1 K(x)

$$\beta = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$$

$$\epsilon(q) = \epsilon_1\epsilon_0\frac{1+(q\xi)^2}{\epsilon_1+\epsilon_0(q\xi)^2}$$

$$K(x) = \frac{1}{2\pi}\epsilon_0\epsilon_1\int\limits_{-\infty}^{+\infty}\frac{1+(q\xi)^2}{\epsilon_1+\epsilon_0(q\xi)}e^{iqx}dq =$$

$$= \frac{\epsilon_1}{2\pi}\int e^{iqx}(1+\frac{\epsilon_0-\epsilon_1}{\epsilon_1+\epsilon_0(q\xi)^2})dq =$$

$$= \epsilon_1\delta(x) + (\epsilon_0-\epsilon_1)\frac{\beta}{2\xi}e^{-\beta\frac{|x|}{\xi}}$$

2 D(x)

Let's pretend that $E(x) = E_0 + E_1 e^{\frac{x-h}{\xi}} + E_1 e^{\frac{-x-h}{\xi}} = E_0 + 2E_1 e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi})$

$D(x) = \int\limits_{-h}^{h} K(x-x')E(x')dx' = \int\limits_{-h}^{h}\left(\epsilon_1\delta(x-x') + \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)e^{-\beta\frac{|x-x'|}{\epsilon}}\right)\left(E_0 + 2E_1e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi})\right)$

2.1 D_1

$D_1(x) = \int \epsilon_1\delta(x-x')\left(E_0 + E_1e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi})\right) = \epsilon_1E_0 + \epsilon_1E_1e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi})$

2.2 D_2

$$D_2(x) = \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0\int e^{-\beta\frac{|x-x'|}{\epsilon}}dx' =$$

$$= \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0\left[\int\limits_{-h}^{x}\exp(-\beta\frac{x-x'}{\xi})dx' + \int\limits_{x}^{h}\exp(\beta\frac{x-x'}{\xi})\right] =$$

$$= \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0\left[e^{-\beta\frac{x}{\xi}}\frac{\xi}{\beta}\left(e^{\beta\frac{x}{\xi}} - e^{-\beta\frac{h}{\xi}}\right) - e^{\beta\frac{x}{\xi}}\frac{\xi}{\beta}\left(e^{-\beta\frac{h}{\xi}} - e^{-\beta\frac{x}{\xi}}\right)\right] =$$

$$= (\epsilon_0-\epsilon_1)E_0\left(1 - e^{-\beta\frac{h}{\xi}}\,\mathrm{ch}\left(\beta\frac{x}{\xi}\right)\right)$$

2.3 D_3

$$D_3(x) = \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \int\limits_{-h}^{+h} e^{-\beta\frac{|x-x'|}{\xi}} \operatorname{ch}\left(\frac{x'}{\xi}\right) dx' =$$

$$= \frac{\beta}{4\xi}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[e^{-\beta\frac{x}{\xi}} \int\limits_{-h}^{x} \left(e^{\frac{x'}{\xi}(1+\beta)} + e^{\frac{x'}{\xi}(\beta-1)} \right) dx' + e^{\beta\frac{x}{\xi}} \int\limits_{x}^{h} \left(e^{\frac{x'}{\xi}(1-\beta)} + e^{-\frac{x'}{\xi}(1+\beta)} \right) dx' \right] =$$

$$= \frac{\beta}{4\xi}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \Bigg[e^{-\beta\frac{x}{\xi}} \left(\frac{\xi}{1+\beta} \left\{ e^{\frac{x}{\xi}(1+\beta)} - e^{-\frac{h}{\xi}(1+\beta)} \right\} + \frac{\xi}{\beta-1} \left\{ e^{\frac{x}{\xi}(\beta-1)} - e^{-\frac{h}{\xi}(\beta-1)} \right\} \right) +$$

$$+ e^{\beta\frac{x}{\xi}} \left(\frac{\xi}{1-\beta} \left\{ e^{\frac{h}{\xi}(1-\beta)} - e^{\frac{x}{\xi}(1-\beta)} \right\} - \frac{\xi}{\beta+1} \left\{ e^{-\frac{h}{\xi}(1+\beta)} - e^{-\frac{x}{\xi}(1+\beta)} \right\} \right) \Bigg] =$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{1}{1+\beta} \left(e^{\frac{x}{\xi}} - e^{-\frac{h}{\xi}(1+\beta)-\beta\frac{x}{\xi}} - e^{-\frac{h}{\xi}(1+\beta)+\beta\frac{x}{\xi}} + e^{-\frac{x}{\xi}} \right) + \frac{1}{\beta-1} \left(e^{-\frac{x}{\xi}} - e^{-\frac{h}{\xi}(\beta-1)-\beta\frac{x}{\xi}} - e^{-\frac{h}{\xi}(\beta-1)+\beta\frac{x}{\xi}} + e^{\frac{x}{\xi}} \right) \right]$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{2}{1+\beta} \left(\operatorname{ch}(\frac{x}{\xi}) - \operatorname{ch}(\beta\frac{x}{\xi}) e^{-\frac{h}{\xi}(1+\beta)} \right) - \frac{2}{1-\beta} \left(\operatorname{ch}(\frac{x}{\xi}) - \operatorname{ch}(\beta\frac{x}{\xi}) e^{-\frac{h}{\xi}(\beta-1)} \right) \right] =$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\frac{4\beta}{1-\beta^2} \operatorname{ch}(\frac{x}{\xi}) - 2\operatorname{ch}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\frac{e^{-\frac{h}{\xi}}}{1+\beta} - \frac{e^{\frac{h}{\xi}}}{1-\beta} \right) \right] =$$

$$= \frac{1}{4}\beta(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\frac{4\beta}{1-\beta^2} \operatorname{ch}(\frac{x}{\xi}) - 2\operatorname{ch}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\frac{-2\operatorname{sh}(\frac{h}{\xi}) - 2\beta\operatorname{ch}(\frac{h}{\xi})}{1-\beta^2} \right) \right] =$$

$$= \frac{\beta}{1-\beta^2}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\beta\operatorname{ch}(\frac{x}{\xi}) + \operatorname{ch}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\operatorname{sh}(\frac{h}{\xi}) + \beta\operatorname{ch}(\frac{h}{\xi}) \right) \right]$$

