Homework 6

12.1 #6

a) df for All Groups

$n_T = 3 \times 12 = 36$

$df_{ss} = 36 - 1 = 35$

Sales by Strategy (Millions of Dollars)		
Strategy 1	Strategy 2	Strategy 3
3	2	4
6	5	2
7	5	5
4	3	6
6	7	6
7	8	7
10	6	9
6	4	8
15	10	14
8	6	8
9	9	7
16	12	16

b) df with sum of squares

$k = 3$

$df_{SST} = 2$

c) Mean square of treatments

$MST = \frac{SST}{k-1}$

$\bar{\bar{x}} = 7.39$ $\bar{x}_1 = 8.08$

$MST = \frac{\sum n_g (\bar{x}_g - \bar{\bar{x}})^2}{k-1}$

$\bar{x}_2 = 6.42$

$\bar{x}_3 = 7.67$

$\frac{18.06}{2} = 9.03 = MST$

d) Find MSE

$\frac{SSE}{N_T - k} = \frac{\sum (x_g - \bar{x}_g)^2}{N_T - k}$

d) Find MSE

$$\frac{SSE}{N_T - k} = \frac{\Sigma(X_g - \bar{X}_g)^2}{N_T - k}$$

findMSE definition

```
def findMSE (df: pd.DataFrame):
    numeratorSum = 0
    denominator = (df.size - df.shape[1])
    for columnName in df.columns:
        columnMean = df[columnName].mean()
        for element in df[columnName]:
            numeratorSum = ((element - columnMean) ** 2) + numeratorSum
    return numeratorSum / denominator
```

[ ] Python

```
df1216 = pd.DataFrame({
    "strategy1" : [3,6,7,4,6,7,10,6,15,8,9,16],
    "strategy2" : [2,5,5,3,7,8,6,4,10,6,9,12],
    "strategy3" : [4,2,5,6,6,7,9,8,14,8,7,16]
})
print(findMSE(df1216))
```

[6] ✓ 0.0s Python

```
13.287878787878784
```

MSE = 13.29

12.1 # 20↓

c) grand mean table →

```
df = pd.DataFrame({
    'day' : ['Monday','Tuesday','Wednesday','Thursday','Friday','Saturday','Sunday'],
    'firstShift' : [140,181,127,172,161,152,173],
    'secondShift' : [168,224,162,182,219,171,217],
    'thirdShift' : [77,123,77,101,147,145,111]
}).set_index('day')
```

[11] ✓ 0.0s

$\bar{\bar{X}} =$

```
sum = 0
for columnName in df.columns:
    sum = df[columnName].mean() + sum
print(str(sum/df.shape[1]))
```

[12] ✓ 0.0s

```
153.80952380952382
```

b) Value of n_j

```
print(df.shape[0])
```

✓ 0.0s

$n_j = 7$

C) Value of k

```
print(df.shape[1])
✓ 0.0s
```

$k = 3$

d) Value of n_T

```
df.size
✓ 0.0s
```

$n_T = 21$

e) Find df_T, df_{SST}, df_{SSE}

$df_{T_{SS}} = 21-1 = 20$ $df_{SST} = 3-1 = 2$ $df_{SSE} = 21-3 = 18$

$18+2 = 20 = df_{T_{SS}}$

12.3 #10 Table →

```
df = pd.DataFrame({
    'minivanA': [150,152,151,149,153],
    'minivanB': [153,150,156,151,155],
    'minivanC' : [155,150,157,158,155],
    'minivanD': [167,164,169,162,173]
})
✓ 0.0s
```

A) $\alpha = 0.10$

12.3 # 10 cont.

12.3 Problem 10

Generate | + Code | + Markdown

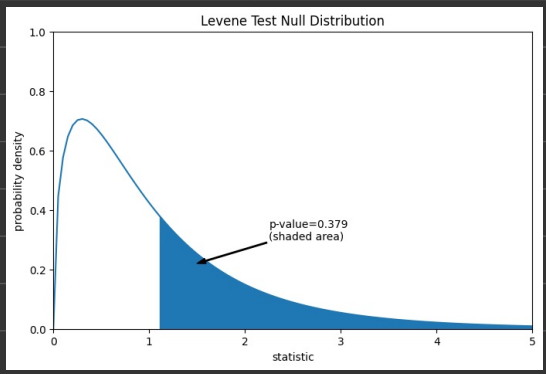
```
df = pd.DataFrame({
    'minivanA': [150,152,151,149,153],
    'minivanB': [153,150,156,151,155],
    'minivanC' : [155,150,157,158,155],
    'minivanD': [167,164,169,162,173]
})

print(generateANOVAFrame(df))
print(makeDetermination(df,0.10))
```

