = 0.5$$

$$\text{Strength of the connectedness between ‘a’ and ‘d’} = \max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$$

$$= \max \{ 0.5, 0.5, 0.7 \}$$

$$= 0.7$$

$$\text{ie) } \text{CONN}_G(a,d) = 0.7$$

Strength of the connectedness between ‘b’ and ‘d’:

There are three paths between ‘b’ and ‘d’

$$P_1 : b - c - d$$

$$P_2 : b - a - d$$

$$P_3 : b - a - c - d$$

Strength of the path $P_1 = \min \{ \mu(b,c) ; \mu(c,d) \}$

$$= \min \{ 0.7, 0.5 \}$$

$$P_1 = 0.5$$

Strength of the path $P_2 = \min \{ \mu(b,a) ; \mu(a,d) \}$

$$= \min \{ 1, 0.7 \}$$

$$P_2 = 0.7$$

Strength of the path $P_3 = \min \{ \mu(b,a) ; \mu(a,c) ; \mu(c,d) \}$

$$= \min \{ 1, 0.8, 0.5 \}$$

$$P_3 = 0.5$$

Strength of the connectedness between 'b' and 'd' = $\max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$

$$= \max \{ 0.5, 0.7, 0.5 \}$$

$$= 0.7$$

$$\text{ie) } \text{CONN}_G(b,d) = 0.7$$

Strength of the connectedness between 'b' and 'c':

There are three paths between 'b' and 'c'

$$P_1 : b - c$$

$$P_2 : b - a - c$$

$$P_3 : b - a - d - c$$

Strength of the path $P_1 = \min \{ \mu(b,c) \}$

$$= \min \{ 0.7 \}$$

$$P_1 = 0.7$$

Strength of the path $P_2 = \min \{ \mu(b,a) ; \mu(a,c) \}$

$$= \min \{ 1, 0.7 \}$$

$$P_2 = 0.7$$

Strength of the path $P_3 = \min \{ \mu(b,a) ; \mu(a,d) ; \mu(d,c) \}$

$$= \min \{ 1, 0.7, 0.5 \}$$

$$P_3 = 0.5$$

Strength of the connectedness between 'b' and 'c' = $\max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$

$$= \max \{ 0.7, 0.7, 0.5 \}$$

$$= 0.7$$

$$\text{ie) } \text{CONN}_G(b,c) = 0.7$$

Strength of the connectedness between 'c' and 'd':

There are three paths between 'c' and 'd'

$$P_1 : c - d$$

$$P_2 : c - a - d$$

$$P_3 : c - b - a - d$$

Strength of the path $P_1 = \min \{ \mu(c,d) \}$

$$P_1 = 0.5$$

Strength of the path $P_2 = \min \{ \mu(c,a) ; \mu(a,d) \}$

$$= \min \{ 0.8, 0.7 \}$$

$$P_2 = 0.7$$

Strength of the path $P_3 = \min \{ \mu(c,b) ; \mu(b,a) ; \mu(a,d) \}$

$$= \min \{ 0.7, 1, 0.7 \}$$

$$P_3 = 0.7$$

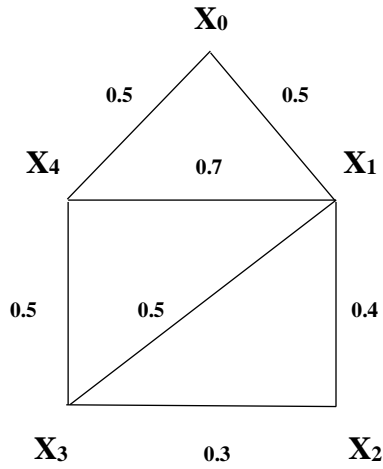
Strength of the connectedness between 'c' and 'd' = $\max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$

$$= \max \{ 0.5, 0.7, 0.7 \}$$

$$= 0.7$$

$$\text{ie) } \text{CONN}_G(c,d) = 0.7$$

Example: 2.20



Path in the given graph is $P = X_0 - X_1 - X_2 - X_3 - X_4$

$$\mu(x_0, x_1) = 0.5 ; \mu(x_1, x_2) = 0.4 ; \mu(x_2, x_3) = 0.3 ; \mu(x_3, x_4) = 0.5$$

$$\text{Strength of the path} = \min \{ \mu(x_0, x_1) ; \mu(x_1, x_2) ; \mu(x_2, x_3) ; \mu(x_3, x_4) \}$$

$$= \min \{ 0.5, 0.4, 0.3, 0.5 \}$$

$$= 0.3$$

$$\text{Strength of the path} = 0.3$$

Strength of the connectedness between ' x_0 ' and ' x_1 ':

There are four paths between ' x_0 ' and ' x_1 '

$$P_1 : x_0 - x_1$$

$$P_2 : x_0 - x_4 - x_1$$

$$P_3 : x_0 - x_4 - x_3 - x_1$$

$$P_4 : x_0 - x_4 - x_3 - x_2 - x_1$$

$$\text{Strength of the path } P_1 = \min \{ \mu(x_0, x_1) \}$$

$$P_1 = 0.5$$

$$\text{Strength of the path } P_2 = \min \{ \mu(x_0, x_4) ; \mu(x_4, x_1) \}$$

$$= \min \{0.5, 0.7\}$$

$$P_2 = 0.5$$

$$\text{Strength of the path } P_3 = \min \{\mu(x_0, x_4) ; \mu(x_4, x_3) ; \mu(x_3, x_1)\}$$

$$= \min \{0.5, 0.5, 0.5\}$$

$$P_3 = 0.5$$

$$\text{Strength of the path } P_4 = \min \{\mu(x_0, x_4) ; \mu(x_4, x_3) ; \mu(x_3, x_2) ; \mu(x_2, x_1)\}$$

$$= \min \{0.5, 0.5, 0.3, 0.4\}$$

$$P_4 = 0.3$$

$$\text{Strength of the connectedness between 'x}_0\text{' and 'x}_1\text{' = max \{strength of the paths}$$

$$P_1, P_2, P_3 \text{ and } P_4\}$$

$$= \max \{0.5, 0.5, 0.5, 0.3\}$$

$$= 0.5$$

$$\text{ie) } \text{CONN}_G(x_0, x_1) = 0.5$$

Strength of the connectedness between 'x₀' and 'x₂':

There are four paths between 'x₀' and 'x₂'

$$P_1 : x_0 - x_1 - x_2$$

$$P_2 : x_0 - x_4 - x_1 - x_2$$

$$P_3 : x_0 - x_1 - x_3 - x_2$$

$$P_4 : x_0 - x_4 - x_3 - x_2$$

$$\text{Strength of the path } P_1 = \min \{\mu(x_0, x_1) ; \mu(x_1, x_2)\}$$

$$= \min \{0.5, 0.4\}$$

$$P_1 = 0.4$$

$$\text{Strength of the path } P_2 = \min \{\mu(x_0, x_4) ; \mu(x_4, x_1) ; \mu(x_1, x_2)\}$$

$$= \min \{0.5, 0.7, 0.4\}$$

$$P_2 = 0.4$$

$$\text{Strength of the path } P_3 = \min \{\mu(x_0, x_1) ; \mu(x_1, x_3) ; \mu(x_3, x_2)\}$$

$$= \min \{0.5, 0.5, 0.3\}$$

$$P_3 = 0.3$$

$$\text{Strength of the path } P_4 = \min \{\mu(x_0, x_4) ; \mu(x_4, x_3) ; \mu(x_3, x_2)\}$$

$$= \min \{0.5, 0.5, 0.3\}$$

$$P_4 = 0.3$$

$$\text{Strength of the connectedness between 'x}_0\text{' and 'x}_2\text{' = max \{strength of the paths}$$

$$P_1, P_2, P_3 \text{ and } P_4\}$$

$$= \max \{0.4, 0.4, 0.3, 0.3\}$$

$$= 0.4$$

$$\text{ie) } \text{CONN}_G(x_0, x_2) = 0.4$$

Strength of the connectedness between 'x₀' and 'x₃':

