

1 K(x)

$$\begin{aligned}
\beta &= \sqrt{\frac{\epsilon_1}{\epsilon_0}} \\
\epsilon(q) &= \epsilon_1 \epsilon_0 \frac{1 + (q\xi)^2}{\epsilon_1 + \epsilon_0 (q\xi)^2} \\
K(x) &= \frac{1}{2\pi} \epsilon_0 \epsilon_1 \int_{-\infty}^{+\infty} \frac{1 + (q\xi)^2}{\epsilon_1 + \epsilon_0 (q\xi)^2} e^{iqx} dq = \\
&= \frac{\epsilon_1}{2\pi} \int e^{iqx} \left(1 + \frac{\epsilon_0 - \epsilon_1}{\epsilon_1 + \epsilon_0 (q\xi)^2}\right) dq = \\
&= \epsilon_1 \delta(x) + (\epsilon_0 - \epsilon_1) \frac{\beta}{2\xi} e^{-\beta \frac{|x|}{\xi}}
\end{aligned}$$

2 D(x)

Let's pretend that $E(x) = E_0 + E_1 e^{\frac{x-h}{\xi}} + E_1 e^{-\frac{x-h}{\xi}} = E_0 + 2E_1 e^{-\frac{h}{\xi}} \text{ch}\left(\frac{x}{\xi}\right)$

$$D(x) = \int_{-h}^h K(x-x') E(x') dx' = \int_{-h}^h \left(\epsilon_1 \delta(x-x') + \frac{\beta}{2\xi} (\epsilon_0 - \epsilon_1) e^{-\beta \frac{|x-x'|}{\xi}} \right) \left(E_0 + 2E_1 e^{-\frac{h}{\xi}} \text{ch}\left(\frac{x}{\xi}\right) \right)$$

2.1 D_1

$$D_1(x) = \int \epsilon_1 \delta(x-x') \left(E_0 + E_1 e^{-\frac{h}{\xi}} \text{ch}\left(\frac{x}{\xi}\right) \right) = \epsilon_1 E_0 + \epsilon_1 E_1 e^{-\frac{h}{\xi}} \text{ch}\left(\frac{x}{\xi}\right)$$

2.2 D_2

$$\begin{aligned}
D_2(x) &= \frac{\beta}{2\xi} (\epsilon_0 - \epsilon_1) E_0 \int e^{-\beta \frac{|x-x'|}{\xi}} dx' = \\
&= \frac{\beta}{2\xi} (\epsilon_0 - \epsilon_1) E_0 \left[\int_{-h}^x \exp\left(-\beta \frac{x-x'}{\xi}\right) dx' + \int_x^h \exp\left(\beta \frac{x-x'}{\xi}\right) dx' \right] = \\
&= \frac{\beta}{2\xi} (\epsilon_0 - \epsilon_1) E_0 \left[e^{-\beta \frac{x}{\xi}} \frac{\xi}{\beta} \left(e^{\beta \frac{x}{\xi}} - e^{-\beta \frac{h}{\xi}} \right) - e^{\beta \frac{x}{\xi}} \frac{\xi}{\beta} \left(e^{-\beta \frac{h}{\xi}} - e^{-\beta \frac{x}{\xi}} \right) \right] = \\
&= (\epsilon_0 - \epsilon_1) E_0 \left(1 - e^{-\beta \frac{h}{\xi}} \text{ch}\left(\beta \frac{x}{\xi}\right) \right)
\end{aligned}$$