[425] ✓ 0.0s

```
            df  Sum of Squares  Mean Squares       F  P-value
source
Treatments   3           775.0       258.333  27.928      0.0
Error       16           148.0         9.250     NaN      NaN
Total       19           923.0           NaN     NaN      NaN
0.1 > 0.0: Reject Null Hypothesis at α=0.1
```

ANOVA Results

Levene Test →



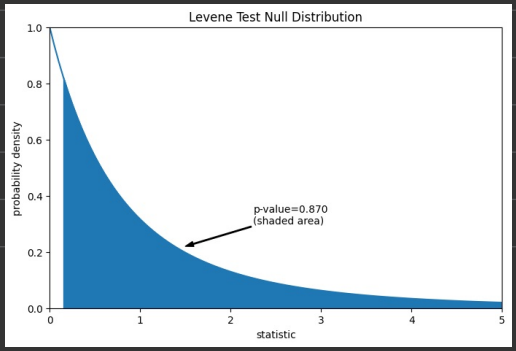
12.3 # 13

ANOVA Frame →

```
df1213 = pd.DataFrame({
    'Workout#1' : [180,185,170],
    'Workout#2': [160,170,175],
    'Workout#3': [185,190,180]
})
print(generateANOVAFrame(df1213))
plotLevene(df1213)
```

[495] ✓ 0.1s

```
            df  Sum of Squares  Mean Squares      F  P-value
source
Treatments   2         422.222       211.111  4.471  0.06476
Error        6         283.333        47.222    NaN      NaN
Total        8         705.556           NaN    NaN      NaN
```



Levene Test

12.3 # 15

$\alpha = 0.10$

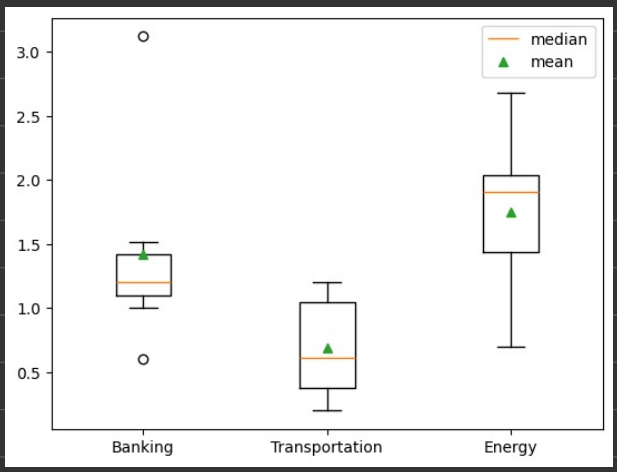
ANOVA →

```
df12315 = pd.DataFrame({
    'Banking' : [1.52, 3.12,1.32,0.6,1.2,1,1.19],
    'Transportation' : [1,1.2,0.2,0.4,1.09,0.61,0.35],
    'Energy' : [2.08,2.68,0.7,2,1.91,1.6,1.28]
})

print(generateANOVAFrame(df12315))
print(makeDetermination(df12315,alpha=0.10))
bp = plt.boxplot(df12315, showmeans=True, labels=df12315.columns)
plt.legend([bp['medians'][0], bp['means'][0]], ['median', 'mean'])
plotLevene(df12315, alpha=0.10)
```

[27] ✓ 0.2s

```
            df  Sum of Squares  Mean Squares      F  P-value
source
Treatments   2           4.098         2.049  5.101  0.01757
Error       18           7.230         0.402    NaN      NaN
Total       20          11.328           NaN    NaN      NaN
0.1 > 0.01757: Reject Null Hypothesis at α=0.1
```