There are four paths between 'x₀' and 'x₃'

$$P_1 : x_0 - x_1 - x_3$$

$$P_2 : x_0 - x_1 - x_2 - x_3$$

$$P_3 : x_0 - x_4 - x_3$$

$$P_4 : x_0 - x_4 - x_1 - x_3$$

$$\text{Strength of the path } P_1 = \min \{\mu(x_0, x_1) ; \mu(x_1, x_3)\}$$

$$= \min \{0.5, 0.5\}$$

$$P_1 = 0.5$$

$$\text{Strength of the path } P_2 = \min \{\mu(x_0, x_1) ; \mu(x_1, x_2) ; \mu(x_2, x_3)\}$$

$$= \min \{0.5, 0.4, 0.3\}$$

$$P_2 = 0.3$$

$$\text{Strength of the path } P_3 = \min \{\mu(x_0, x_4) ; \mu(x_4, x_3)\}$$

$$= \min \{0.5, 0.5\}$$

$$P_3 = 0.5$$

$$\text{Strength of the path } P_4 = \min \{\mu(x_0, x_4) ; \mu(x_4, x_1) ; \mu(x_1, x_3)\}$$

$$= \min \{0.5, 0.7, 0.5\}$$

$$P_4 = 0.5$$

$$\text{Strength of the connectedness between 'x}_0\text{' and 'x}_3\text{' = max \{strength of the paths}$$

$$P_1, P_2, P_3 \text{ and } P_4\}$$

$$= \max \{0.5, 0.3, 0.5, 0.5\}$$

$$= 0.5$$

$$\text{ie) } \text{CONN}_G = (x_0, x_3) = 0.5$$

Strength of the connectedness between 'x₀' and 'x₄':

There are four paths between 'x₀' and 'x₄'

$$P_1 : x_0 - x_1 - x_4$$

$$P_2 : x_0 - x_1 - x_3 - x_4$$

$$P_3 : x_0 - x_1 - x_2 - x_3 - x_4$$

$$P_4 : x_0 - x_4$$

$$\text{Strength of the path } P_1 = \min \{\mu(x_0, x_1) ; \mu(x_1, x_4)\}$$

$$= \min \{0.5, 0.7\}$$

$$P_1 = 0.5$$

$$\text{Strength of the path } P_2 = \min \{\mu(x_0, x_1) ; \mu(x_1, x_3) ; \mu(x_3, x_4)\}$$

$$= \min \{0.5, 0.5, 0.5\}$$

$$P_2 = 0.5$$

$$\text{Strength of the path } P_3 = \min \{ \mu(x_0, x_1) ; \mu(x_1, x_2) ; \mu(x_2, x_3) ; \mu(x_3, x_4) \}$$

$$= \min \{0.5, 0.4, 0.3, 0.5\}$$

$$P_3 = 0.3$$

$$\text{Strength of the path } P_4 = \min \{ \mu(x_0, x_4) \}$$

$$P_4 = 0.5$$

$$\text{Strength of the connectedness between 'x}_0\text{' and 'x}_4\text{' = max \{strength of the paths}$$

$$P_1, P_2, P_3 \text{ and } P_4\}$$

$$= \max \{0.5, 0.5, 0.3, 0.5\}$$

$$= 0.5$$

$$\text{ie) } \text{CONN}_G(x_0, x_4) = 0.5$$

Strength of the connectedness between 'x₁' and 'x₂':

There are four paths between 'x₁' and 'x₂'

$$P_1 : x_1 - x_2$$

$$P_2 : x_1 - x_3 - x_2$$

$$P_3 : x_1 - x_4 - x_3 - x_2$$

$$P_4 : x_1 - x_0 - x_4 - x_3 - x_2$$

$$\text{Strength of the path } P_1 = \min \{ \mu(x_1, x_2) \}$$

$$P_1 = 0.4$$

$$\text{Strength of the path } P_2 = \min \{ \mu(x_1, x_3) ; \mu(x_3, x_2) \}$$

$$= \min \{0.5, 0.3\}$$

$$P_2 = 0.3$$

Strength of the path $P_3 = \min \{ \mu(x_1, x_4) ; \mu(x_4, x_3) ; \mu(x_3, x_2) \}$

$$= \min \{ 0.7, 0.5, 0.3 \}$$

$$P_3 = 0.3$$

Strength of the path $P_4 = \min \{ \mu(x_1, x_0) ; \mu(x_0, x_4) ; \mu(x_4, x_3) ; \mu(x_3, x_2) \}$

$$= \min \{ 0.5, 0.5, 0.5, 0.3 \}$$

$$P_4 = 0.3$$

Strength of the connectedness between 'x₁' and 'x₂' = max {strength of the paths

P_1, P_2, P_3 and P_4 }

$$= \max \{ 0.4, 0.3, 0.3, 0.3 \}$$

$$= 0.4$$

$$\text{ie) } \text{CONN}_G(x_1, x_2) = 0.4$$

Strength of the connectedness between 'x₁' and 'x₃':

There are four paths between 'x₁' and 'x₃'

$$P_1 : x_1 - x_3$$

$$P_2 : x_1 - x_2 - x_3$$

$$P_3 : x_1 - x_4 - x_3$$

$$P_4 : x_1 - x_0 - x_4 - x_3$$

Strength of the path $P_1 = \min \{ \mu(x_1, x_3) \}$

$$P_1 = 0.5$$

Strength of the path $P_2 = \min \{ \mu(x_1, x_2) ; \mu(x_2, x_3) \}$

$$= \min \{ 0.4, 0.3 \}$$

$$P_2 = 0.3$$

Strength of the path $P_3 = \min \{ \mu(x_1, x_4) ; \mu(x_4, x_3) \}$

$$= \min \{0.7, 0.5\}$$

$$P_3 = 0.5$$

Strength of the path $P_4 = \min \{\mu(x_1, x_0) ; \mu(x_0, x_4) ; \mu(x_4, x_3)\}$

$$= \min \{0.5, 0.5, 0.5\}$$

$$P_4 = 0.5$$

Strength of the connectedness between 'x₁' and 'x₃' = max {strength of the paths

P_1, P_2, P_3 and P_4 }

$$= \max \{0.5, 0.3, 0.5, 0.5\}$$

$$= 0.5$$

$$\text{ie) } \text{CONN}_G(x_1, x_3) = 0.5$$

Strength of the connectedness between 'x₁' and 'x₄':

