1 P(x)

Будем решать систему записанную в (1).

$$\begin{cases} \frac{d(E+4\pi P)}{dx} = 0 \\ \frac{1}{\alpha}E = P - \xi_1^2 \frac{\partial^2 P}{\partial x^2} \qquad \text{где } \xi_1 = \sqrt{\frac{\delta}{\alpha}}, \alpha = \frac{1}{\epsilon_0} \\ D = 4\pi\sigma \\ P(-h) = P(h) = 0 \end{cases} \tag{1.1}$$

$$E = D - 4\pi P \tag{1.2}$$

$$\frac{\partial^2 P}{\partial x^2} - \frac{(4\pi/\alpha + 1)}{\xi_1^2} P = -\frac{1}{\alpha \xi_1^2} D \tag{1.3}$$

$$P(x) = A_1 e^{\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}x} + A_2 e^{-\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}x} + \frac{1}{4\pi+\alpha}D \tag{1.4}$$

$$\begin{cases} P(-h) = 0 = A_1 e^{-\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h} + A_2 e^{\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h} + \frac{1}{4\pi+\alpha}D \\ P(h) = 0 = A_1 e^{\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h} + A_2 e^{-\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h} + \frac{1}{4\pi+\alpha}D \end{cases} \tag{1.5}$$

$$A_1 = A_2 = -\frac{D}{(4\pi+\alpha)\,\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h\right)}$$

$$P(x) = \frac{D}{4\pi+\alpha}\left[1 - \frac{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}x\right)}{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h\right)}\right] \tag{1.6}$$

$$E(x) = D - 4\pi P = \frac{4\pi D}{4\pi+\alpha}\left[\frac{\alpha}{4\pi} + \frac{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}x\right)}{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h\right)}\right] \tag{1.7}$$

2 ϵ_{eff}

$$W = \frac{1}{8\pi}\int_{-h}^{h} E(x) D dx = \frac{4\pi(4\pi\sigma)^2}{8\pi(4\pi+\alpha)}\left[2\alpha h\frac{1}{4\pi} + \frac{1}{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h\right)}\int_{-h}^{h}\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}x\right)dx\right] \tag{2.1}$$

$$= \frac{(4\pi\sigma)^2}{(4\pi+\alpha)}\left[\alpha h/4\pi + \frac{\xi_1}{\sqrt{4\pi/\alpha+1}}\,\mathrm{th}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h\right)\right]$$

С другой стороны (2.2)

$$W = \frac{4\pi\sigma^2}{\epsilon_{eff}}h \tag{2.3}$$

$$\epsilon_{eff} = \frac{1+\frac{\alpha}{4\pi}}{\alpha/4\pi + \frac{\xi_1}{h}\frac{4\pi}{\sqrt{4\pi/\alpha+1}}\,\mathrm{th}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_1}h\right)} \approx \frac{1+\frac{\alpha}{4\pi}}{\frac{\alpha}{4\pi} + \frac{\xi_1}{h}\frac{4\pi}{\sqrt{4\pi/\alpha+1}}} = \frac{1+\frac{1}{4\pi\epsilon_0}}{\frac{1}{4\pi\epsilon_0} + \frac{\xi_1}{h}\frac{4\pi}{\sqrt{4\pi\epsilon_0+1}}} \approx \frac{\epsilon_0}{\frac{1}{4\pi} + \frac{\xi_1}{h}\sqrt{\frac{\epsilon_0}{4\pi}}} \tag{2.4}$$

Вычисленный прошлым способом ϵ_{eff}

$$\epsilon_{eff} = \frac{\epsilon_0}{1 + \frac{\xi}{h}\frac{\epsilon_0 - \epsilon_1}{\sqrt{\epsilon_0\epsilon_1}\left[\beta\ \ \mathrm{th}^{-1}\left(\frac{h}{\xi}\right)+1\right]}} \approx \frac{\epsilon_0}{1 + \frac{\xi}{h}\sqrt{\frac{\epsilon_0}{\epsilon_1}}} \tag{2.5}$$

$$\beta = \sqrt{\frac{\epsilon_1}{\epsilon_0}} \tag{2.6}$$

Видимо, лишнее 4π получились из перехода из одной системы в другую, и то что у Таганцева записано в СИ как $\kappa = (\alpha\epsilon_0)^{-1}$ в СГС выглядит как $\kappa = 4\pi/\alpha$. Тогда все хорошо и имеем две схожие формулы в пределе $\frac{h}{\xi} \gg \beta \gg 1$

$$\epsilon_{eff} \approx \frac{\epsilon_0}{1 + \frac{\xi_1}{h} 4\pi\sqrt{\epsilon_0}} \tag{2.7}$$

$$\epsilon_{eff} \approx \frac{\epsilon_0}{1 + \frac{\xi}{h}\sqrt{\frac{\epsilon_0}{\epsilon_1}}} \tag{2.8}$$

$$\text{откуда } \xi_1 = \frac{\xi}{4\pi\sqrt{\epsilon_1}}$$

Точные формулы

$$\epsilon_{eff} = \frac{1 + \frac{1}{\epsilon_0}}{\frac{1}{\epsilon_0} + \frac{\xi}{h}\frac{1}{\sqrt{\epsilon_1\epsilon_0 + \epsilon_1}}\,\mathrm{th}\left(4\pi\frac{\sqrt{\epsilon_1\epsilon_0+\epsilon_1}}{\xi}h\right)} \tag{2.9}$$

$$\epsilon_{eff} = \frac{\epsilon_0}{1 + \frac{\xi}{h}\frac{\epsilon_0 - \epsilon_1}{\sqrt{\epsilon_0\epsilon_1}\left[\beta\ \ \mathrm{th}^{-1}\left(\frac{h}{\xi}\right)+1\right]}} \tag{2.10}$$