Determination ↑



12.1 # 6

In [7]:
```
import pandas as pd
```

In [ ]:
```
from scipy.stats import f_oneway
import scipy.stats as stats
import matplotlib.pyplot as plt
import numpy as np
# -- origin calculation
def findSST(df: pd.DataFrame):
    sum = 0
    grandMean = findGrandMean(df)
    for columnName in df.columns:
        column = df[columnName]
        sum = column.size*(column.mean() - grandMean)**2 + sum
    return sum

# -- origin calculation
def findTSS (df: pd.DataFrame):
    sum = 0
    grandMean = findGrandMean(df)
    for columnName in df.columns:
        for element in df[columnName]:
            sum = (element - grandMean)**2 + sum
    return sum

# -- origin calculation
def findMSE (df: pd.DataFrame):
    numeratorSum = 0
    denominator = (df.size - df.shape[1])
    for columnName in df.columns:
        columnMean = df[columnName].mean()
        for element in df[columnName]:
            numeratorSum = ((element - columnMean) ** 2) + numeratorSum
    return numeratorSum / denominator

def findSSE( df: pd.DataFrame):
    mse=findMSE(df)
    return mse*(df.size - df.shape[1])

def findMST (df: pd.DataFrame):
    return findSST(df) / (df.shape[1] - 1)

def findF (df: pd.DataFrame):
    return findMST(df) / findMSE(df)

def findGrandMean(df: pd.DataFrame):
    sum = 0
    for columnName in df.columns:
        for element in df[columnName]:
            sum = element + sum
    return sum / df.size

def generateANOVAFrame (df: pd.DataFrame):
```

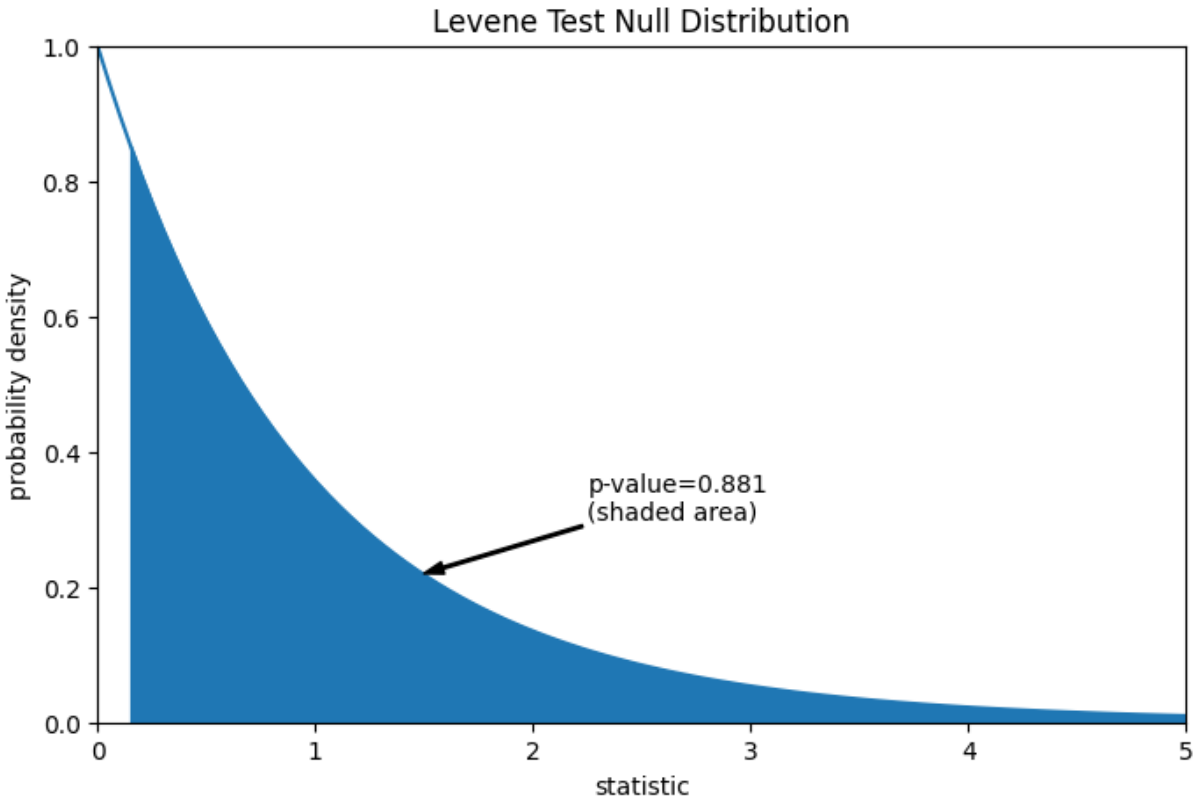
```
    treaments = df.columns.size - 1
    total = df.size - 1
    error = total - treaments
    sst = round(findSST(df),3)
    sse = round(findSSE(df),3)
    tss = round(findTSS(df),3)
    mst = round(findMST(df),3)