There are four paths between 'x₁' and 'x₄'

$$P_1 : x_1 - x_4$$

$$P_2 : x_1 - x_0 - x_4$$

$$P_3 : x_1 - x_3 - x_4$$

$$P_4 : x_1 - x_2 - x_3 - x_4$$

Strength of the path $P_1 = \min \{\mu(x_1, x_4)\}$

$$P_1 = 0.7$$

Strength of the path $P_2 = \min \{\mu(x_1, x_0) ; \mu(x_0, x_4)\}$

$$= \min \{0.5, 0.5\}$$

$$P_2 = 0.5$$

Strength of the path $P_3 = \min \{\mu(x_1, x_3) ; \mu(x_3, x_4)\}$

$$= \min \{0.5, 0.5\}$$

$$P_3 = 0.5$$

Strength of the path $P_4 = \min \{ \mu(x_1, x_2) ; \mu(x_2, x_3) ; \mu(x_3, x_4) \}$

$$= \min \{ 0.4, 0.3, 0.5 \}$$

$$P_4 = 0.3$$

Strength of the connectedness between 'x₁' and 'x₄' = max {strength of the paths

P_1, P_2, P_3 and P_4 }

$$= \max \{ 0.7, 0.5, 0.5, 0.3 \}$$

$$= 0.7$$

$$\text{ie) } \text{CONN}_G(x_1, x_4) = 0.7$$

Strength of the connectedness between 'x₂' and 'x₃':

There are four paths between 'x₂' and 'x₃'

$$P_1 : x_2 - x_3$$

$$P_2 : x_2 - x_1 - x_3$$

$$P_3 : x_2 - x_1 - x_4 - x_3$$

$$P_4 : x_2 - x_1 - x_0 - x_4 - x_3$$

Strength of the path $P_1 = \min \{ \mu(x_2, x_3) \}$

$$P_1 = 0.3$$

Strength of the path $P_2 = \min \{ \mu(x_2, x_1) ; \mu(x_1, x_3) \}$

$$= \min \{ 0.4, 0.5 \}$$

$$P_2 = 0.4$$

Strength of the path $P_3 = \min \{ \mu(x_2, x_1) ; \mu(x_1, x_4) ; \mu(x_4, x_3) \}$

$$= \min \{ 0.4, 0.7, 0.5 \}$$

$$P_3 = 0.4$$

Strength of the path $P_4 = \min \{ \mu(x_2, x_1) ; \mu(x_1, x_0) ; \mu(x_0, x_4) ; \mu(x_4, x_3) \}$

$$= \min \{0.4, 0.5, 0.5, 0.5\}$$

$$P_4 = 0.4$$

Strength of the connectedness between 'x₂' and 'x₃' = max {strength of the paths

P₁, P₂, P₃ and P₄}

$$= \max \{0.3, 0.4, 0.4, 0.4\}$$

$$= 0.4$$

$$\text{ie) } \text{CONN}_G(x_2, x_3) = 0.4$$

Strength of the connectedness between 'x₂' and 'x₄':

There are four paths between 'x₂' and 'x₄'

$$P_1 : x_2 - x_1 - x_4$$

$$P_2 : x_2 - x_3 - x_4$$

$$P_3 : x_2 - x_1 - x_3 - x_4$$

$$P_4 : x_2 - x_1 - x_0 - x_4$$

Strength of the path P₁ = min {μ(x₂, x₁) ; μ(x₁, x₄)}

$$= \min \{0.4, 0.7\}$$

$$P_1 = 0.4$$

Strength of the path P₂ = min {μ(x₂, x₃) ; μ(x₃, x₄)}

$$= \min \{0.3, 0.5\}$$

$$P_2 = 0.3$$

Strength of the path P₃ = min {μ(x₂, x₁) ; μ(x₁, x₃) ; μ(x₃, x₄)}

$$= \min \{0.4, 0.5, 0.5\}$$

$$P_3 = 0.4$$

Strength of the path P₄ = min {μ(x₂, x₁) ; μ(x₁, x₀) ; μ(x₀, x₄)}

$$= \min \{0.4, 0.5, 0.5\}$$

$$P_4 = 0.4$$

Strength of the connectedness between 'x₂' and 'x₄' = max {strength of the paths

P₁, P₂, P₃ and P₄}

$$= \max \{0.4, 0.3, 0.4, 0.4\}$$

$$= 0.4$$

$$\text{ie) CONNG}(x_2, x_4) = 0.4$$

Strength of the connectedness between 'x₃' and 'x₄':

There are four paths between 'x₃' and 'x₄'

$$P_1 : x_3 - x_4$$

$$P_2 : x_3 - x_1 - x_4$$

$$P_3 : x_3 - x_1 - x_0 - x_4$$

$$P_4 : x_3 - x_2 - x_1 - x_0 - x_4$$

Strength of the path P₁ = min {μ(x₃, x₄)}

$$P_1 = 0.5$$

Strength of the path P₂ = min {μ(x₃, x₁) ; μ(x₁, x₄)}

$$= \min \{0.5, 0.7\}$$

$$P_2 = 0.5$$

Strength of the path P₃ = min {μ(x₃, x₁) ; μ(x₁, x₀) ; μ(x₀, x₄)}

$$= \min \{0.5, 0.5, 0.5\}$$

$$P_3 = 0.5$$

Strength of the path P₄ = min {μ(x₃, x₂) ; μ(x₂, x₁) ; μ(x₁, x₀) ; μ(x₀, x₄)}

$$= \min \{0.3, 0.4, 0.5, 0.5\}$$

$$P_4 = 0.3$$

Strength of the connectedness between 'x₃' and 'x₄' = max {strength of the paths

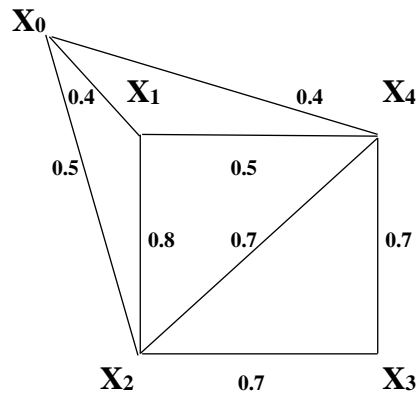
P₁, P₂, P₃ and P₄}

= max {0.5, 0.5, 0.5, 0.3}

= 0.5

ie) CONN_G (x₃, x₄) = 0.5

Example: 2.21



Path in the given graph is P = X₀ – X₁ – X₂ – X₃ – X₄

Strength of the path P = min {μ(x₀, x₁) ; μ(x₁, x₂) ; μ(x₂, x₃) ; μ(x₃, x₄)}

= min {0.4, 0.8, 0.7, 0.7}

= 0.4

Strength of the path = 0.4

Strength of the connectedness between 'x₀' and 'x₁':

There are five paths between 'x₀' and 'x₁'

P₁ : x₀ – x₁

P₂ : x₀ – x₄ – x₁

P₃ : x₀ – x₂ – x₁

P₄ : x₀ – x₂ – x₄ – x₁

P₅ : x₀ – x₂ – x₃ – x₄ – x₁

Strength of the path $P_1 = \min \{\mu(x_0, x_1)\}$

$$P_1 = 0.4$$

Strength of the path $P_2 = \min \{\mu(x_0, x_4) ; \mu(x_4, x_1)\}$

$$= \min \{0.4, 0.5\}$$

$$P_2 = 0.4$$

Strength of the path $P_3 = \min \{\mu(x_0, x_2) ; \mu(x_2, x_1)\}$

$$= \min \{0.5, 0.8\}$$

$$P_3 = 0.5$$

Strength of the path $P_4 = \min \{\mu\mu(x_0, x_2) ; \mu(x_2, x_4) ; \mu(x_4, x_1)\}$

$$= \min \{0.5, 0.7, 0.5\}$$

$$P_4 = 0.5$$

Strength of the path $P_5 = \min \{\mu(x_0, x_2) ; \mu(x_2, x_3) ; \mu(x_3, x_4) ; \mu(x_4, x_1)\}$

$$= \min \{0.5, 0.7, 0.7, 0.5\}$$

$$P_5 = 0.5$$

Strength of the connectedness between 'x₀' and 'x₁' = max {strength of the paths

$$P_1, P_2, P_3, P_4 \text{ and } P_5\}$$

$$= \max \{0.4, 0.4, 0.5, 0.5, 0.5\}$$

$$= 0.5$$

$$\text{ie) } \text{CONN}_G(x_0, x_1) = 0.5$$

Strength of the connectedness between 'x₀' and 'x₂':

