Solving Problems by Searching

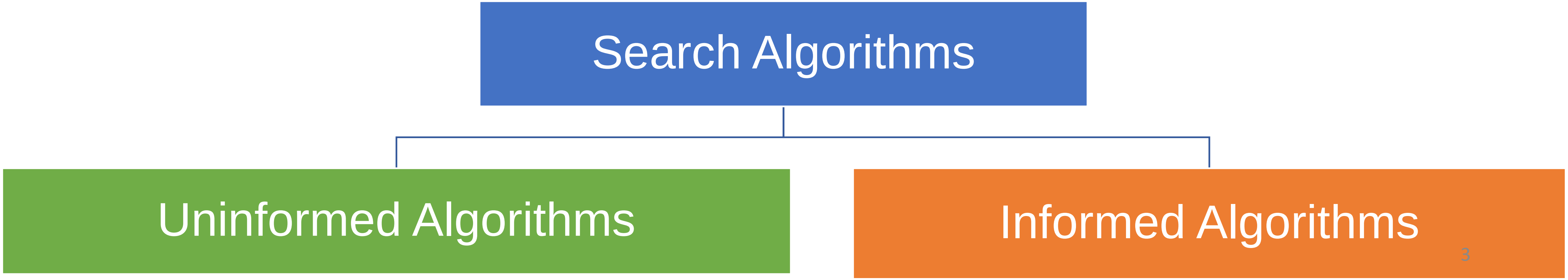
Dr. Sandareka Wickramanayake

Outline

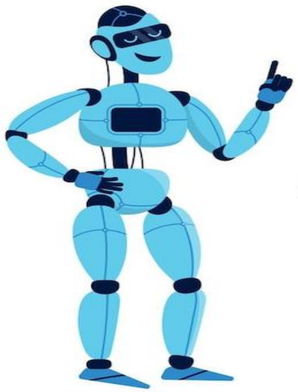
- Problem-solving agents
- Problem formulation
- Example problems
- Uninformed Search Algorithms
- Informed Search Algorithms
- Heuristic Functions

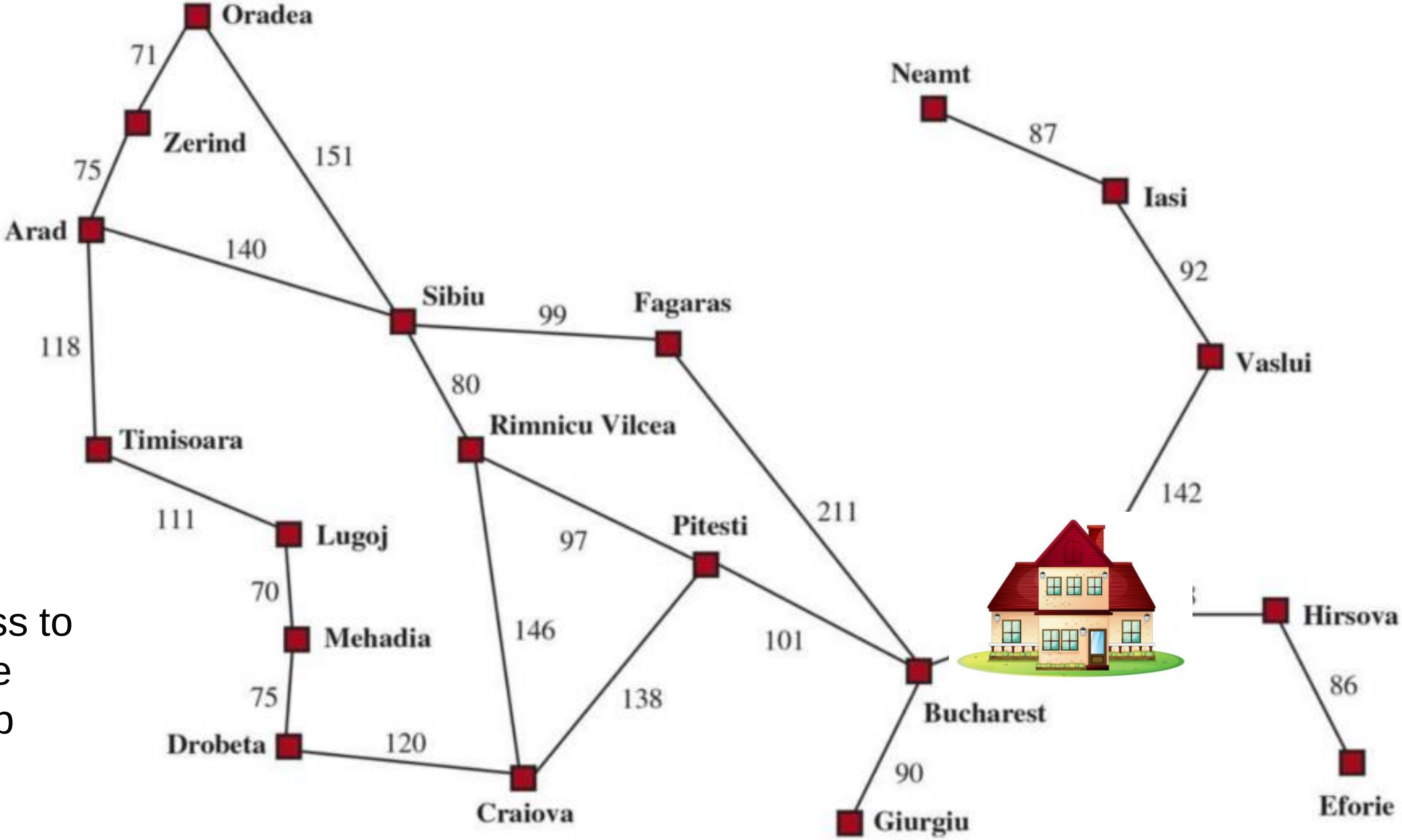
Problem-Solving Agents

- Problem-Solving Agent - An agent that plans ahead: considers a sequence of actions that form a path to a goal state.
- Search – The computational process undertaken by a problem-solving agent.
- Use atomic representations.
- Only the simplest environments: episodic, single agent, fully observable, deterministic, static, and discrete.



A Vacation in Romania





The agent has access to information about the world, such as a map

A simplified road map of part of Romania, with road distances in miles.

The Problem-Solving Process

- GOAL FORMULATION: Goals organize behavior by limiting the objectives and hence the actions to be considered.
- PROBLEM FORMULATION: The agent devises a description of the states and actions necessary to reach the goal.
- SEARCH: Before taking any action in the real world, the agent simulates sequences of actions in its model, searching until it finds a sequence of actions that reaches the goal (*solution*).
- EXECUTION: The agent can now execute the actions in the solution, one at a time.

A Vacation in Romania

- Agent on holiday in Romania; currently in Arad.
- Needs to catch a flight taking off from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest.

Search Problems and Solutions

- A **search problem** has the following components:

- **State space -** A set of possible states that the environment can be in.
- **Initial state** – The state that the agent starts in.
  - E.g., Arad
- **Goal states**
  - One goal state (e.g., Bucharest)
  - A small set of alternative goal states (e.g., The goal of a vacuum cleaner is to have no dirt in any location.)
  - The goal is defined by a property that applies to many states

Search Problems and Solutions

- A **search problem** has the following components:

- **Actions** - Actions available to the agent. Given a state s, $Action(s)$ returns a finite set of actions that can be executed in s. We say that each of these actions is **applicable** in s.
  - $ACTIONS(Arad) = \{ToSibiu, ToTimisoara, ToZerind\}$
- **Transition model** – Describes what each action does. $RESULT(s, a)$ returns the state that results from doing action a in state s.
  - E.g., $RESULT(Arad, ToZerind) = Zerind$

Search Problems and Solutions

- A search **problem** has the following components:

- **Action cost function** – The numeric cost of applying action a in state s to reach $s', ACTION\_COST(s, a, s')$.
  - E.g., lengths in miles/ time it takes to complete the action
- **Path** – A sequence of states connected by a sequence of actions.
- **Solution** – A path from the initial state to a goal state.
- **Optimal solution** – The lowest path cost among all solutions.
- **Graph** – A representation of state space in which vertices are states and the directed edges between them are actions.