    mse = round(findMSE(df),3)
    f_value = round(findF(df),3)
    p_value = round(f_oneway(*[df[column] for column in df.columns]).pvalue,

    return pd.DataFrame({
        'source' : ['Treatments' , 'Error' , 'Total'],
        'df' : [treaments, error, total],
        'Sum of Squares' : [sst, sse, tss ],
        'Mean Squares' : [mst, mse , None],
        'F' : [f_value, None, None],
        'P-value' : [p_value,None, None]
    }).set_index('source')

def makeDetermination(df: pd.DataFrame, alpha = 0.05 ):
    p_value = round(f_oneway(*[df[column] for column in df.columns]).pvalue,
    if p_value < alpha:
        return (str(alpha) + " > " + str(p_value) +  ": Reject Null Hypothes
    return (str(alpha) + " < " + str(p_value) +  ": Fail to Reject Null Hypo

def plotLevene (df: pd.DataFrame , alpha = 0.05):
    # Calculate levene test
    res = stats.levene(*[df[columnName] for columnName in df.columns], propo

    def plot(ax):  # we'll reuse this
        ax.plot(val, pdf, color='C0')
        ax.set_title("Levene Test Null Distribution")
        ax.set_xlabel("statistic")
        ax.set_ylabel("probability density")
        ax.set_xlim(0, 5)
        ax.set_ylim(0, 1)

    k, n = df.shape[1], df.size   # number of samples, total number of obser
    dist = stats.f(dfn=k-1, dfd=n-k)
    val = np.linspace(0, 5, 100)
    pdf = dist.pdf(val)
    fig, ax = plt.subplots(figsize=(8, 5))

    plot(ax)
    pvalue = dist.sf(res.statistic)
    annotation = (f'p-value={pvalue:.3f}\n(shaded area)')
    props = dict(facecolor='black', width=1, headwidth=5, headlength=8)
    _ = ax.annotate(annotation, (1.5, 0.22), (2.25, 0.3), arrowprops=props)
    i = val >= res.statistic
    ax.fill_between(val[i], y1=0, y2=pdf[i], color='C0')
    plt.show()
```