2.4 $D(x)$

$$D(x) = D_1(x) + D_2(x) + D_3(x) =$$

$$= \epsilon_1 E_0 + \epsilon_1 E_1 e^{-\frac{h}{\xi}} \operatorname{ch}(\frac{x}{\xi}) + (\epsilon_0-\epsilon_1)E_0 \left(1 - e^{-\beta\frac{h}{\xi}} \operatorname{ch}\left(\beta\frac{x}{\xi}\right) \right) +$$

$$+ \frac{\beta}{1-\beta^2}(\epsilon_0-\epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\beta\operatorname{ch}(\frac{x}{\xi}) + \operatorname{ch}(\beta\frac{x}{\xi}) e^{-\beta\frac{h}{\xi}} \left(\operatorname{sh}(\frac{h}{\xi}) + \beta\operatorname{ch}(\frac{h}{\xi}) \right) \right]$$

$$= \epsilon_0 E_0 + \operatorname{ch}\left(\beta\frac{x}{\xi}\right) e^{-\beta\frac{h}{\xi}} \left[-(\epsilon_0-\epsilon_1)E_0 + \frac{E_1\epsilon_1}{\beta} \left(\operatorname{sh}(\frac{h}{\xi}) + \beta\operatorname{ch}(\frac{h}{\xi}) \right) \right]$$

3 $\frac{d}{dx}D(x) = 0$

$E_1 = E_0\beta\frac{\epsilon_0-\epsilon_1}{\epsilon_1\left[\operatorname{sh}\left(\frac{h}{\xi}\right)+\beta\ \operatorname{ch}\left(\frac{h}{\xi}\right)\right]}$

Итого мы имеем

$D = E_0\epsilon_0 = 4\pi\sigma$

$E(x) = E_0\left(1 + \frac{(\epsilon_0-\epsilon_1)\beta}{\epsilon_1\left(\operatorname{sh}(\frac{h}{\xi})+\beta\operatorname{ch}(\frac{h}{\xi})\right)} e^{-\frac{h}{\xi}} \operatorname{ch}(\frac{x}{\xi})\right)$

Что полностью соответствует написанному в заметке. Видимо, отсутствие $(\epsilon_0-\epsilon_1)$ перед вторым слагаемым в формуле (6) в заметке - это просто опечатка, а не неправильное решение. Выражая P(x)

$P(x) = \frac{E_0}{4\pi}\left((\epsilon_0-1) - \frac{(\epsilon_0-\epsilon_1)\beta}{\epsilon_1\left(\operatorname{sh}(\frac{h}{\xi})+\beta\operatorname{ch}(\frac{h}{\xi})\right)} e^{-\frac{h}{\xi}} \operatorname{ch}(\frac{x}{\xi})\right)$

4 ϵ_{eff}

$$W = \frac{(4\pi\sigma)^2}{8\pi\epsilon_0}\int_{-h}^{h}\left(1+\frac{(\epsilon_0-\epsilon_1)\beta}{\epsilon_1\left(\mathrm{sh}(\frac{h}{\xi})+\beta\,\mathrm{ch}(\frac{h}{\xi})\right)}e^{-\frac{h}{\xi}}\,\mathrm{ch}(\frac{x}{\xi})\right)dx = \frac{(4\pi\sigma)^2}{\epsilon_0}\left(2h+2\xi\frac{(\epsilon_0-\epsilon_1)\beta}{\epsilon_1\left(\mathrm{sh}(\frac{h}{\xi})+\beta\,\mathrm{ch}(\frac{h}{\xi})\right)}e^{-\frac{h}{\xi}}\,\mathrm{sh}(\frac{h}{\xi})\right)dx \tag{4.1}$$

$$W = \frac{4\pi\sigma^2}{\epsilon_{eff}}h \tag{4.2}$$

$$\epsilon_{eff} = \frac{\epsilon_0}{1+\frac{\xi}{h}\frac{(\epsilon_0-\epsilon_1)\beta}{\epsilon_1\left(\mathrm{sh}(\frac{h}{\xi})+\beta\,\mathrm{ch}(\frac{h}{\xi})\right)}e^{-\frac{h}{\xi}}\,\mathrm{sh}(\frac{h}{\xi})} \approx \frac{\epsilon_0}{1+\frac{\xi}{h}\frac{(\epsilon_0-\epsilon_1)\beta}{\epsilon_1\left(\mathrm{sh}(\frac{h}{\xi})+\beta\,\mathrm{ch}(\frac{h}{\xi})\right)}e^{-\frac{h}{\xi}}\,\mathrm{sh}(\frac{h}{\xi})} \approx \frac{\epsilon_0}{1+\sqrt{\frac{\epsilon_0}{\epsilon_1}}\frac{\xi}{h}e^{-\frac{h}{\xi}}} \tag{4.3}$$