2.3 D_3

$$\begin{aligned}
D_3(x) &= \frac{\beta}{2\xi}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \int_{-h}^{+h} e^{-\beta \frac{|x-x'|}{\xi}} \operatorname{ch}\left(\frac{x'}{\xi}\right) dx' = \\
&= \frac{\beta}{4\xi}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[e^{-\beta \frac{x}{\xi}} \int_{-h}^x \left(e^{\frac{x'}{\xi}(1+\beta)} + e^{\frac{x'}{\xi}(\beta-1)} \right) dx' + e^{\beta \frac{x}{\xi}} \int_x^h \left(e^{\frac{x'}{\xi}(1-\beta)} + e^{-\frac{x'}{\xi}(1+\beta)} \right) dx' \right] = \\
&= \frac{\beta}{4\xi}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[e^{-\beta \frac{x}{\xi}} \left(\frac{\xi}{1+\beta} \left\{ e^{\frac{x}{\xi}(1+\beta)} - e^{-\frac{h}{\xi}(1+\beta)} \right\} + \frac{\xi}{\beta-1} \left\{ e^{\frac{x}{\xi}(\beta-1)} - e^{-\frac{h}{\xi}(\beta-1)} \right\} \right) + \right. \\
&\quad \left. + e^{\beta \frac{x}{\xi}} \left(\frac{\xi}{1-\beta} \left\{ e^{\frac{h}{\xi}(1-\beta)} - e^{\frac{x}{\xi}(1-\beta)} \right\} - \frac{\xi}{\beta+1} \left\{ e^{-\frac{h}{\xi}(1+\beta)} - e^{-\frac{x}{\xi}(1+\beta)} \right\} \right) \right] = \\
&= \frac{1}{4}\beta(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{1}{1+\beta} \left(e^{\frac{x}{\xi}} - e^{-\frac{h}{\xi}(1+\beta)-\beta \frac{x}{\xi}} - e^{-\frac{h}{\xi}(1+\beta)+\beta \frac{x}{\xi}} + e^{-\frac{x}{\xi}} \right) + \frac{1}{\beta-1} \left(e^{-\frac{x}{\xi}} - e^{-\frac{h}{\xi}(\beta-1)-\beta \frac{x}{\xi}} - e^{-\frac{h}{\xi}(\beta-1)+\beta \frac{x}{\xi}} + e^{\frac{x}{\xi}} \right) \right] \\
&= \frac{1}{4}\beta(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{2}{1+\beta} \left(\operatorname{ch}\left(\frac{x}{\xi}\right) - \operatorname{ch}\left(\beta \frac{x}{\xi}\right) e^{-\frac{h}{\xi}(1+\beta)} \right) - \frac{2}{1-\beta} \left(\operatorname{ch}\left(\frac{x}{\xi}\right) - \operatorname{ch}\left(\beta \frac{x}{\xi}\right) e^{-\frac{h}{\xi}(\beta-1)} \right) \right] = \\
&= \frac{1}{4}\beta(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\frac{4\beta}{1-\beta^2} \operatorname{ch}\left(\frac{x}{\xi}\right) - 2 \operatorname{ch}\left(\beta \frac{x}{\xi}\right) e^{-\beta \frac{h}{\xi}} \left(\frac{e^{-\frac{h}{\xi}}}{1+\beta} - \frac{e^{\frac{h}{\xi}}}{1-\beta} \right) \right] = \\
&= \frac{1}{4}\beta(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\frac{4\beta}{1-\beta^2} \operatorname{ch}\left(\frac{x}{\xi}\right) - 2 \operatorname{ch}\left(\beta \frac{x}{\xi}\right) e^{-\beta \frac{h}{\xi}} \left(\frac{-2 \operatorname{sh}\left(\frac{h}{\xi}\right) - 2\beta \operatorname{ch}\left(\frac{h}{\xi}\right)}{1-\beta^2} \right) \right] = \\
&= \frac{\beta}{1-\beta^2}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\beta \operatorname{ch}\left(\frac{x}{\xi}\right) + \operatorname{ch}\left(\beta \frac{x}{\xi}\right) e^{-\beta \frac{h}{\xi}} \left(\operatorname{sh}\left(\frac{h}{\xi}\right) + \beta \operatorname{ch}\left(\frac{h}{\xi}\right) \right) \right]
\end{aligned}$$

2.4 $D(x)$

$$\begin{aligned}
D(x) &= D_1(x) + D_2(x) + D_3(x) = \\
&= \epsilon_1 E_0 + \epsilon_1 E_1 e^{-\frac{h}{\xi}} \operatorname{ch}\left(\frac{x}{\xi}\right) + (\epsilon_0 - \epsilon_1) E_0 \left(1 - e^{-\beta \frac{h}{\xi}} \operatorname{ch}\left(\beta \frac{x}{\xi}\right) \right) + \\
&\quad + \frac{\beta}{1-\beta^2}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\beta \operatorname{ch}\left(\frac{x}{\xi}\right) + \operatorname{ch}\left(\beta \frac{x}{\xi}\right) e^{-\beta \frac{h}{\xi}} \left(\operatorname{sh}\left(\frac{h}{\xi}\right) + \beta \operatorname{ch}\left(\frac{h}{\xi}\right) \right) \right] \\
&= \epsilon_0 E_0 + \operatorname{ch}\left(\beta \frac{x}{\xi}\right) e^{-\beta \frac{h}{\xi}} \left[-(\epsilon_0 - \epsilon_1)E_0 + \frac{E_1 \epsilon_1}{\beta} \left(\operatorname{sh}\left(\frac{h}{\xi}\right) + \beta \operatorname{ch}\left(\frac{h}{\xi}\right) \right) \right]
\end{aligned}$$

