1 P(x)

Приведём константы в соответствие

$$D = E + 4\pi P = \epsilon_0 E,\ \epsilon_0 \gg 1, \Rightarrow E \approx \frac{4\pi}{\epsilon_0} P \Rightarrow \alpha = \frac{4\pi}{\epsilon_0}$$

$$\xi_T = \xi_T^1 \frac{1}{\beta} = \xi \sqrt{\frac{\epsilon_0}{\epsilon_1}}$$

$$\begin{cases} \xi_T = \xi \sqrt{\frac{\epsilon_0}{\epsilon_1}} \\ \alpha = \frac{4\pi}{\epsilon_0} \ll 1 \\ \epsilon_1 \sim 1 \\ \frac{h}{\xi} \gg 1 \end{cases}$$

Переходя собственно к решению:

$$\begin{cases} \frac{d(E+4\pi P)}{dx} = 0 \\ \frac{1}{\alpha} E = P - \xi_T^2 \frac{\partial^2 P}{\partial x^2} \\ D = 4\pi\sigma \\ P(-h) = P(h) = 0 \end{cases} \tag{1.1}$$

$$E = D - 4\pi P \tag{1.2}$$

$$\frac{\partial^2 P}{\partial x^2} - \frac{(4\pi/\alpha + 1)}{\xi_T^2} P = -\frac{1}{\alpha \xi_T^2} D \tag{1.3}$$

$$P(x) = A_1 e^{\frac{\sqrt{4\pi/\alpha+1}}{\xi_T} x} + A_2 e^{-\frac{\sqrt{4\pi/\alpha+1}}{\xi_T} x} + \frac{1}{4\pi + \alpha} D \tag{1.4}$$

$$\begin{cases} P(-h) = 0 = A_1 e^{-\frac{\sqrt{4\pi/\alpha+1}}{\xi_T} h} + A_2 e^{\frac{\sqrt{4\pi/\alpha+1}}{\xi_T} h} + \frac{1}{4\pi+\alpha} D \\ P(h) = 0 = A_1 e^{\frac{\sqrt{4\pi/\alpha+1}}{\xi_T} h} + A_2 e^{-\frac{\sqrt{4\pi/\alpha+1}}{\xi_T} h} + \frac{1}{4\pi+\alpha} D \end{cases} \tag{1.5}$$

$$A_1 = A_2 = -\frac{D}{(4\pi + \alpha)\,\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T} h\right)}$$

$$P(x) = \frac{D}{4\pi + \alpha} \left[1 - \frac{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T} x\right)}{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T} h\right)} \right] = \frac{4\pi\sigma}{4\pi + \alpha} \left[1 - e^{\frac{x-h}{\sqrt{\epsilon_1}\xi}} \right] \tag{1.6}$$

2 ϵ_{eff} energy

$$E(x) = D - 4\pi P = \frac{4\pi D}{4\pi+\alpha}\left[\frac{\alpha}{4\pi} + \frac{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}x\right)}{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)}\right] \tag{2.1}$$

$$W = \frac{1}{8\pi}\int\limits_{-h}^{h} E(x)D dx = \frac{4\pi(4\pi\sigma)^2}{8\pi(4\pi+\alpha)}\left[2\alpha h\frac{1}{4\pi} + \frac{1}{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)}\int\limits_{-h}^{h}\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}x\right)dx\right] \tag{2.2}$$

$$= \frac{(4\pi\sigma)^2}{(4\pi+\alpha)}\left[\alpha h/4\pi + \frac{\xi_T}{\sqrt{4\pi/\alpha+1}}\,\mathrm{th}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)\right]$$