Assumptions - Action costs are additive, and all action costs will be positive

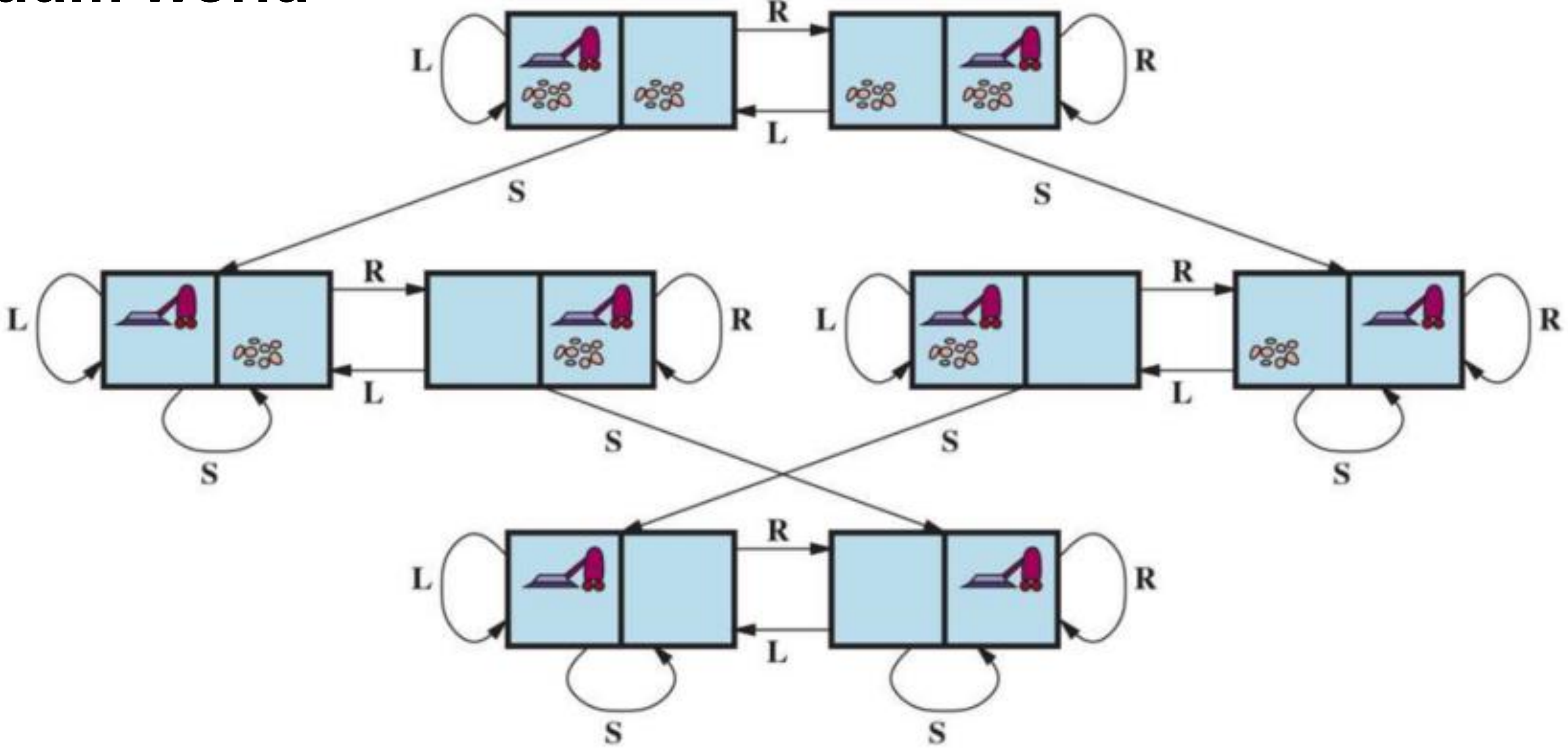
Search Problems and Solutions

- A search **problem** has the following components:

- **Model -** An abstract mathematical description.
  - E.g., our formulation of the problem of getting to Bucharest.
- **Abstraction** – Removing details from a representation.
  - Real world is quite complex.
  - A good problem formulation has the right level of detail.

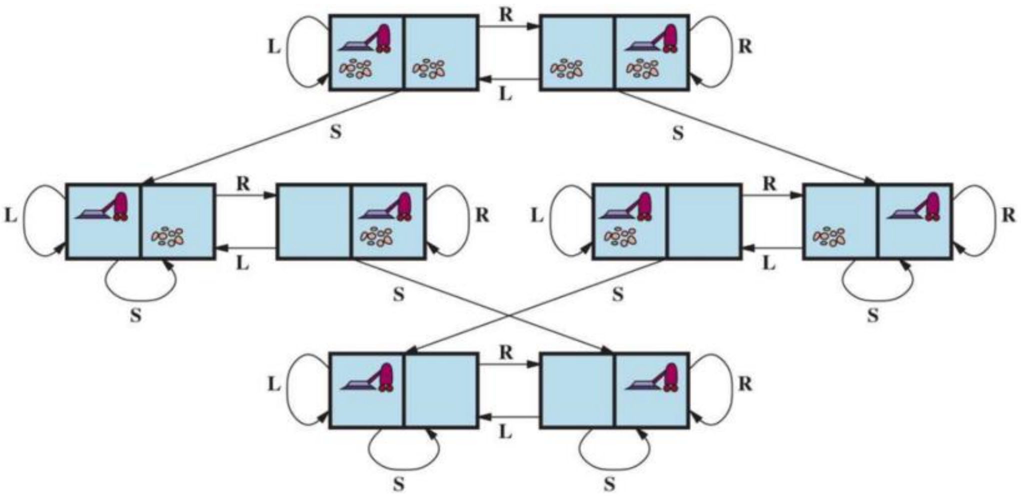
Example Problems

- **Vacuum world**



The state-space graph for the two-cell vacuum world. There are 8 states and three actions for each state: L = *Left*, R = *Right*, S = *Suck*.

Example Problems



The state-space graph for the two-cell vacuum world. There are 8 states and three actions for each state: L = *Left*, R = *Right*, S = *Suck*.

- **Vacuum world**
  - **STATES –** 8 states (Agent in cell 1, cell 1 has dirt, cell 2 has dirt, etc.)
  - **INITIAL STATE** - Any state can be designated as the initial state.
  - **ACTIONS** - *Suck, move Left*, and *move Right*.
  - **TRANSITION MODEL** - Suck removes any dirt from the agent's cell; *Forward moves* the agent ahead of one cell in the direction it is facing unless it hits a wall, in which case the action has no effect. *Backward* moves the agent in the opposite direction, while *TurnRight* and *TurnLeft* change the direction it is facing by 90^0.
  - **GOAL STATES**: The states in which every cell is clean.
  - **ACTION COST:** Each action costs 1.

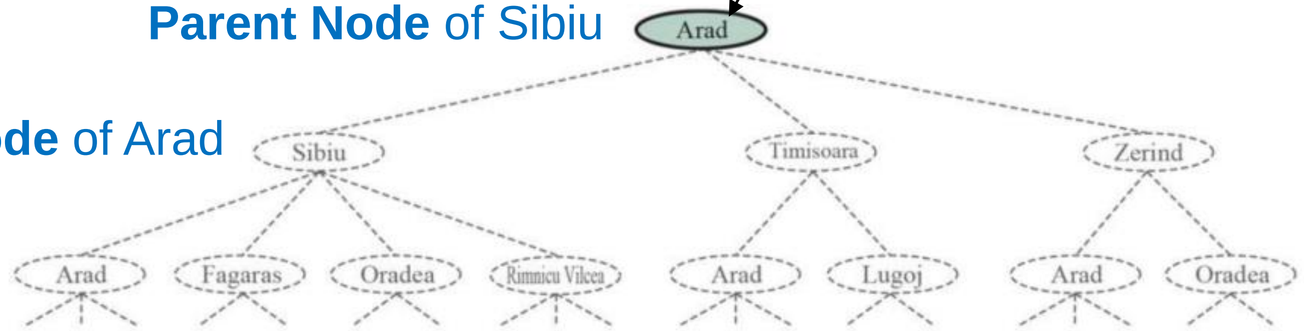
Example Problems

- **Route-finding problem – Travel-planning website**
  - **STATES** - Each state includes a location (e.g., an airport) and the current time.
  - **INITIAL STATE** - The user’s home airport.
  - **ACTIONS** - Take any flight from the current location, in any seating class, leaving after the current time, leaving enough time for within-airport transfer if needed.
  - **TRANSITION MODEL**: The state resulting from taking a flight will have the flight’s destination as the new location and the flight’s arrival time as the new time.
  - **GOAL STATE**: A destination city. Sometimes the goal can be more complex, such as “arrive at the destination on a nonstop flight.”
  - **ACTION COST**: Monetary cost, waiting time, flight time, customs and immigration procedures, seat quality, time of day, type of airplane, frequent-flyer reward points, etc.

