Effective dielectric response of thin water

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I. INTRODUCTION

One may consider, that $\vec{D}$ depends not only on $\vec{E}$ in specific point but on some area around. Lets take in consideration one-dimensional case.

$$D(x) = \int_{-\infty}^{+\infty} K(x - x')E(x')dx' \tag{1}$$

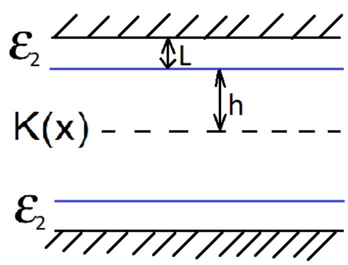
Here $K(x)$ is the non-local dielectric response function given by the inverse Fourier-transform of momentum-space dielectric function $\epsilon^{-1}(q)$ proposed in Ref. [1]:

$$\epsilon(q) = \epsilon_1\epsilon_0 \frac{1 + (q\xi)^2}{\epsilon_1 + \epsilon_0(q\xi)^2} \tag{2}$$

Then $K(x)$ is just a sum of two exponential functions plus a $\delta-$ function term.

II. ELECTRIC FIELD

We will consider that water consists of 3 layers as described on the picture below.



Two "dead" layers with constant dielectric constant $\epsilon_2 = 1.8$ and one layer with dispersion and K(x) described above. We will consider $\epsilon_1 = 5$ and $\epsilon_0 = 80$ as in Ref. [1] Our equation could be written as

$$\frac{d}{dx}D(x) = 4\pi\rho(x) = 4\pi\sigma[\delta(x - h - l) - \delta(x + h + l)] \tag{3}$$

Where

$$D(x) = \int_{-h}^{h} K(x - x')E(x')dx' \tag{4}$$

As in Ref. [1] one could expect electric field in next form

$$E(x) = E_0 + E_1 e^{\frac{x-h-l}{\xi}} + E_1 e^{\frac{-x-h-l}{\xi}} = E_0 + E_1 e^{-\frac{h+l}{\xi}} Cosh(\frac{x}{\xi}) \tag{5}$$

Doing the necessary calculations, we have

$$K(x-x') = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\epsilon(q)e^{iq(x-x')} = \epsilon_1\delta(x-x') + \frac{\beta}{2\xi}exp(-\beta\frac{|x-x'|}{\xi}) \tag{6}$$

$$D(x) = \epsilon_0 E_0 + e^{-\beta\frac{h}{\xi}}Cosh(\beta\frac{x}{\xi})\Big[-(\epsilon_0-\epsilon_1)E_0 + E_1\sqrt{\epsilon_0\epsilon_1}e^{-\frac{h+l}{\xi}}(\beta Cosh(\frac{h}{\xi}) + Sinh(\frac{h}{\xi}))\Big] \tag{7}$$

Obviously from equality above and $\frac{d}{dx}D(x) = 0$(as $x \in \{-h, h\}$) one can find ratio between E_0 and E_1, and then the function $E(x)$:

$$E(x) = E_0\Big(1 + \frac{\epsilon_0-\epsilon_1}{\sqrt{\epsilon_0\epsilon_1}(\beta Cosh(\frac{h}{\xi} + Sinh(\frac{h}{\xi}))}Cosh(\frac{x}{\xi})\Big) \tag{8}$$

We will define

$$\gamma = \frac{\epsilon_0-\epsilon_1}{\sqrt{\epsilon_0\epsilon_1}(\beta Cosh(\frac{h}{\xi} + Sinh(\frac{h}{\xi}))} \tag{9}$$

On the border of "dead" layer of water $D(h) = 4\pi\sigma$, from where we have:

$$E_0 = \frac{4\pi\sigma}{\epsilon_0} \tag{10}$$

One can find ϵ_{eff} using formal equality of energies

$$F = \frac{1}{8\pi}\int\limits_{-h}^{h}D(x)E(x)dx + \frac{2\pi\sigma^2}{\epsilon_2}2l \tag{11}$$

