

Name

Hypothesis Testing: Using Evidence to Decide if a Coin is Unfair.

This activity is designed to give you some ideas about hypothesis testing. This is a process based on probability which allows to decide between two separate and competing hypotheses. In this activity you will be testing to see if you have evidence to say a coin is unfair. To do this you will need a coin.

1. Begin by stating the null (H_0) and Alternative (H_a) hypotheses both in words and in mathematical symbolism.

We will assume that the coin is fair as our initial (null) hypothesis. If a coin is fair what is the probability of getting heads or tails? What would be true about this probability if the coin were not fair?

2. Now to begin the experiment choose your variable (either # or heads or # of tails). Then choose the number of times you would like to flip the coin or the sample size (it is easy to choose either $n = 10$ or $n = 20$). Once you chose the sample size decide what you think an extreme number of heads or tails would be given the experiment's sample size. For example I think it would be extreme if I tossed the coin 10 times and got 10 heads or 0 heads. Once this is done begin flipping the coin and record the number of heads or tails you get in your sample.
3. Using the x-values you choose as extreme, decide if your experiment produced extreme results. Use the language "fail to reject the null hypothesis" or "reject the null hypothesis".
4. State the conclusion. "The evidence supports..." or "The evidence does not support..."
5. Now we will consider how extreme (or not extreme) your sample was using the test statistic. To calculate the test statistic we will need to make a few calculations. The formula for the test statistic given a normal sampling distribution is given below. You will need to find the values to put into the test statistic. Make sure to list your values for x, n, \hat{p}, p , and $(1 - p)$ before putting them into your test statistic formula.

$$x = 11$$

$$n = 20$$

$$\hat{p} = \frac{11}{20} = 0.55$$

$$p = 0.5$$

$$1 - p = 0.5$$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{0.55 - 0.5}{\sqrt{\frac{0.5 \cdot 0.5}{20}}}$$

$$z = 0.446$$

1) Probability for both heads and tails would be 0.5. An unfair coin would have uneven probabilities.

$$2) \frac{3}{20} \text{ or } \frac{17}{20}$$

$$\sigma = \sqrt{\frac{0.5 \cdot 0.5}{20}}$$
$$\sigma = 0.112$$

$$z = \frac{\frac{3}{20} - 0.5}{0.112} = -3.125$$

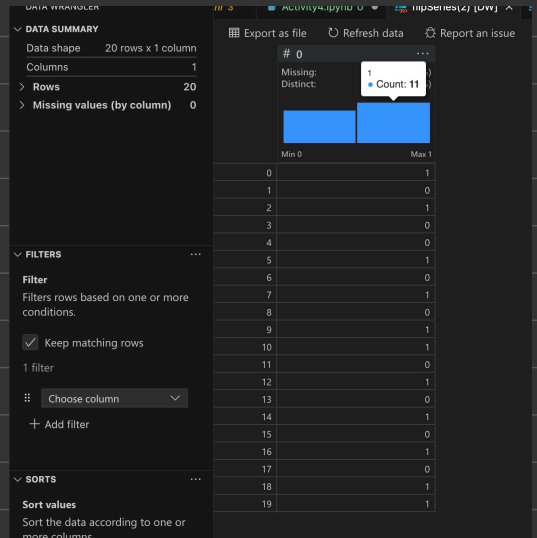
$$z = 3.125$$

$$\alpha = 1.563$$

Using python "random" library

$$n = 20$$

$$\text{Heads} = 11 \quad \text{Tails} = 9$$

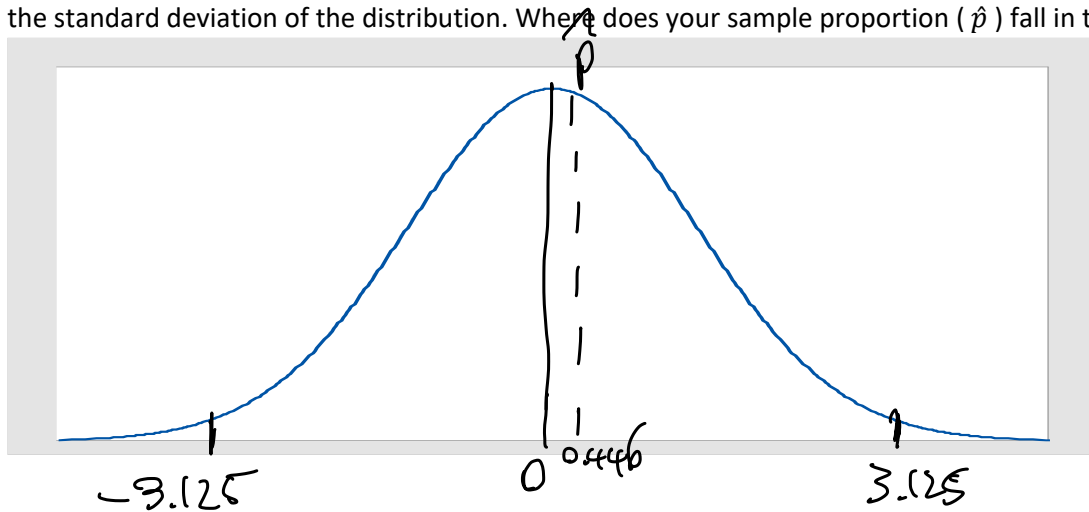


3) At the $\alpha = 1.563$ level of significance, there is not sufficient evidence to suggest that the alternative hypothesis is accurate.

4) The evidence does not support that the coin is unfair.

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6. Sketch a normal sampling distribution with p in the middle and use your calculation of the standard error $\left(\sqrt{\frac{p \cdot q}{n}}\right)$ for the standard deviation of the distribution. Where does your sample proportion (\hat{p}) fall in the distribution?



$$0.5 \pm 0.446 \left(\frac{0.25}{20} \right)^{\frac{1}{2}}$$

7. Show the place in the standard normal distribution where the test statistic you calculated in number 5 falls. While the numbers are different the test statistic should fall in the same place as your sample proportion \hat{p} .