Search Algorithms

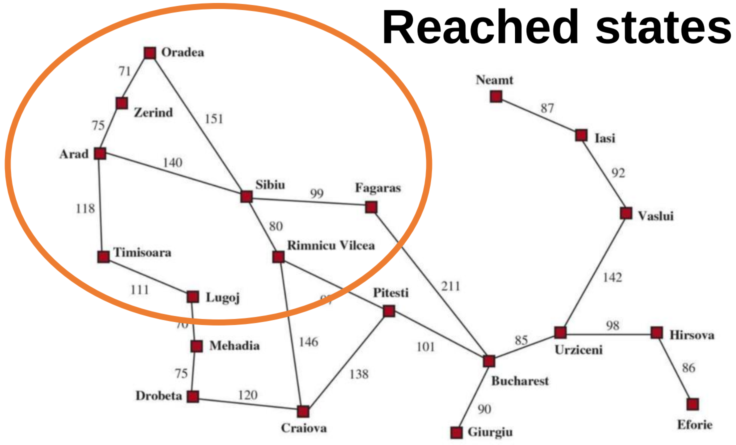
Root (Corresponds to the initial state)

Parent Node of Sibiu



Child Node of Arad

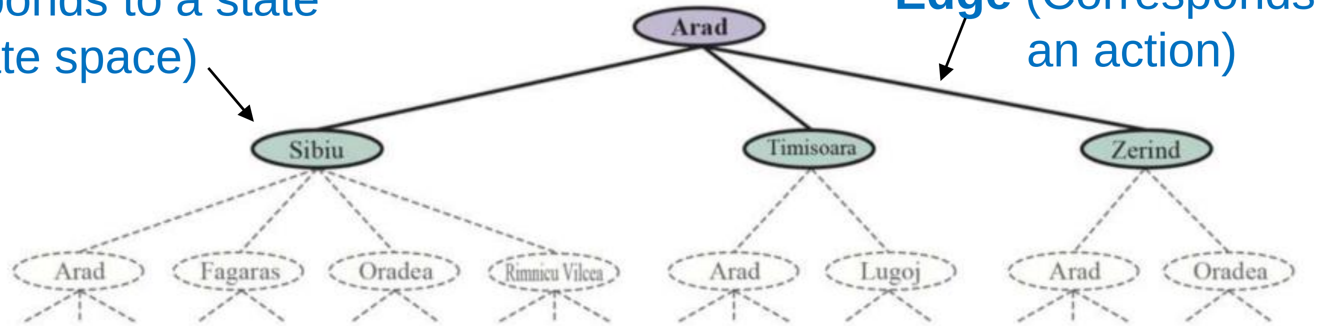
Reached states



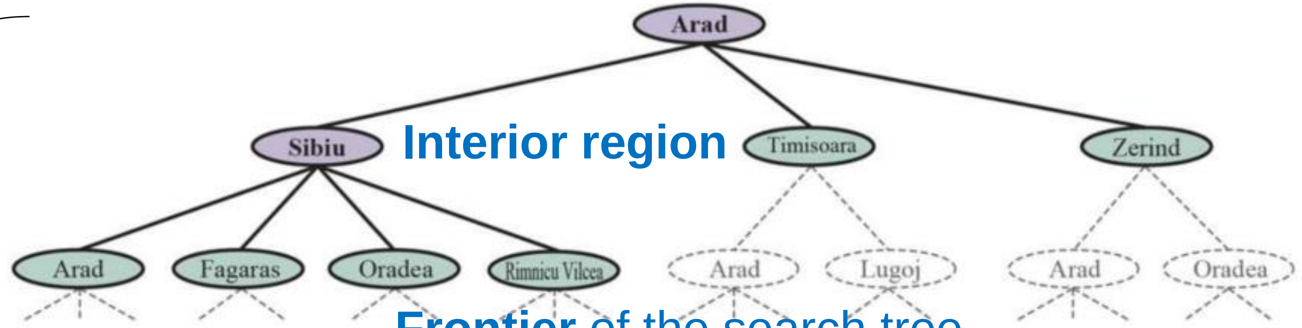
State Space
All possible transitions among all the states.

Node (Corresponds to a state in the state space)

Edge (Corresponds to an action)



Search Tree
Describes paths between the states toward the goal state.



Interior region

Frontier of the search tree

Exterior region

Expand – Expand the node considering available actions.

Redundant Paths

- How do we decide which node from the frontier to expand next?
- Redundant paths
  - **Repeated state –** Meeting a top node again.
  - **Redundant path**
    - We can get to Sibiu via the path Arad–Sibiu (140 miles long) or ~~the path Arad–Zerind–Oradea–Sibiu (297 miles long).~~
    - Eliminating redundant paths leads to faster solutions.

Measuring Problem-Solving Performance

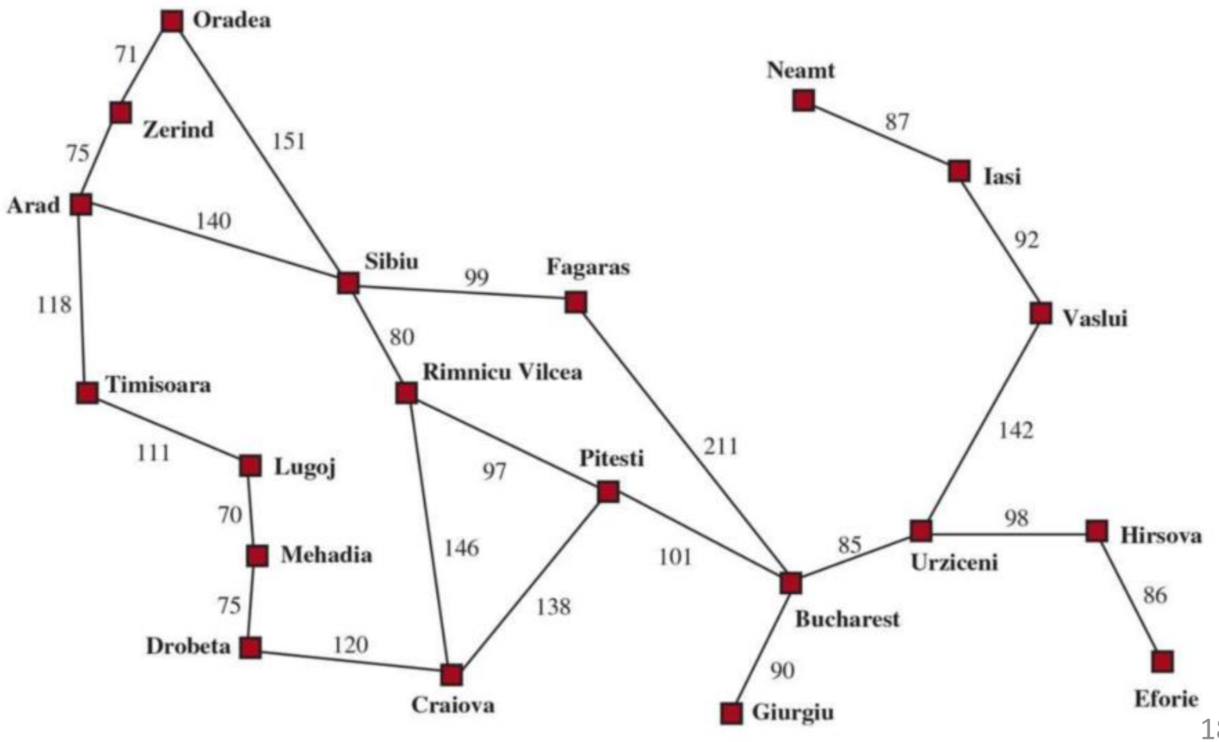
- Algorithms are evaluated along the following dimensions:
  - **Completeness**: Is the algorithm guaranteed to find a solution when there is one, and to correctly report failure when there is not?
  - **Cost optimality:** Does it find a solution with the lowest path cost of all solutions?
    - Also referred to as admissibility or optimality.
  - **Time complexity:** How long does it take to find a solution?
    - Can be measured in seconds, or more abstractly by the number of states and actions considered.
  - **Space complexity**: How much memory is needed to perform the search?

Measuring Problem-Solving Performance

- Time and space complexity are measured in terms of
  - ***b***: maximum branching factor of the search tree (number of successors of a node that need to be considered)
  - ***d***: depth of the least-cost solution
  - ***m***: maximum number of actions in any path (maybe ∞)

Uninformed Search Algorithms

- Have access only to the problem definition.
- No clue about how close a state is to the goal(s).
- Build a search tree to find a solution.



Uninformed Search Algorithms

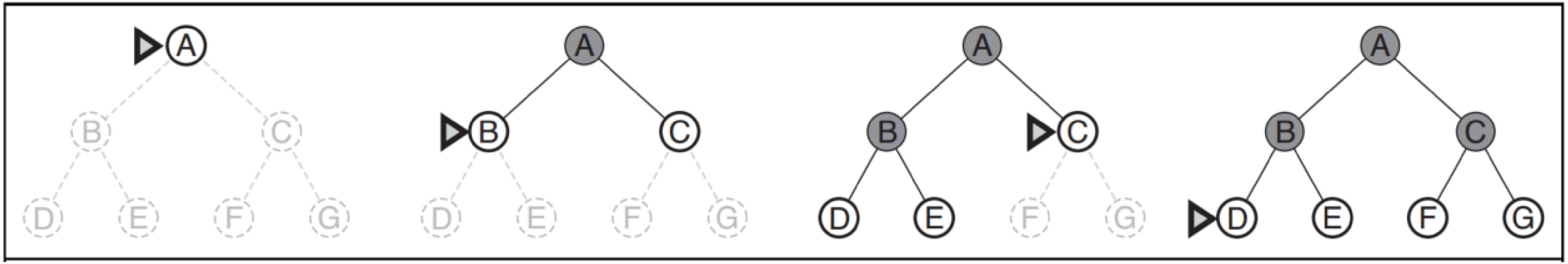
- Algorithms differ based on which node they expand first.
- Algorithms
  - **Breadth-first search** – Expands the shallowest nodes first.
    - Complete
    - Optimal for unit action costs.
    - Exponential space complexity.
  - **Uniform-cost search** – Expands the node with the lowest path cost.
    - Optimal for general action costs.