On the other hand

$$F = \frac{2\pi\sigma^2}{\epsilon_{eff}}(2h+2l) \tag{12}$$

There we find effective permeability as

$$\epsilon_{eff} = \frac{h+l}{\frac{1}{\epsilon_0}(h + \xi\gamma Sinh\frac{h}{\xi}) + \frac{1}{\epsilon_2}l} \tag{13}$$

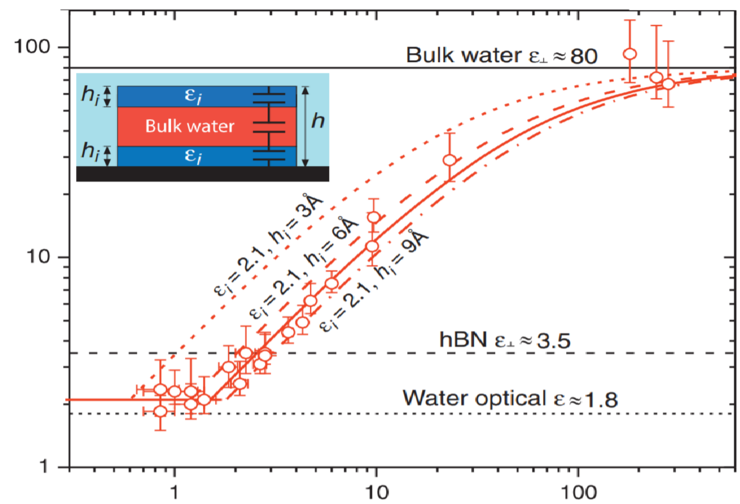
Now we change notations of lengths to those of the paper [2] and obtain:

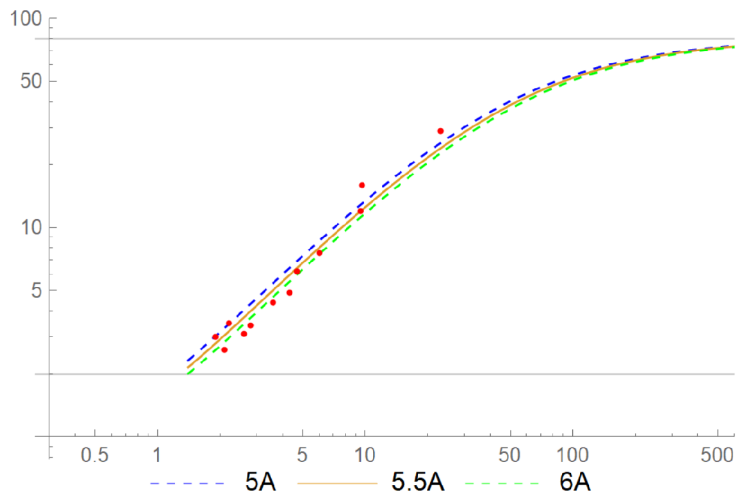
$$\epsilon_{eff} = \frac{h}{\frac{1}{\epsilon_0}(h - 2l + 2\xi\gamma Sinh\frac{h-2l}{\xi}) + \frac{1}{\epsilon_2}2l} \tag{14}$$

and

$$\gamma = \frac{\epsilon_0-\epsilon_1}{\sqrt{\epsilon_0\epsilon_1}(\beta Cosh(\frac{h/2-l}{\xi} + Sinh(\frac{h/2-l}{\xi}))} \tag{15}$$

Comparing graphs with different values of l one may make sure that the choice of $l = 5.5 \pm 0.5$A prvides good approximation of the experimental data [2], as shown in the next figure (full line corresponds to $l = 5.5$A, dashed lines correspond to 5 and 6A).





So that $l = 5.5 \pm 0.5A$.

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[1] Belaya, Levadnyi, Feigel'man, Zh. Eksp. Teor. Fiz. 91, 1336-1345 (1986)

[2] Fumagalli, Esfandiar, Fabregas, Hu, Ares, Janardanan, Yang, Radha, Taniguchi, Watanabe, Gomila, Novoselov, Geim, Science 360 (6395), 1339-1342.