There are five paths between 'x₀' and 'x₂'

$$P_1 : x_0 - x_2$$

$$P_2 : x_0 - x_1 - x_2$$

$$P_3 : x_0 - x_4 - x_2$$

$$P_4 : x_0 - x_4 - x_3 - x_2$$

$$P_5 : x_0 - x_4 - x_1 - x_2$$

$$\text{Strength of the path } P_1 = \min \{ \mu(x_0, x_2) \}$$

$$P_1 = 0.5$$

$$\text{Strength of the path } P_2 = \min \{ \mu(x_0, x_1) ; \mu(x_1, x_2) \}$$

$$= \min \{ 0.4, 0.8 \}$$

$$P_2 = 0.4$$

$$\text{Strength of the path } P_3 = \min \{ \mu(x_0, x_4) ; \mu(x_4, x_2) \}$$

$$= \min \{ 0.4, 0.7 \}$$

$$P_3 = 0.4$$

$$\text{Strength of the path } P_4 = \min \{ \mu(x_0, x_4) ; \mu(x_4, x_3) ; \mu(x_3, x_2) \}$$

$$= \min \{ 0.4, 0.7, 0.7 \}$$

$$P_4 = 0.4$$

$$\text{Strength of the path } P_5 = \min \{ \mu(x_0, x_4) ; \mu(x_4, x_1) ; \mu(x_1, x_2) \}$$

$$= \min \{ 0.4, 0.5, 0.8 \}$$

$$P_5 = 0.5$$

$$\text{Strength of the connectedness between 'x}_0\text{' and 'x}_2\text{' = max \{ strength of the paths } P_1,$$

$$P_2, P_3, P_4 \text{ and } P_5 \}$$

$$= \max \{ 0.5, 0.4, 0.4, 0.4, 0.5 \}$$

$$= 0.5$$

$$\text{ie) } \text{CONN}_G(x_0, x_2) = 0.5$$

Strength of the connectedness between 'x₀' and 'x₃':

There are five paths between 'x₀' and 'x₃'

$$P_1 : x_0 - x_2 - x_3$$

$$P_2 : x_0 - x_1 - x_2 - x_3$$

$$P_3 : x_0 - x_4 - x_3$$

$$P_4 : x_0 - x_1 - x_2 - x_4 - x_3$$

$$P_5 : x_0 - x_1 - x_4 - x_3$$

$$\text{Strength of the path } P_1 = \min \{ \mu(x_0, x_2) ; \mu(x_2, x_3) \}$$

$$= \min \{ 0.5, 0.7 \}$$

$$P_1 = 0.5$$

$$\text{Strength of the path } P_2 = \min \{ \mu(x_0, x_1) ; \mu(x_1, x_2) ; \mu(x_2, x_3) \}$$

$$= \min \{ 0.4, 0.8, 0.7 \}$$

$$P_2 = 0.4$$

$$\text{Strength of the path } P_3 = \min \{ \mu(x_0, x_4) ; \mu(x_4, x_3) \}$$

$$= \min \{ 0.4, 0.7 \}$$

$$P_3 = 0.4$$

$$\text{Strength of the path } P_4 = \min \{ \mu(x_0, x_1) ; \mu(x_1, x_2) ; \mu(x_2, x_4) ; \mu(x_4, x_3) \}$$

$$= \min \{ 0.4, 0.8, 0.7, 0.7 \}$$

$$P_4 = 0.4$$

$$\text{Strength of the path } P_5 = \min \{ \mu(x_0, x_1) ; \mu(x_1, x_4) ; \mu(x_4, x_3) \}$$

$$= \min \{ 0.4, 0.5, 0.7 \}$$

$$P_5 = 0.4$$

$$\text{Strength of the connectedness between 'x}_0\text{' and 'x}_3\text{' = max \{ strength of the paths$$

$$P_1, P_2, P_3, P_4 \text{ and } P_5 \}$$

$$= \max \{ 0.5, 0.4, 0.4, 0.4, 0.4 \}$$

$$= 0.5$$

$$\text{ie) } \text{CONN}_G(x_0, x_3) = 0.5$$

Strength of the connectedness between 'x₀' and 'x₄':

There are five paths between 'x₀' and 'x₄'

$$P_1 : x_0 - x_4$$

$$P_2 : x_0 - x_1 - x_4$$

$$P_3 : x_0 - x_2 - x_4$$

$$P_4 : x_0 - x_1 - x_2 - x_3 - x_4$$

$$P_5 : x_0 - x_2 - x_3 - x_4$$

Strength of the path $P_1 = \min \{ \mu(x_0, x_4) \}$

$$P_1 = 0.4$$

Strength of the path $P_2 = \min \{ \mu(x_0, x_1) ; \mu(x_1, x_4) \}$

$$= \min \{ 0.4, 0.5 \}$$

$$P_2 = 0.4$$

Strength of the path $P_3 = \min \{ \mu(x_0, x_2) ; \mu(x_2, x_4) \}$

$$= \min \{ 0.5, 0.7 \}$$

$$P_3 = 0.5$$

Strength of the path $P_4 = \min \{ \mu(x_0, x_1) ; \mu(x_1, x_2) ; \mu(x_2, x_3) ; \mu(x_3, x_4) \}$

$$= \min \{ 0.4, 0.8, 0.7, 0.7 \}$$

$$P_4 = 0.4$$

Strength of the path $P_5 = \min \{ \mu(x_0, x_2) ; \mu(x_2, x_3) ; \mu(x_3, x_4) \}$

$$= \min \{ 0.5, 0.7, 0.7 \}$$

$$P_5 = 0.5$$

Strength of the connectedness between 'x₀' and 'x₄' = $\max \{ \text{strength of the paths}$

$$P_1, P_2, P_3, P_4 \text{ and } P_5 \}$$

$$= \max \{0.4, 0.4, 0.5, 0.4, 0.5\}$$

$$= 0.5$$

$$\text{ie) } \text{CONN}_G(x_0, x_4) = 0.5$$

Strength of the connectedness between ‘x₁’ and ‘x₂’:

There are five paths between ‘x₁’ and ‘x₂’

$$P_1 : x_1 - x_2$$

$$P_2 : x_1 - x_4 - x_2$$

$$P_3 : x_1 - x_0 - x_2$$

$$P_4 : x_1 - x_4 - x_3 - x_2$$

$$P_5 : x_1 - x_0 - x_4 - x_2$$

Strength of the path $P_1 = \min \{\mu(x_1, x_2)\}$

$$P_1 = 0.8$$

Strength of the path $P_2 = \min \{\mu(x_1, x_4) ; \mu(x_4, x_2)\}$

$$= \min \{0.5, 0.7\}$$

$$P_2 = 0.5$$

Strength of the path $P_3 = \min \{\mu(x_1, x_0) ; \mu(x_0, x_2)\}$

$$= \min \{0.4, 0.5\}$$

$$P_3 = 0.4$$

Strength of the path $P_4 = \min \{\mu(x_1, x_4) ; \mu(x_4, x_3) ; \mu(x_3, x_2)\}$

$$= \min \{0.5, 0.7, 0.7\}$$

$$P_4 = 0.5$$

Strength of the path $P_5 = \min \{\mu(x_1, x_0) ; \mu(x_0, x_4) ; \mu(x_4, x_2)\}$

$$= \min \{0.4, 0.4, 0.7\}$$

$$P_5 = 0.4$$

Strength of the connectedness between 'x₁' and 'x₂' = max {strength of the paths

P₁, P₂, P₃, P₄ and P₅}

= max {0.8, 0.5, 0.4, 0.5, 0.4}

= 0.8

ie) CONN_G (x₁, x₂) = 0.8

Strength of the connectedness between 'x₁' and 'x₃':