Uninformed Search Algorithms

- Algorithms
  - **Depth-first Search** - Expands the deepest unexpanded node first.
    - Neither complete nor optimal
    - Linear time complexity
  - **Iterative Deepening Search** - Calls DFS with increasing depth limits until a goal is found.
    - Complete when full cycle checking is done
    - Optimal for unit action costs
    - Time complexity is comparable to BFS
    - Space complexity is linear.
  - **Bidirectional Search** - Expands two frontiers, one around the initial state and one around the goal, stopping when the two frontiers meet.

Breadth-first Search

- Appropriate when all the actions have the same cost.



Breadth-first Search - Evaluation

- **Complete?** Yes (if b is finite)
- **Time?** $1+b+b^2+ b^3 +\ldots + b^d = O(b^d)$, where d is the depth of the solution.
- **Space?** $O(b^d)$ (keeps every node in memory)
- **Cost Optimal?** Yes (Only if path costs are identical)
- Space is the bigger problem (more than time)

- *Exponential complexity search problems cannot be solved by uninformed search for any but the smallest instances.*

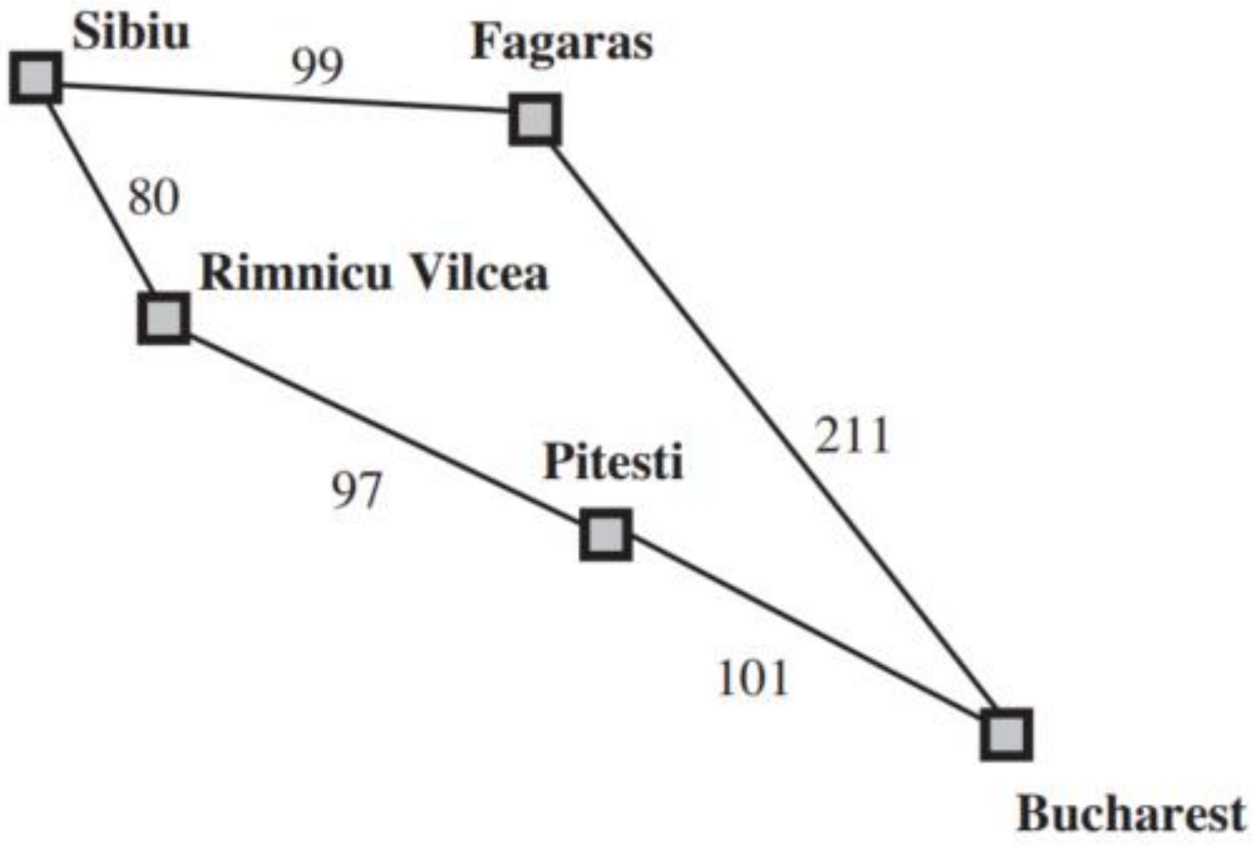
Breadth-first Search - Evaluation

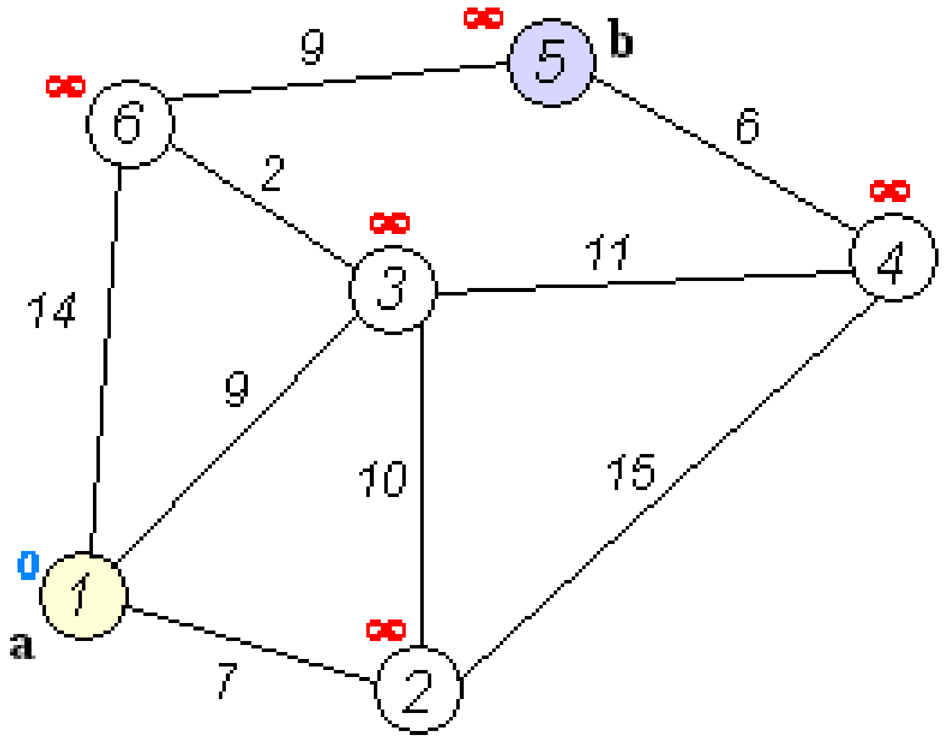
Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

Figure 3.13 Time and memory requirements for breadth-first search. The numbers shown assume branching factor $b = 10$; 1 million nodes/second; 1000 bytes/node.

Uniform-cost Search or Dijkstra's Algorithm

- Appropriate when the actions have different costs.