3 $\frac{d}{dx}D(x) = 0$

$$E_1 = E_0 \beta \frac{\epsilon_0 - \epsilon_1}{\epsilon_1 [\operatorname{sh}\left(\frac{h}{\xi}\right) + \beta \operatorname{ch}\left(\frac{h}{\xi}\right)]}$$

Итого мы имеем

$$D = E_0 \epsilon_0 = 4\pi\sigma$$

$$E(x) = E_0 \left(1 + \frac{(\epsilon_0 - \epsilon_1)\beta}{\epsilon_1 (\operatorname{sh}\left(\frac{h}{\xi}\right) + \beta \operatorname{ch}\left(\frac{h}{\xi}\right))} e^{-\frac{h}{\xi}} \operatorname{ch}\left(\frac{x}{\xi}\right) \right)$$

Что полностью соответствует написанному в заметке. Видимо, отсутствие $(\epsilon_0 - \epsilon_1)$ перед вторым слагаемым в формуле (6) в заметке - это просто опечатка, а не неправильное решение. Выражая $P(x)$

$$P(x) = \frac{E_0}{4\pi} \left((\epsilon_0 - 1) - \frac{(\epsilon_0 - \epsilon_1)\beta}{\epsilon_1 (\operatorname{sh}\left(\frac{h}{\xi}\right) + \beta \operatorname{ch}\left(\frac{h}{\xi}\right))} e^{-\frac{h}{\xi}} \operatorname{ch}\left(\frac{x}{\xi}\right) \right)$$

4 ϵ_{eff}

$$W = \frac{(4\pi\sigma)^2}{8\pi\epsilon_0} \int_{-h}^h \left(1 + \frac{(\epsilon_0 - \epsilon_1)\beta}{\epsilon_1 \left(\text{sh}\left(\frac{h}{\xi}\right) + \beta \text{ch}\left(\frac{h}{\xi}\right) \right)} e^{-\frac{h}{\xi} \text{ch}\left(\frac{x}{\xi}\right)} \right) dx = \frac{(4\pi\sigma)^2}{\epsilon_0} \left(2h + 2\xi \frac{(\epsilon_0 - \epsilon_1)\beta}{\epsilon_1 \left(\text{sh}\left(\frac{h}{\xi}\right) + \beta \text{ch}\left(\frac{h}{\xi}\right) \right)} e^{-\frac{h}{\xi} \text{sh}\left(\frac{h}{\xi}\right)} \right) dx \quad (4.1)$$

$$W = \frac{4\pi\sigma^2}{\epsilon_{eff}} h \quad (4.2)$$

$$\epsilon_{eff} = \frac{\epsilon_0}{1 + \frac{\xi}{h} \frac{(\epsilon_0 - \epsilon_1)\beta}{\epsilon_1 \left(\text{sh}\left(\frac{h}{\xi}\right) + \beta \text{ch}\left(\frac{h}{\xi}\right) \right)} e^{-\frac{h}{\xi} \text{sh}\left(\frac{h}{\xi}\right)}} \approx \frac{\epsilon_0}{1 + \frac{\xi}{h} \frac{(\epsilon_0 - \epsilon_1)\beta}{\epsilon_1 \left(\text{sh}\left(\frac{h}{\xi}\right) + \beta \text{ch}\left(\frac{h}{\xi}\right) \right)} e^{-\frac{h}{\xi} \text{sh}\left(\frac{h}{\xi}\right)}} \approx \frac{\epsilon_0}{1 + \frac{\xi}{h} \frac{1}{\beta} e^{-\frac{h}{\xi}}} \quad (4.3)$$