There are five paths between 'x₁' and 'x₃'

P₁ : x₁ – x₂ – x₃

P₂ : x₁ – x₄ – x₃

P₃ : x₁ – x₀ – x₂ – x₃

P₄ : x₁ – x₀ – x₄ – x₃

P₅ : x₁ – x₄ – x₂ – x₃

Strength of the path P₁ = min {μ(x₁, x₂) ; μ(x₂, x₃)}

= min {0.8, 0.7}

P₁ = 0.7

Strength of the path P₂ = min {μ(x₁, x₄) ; μ(x₄, x₃)}

= min {0.5, 0.7}

P₂ = 0.5

Strength of the path P₃ = min {μ(x₁, x₀) ; μ(x₀, x₂) ; μ(x₂, x₃)}

= min {0.4, 0.5, 0.7}

P₃ = 0.4

Strength of the path P₄ = min {μ(x₁, x₀) ; μ(x₀, x₄) ; μ(x₄, x₃)}

= min {0.4, 0.4, 0.7}

$$P_4 = 0.4$$

Strength of the path $P_5 = \min \{ \mu(x_1, x_4) ; \mu(x_4, x_2) ; \mu(x_2, x_3) \}$

$$= \min \{ 0.5, 0.7, 0.7 \}$$

$$P_5 = 0.5$$

Strength of the connectedness between 'x₁' and 'x₃' = max {strength of the paths P₁,

P₂, P₃, P₄ and P₅}

$$= \max \{ 0.7, 0.5, 0.4, 0.4, 0.5 \}$$

$$= 0.7$$

$$\text{ie) } \text{CONN}_G(x_1, x_3) = 0.7$$

Strength of the connectedness between 'x₂' and 'x₃':

There are five paths between 'x₂' and 'x₃'

$$P_1 : x_2 - x_3$$

$$P_2 : x_2 - x_4 - x_3$$

$$P_3 : x_2 - x_1 - x_4 - x_3$$

$$P_4 : x_2 - x_0 - x_4 - x_3$$

$$P_5 : x_2 - x_1 - x_0 - x_4 - x_3$$

Strength of the paths $P_1 = \min \{ \mu(x_2, x_3) \}$

$$P_1 = 0.7$$

Strength of the path $P_2 = \min \{ \mu(x_2, x_4) ; \mu(x_4, x_3) \}$

$$= \min \{ 0.7, 0.7 \}$$

$$P_2 = 0.7$$

Strength of the path $P_3 = \min \{ \mu(x_2, x_1) ; \mu(x_1, x_4) ; \mu(x_4, x_3) \}$

$$= \min \{ 0.8, 0.5, 0.7 \}$$

$$P_3 = 0.5$$

Strength of the path $P_4 = \min \{ \mu(x_2, x_0) ; \mu(x_0, x_4) ; \mu(x_4, x_3) \}$

$$= \min \{ 0.5, 0.4, 0.7 \}$$

$$= 0.4$$

Strength of the path $P_5 = \min \{ \mu(x_2, x_1) ; \mu(x_1, x_0) ; \mu(x_0, x_4) ; \mu(x_4, x_3) \}$

$$= \min \{ 0.8, 0.4, 0.4, 0.7 \}$$

$$P_5 = 0.4$$

Strength of the connectedness between 'x₂' and 'x₃' = max {strength of the paths

$$P_1, P_2, P_3, P_4 \text{ and } P_5 \}$$

$$= \max \{ 0.7, 0.7, 0.5, 0.4, 0.4 \}$$

$$= 0.7$$

$$\text{ie) } \text{CONN}_G(x_2, x_3) = 0.7$$

Strength of the connectedness between 'x₂' and 'x₄':

There are five paths between 'x₂' and 'x₄'

$$P_1 : x_2 - x_4$$

$$P_2 : x_2 - x_1 - x_4$$

$$P_3 : x_2 - x_3 - x_4$$

$$P_4 : x_2 - x_0 - x_4$$

$$P_5 : x_2 - x_0 - x_1 - x_4$$

Strength of the path $P_1 = \min \{ \mu(x_2, x_4) \}$

$$P_1 = 0.7$$

Strength of the path $P_2 = \min \{ \mu(x_2, x_1) ; \mu(x_1, x_4) \}$

$$= \min \{ 0.8, 0.5 \}$$

$$P_2 = 0.5$$

Strength of the path $P_3 = \min \{ \mu(x_2, x_3) ; \mu(x_3, x_4) \}$

$$= \min \{ 0.7, 0.7 \}$$

$$P_3 = 0.7$$

Strength of the path $P_4 = \min \{ \mu(x_2, x_0) ; \mu(x_0, x_4) \}$

$$= \min \{ 0.5, 0.4 \}$$

$$P_4 = 0.4$$

Strength of the path $P_5 = \min \{ \mu(x_2, x_0) ; \mu(x_0, x_1) ; \mu(x_1, x_4) \}$

$$= \min \{ 0.5, 0.4, 0.5 \}$$

$$P_5 = 0.4$$

Strength of the connectedness between 'x₂' and 'x₄' = max {strength of the paths

$$P_1, P_2, P_3, P_4 \text{ and } P_5 \}$$

$$= \max \{ 0.7, 0.5, 0.7, 0.4, 0.4 \}$$

$$= 0.7$$

$$\text{ie) } \text{CONN}_G(x_2, x_4) = 0.7$$

Strength of the connectedness between 'x₃' and 'x₄':

There are five paths between 'x₃' and 'x₄'

$$P_1 : x_3 - x_4$$

$$P_2 : x_3 - x_2 - x_4$$

$$P_3 : x_3 - x_2 - x_1 - x_4$$

$$P_4 : x_3 - x_2 - x_0 - x_4$$

$$P_5 : x_3 - x_2 - x_0 - x_1 - x_4$$

Strength of the path $P_1 = \min \{ \mu(x_3, x_4) \}$

$$P_1 = 0.7$$

Strength of the path $P_2 = \min \{ \mu(x_3, x_2) ; \mu(x_2, x_4) \}$

$$= \min \{ 0.7, 0.7 \}$$

$$P_2 = 0.7$$

Strength of the path $P_3 = \min \{ \mu(x_3, x_2) ; \mu(x_2, x_1) ; \mu(x_1, x_4) \}$

$$= \min \{ 0.7, 0.8, 0.5 \}$$

$$P_3 = 0.5$$

Strength of the path $P_4 = \min \{ \mu(x_3, x_2) ; \mu(x_2, x_0) ; \mu(x_0, x_4) \}$

$$= \min \{ 0.7, 0.5, 0.4 \}$$

$$P_4 = 0.4$$

Strength of the path $P_5 = \min \{ \mu(x_3, x_2) ; \mu(x_2, x_0) ; \mu(x_0, x_1) ; \mu(x_1, x_4) \}$