In [ ]:

```
df1216 = pd.DataFrame({
    "strategy1" : [3,6,7,4,6,7,10,6,15,8,9,16],
    "strategy2" : [2,5,5,3,7,8,6,4,10,6,9,12],
```

```
    "strategy3" : [4,2,5,6,6,7,9,8,14,8,7,16]
})
print(findMSE(df1216))
plotLevene(df1216)
```

```
13.287878787878784
```



12.1 Problem 20

In [10]:
```
df12120 = pd.DataFrame({
    'day' : ['Monday','Tuesday','Wednesday','Thursday','Friday','Saturday','
    'firstShift' : [140,181,127,172,161,152,173],
    'secondShift' : [168,224,162,182,219,171,217],
    'thirdShift' : [77,123,77,101,147,145,111]
}).set_index('day')
```

In [11]:
```
sum = 0
for columnName in df12120.columns:
    sum = df12120[columnName].mean() + sum
print(str(sum/df12120.shape[1]))
```

```
153.80952380952382
```

In [12]:
```
print(df12120.shape[0])
```

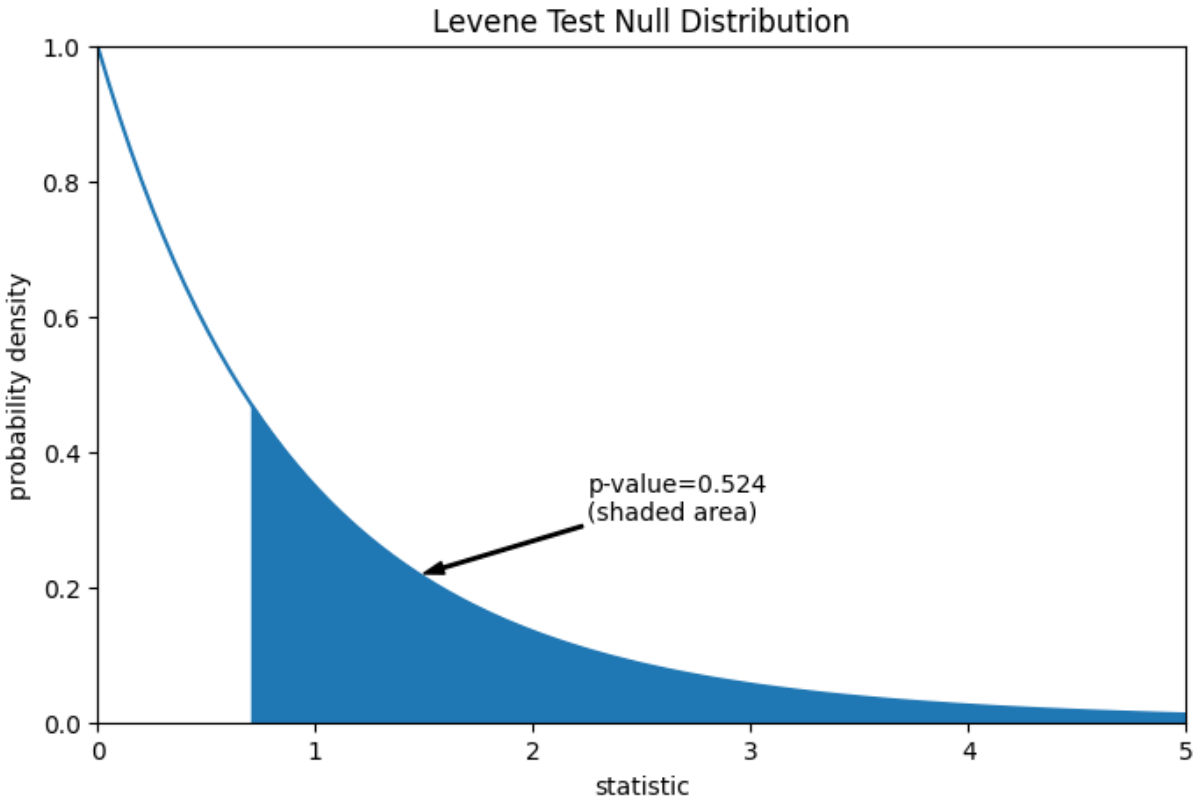
```
7
```

