1 K(x)

$$
\begin{aligned}
\beta &= \sqrt{\frac{\epsilon_1}{\epsilon_0}} \\
\epsilon(q) &= \epsilon_1\epsilon_0\frac{1+(q\xi)^2}{\epsilon_1+\epsilon_0(q\xi)^2} \\
K(x) &= \frac{1}{2\pi}\epsilon_0\epsilon_1\int\limits_{-\infty}^{+\infty}\frac{1+(q\xi)^2}{\epsilon_1+\epsilon_0(q\xi)}e^{iqx}dq = \\
&= \frac{\epsilon_1}{2\pi}\int e^{iqx}(1+\frac{\epsilon_0-\epsilon_1}{\epsilon_1+\epsilon_0(q\xi)^2})dq = \\
&= \epsilon_1\delta(x)+(\epsilon_0-\epsilon_1)\frac{\beta}{2\xi}e^{-\beta\frac{|x|}{\epsilon}}
\end{aligned}
$$

2 D(x)

Let's pretend that $E(x) = E_0 + E_1e^{\frac{x-h}{\xi}} + E_1e^{\frac{-x-h}{\xi}} = E_0 + 2E_1e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi})$

$D(x) = \int\limits_{-h}^{h} K(x-x')E(x')dx' = \int\limits_{-h}^{h}\left(\epsilon_1\delta(x-x') + \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)e^{-\beta\frac{|x-x'|}{\epsilon}}\right)\left(E_0+2E_1e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi})\right)$

2.1 D_1

$D_1(x) = \int\epsilon_1\delta(x-x')\left(E_0+E_1e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi})\right) = \epsilon_1E_0 + \epsilon_1E_1e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi})$

2.2 D_2

$$
\begin{aligned}
D_2(x) &= \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0\int\mathrm{sgn}(x-x')e^{-\beta\frac{|x-x'|}{\epsilon}}dx' = \\
&= \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0\left[\int\limits_{-h}^{x}\exp(-\beta\frac{x-x'}{\xi})dx' + \int\limits_{x}^{h}\exp(\beta\frac{x-x'}{\xi})\right] = \\
&= \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0\left[e^{-\beta\frac{x}{\xi}}\frac{\xi}{\beta}\left(e^{\beta\frac{x}{\xi}}-e^{-\beta\frac{h}{\xi}}\right) - e^{\beta\frac{x}{\xi}}\frac{\xi}{\beta}\left(e^{-\beta\frac{h}{\xi}}-e^{-\beta\frac{x}{\xi}}\right)\right] = \\
&= (\epsilon_0-\epsilon_1)E_0\left(1-e^{-\beta\frac{h}{\xi}}\,\mathrm{ch}\left(\beta\frac{x}{\xi}\right)\right)
\end{aligned}
$$

2.3 D_3

$$D_3(x) = \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \int\limits_{-h}^{+h} e^{-\beta\frac{|x-x'|}{\xi}} \,\mathrm{ch}\left(\frac{x'}{\xi}\right) dx' =$$

$$= \frac{\beta}{4\xi}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[e^{-\beta\frac{x}{\xi}} \int\limits_{-h}^{x} \left(e^{\frac{x'}{\xi}(1+\beta)} + e^{\frac{x'}{\xi}(\beta-1)}\right) dx' + e^{\beta\frac{x}{\xi}} \int\limits_{x}^{h} \left(e^{\frac{x'}{\xi}(1-\beta)} + e^{-\frac{x'}{\xi}(1+\beta)}\right) dx' \right] =$$

$$= \frac{\beta}{4\xi}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \Bigg[e^{-\beta\frac{x}{\xi}} \left(\frac{\xi}{1+\beta}\left\{e^{\frac{x}{\xi}(1+\beta)} - e^{-\frac{h}{\xi}(1+\beta)}\right\} + \frac{\xi}{\beta-1}\left\{e^{\frac{x}{\xi}(\beta-1)} - e^{-\frac{h}{\xi}(\beta-1)}\right\} \right) +$$

$$+ e^{\beta\frac{x}{\xi}} \left(\frac{\xi}{1-\beta}\left\{e^{\frac{h}{\xi}(1-\beta)} - e^{\frac{x}{\xi}(1-\beta)}\right\} - \frac{\xi}{\beta+1}\left\{e^{-\frac{h}{\xi}(1+\beta)} - e^{-\frac{x}{\xi}(1+\beta)}\right\} \right) \Bigg] =$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{1}{1+\beta}\left(e^{\frac{x}{\xi}} - e^{-\frac{h}{\xi}(1+\beta)-\beta\frac{x}{\xi}} - e^{-\frac{h}{\xi}(1+\beta)+\beta\frac{x}{\xi}} + e^{-\frac{x}{\xi}}\right) + \frac{1}{\beta-1}\left(e^{-\frac{x}{\xi}} - e^{-\frac{h}{\xi}(\beta-1)-\beta\frac{x}{\xi}} - e^{-\frac{h}{\xi}(\beta-1)+\beta\frac{x}{\xi}} + e^{\frac{x}{\xi}}\right) \right]$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{2}{1+\beta}\left(\mathrm{ch}(\frac{x}{\xi}) - \mathrm{ch}(\beta\frac{x}{\xi})e^{-\frac{h}{\xi}(1+\beta)}\right) - \frac{2}{1-\beta}\left(\mathrm{ch}(\frac{x}{\xi}) - \mathrm{ch}(\beta\frac{x}{\xi})e^{-\frac{h}{\xi}(\beta-1)}\right) \right] =$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\frac{4\beta}{1-\beta^2}\,\mathrm{ch}(\frac{x}{\xi}) - 2\,\mathrm{ch}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}} \left(\frac{e^{-\frac{h}{\xi}}}{1+\beta} - \frac{e^{\frac{h}{\xi}}}{1-\beta}\right) \right] =$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\frac{4\beta}{1-\beta^2}\,\mathrm{ch}(\frac{x}{\xi}) - 2\,\mathrm{ch}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}} \left(\frac{-2\,\mathrm{sh}(\frac{h}{\xi}) - 2\beta\,\mathrm{ch}(\frac{h}{\xi})}{1-\beta^2}\right) \right] =$$