$$= \min \{ 0.7, 0.5, 0.4, 0.5 \}$$

$$P_5 = 0.4$$

Strength of the connectedness between 'x₃' and 'x₄' = max {strength of the paths

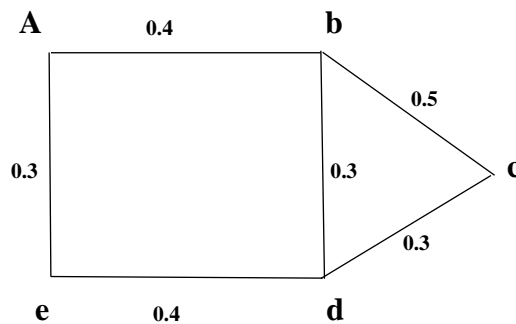
$$P_1, P_2, P_3, P_4 \text{ and } P_5 \}$$

$$= \max \{ 0.7, 0.7, 0.5, 0.4, 0.4 \}$$

$$= 0.7$$

$$\text{ie) } \text{CONN}_G(x_3, x_4) = 0.7$$

Example: 2.22



Path in the given graph is **a – b – c – d – e**

$$\begin{aligned}
\text{Strength of the path} &= \min \{ \mu(a,b) ; \mu(b,c) ; \mu(c,d) ; \mu(d,e) \} \\
&= \min \{ 0.4, 0.5, 0.3, 0.4 \} \\
&= 0.3
\end{aligned}$$

$$\text{Strength of the path} = 0.3$$

Strength of the connectedness between ‘a’ and ‘b’:

There are three paths between ‘a’ and ‘b’

$$P_1 : a - b$$

$$P_2 : a - e - d - b$$

$$P_3 : a - e - d - c - b$$

$$\text{Strength of the path } P_1 = \min \{ \mu(a,b) \}$$

$$P_1 = 0.4$$

$$\text{Strength of the path } P_2 = \min \{ \mu(a,e) ; \mu(e,d) ; \mu(d,b) \}$$

$$= \min \{ 0.3, 0.4, 0.3 \}$$

$$P_2 = 0.3$$

$$\text{Strength of the path } P_3 = \min \{ \mu(a,e) ; \mu(e,d) ; \mu(d,c) ; \mu(c,b) \}$$

$$= \min \{ 0.3, 0.4, 0.3, 0.5 \}$$

$$P_3 = 0.3$$

$$\text{Strength of the connectedness between ‘a’ and ‘b’} = \max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$$

$$= \max \{ 0.4, 0.3, 0.3 \}$$

$$= 0.4$$

$$\text{ie) } \text{CONN}_G(a,b) = 0.4$$

Strength of the connectedness between ‘a’ and ‘c’:

There are three paths between ‘a’ and ‘c’

$$P_1 : a - b - c$$

$$P_2 : a - e - d - c$$

$$P_3 : a - e - d - b - c$$

$$\text{Strength of the path } P_1 = \min \{ \mu(a,b) ; \mu(b,c) \}$$

$$= \min \{ 0.4, 0.5 \}$$

$$P_1 = 0.4$$

$$\text{Strength of the path } P_2 = \min \{ \mu(a,e) ; \mu(e,d) ; \mu(d,c) \}$$

$$= \min \{ 0.3, 0.4, 0.3 \}$$

$$P_2 = 0.3$$

$$\text{Strength of the path } P_3 = \min \{ \mu(a,e) ; \mu(e,d) ; \mu(d,b) ; \mu(b,c) \}$$

$$= \min \{ 0.3, 0.4, 0.3, 0.5 \}$$

$$P_3 = 0.3$$

$$\text{Strength of the connectedness between 'a' and 'c'} = \max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$$

$$= \max \{ 0.4, 0.3, 0.3 \}$$

$$= 0.4$$

$$\text{ie) } \text{CONN}_G(a,c) = 0.4$$

Strength of the connectedness between 'a' and 'd':

There are three paths between 'a' and 'd'

$$P_1 : a - b - d$$

$$P_2 : a - e - d$$

$$P_3 : a - b - c - d$$

$$\text{Strength of the path } P_1 = \min \{ \mu(a,b) ; \mu(b,d) \}$$

$$= \min \{ 0.4, 0.3 \}$$

$$P_1 = 0.3$$

$$\text{Strength of the path } P_2 = \min \{ \mu(a,e) ; \mu(e,d) \}$$

$$= \min \{0.3, 0.4\}$$

$$P_2 = 0.3$$

$$\text{Strength of the path } P_3 = \min \{ \mu(a,b) ; \mu(b,c) ; \mu(c,d) \}$$

$$= \min \{0.4, 0.5, 0.3\}$$

$$P_3 = 0.3$$

$$\text{Strength of the connectedness between 'a' and 'd'} = \max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$$

$$= \max \{0.3, 0.3, 0.3\}$$

$$= 0.3$$

$$\text{ie) } \text{CONN}_G(a,d) = 0.3$$

Strength of the connectedness between 'a' and 'e':

There are three paths between 'a' and 'e'

$$P_1 : a - e$$

$$P_2 : a - b - d - e$$

$$P_3 : a - b - c - d - e$$

$$\text{Strength of the path } P_1 = \min \{ \mu(a,e) \}$$

$$P_1 = 0.3$$

$$\text{Strength of the path } P_2 = \min \{ \mu(a,b) ; \mu(b,d) ; \mu(d,e) \}$$

$$= \min \{0.4, 0.3, 0.4\}$$

$$P_2 = 0.3$$

$$\text{Strength of the path } P_3 = \min \{ \mu(a,b) ; \mu(b,c) ; \mu(c,d) ; \mu(d,e) \}$$

$$= \min \{0.4, 0.5, 0.3, 0.4\}$$

$$P_3 = 0.3$$

$$\text{Strength of the connectedness between 'a' and 'e'} = \max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$$

$$= \max \{0.3, 0.3, 0.3\}$$

$$= 0.3$$

$$\text{ie) } \text{CONN}_G(a,e) = 0.3$$

Strength of the connectedness between ‘b’ and ‘c’:

There are three paths between ‘b’ and ‘c’

$$P_1 : b - c$$

$$P_2 : b - d - c$$

$$P_3 : b - a - e - d - c$$

Strength of the path $P_1 = \min \{ \mu(b,c) \}$

$$P_1 = 0.5$$

Strength of the path $P_2 = \min \{ \mu(b,d) ; \mu(d,c) \}$

$$= \min \{ 0.3, 0.3 \}$$

$$P_2 = 0.3$$

Strength of the path $P_3 = \min \{ \mu(b,a) ; \mu(a,e) ; \mu(e,d) ; \mu(d,c) \}$

$$= \min \{ 0.4, 0.3, 0.4, 0.3 \}$$

$$P_3 = 0.3$$

Strength of the connectedness between ‘b’ and ‘c’ = $\max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$

$$= \max \{ 0.5, 0.3, 0.3 \}$$

$$= 0.5$$

$$\text{ie) } \text{CONN}_G(b,c) = 0.5$$

Strength of the connectedness between ‘b’ and ‘d’:

There are three paths between ‘b’ and ‘d’

$$P_1 : b - d$$

$$P_2 : b - c - d$$

$$P_3 : b - a - e - d$$

Strength of the path $P_1 = \min \{ \mu(b,d) \}$

$$P_1 = 0.3$$

Strength of the path $P_2 = \min \{ \mu(b,c) ; \mu(c,d) \}$

$$= \min \{ 0.5, 0.3 \}$$

$$P_2 = 0.3$$

Strength of the path $P_3 = \min \{ \mu(b,a) ; \mu(a,e) ; \mu(e,d) \}$

$$= \min \{ 0.4, 0.3, 0.4 \}$$

$$P_3 = 0.3$$

Strength of the connectedness between 'b' and 'd' = $\max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$

$$= \max \{ 0.3, 0.3, 0.3 \}$$

$$= 0.3$$

$$\text{ie) } \text{CONN}_G(b,d) = 0.3$$

Strength of the connectedness between 'b' and 'e':

