1 K(x)

$$\beta = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$$

$$\epsilon(q) = \epsilon_1\epsilon_0\frac{1+(q\xi)^2}{\epsilon_1+\epsilon_0(q\xi)^2}$$

$$K(x) = \frac{1}{2\pi}\epsilon_0\epsilon_1\int\limits_{-\infty}^{+\infty}\frac{1+(q\xi)^2}{\epsilon_1+\epsilon_0(q\xi)}e^{iqx}dq =$$

$$= \frac{\epsilon_1}{2\pi}\int e^{iqx}(1+\frac{\epsilon_0-\epsilon_1}{\epsilon_1+\epsilon_0(q\xi)^2})dq =$$

$$= \epsilon_1\delta(x) + (\epsilon_0-\epsilon_1)\frac{\beta}{2\xi}\operatorname{sgn}(x)e^{-\beta\frac{|x|}{\epsilon}}$$

2 D(x)

Let's pretend that $E(x) = E_0 + E_1 e^{\frac{x-h}{\xi}} + E_1 e^{\frac{-x-h}{\xi}} = E_0 + 2E_1 e^{-\frac{h}{\xi}}\operatorname{ch}(\frac{x}{\xi})$

$D(x) = \int\limits_{-h}^{h} K(x-x')E(x')dx' = \int\limits_{-h}^{h}\left(\epsilon_1\delta(x-x') + \frac{\beta}{2\xi}\operatorname{sgn}(x-x')(\epsilon_0-\epsilon_1)e^{-\beta\frac{|x-x'|}{\epsilon}}\right)\left(E_0 + 2E_1 e^{-\frac{h}{\xi}}\operatorname{ch}(\frac{x}{\xi})\right)$

2.1 D_1

$D_1(x) = \int \epsilon_1\delta(x-x')\left(E_0 + E_1 e^{-\frac{h}{\xi}}\operatorname{ch}(\frac{x}{\xi})\right) = \epsilon_1 E_0 + \epsilon_1 E_1 e^{-\frac{h}{\xi}}\operatorname{ch}(\frac{x}{\xi})$

2.2 D_2

$$D_2(x) = \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0\int \operatorname{sgn}(x-x')e^{-\beta\frac{|x-x'|}{\epsilon}}dx' =$$

$$= \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0\left[\int\limits_{-h}^{x}\exp(-\beta\frac{x-x'}{\xi})dx' - \int\limits_{x}^{h}\exp(\beta\frac{x-x'}{\xi})\right] =$$

$$= \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0\left[e^{-\beta\frac{x}{\xi}}\frac{\xi}{\beta}\left(e^{\beta\frac{x}{\xi}} - e^{-\beta\frac{h}{\xi}}\right) + e^{\beta\frac{x}{\xi}}\frac{\xi}{\beta}\left(e^{-\beta\frac{h}{\xi}} - e^{-\beta\frac{x}{\xi}}\right)\right] =$$

$$= (\epsilon_0-\epsilon_1)E_0 e^{-\beta\frac{x}{\xi}}\operatorname{ch}\left(\beta\frac{x}{\xi}\right)$$

2.3 D_3

$$D_3(x) = \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \int_{-h}^{+h} \operatorname{sgn}(x-x') e^{-\beta\frac{|x-x'|}{\xi}} \operatorname{ch}\left(\frac{x'}{\xi}\right) dx' =$$

$$= \frac{\beta}{4\xi}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[e^{-\beta\frac{x}{\xi}} \int_{-h}^{x} \left(e^{\frac{x'}{\xi}(1+\beta)} + e^{\frac{x'}{\xi}(\beta-1)} \right) dx' - e^{\beta\frac{x}{\xi}} \int_{x}^{h} \left(e^{\frac{x'}{\xi}(1-\beta)} + e^{-\frac{x'}{\xi}(1+\beta)} \right) dx' \right] =$$

$$= \frac{\beta}{4\xi}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \Bigg[e^{-\beta\frac{x}{\xi}} \left(\frac{\xi}{1+\beta} \left\{ e^{\frac{x}{\xi}(1+\beta)} - e^{-\frac{h}{\xi}(1+\beta)} \right\} + \frac{\xi}{\beta-1} \left\{ e^{\frac{x}{\xi}(\beta-1)} - e^{-\frac{h}{\xi}(\beta-1)} \right\} \right) -$$