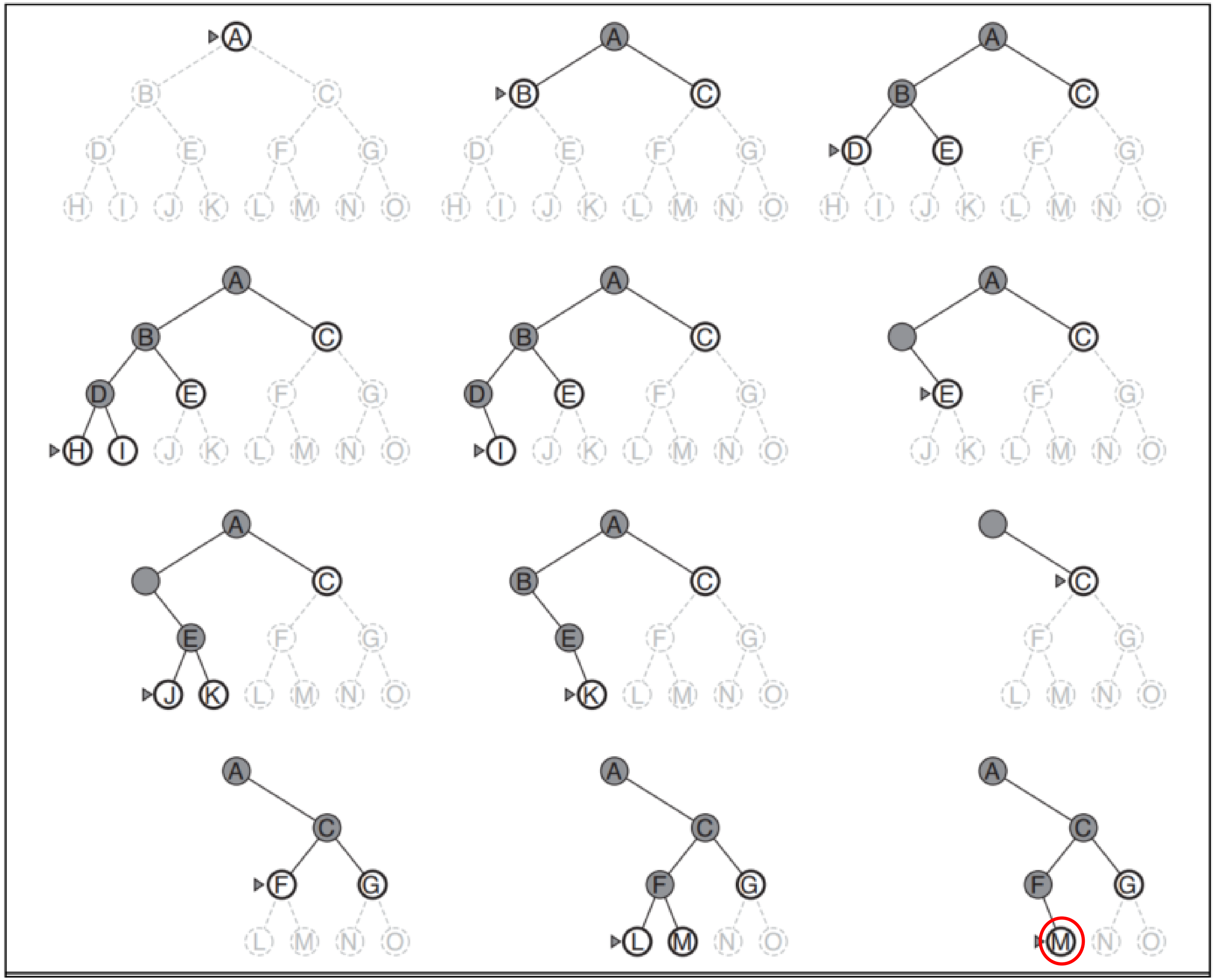


Source - https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm

Uniform-cost Search - Evaluation

- **Complete?** Yes (if b is finite)
- **Time?** $O(b^{1+\lfloor C^*/\epsilon \rfloor})$
  - Where C^* - The cost of the optimal solution and ϵ – lower bound on the cost of each action, with $\epsilon > 0$.
- **Space?** $O(b^{1+\lfloor C^*/\epsilon \rfloor})$
- **Cost Optimal?** Yes

Depth-first Search



Depth-first Search - Evaluation

- **Complete?** Yes, for finite state spaces. No for infinite state spaces and spaces with loops.
- **Time?** $O(b^m)$, where m is the maximum depth.
- **Space?** $O(bm)$
- **Cost Optimal?** No: It returns the first solution it finds, even if it is not the cheapest.

Depth-limited Search

- Keep DFS from wandering down an infinite path.
- A version of DFS which has a depth limit, l, and treats all nodes at depth l as if they had no successors.
- Evaluation
  - **Complete?** No, a poor choice for l makes the algorithm fail to reach the solution.
  - **Time?** $O(b^l)$
  - **Space?** $O(bl)$
  - **Cost Optimal?** No: It returns the first solution it finds, even if it is not the cheapest.

Uninformed Search Algorithms Comparison

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes^{a}	Yes^{a,b}	No	No	Yes^{a}	Yes^{a,d}
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes^{c}	Yes	No	No	Yes^{c}	Yes^{c,d}

Figure 3.21 Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: ^{a} complete if b is finite; ^{b} complete if step costs $\geq \epsilon$ for positive ϵ; ^{c} optimal if step costs are all identical; ^{d} if both directions use breadth-first search.

Informed Search Algorithms

- Uses domain-specific hints about the location of goals.
- Finds solutions more efficiently than an uninformed strategy.
- The hints come in the form of a **heuristic function**, $h(n)$.
- $h(n)$ = estimated cost of the cheapest path from the state at node n to a goal state.
  - In route-finding problems - the straight-line distance on the map between the current state and a goal.

Informed Search Algorithms

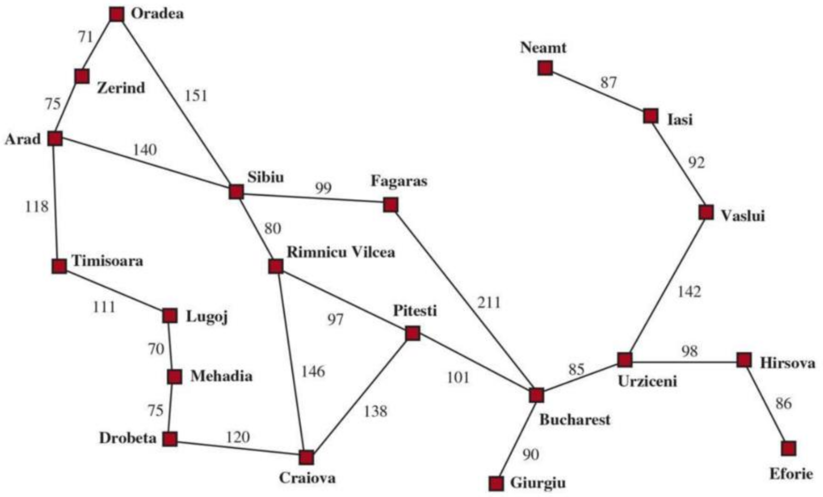
- Algorithms
  - **Greedy Best-First Search** – Expands nodes with minimal $h(n)$
    - Not optimal
    - Efficient
  - **A* Search** – Expands nodes with minimal $f(n) = g(n) + h(n)$
    - Complete and optimal provided that $h(n)$ is admissible.
    - Bad space complexity.
  - **Bidirectional A* Search**
    - More efficient than A*
  - **Iterative Deepening A* Search** – An Iterative version of A*
    - Address the space complexity issue.
  - **Beam Search –** Puts a limit on the size of the frontier.
    - Incomplete and suboptimal
    - Efficient with reasonably good solutions.

Greedy Best-First Search

- **Best-First Search**
  - Idea: use an evaluation function *f* for each node *n*
    - *f(n)* estimates the "desirability“ of node *n*
    - Expand the most desirable unexpanded node

- **Greedy Best-First Search**
  - Evaluation function $f(n) = h(n)$
    - where $h(n)$ is some heuristic estimate of cost from *n* to *goal*
    - e.g., $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest
  - Expands the node that **appears** to be closest to the goal
    - E.g., node *n*, such that $h_{SLD}(n)$ is minimum

Greedy Best-First Search

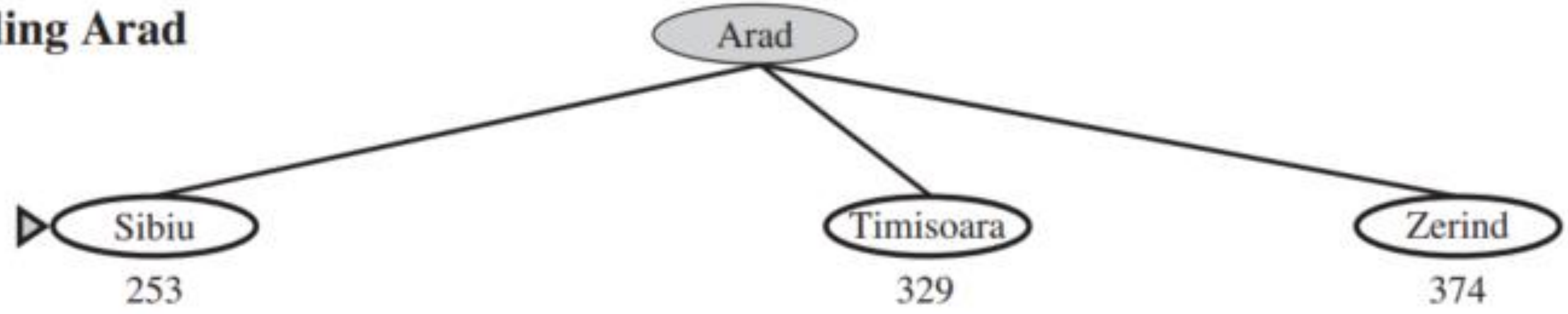


Straight-line distances to Bucharest.