There are three paths between 'b' and 'e'

$$P_1 : b - a - e$$

$$P_2 : b - d - e$$

$$P_3 : b - c - d - e$$

Strength of the path $P_1 = \min \{ \mu(b,a) ; \mu(a,e) \}$

$$= \min \{ 0.4, 0.3 \}$$

$$P_1 = 0.3$$

Strength of the path $P_2 = \min \{ \mu(b,d) ; \mu(d,e) \}$

$$= \min \{ 0.3, 0.4 \}$$

$$P_2 = 0.3$$

Strength of the path $P_3 = \min \{ \mu(b,c) ; \mu(c,d) ; \mu(d,e) \}$

$$= \min \{0.5, 0.3, 0.4\}$$

$$P_3 = 0.3$$

Strength of the connectedness between 'b' and 'e' = $\max \{\text{strength of the paths } P_1, P_2 \text{ and } P_3\}$

$$= \max \{0.3, 0.3, 0.3\}$$

$$= 0.3$$

$$\text{ie) } \text{CONN}_G(b,e) = 0.3$$

Strength of the connectedness between 'c' and 'd':

There are three paths between 'c' and 'd'

$$P_1 : c - d$$

$$P_2 : c - b - d$$

$$P_3 : c - b - a - e - d$$

Strength of the path $P_1 = \min \{\mu(c,d)\}$

$$P_1 = 0.3$$

Strength of the path $P_2 = \min \{\mu(c,b) ; \mu(b,d)\}$

$$= \min \{0.5, 0.3\}$$

$$P_2 = 0.3$$

Strength of the path $P_3 = \min \{\mu(c,b) ; \mu(b,a) ; \mu(a,e) ; \mu(e,d)\}$

$$= \min \{0.5, 0.4, 0.3, 0.4\}$$

$$P_3 = 0.3$$

Strength of the connectedness between 'c' and 'd' = $\max \{\text{strength of the paths } P_1, P_2 \text{ and } P_3\}$

$$= \max \{0.3, 0.3, 0.3\}$$

$$= 0.3$$

$$\text{ie) } \text{CONN}_G(c,d) = 0.3$$

Strength of the connectedness between ‘c’ and ‘e’:

There are three paths between ‘c’ and ‘e’

$$P_1 : c - d - e$$

$$P_2 : c - b - a - e$$

$$P_3 : c - b - d - e$$

Strength of the path $P_1 = \min \{ \mu(c,d) ; \mu(d,e) \}$

$$= \min \{ 0.3, 0.4 \}$$

$$P_1 = 0.3$$

Strength of the path $P_2 = \min \{ \mu(c,b) ; \mu(b,a) ; \mu(a,e) \}$

$$= \min \{ 0.5, 0.4, 0.3 \}$$

$$P_2 = 0.3$$

Strength of the path $P_3 = \min \{ \mu(c,b) ; \mu(b,d) ; \mu(d,e) \}$

$$= \min \{ 0.5, 0.3, 0.4 \}$$

$$P_3 = 0.3$$

Strength of the connectedness between ‘c’ and ‘e’ = $\max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$

$$= \max \{ 0.3, 0.3, 0.3 \}$$

$$= 0.3$$

$$\text{ie) } \text{CONN}_G(c,e) = 0.3$$

Strength of the connectedness between ‘d’ and ‘e’:

There are three paths between ‘d’ and ‘e’

$$P_1 : d - e$$

$$P_2 : d - b - a - e$$

$$P_3 : d - c - b - a - e$$

Strength of the path $P_1 = \min \{ \mu(d,e) \}$

$$P_1 = 0.4$$

Strength of the path $P_2 = \min \{ \mu(d,b) ; \mu(b,a) ; \mu(a,e) \}$

$$= \min \{ 0.3, 0.4, 0.3 \}$$

Strength of the path $P_3 = \min \{ \mu(d,c) ; \mu(c,b) ; \mu(b,a) ; \mu(a,e) \}$

$$= \min \{ 0.3, 0.5, 0.4, 0.3 \}$$

Strength of the connectedness between 'd' and 'e' = $\max \{ \text{strength of the paths } P_1, P_2 \text{ and } P_3 \}$

$$= \max \{ 0.4, 0.3, 0.3 \}$$

$$= 0.4$$

$$\text{ie) } \text{CONN}_G(d,e) = 0.4$$

2.2 METHODS

Symptoms for diabetic general (initial stage):

Fatigue, Weight loss, Polydipsia, Polyuria, Altered mental status, Polyphagia.

Symptoms of diabetic Ketoacidosis:

Nausea and Vomiting, Dehydration, abdominal pain, Low blood pressure, Polyuria, Thirsty, Loss of appetite, Dry skin, Dry mouth.

Symptoms of Diabetic Nephropathy:

Loss of appetite, Nausea and Vomiting, Polyuria, Swelling of legs and Puffiness around the eyes, Itching, Easy bruising, Pale skin, Head aches, Numbness in the feet or hands, Disturbed sleep, Bleeding, High blood pressure, Bone pain, Decreased sexual interest and Erectile dysfunction.

Symptoms of Diabetic Retinopathy:

All the symptoms similar to starting stage of diabetic, Mild to severe blurring or vision loss, Cataract, Glaucoma.

Let S denote the crisp universal set of all symptoms $S = \{s_1, s_2, s_3, s_4\}$

Let D denote the crisp universal set of all diseases $D = \{d_1, d_2, d_3, d_4\}$

Let P denote the crisp universal set of all patients $P = \{p_1, p_2, p_3, p_4\}$

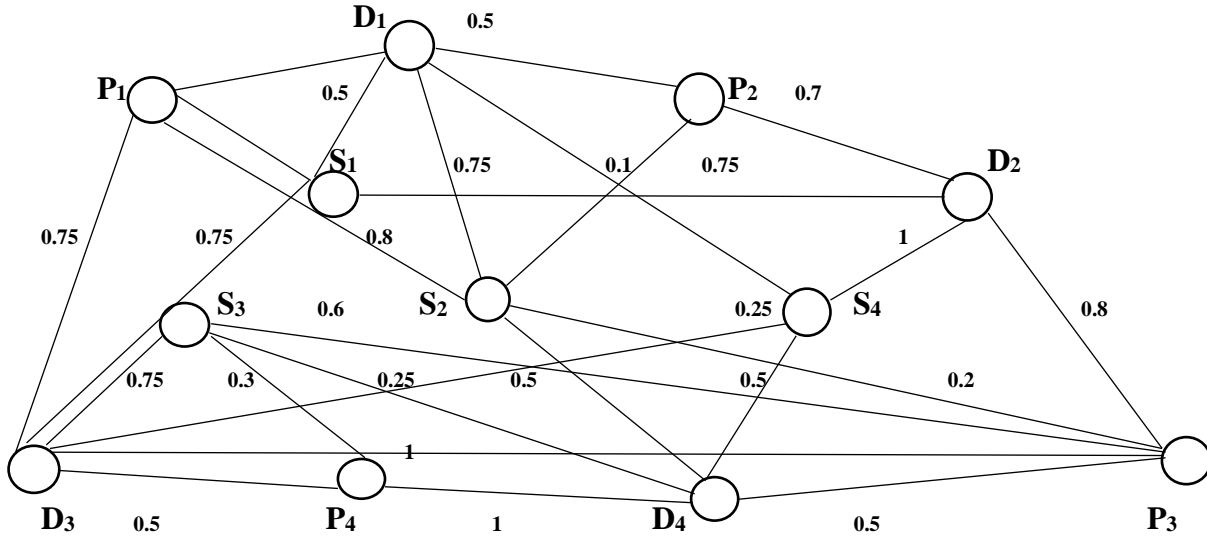
☼ D_1 = DIABETIC GENERAL

☼ D_2 = DIABETIC NEPHROPATHY

☼ D_3 = DIABETIC KETOACIDOSIS

☼ D_4 = DIABETIC RETINOPATHY

Fuzzy network graph between Patients, Symptoms and Diseases:



From the figure, we are going to find the strength of the following paths.