$$- e^{\beta\frac{x}{\xi}} \left(\frac{\xi}{1-\beta} \left\{ e^{\frac{h}{\xi}(1-\beta)} - e^{\frac{x}{\xi}(1-\beta)} \right\} - \frac{\xi}{\beta+1} \left\{ e^{-\frac{h}{\xi}(1+\beta)} - e^{-\frac{x}{\xi}(\beta+1)} \right\} \right) \Bigg] =$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{1}{1+\beta} \left(e^{\frac{x}{\xi}} - e^{-\frac{h}{\xi}(1+\beta)-\beta\frac{x}{\xi}} + e^{-\frac{h}{\xi}(1+\beta)+\beta\frac{x}{\xi}} - e^{-\frac{x}{\xi}} \right) + \frac{1}{\beta-1} \left(e^{-\frac{x}{\xi}} - e^{-\frac{h}{\xi}(\beta-1)-\beta\frac{x}{\xi}} + e^{-\frac{h}{\xi}(\beta-1)+\beta\frac{x}{\xi}} - e^{\frac{x}{\xi}} \right) \right]$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{2}{1+\beta} \left(\operatorname{sh}(\frac{x}{\xi}) - \operatorname{sh}(\beta\frac{x}{\xi}) e^{-\frac{h}{\xi}(1+\beta)} \right) + \frac{2}{1-\beta} \left(\operatorname{sh}(\frac{x}{\xi}) - \operatorname{sh}(\beta\frac{x}{\xi}) e^{-\frac{h}{\xi}(\beta-1)} \right) \right] =$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{4}{1-\beta^2} \operatorname{sh}(\frac{x}{\xi}) - 2\operatorname{sh}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\frac{e^{-\frac{h}{\xi}}}{1+\beta} + \frac{e^{\frac{h}{\xi}}}{1-\beta} \right) \right] =$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{4}{1-\beta^2} \operatorname{sh}(\frac{x}{\xi}) - 2\operatorname{sh}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\frac{2\operatorname{ch}(\frac{h}{\xi}) + 2\beta\operatorname{sh}(\frac{h}{\xi})}{1-\beta^2} \right) \right] =$$

$$= \frac{\beta}{1-\beta^2}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\operatorname{sh}(\frac{x}{\xi}) - \operatorname{sh}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\operatorname{ch}(\frac{h}{\xi}) + \beta\operatorname{sh}(\frac{h}{\xi}) \right) \right]$$

2.4 $D(x)$

$$D(x) = D_1(x) + D_2(x) + D_3(x) =$$

$$= \epsilon_1 E_0 + \epsilon_1 E_1 e^{-\frac{h}{\xi}} \operatorname{ch}(\frac{x}{\xi}) + (\epsilon_0-\epsilon_1)E_0 e^{-\beta\frac{x}{\xi}} \operatorname{ch}\left(\beta\frac{x}{\xi}\right) + \frac{\beta}{1-\beta^2}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\operatorname{sh}(\frac{x}{\xi}) - \operatorname{sh}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\operatorname{ch}(\frac{h}{\xi}) + \beta\operatorname{sh}(\frac{h}{\xi}) \right) \right]$$

3 $\frac{d}{dx}D(x) = 0$

$$0 = \epsilon_1 E_1 e^{-\frac{h}{\xi}} \operatorname{sh}(\frac{x}{\xi}) - \beta(\epsilon_0-\epsilon_1)E_0 e^{-\beta\frac{x}{\xi}} \operatorname{ch}\left(\beta\frac{x}{\xi}\right) + (\epsilon_0-\epsilon_1)E_0 e^{-\beta\frac{x}{\xi}} \operatorname{sh}\left(\beta\frac{x}{\xi}\right) +$$

$$+ \frac{\beta}{1-\beta^2}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\operatorname{ch}(\frac{x}{\xi}) - \beta\operatorname{ch}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\operatorname{ch}(\frac{h}{\xi}) + \beta\operatorname{sh}(\frac{h}{\xi}) \right) \right]$$

$$\frac{\beta}{1-\beta^2} = \frac{\sqrt{\frac{\epsilon_1}{\epsilon_0}}}{1-\frac{\epsilon_1}{\epsilon_0}} = \frac{\sqrt{\epsilon_0\epsilon_1}}{\epsilon_0-\epsilon_1}$$

$$0 = (\epsilon_0-\epsilon_1)E_0 e^{-\beta\frac{x}{\xi}} \left(-\beta\ \operatorname{ch}\left(\beta\frac{x}{\xi}\right) + \operatorname{sh}\left(\beta\frac{x}{\xi}\right) \right) + E_1 e^{-\frac{h}{\xi}} \left[\sqrt{\epsilon_0\epsilon_1}\operatorname{ch}(\frac{x}{\xi}) - \epsilon_1\operatorname{ch}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\operatorname{ch}(\frac{h}{\xi}) + \beta\operatorname{sh}(\frac{h}{\xi}) \right) + \epsilon_1\operatorname{sh}\left(\frac{x}{\xi}\right) \right]$$