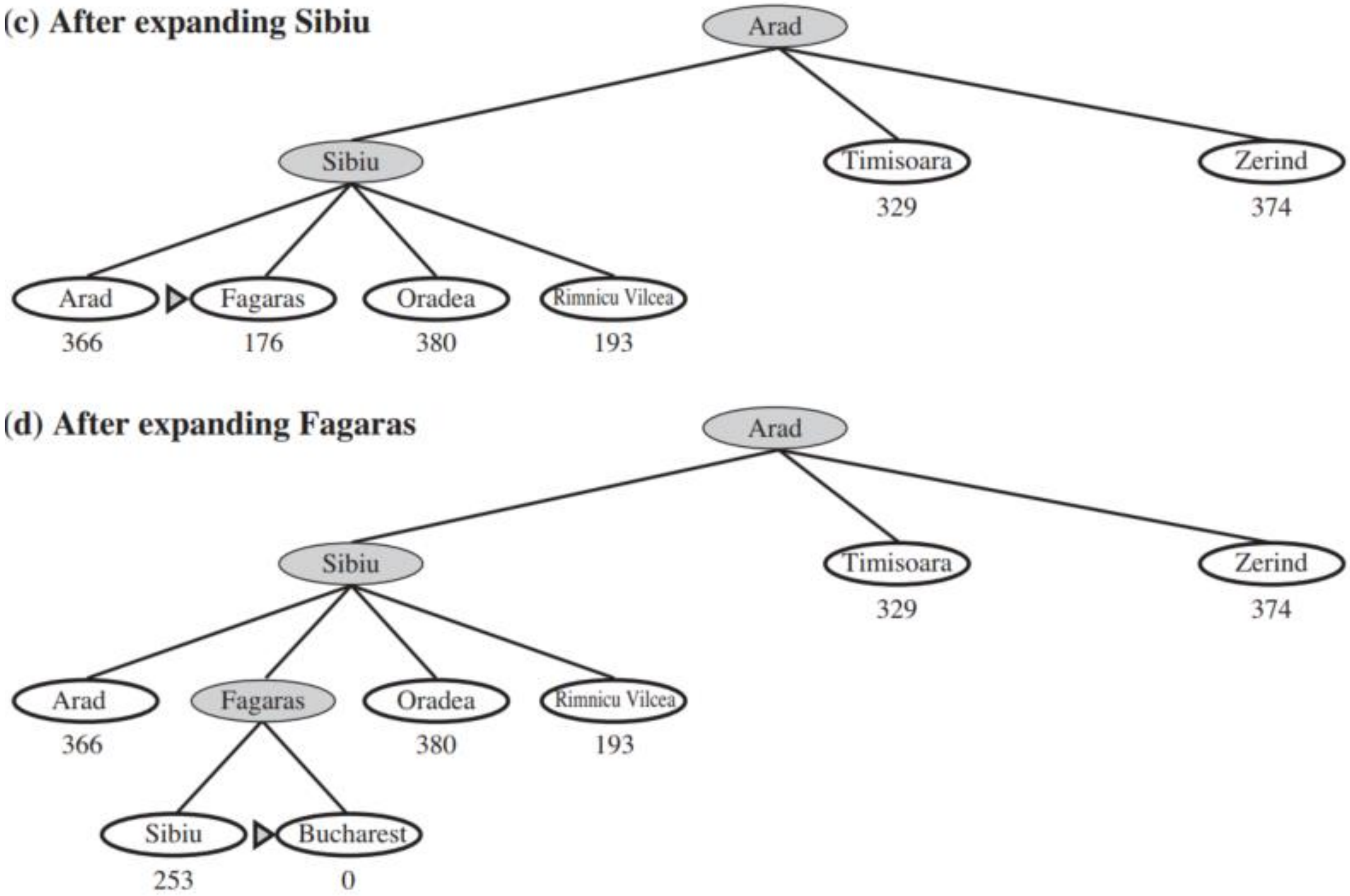
Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

(a) The initial state

(b) After expanding Arad

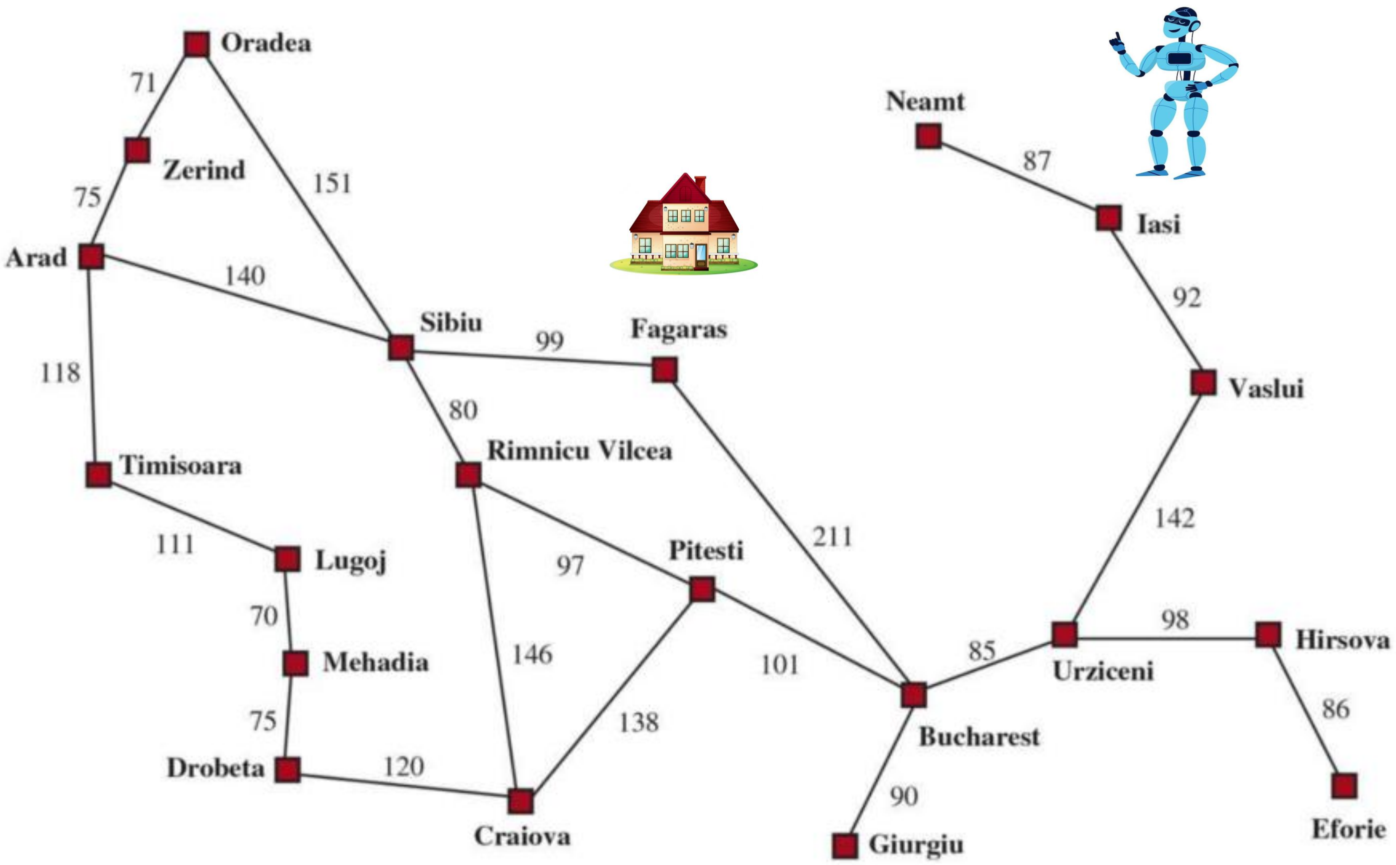


Greedy Best-First Search



Greedy Best-First Search - Evaluation

- **Complete**? No. Can lead to dead ends and the tree search version (not the graph search version) can go into infinite loops.