С другой стороны (2.3)

$$W = \frac{4\pi\sigma^2}{\epsilon_{eff}}h \tag{2.4}$$

$$\frac{4\pi\sigma^2}{\epsilon_{eff}}h = \frac{(4\pi\sigma)^2}{(4\pi+\alpha)}\left[\alpha h/4\pi + \frac{\xi_T}{\sqrt{4\pi/\alpha+1}}\,\mathrm{th}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)\right] \tag{2.5}$$

$$\frac{1}{\epsilon_{eff}} = \frac{4\pi}{(4\pi+\alpha)}\left[\alpha/4\pi + \frac{\xi_T/h}{\sqrt{4\pi/\alpha+1}}\,\mathrm{th}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)\right] \tag{2.6}$$

$$\frac{1}{\epsilon_{eff}} = \frac{1}{(1+\alpha/4\pi)}\left[\alpha/4\pi + \frac{\xi_T/h}{\sqrt{4\pi/\alpha+1}}\,\mathrm{th}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)\right] \tag{2.7}$$

$$\epsilon_{eff} = \frac{1+\frac{\alpha}{4\pi}}{\frac{\alpha}{4\pi} + \frac{\xi_T}{h}\frac{1}{\sqrt{4\pi/\alpha+1}}\,\mathrm{th}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)} \approx \frac{\epsilon_0}{1+\epsilon_0\frac{\xi_T}{h}\frac{1}{\sqrt{\epsilon_0}}\,\mathrm{th}\left(\frac{\sqrt{\epsilon_0}}{\xi_T}h\right)} \approx \frac{\epsilon_0}{1+\epsilon_0\frac{\xi}{h}\frac{1}{\sqrt{\epsilon_1}}\,\mathrm{th}\left(\frac{\sqrt{\epsilon_1}}{\xi}h\right)} \tag{2.8}$$

$$\epsilon_{eff} \approx \frac{\epsilon_0}{1+\frac{\epsilon_0}{\sqrt{\epsilon_1}}\frac{\xi}{h}} \tag{2.9}$$

3 ϵ_{eff} <**P**>

$$< P > = \frac{1}{2h}\int\limits_{-h}^{h}\frac{D}{4\pi+\alpha}\left[1 - \frac{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}x\right)}{\mathrm{ch}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)}\right]dx = \frac{4\pi\sigma}{4\pi+\alpha}\left(1 - \frac{\xi_T}{h\sqrt{4\pi/\alpha+1}}\tanh\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)\right) \approx \tag{3.1}$$

$$\approx \frac{4\pi\sigma}{4\pi+\alpha}\left[1 - \frac{\xi_T}{h\sqrt{4\pi/\alpha+1}}\right] = \frac{4\pi\sigma}{4\pi+\alpha}\left[1 - \frac{\xi\sqrt{\frac{\epsilon_0}{\epsilon_1}}}{h\sqrt{\epsilon_0+1}}\right] \approx \frac{4\pi\sigma}{4\pi+\alpha}\left[1 - \frac{\xi}{h\sqrt{\epsilon_1}}\right] \tag{3.2}$$

$$\begin{cases} < D > = < E > + 4\pi < P > \\ < D > = \epsilon_{eff} < E > \end{cases} \tag{3.3}$$

$$\epsilon_{eff} = \frac{D}{D - 4\pi < P >} = \frac{4\pi\sigma}{4\pi\sigma - 4\pi\frac{4\pi\sigma}{4\pi+\alpha}\left[1 - \frac{\xi_T}{h\sqrt{4\pi/\alpha+1}}\tanh\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)\right]} = \frac{4\pi+\alpha}{\alpha + 4\pi\frac{\xi_T}{h\sqrt{4\pi/\alpha+1}}\tanh\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)} = \tag{3.4}$$

$$= \frac{1+\frac{\alpha}{4\pi}}{\frac{\alpha}{4\pi} + \frac{\xi_T}{h\sqrt{4\pi/\alpha+1}}\,\mathrm{th}\left(\frac{\sqrt{4\pi/\alpha+1}}{\xi_T}h\right)} \text{что совпадает с ответом через энгергию} \tag{3.5}$$