In [13]:
```
print(df12120.shape[1])
```

```
3
```

In [14]:
```
print(df12120.size)
plotLevene(df12120)
```

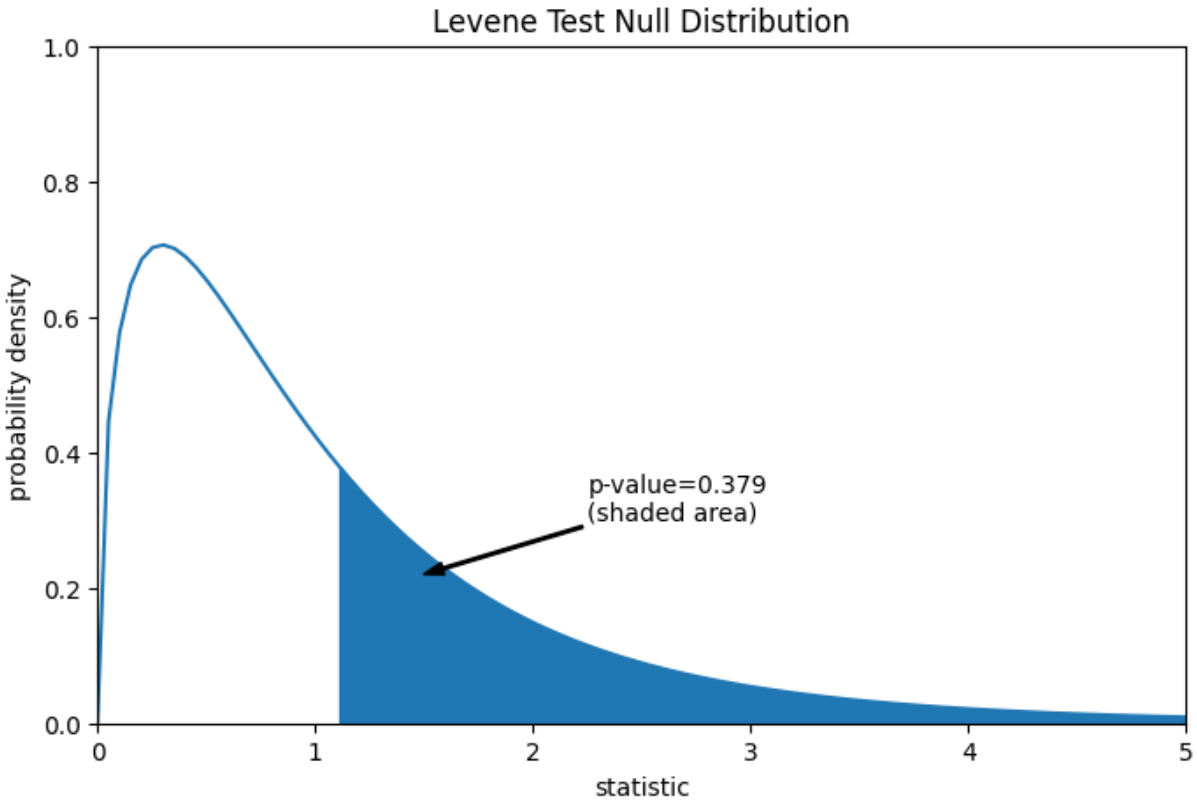
```
21
```



12.3 Problem 10

In [15]:
```
df12310 = pd.DataFrame({
    'minivanA': [150,152,151,149,153],
    'minivanB': [153,150,156,151,155],
    'minivanC' : [155,150,157,158,155],
    'minivanD': [167,164,169,162,173]
})

print(generateANOVAFrame(df12310))
print(makeDetermination(df12310,0.10))
plotLevene(df12310)
```

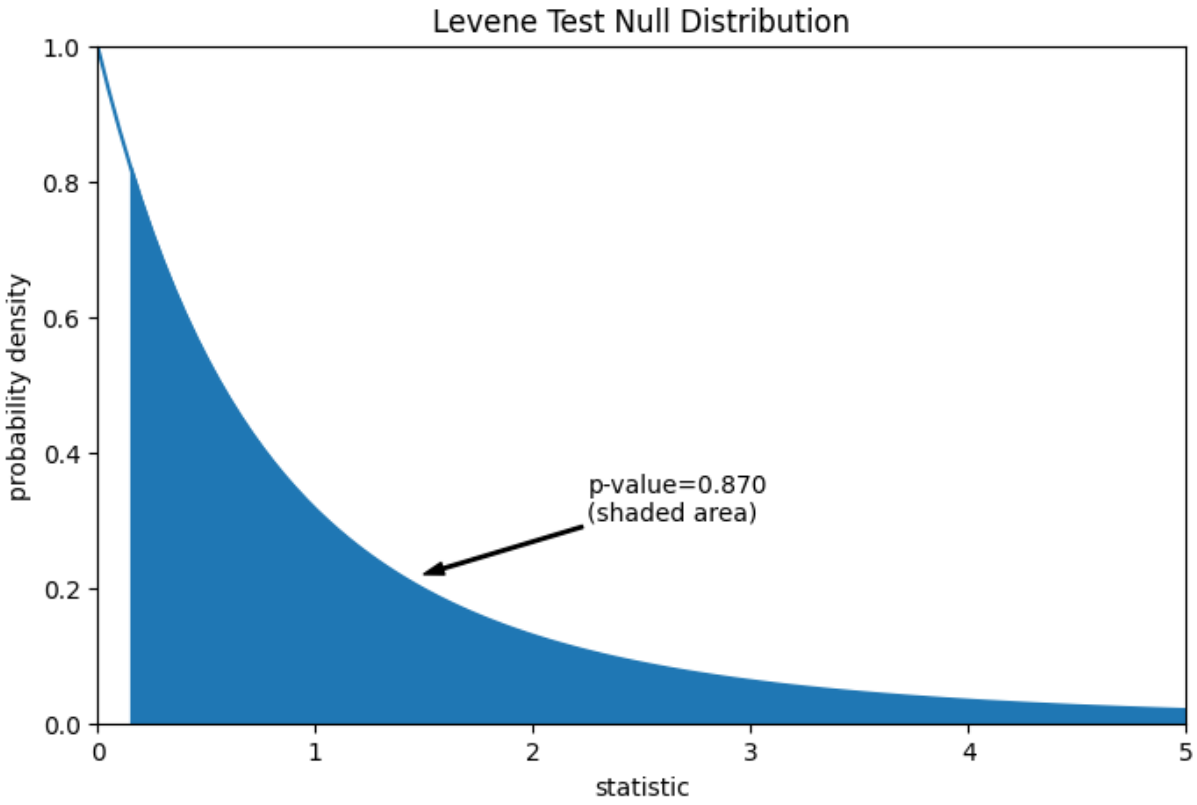
```
            df  Sum of Squares  Mean Squares       F  P-value
source
Treatments   3           775.0       258.333  27.928      0.0
Error       16           148.0         9.250     NaN      NaN
Total       19           923.0           NaN     NaN      NaN
0.1 > 0.0: Reject Null Hypothesis at α=0.1
```