* Consider $S_1 - D_1 - P_1$

$$\text{Strength of the path} = \min \{ \mu(S_1, D_1) ; \mu(D_1, P_1) \}$$

$$= \min \{ 0.5, 0.8 \}$$

$$= 0.5$$

Strength of the path is 0.5

* Consider $S_2 - D_2 - P_1$

This is equal to $S_2 - P_2 - D_2 - S_1 - P_1$

$$\text{Strength of the path} = \min \{ \mu(S_2, P_2) ; \mu(P_2, D_2) ; \mu(D_2, S_1) ; \mu(S_1, P_1) \}$$

$$= \min \{ 0.1, 0.7, 0.75, 0.8 \}$$

$$= 0.1$$

Strength of the path is 0.1

* Consider $S_3 - D_3 - P_1$

$$\text{Strength of the path} = \min \{ \mu(S_3, D_3) ; \mu(D_3, P_1) \}$$

$$= \min \{ 0.75, 0.75 \}$$

$$= 0.75$$

Strength of the path is 0.75

* Consider $S_4 - D_4 - P_1$

This is equal to $S_4 - D_4 - S_2 - P_1$

$$\text{Strength of the path} = \min \{ \mu(S_4, D_4) ; \mu(D_4, S_2) ; \mu(S_2, P_1) \}$$

$$= \min \{ 0.5, 0.5, 0.8 \}$$

$$= 0.5$$

Strength of the path is 0.5

* Consider $S_1 - D_1 - P_2$

$$\text{Strength of the path} = \min \{ \mu(S_1, D_1) ; \mu(D_1, P_2) \}$$

$$= \min \{ 0.5, 0.5 \}$$

$$= 0.5$$

Strength of the path is 0.5

* Consider $S_2 - D_2 - P_2$

This is equal to $S_2 - P_3 - D_2 - P_2$

$$\text{Strength of the path} = \min \{ \mu(S_2, P_3) ; \mu(P_3, D_2) ; \mu(D_2, P_2) \}$$

$$= \min \{ 0.2, 0.8, 0.7 \}$$

$$= 0.2$$

Strength of the path is 0.2

* Consider **S₃ – D₃ – P₂**

This is equal to S₃ – D₃ – P₁ – D₁ – P₂

$$\begin{aligned}\text{Strength of the path} &= \min \{ \mu(S_3, D_3) ; \mu(D_3, P_1) ; \mu(P_1, D_1) ; \mu(D_1, P_2) \} \\ &= \min \{ 0.75, 0.75, 0.8, 0.5 \} \\ &= 0.5\end{aligned}$$

Strength of the path is 0.5

* Consider **S₄ – D₄ – P₂**

This is equal to S₄ – D₄ – S₂ – P₂

$$\begin{aligned}\text{Strength of the path} &= \min \{ \mu(S_4, D_4) ; \mu(D_4, S_2) ; \mu(S_2, P_2) \} \\ &= \min \{ 0.5, 0.5, 0.1 \} \\ &= 0.1\end{aligned}$$

Strength of the path is 0.1

* Consider **S₁ – D₁ – P₃**

This is equal to S₁ – D₁ – S₂ – P₃

$$\begin{aligned}\text{Strength of the path} &= \min \{ \mu(S_1, D_1) ; \mu(D_1, S_2) ; \mu(S_2, P_3) \} \\ &= \min \{ 0.5, 0.75, 0.2 \} \\ &= 0.2\end{aligned}$$

Strength of the path is 0.2

* Consider **S₂ – D₂ – P₃**

This is equal to S₂ – P₂ – D₂ – P₃

$$\begin{aligned}\text{Strength of the path} &= \min \{ \mu(S_2, P_2) ; \mu(P_2, D_2) ; \mu(D_2, P_3) \} \\ &= \min \{ 0.1, 0.7, 0.8 \}\end{aligned}$$

$$= 0.1$$

Strength of the path is 0.1

* Consider **S₃ – D₃ – P₃**

$$\text{Strength of the path} = \min \{ \mu(S_3, D_3) ; \mu(D_3, P_3) \}$$

$$= \min \{ 0.75, 1 \}$$

$$= 0.75$$

Strength of the path is 0.75

* Consider **S₄ – D₄ – P₃**

$$\text{Strength of the path} = \min \{ \mu(S_4, D_4) ; \mu(D_4, P_3) \}$$

$$= \min \{ 0.5, 0.5 \}$$

$$= 0.5$$

Strength of the path is 0.5

* Consider **S₁ – D₁ – P₄**

This is equal to S₁ – D₁ – P₁ – D₃ – P₄

$$\text{Strength of the path} = \min \{ \mu(S_1, D_1) ; \mu(D_1, P_1) ; \mu(P_1, D_3) ; \mu(D_3, P_4) \}$$

$$= \min \{ 0.5, 0.8, 0.75, 0.5 \}$$

$$= 0.5$$

Strength of the path is 0.5

* Consider **S₂ – D₂ – P₄**

This is equal to S₂ – P₂ – D₂ – P₃ – D₁ – P₄

$$\text{Strength of the path} = \min \{ \mu(S_2, P_2) ; \mu(P_2, D_2) ; \mu(D_2, P_3) ; \mu(P_3, D_1) ; \mu(D_1, P_4) \}$$

$$= \min \{ 0.1, 0.7, 0.8, 0.5, 1 \}$$

$$= 0.1$$

Strength of the path is 0.1

* Consider $S_3 - D_3 - P_4$

$$\begin{aligned}\text{Strength of the path} &= \min \{ \mu(S_3, D_3) ; \mu(D_3, P_4) \} \\ &= \min \{ 0.75, 0.5 \} \\ &= 0.5\end{aligned}$$

Strength of the path is 0.5

* Consider $S_4 - D_4 - P_4$

$$\begin{aligned}\text{Strength of the path} &= \min \{ \mu(S_4, D_4) ; \mu(D_4, P_4) \} \\ &= \min \{ 0.5, 1 \} \\ &= 0.5\end{aligned}$$

Strength of the path is 0.5

The maximum strength of all the points obtained above

$$\begin{aligned}&= \max \{ 0.1, 0.2, 0.5, 0.75 \} \\ &= 0.75 = \text{CONN}_G\end{aligned}$$

So we can conclude that, the strongest paths are;

- $S_3 - D_3 - P_1$
- $S_3 - D_3 - P_3$

CHAPTER – 3

Conclusion

From the figure, it clearly shows that **Symptoms 3** for **Patient 1** confirmed **Diseases D₃** and **Symptoms 3** and **Diseases 3** is confirmed for **Patient 4**. Each network is different and has irregularity between them, it shows that it has Fractal.

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