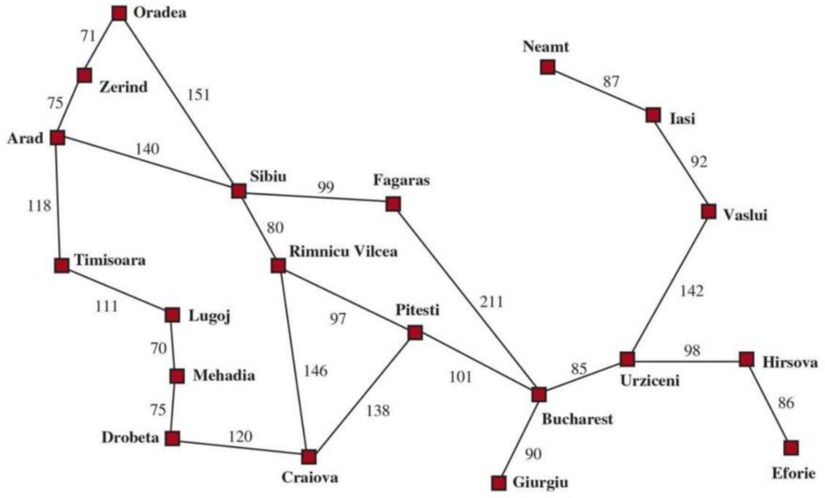
Greedy Best-First Search - Evaluation

- **Worst case time**? $O(b^m)$ - can generate all nodes at depth m before finding the solution.
- **Worst case space**? $O(b^m)$ - can generate all nodes at depth m before finding the solution
  - But a good heuristic can dramatically improve the time and space needed
    - In our example, a solution was found without expanding any node not on the path to goal: Which very efficient in this case
- **Optimal**? No
  - Path found: Arad->Sibiu->Fagaras->Bucharest. Actual cost = 140+99+211=450
  - But the actual cost of, Arad->Sibiu->Rimnicu->Pitesti = 140+80+97+101=418

A* Search

- Idea: avoid expanding paths that are already expensive.
- Evaluation function $f(n) = g(n) + h(n)$, where $g(n)$ is the cost to reach the node n.
- $f(n)$ - Estimated cost of the cheapest solution through n.
- A* is identical to Uniform-cost search except A* uses $g(n) + h(n)$ instead of $g(n)$.

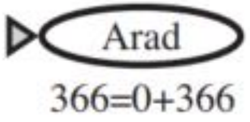
A* Search



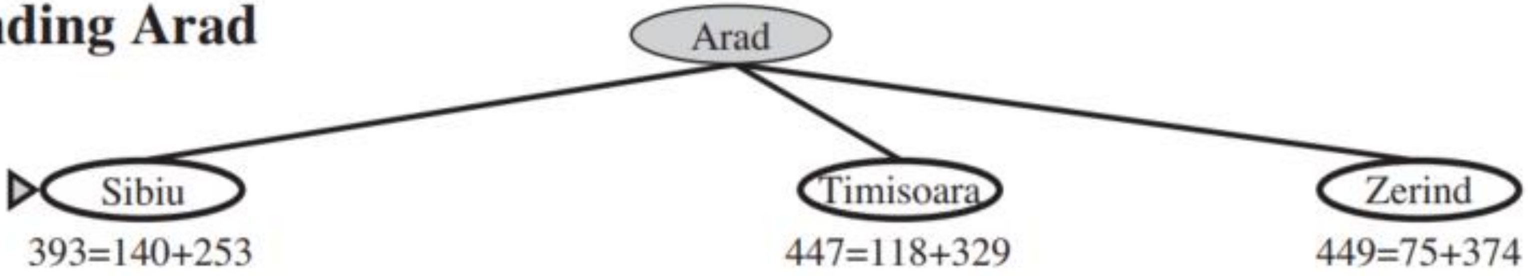
Straight-line distances to Bucharest.

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

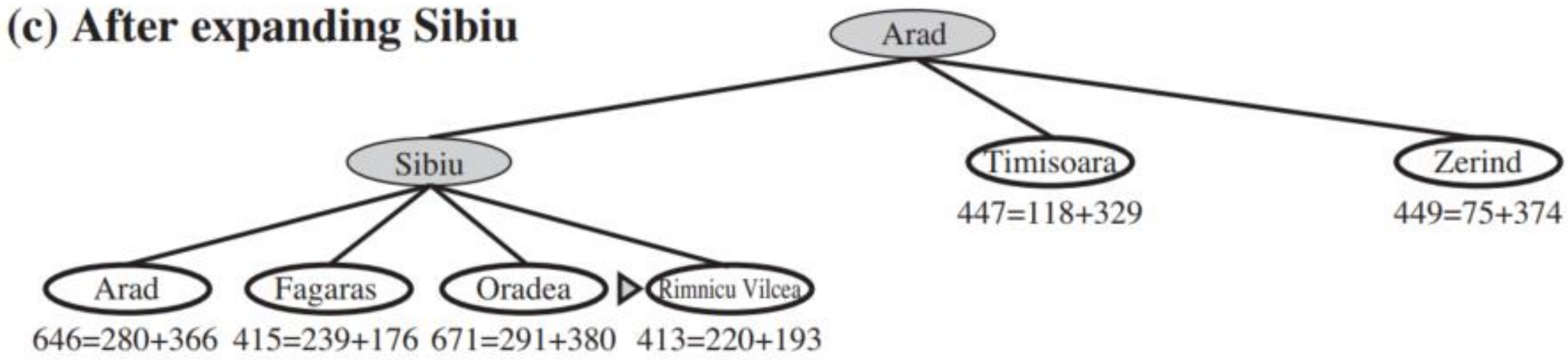
(a) The initial state



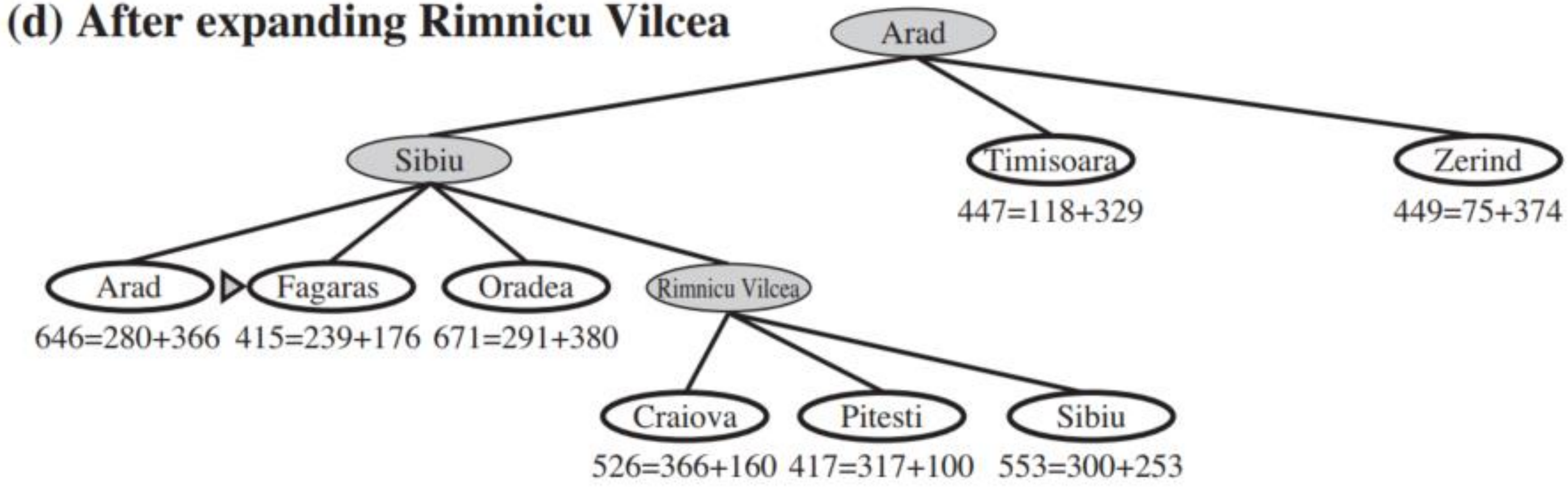
(b) After expanding Arad



A* Search



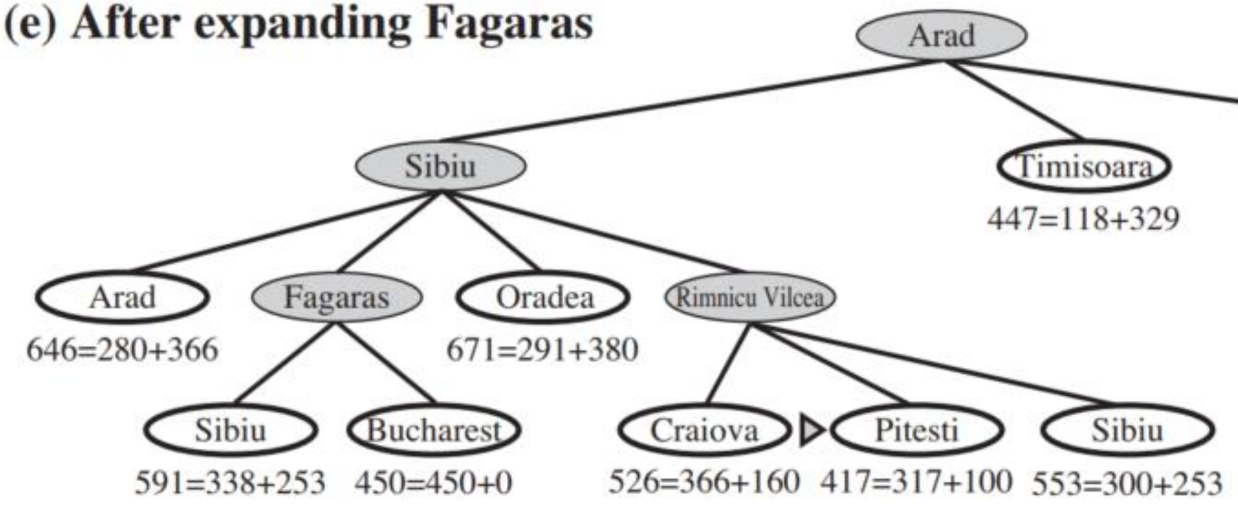
(c) After expanding Sibiu
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
413=220+193