$$= \frac{\beta}{1-\beta^2}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\beta\,\mathrm{ch}(\frac{x}{\xi}) + \mathrm{ch}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}} \left(\mathrm{sh}(\frac{h}{\xi}) + \beta\,\mathrm{ch}(\frac{h}{\xi})\right) \right]$$

2.4 $D(x)$

$$D(x) = D_1(x) + D_2(x) + D_3(x) =$$

$$= \epsilon_1 E_0 + \epsilon_1 E_1 e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi}) + (\epsilon_0-\epsilon_1)E_0\left(1 - e^{-\beta\frac{h}{\xi}}\,\mathrm{ch}\left(\beta\frac{x}{\xi}\right)\right) +$$

$$+ \frac{\beta}{1-\beta^2}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\beta\,\mathrm{ch}(\frac{x}{\xi}) + \mathrm{ch}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}} \left(\mathrm{sh}(\frac{h}{\xi}) + \beta\,\mathrm{ch}(\frac{h}{\xi})\right) \right]$$

3 $\frac{d}{dx}D(x) = 0$

$$0 = \epsilon_1 E_1 e^{-\frac{h}{\xi}} \operatorname{sh}(\frac{x}{\xi}) + (\epsilon_0 - \epsilon_1) E_0 \beta e^{-\beta\frac{h}{\xi}} \operatorname{sh}\left(\beta \frac{x}{\xi}\right) + \frac{\beta}{1-\beta^2}(\epsilon_0 - \epsilon_1) E_1 e^{-\frac{h}{\xi}} \left[-\beta \operatorname{sh}(\frac{x}{\xi}) + \beta \operatorname{sh}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)\right]$$

$$\frac{\beta^2}{1-\beta^2}(\epsilon_0 - \epsilon_1) = \frac{\frac{\epsilon_1}{\epsilon_0}}{1-\frac{\epsilon_1}{\epsilon_0}}(\epsilon_0 - \epsilon_1) = \epsilon_1$$

$$0 = \left[(\epsilon_0 - \epsilon_1) E_0 \beta + \epsilon_1 E_1 \left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)\right] \operatorname{sh}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}}$$

$$E_1 = -\frac{(\epsilon_0 - \epsilon_1)\beta}{\epsilon_1 \left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)} E_0$$

$$D(x) = E_0 \left\{ \epsilon_1 - \frac{(\epsilon_0 - \epsilon_1)\beta}{\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)} e^{-\frac{h}{\xi}} \operatorname{ch}\left(\frac{x}{\xi}\right) + (\epsilon_0 - \epsilon_1)\left(1 - e^{-\beta\frac{h}{\xi}} \operatorname{ch}\left(\beta\frac{x}{\xi}\right)\right) - \right.$$

$$\left. - \frac{(\epsilon_0 - \epsilon_1)\beta}{\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)} e^{-\frac{h}{\xi}} \left[-\beta \operatorname{ch}\left(\frac{x}{\xi}\right) + \operatorname{ch}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)\right]\right\} =$$

$$= E_0 \left\{ \epsilon_1 - \frac{(\epsilon_0 - \epsilon_1)\beta}{\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)} e^{-\frac{h}{\xi}} \operatorname{ch}\left(\frac{x}{\xi}\right)[1-\beta] + (\epsilon_0 - \epsilon_1)\left\{1 - \left[1 + \beta e^{-\frac{h}{\xi}}\right] e^{-\beta\frac{h}{\xi}} \operatorname{ch}\left(\beta\frac{x}{\xi}\right)\right\}\right\}$$

Итого мы имеем

$D(x) = E_0 \left\{ \epsilon_1 - \frac{(\epsilon_0 - \epsilon_1)\beta}{\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)} e^{-\frac{h}{\xi}} \operatorname{ch}\left(\frac{x}{\xi}\right)[1-\beta] + (\epsilon_0 - \epsilon_1)\left\{1 - \left[1 + \beta e^{-\frac{h}{\xi}}\right] e^{-\beta\frac{h}{\xi}} \operatorname{ch}\left(\beta\frac{x}{\xi}\right)\right\}\right\}$

$E(x) = E_0 \left(1 - \frac{2(\epsilon_0 - \epsilon_1)\beta}{\epsilon_1 \left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)} e^{-\frac{h}{\xi}} \operatorname{ch}(\frac{x}{\xi})\right)$

Которые все равно различаются как минимум по порядку ϵ.

А значит $D(h) \neq E(\mathrm{h}) \Rightarrow P(h) = \frac{1}{4\pi}\left(D(h) - E(h)\right) \neq 0$