Problem 12.3 # 13

In [16]:
```
df12313 = pd.DataFrame({
    'Workout#1' : [180,185,170],
    'Workout#2': [160,170,175],
    'Workout#3': [185,190,180]
})
print(generateANOVAFrame(df12313))
plotLevene(df12313)
```

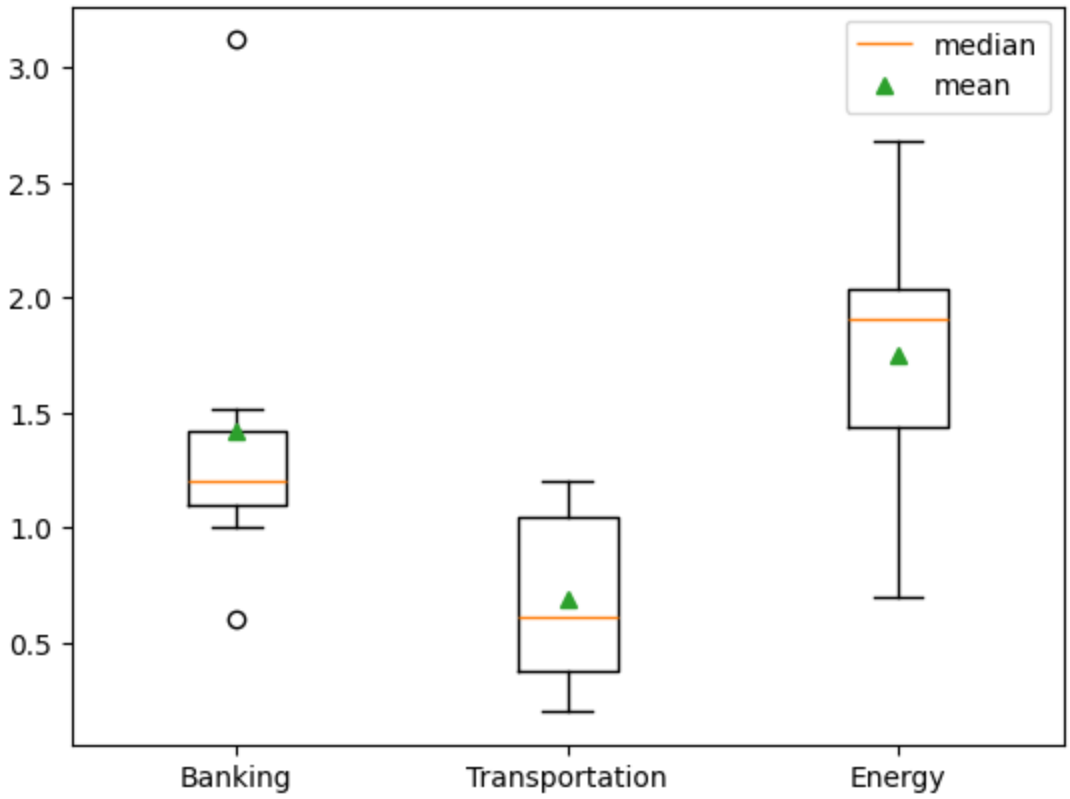
```
            df  Sum of Squares  Mean Squares      F  P-value
source
Treatments   2         422.222       211.111  4.471  0.06476
Error        6         283.333        47.222    NaN      NaN
Total        8         705.556           NaN    NaN      NaN
```