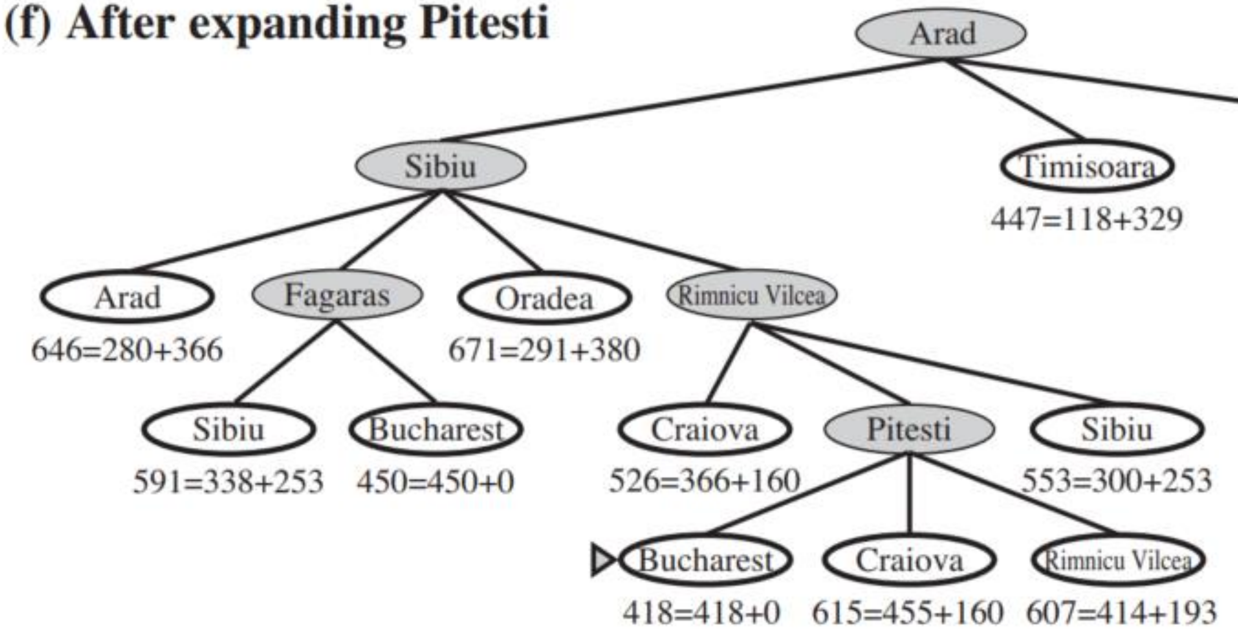
(d) After expanding Rimnicu Vilcea
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
Craiova
526=366+160
Pitesti
417=317+100
Sibiu
553=300+253

A* Search

(e) After expanding Fagaras



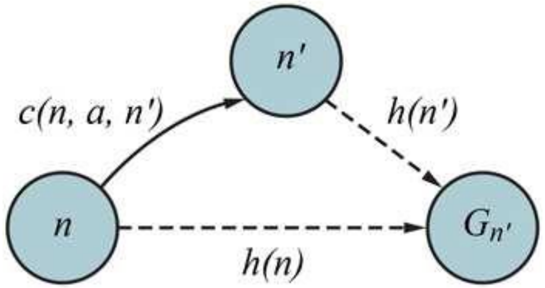
(f) After expanding Pitesti



A* Search - Evaluation

- Complete? Yes
- Optimal?
  - Depends on certain properties of the heuristics
  - **Admissibility**: an admissible heuristic never overestimates the cost of reaching a goal. (An admissible heuristic is therefore optimistic.)
  - *If the heuristic is admissible, A* is optimal.*
  - **Consistency**: A heuristic $h(n)$ is consistent if for every node n and every successor n' of n generated by an action a, we have:

$$h(n) \leq c(n, a, n') + h(n')$$



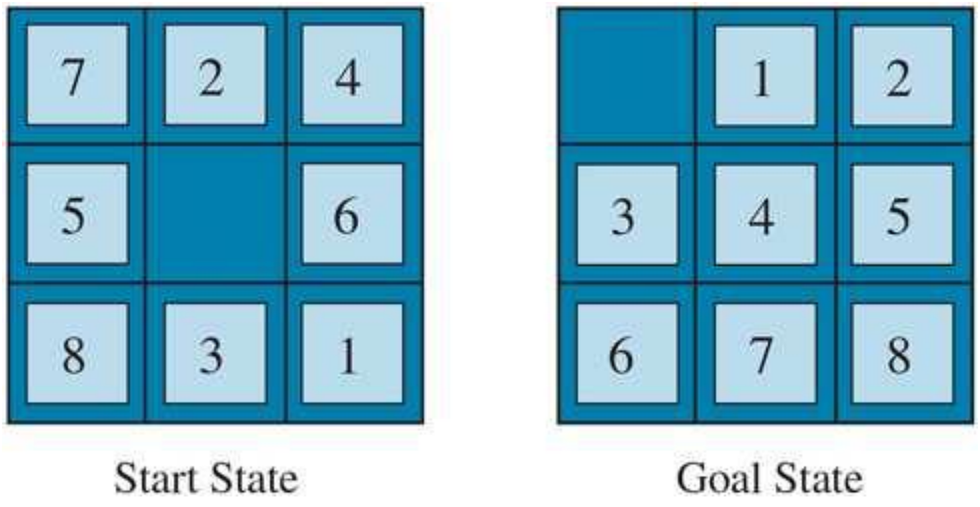
  - Every consistent heuristic is admissible.
  - *If the heuristic is consistent, A* is optimal.*
  - *With an inadmissible heuristic, A* may or may not be cost-optimal.*

A* Search - Evaluation

- Time? Exponential in the worst case.
- Space? Exponential in the worst case.
    - A good heuristic can reduce time and space complexity considerably.

Heuristic Functions

- The performance of heuristic search algorithms depends on the quality of the heuristic function.
- One can sometimes construct good heuristics by
  - Relaxing the problem definition
  - Storing precomputed solution costs for subproblems in a pattern database
  - Defining landmarks
  - Learning from the experience with the problem class



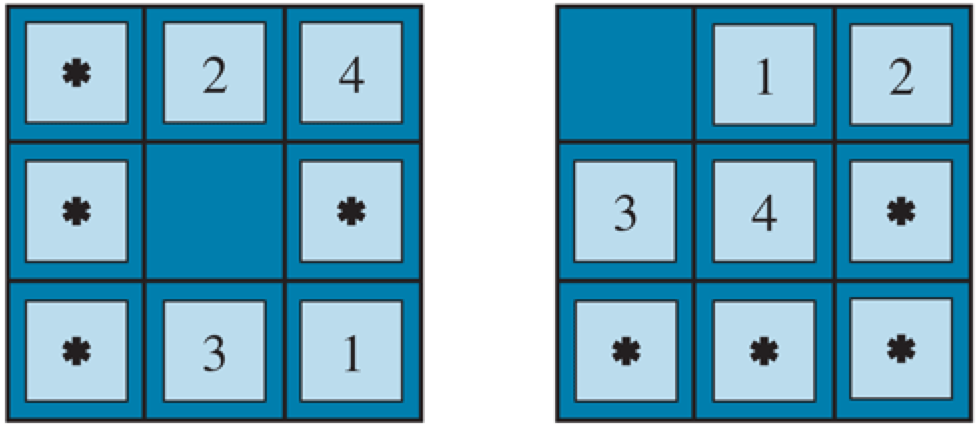
h1 = the number of misplaced tiles (blank not included). (An admissible heuristic)
h2 = the sum of the distances of the tiles from their goal positions. (An admissible heuristic)

Generating Heuristics from Relaxed Problems

- A problem with fewer restrictions on the actions is called a **relaxed problem.**
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem.
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then the shortest solution gives $h_1(n)$

Generating Heuristics from Subproblems: Pattern Databases

- Admissible heuristics can also be derived from the solution cost of a subproblem of a given problem.
- E.g., Subproblem of 8-puzzle example
  - The cost of the optimal solution to this subproblem is a lower bound on the cost of the complete problem.
- **Pattern databases** - Stores these exact solution costs for every possible subproblem instance.



Start State   Goal State