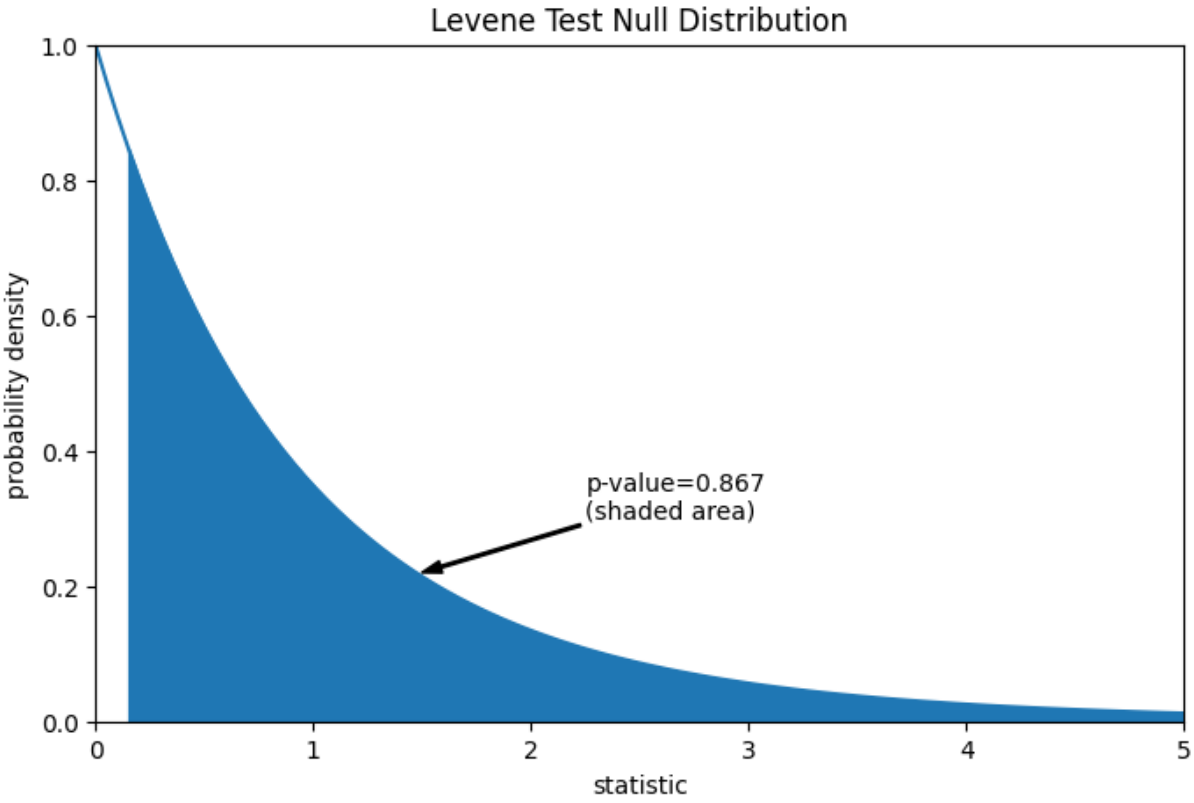


Problem 12.3 # 15

In [ ]:
```
df12315 = pd.DataFrame({
    'Banking' : [1.52, 3.12,1.32,0.6,1.2,1,1.19],
    'Transportation' : [1,1.2,0.2,0.4,1.09,0.61,0.35],
    'Energy' : [2.08,2.68,0.7,2,1.91,1.6,1.28]
})

print(generateANOVAFrame(df12315))
print(makeDetermination(df12315,alpha=0.10))
bp = plt.boxplot(df12315, showmeans=True, labels=df12315.columns)
plt.legend([bp['medians'][0], bp['means'][0]], ['median', 'mean'])
plotLevene(df12315, alpha=0.10)
```

```
             df  Sum of Squares  Mean Squares      F  P-value
source
Treatments    2           4.098         2.049  5.101  0.01757
Error        18           7.230         0.402    NaN      NaN
Total        20          11.328           NaN    NaN      NaN
0.1 > 0.01757: Reject Null Hypothesis at α=0.1
```


median
mean
3.0
2.5
2.0
1.5
1.0
0.5
Banking
Transportation
Energy


Levene Test Null Distribution
probability density
1.0
0.8
0.6
0.4
0.2
0.0
p-value=0.867
(shaded area)
0
1
2
3
4
5
statistic