1 K(x)

$$\beta = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$$

$$\epsilon(q) = \epsilon_1\epsilon_0 \frac{1+(q\xi)^2}{\epsilon_1 + \epsilon_0(q\xi)^2}$$

$$K(x) = \frac{1}{2\pi}\epsilon_0\epsilon_1 \int\limits_{-\infty}^{+\infty} \frac{1+(q\xi)^2}{\epsilon_1 + \epsilon_0(q\xi)} e^{iqx} dq =$$

$$= \frac{\epsilon_1}{2\pi} \int e^{iqx}(1 + \frac{\epsilon_0 - \epsilon_1}{\epsilon_1 + \epsilon_0(q\xi)^2}) dq =$$

$$= \epsilon_1\delta(x) + (\epsilon_0 - \epsilon_1)\frac{\beta}{2\xi}e^{-\beta\frac{|x|}{\epsilon}}$$

2 D(x)

Let's pretend that $E(x) = E_0 + E_1 e^{\frac{x-h}{\xi}} + E_1 e^{\frac{-x-h}{\xi}} = E_0 + 2E_1 e^{-\frac{h}{\xi}} \operatorname{ch}(\frac{x}{\xi})$

$$D(x) = \int\limits_{-h}^{h} K(x-x')E(x')dx' = \int\limits_{-h}^{h} \left(\epsilon_1\delta(x-x') + \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)e^{-\beta\frac{|x-x'|}{\epsilon}}\right)\left(E_0 + 2E_1 e^{-\frac{h}{\xi}}\operatorname{ch}(\frac{x}{\xi})\right)$$

2.1 D_1

$$D_1(x) = \int \epsilon_1\delta(x-x')\left(E_0 + E_1 e^{-\frac{h}{\xi}}\operatorname{ch}(\frac{x}{\xi})\right) = \epsilon_1 E_0 + \epsilon_1 E_1 e^{-\frac{h}{\xi}}\operatorname{ch}(\frac{x}{\xi})$$

2.2 D_2

$$D_2(x) = \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0 \int e^{-\beta\frac{|x-x'|}{\epsilon}}dx' =$$

$$= \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0 \left[\int\limits_{-h}^{x} \exp(-\beta\frac{x-x'}{\xi})dx' + \int\limits_{x}^{h} \exp(\beta\frac{x-x'}{\xi})\right] =$$

$$= \frac{\beta}{2\xi}(\epsilon_0-\epsilon_1)E_0 \left[e^{-\beta\frac{x}{\xi}}\frac{\xi}{\beta}\left(e^{\beta\frac{x}{\xi}} - e^{-\beta\frac{h}{\xi}}\right) - e^{\beta\frac{x}{\xi}}\frac{\xi}{\beta}\left(e^{-\beta\frac{h}{\xi}} - e^{-\beta\frac{x}{\xi}}\right)\right] =$$

$$= (\epsilon_0-\epsilon_1)E_0 \left(1 - e^{-\beta\frac{h}{\xi}}\operatorname{ch}\left(\beta\frac{x}{\xi}\right)\right)$$

2.3 D_3

$$D_3(x) = \frac{\beta}{2\xi}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \int_{-h}^{+h} e^{-\beta\frac{|x-x'|}{\xi}} \operatorname{ch}\left(\frac{x'}{\xi}\right) dx' =$$

$$= \frac{\beta}{4\xi}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[e^{-\beta\frac{x}{\xi}} \int_{-h}^{x} \left(e^{\frac{x'}{\xi}(1+\beta)} + e^{\frac{x'}{\xi}(\beta-1)} \right) dx' + e^{\beta\frac{x}{\xi}} \int_{x}^{h} \left(e^{\frac{x'}{\xi}(1-\beta)} + e^{-\frac{x'}{\xi}(1+\beta)} \right) dx' \right] =$$

$$= \frac{\beta}{4\xi}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[e^{-\beta\frac{x}{\xi}} \left(\frac{\xi}{1+\beta} \left\{ e^{\frac{x}{\xi}(1+\beta)} - e^{-\frac{h}{\xi}(1+\beta)} \right\} + \frac{\xi}{\beta-1} \left\{ e^{\frac{x}{\xi}(\beta-1)} - e^{-\frac{h}{\xi}(\beta-1)} \right\} \right) + \right.$$

$$\left. + e^{\beta\frac{x}{\xi}} \left(\frac{\xi}{1-\beta} \left\{ e^{\frac{h}{\xi}(1-\beta)} - e^{\frac{x}{\xi}(1-\beta)} \right\} - \frac{\xi}{\beta+1} \left\{ e^{-\frac{h}{\xi}(1+\beta)} - e^{-\frac{x}{\xi}(1+\beta)} \right\} \right) \right] =$$

$$= \frac{1}{4}\beta(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{1}{1+\beta} \left(e^{\frac{x}{\xi}} - e^{-\frac{h}{\xi}(1+\beta)-\beta\frac{x}{\xi}} - e^{-\frac{h}{\xi}(1+\beta)+\beta\frac{x}{\xi}} + e^{-\frac{x}{\xi}} \right) + \frac{1}{\beta-1} \left(e^{-\frac{x}{\xi}} - e^{-\frac{h}{\xi}(\beta-1)-\beta\frac{x}{\xi}} - e^{-\frac{h}{\xi}(\beta-1)+\beta\frac{x}{\xi}} + e^{\frac{x}{\xi}} \right) \right]$$

$$= \frac{1}{4}\beta(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[\frac{2}{1+\beta} \left(\operatorname{ch}(\frac{x}{\xi}) - \operatorname{ch}(\beta\frac{x}{\xi})e^{-\frac{h}{\xi}(1+\beta)} \right) - \frac{2}{1-\beta} \left(\operatorname{ch}(\frac{x}{\xi}) - \operatorname{ch}(\beta\frac{x}{\xi})e^{-\frac{h}{\xi}(\beta-1)} \right) \right] =$$

$$= \frac{1}{4}\beta(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\frac{4\beta}{1-\beta^2} \operatorname{ch}(\frac{x}{\xi}) - 2\operatorname{ch}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}} \left(\frac{e^{-\frac{h}{\xi}}}{1+\beta} - \frac{e^{\frac{h}{\xi}}}{1-\beta} \right) \right] =$$

$$= \frac{1}{4}\beta(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\frac{4\beta}{1-\beta^2} \operatorname{ch}(\frac{x}{\xi}) - 2\operatorname{ch}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}} \left(\frac{-2\operatorname{sh}(\frac{h}{\xi}) - 2\beta\operatorname{ch}(\frac{h}{\xi})}{1-\beta^2} \right) \right] =$$

$$= \frac{\beta}{1-\beta^2}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\beta\operatorname{ch}(\frac{x}{\xi}) + \operatorname{ch}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}} \left(\operatorname{sh}(\frac{h}{\xi}) + \beta\operatorname{ch}(\frac{h}{\xi}) \right) \right]$$

2.4 $D(x)$

$$D(x) = D_1(x) + D_2(x) + D_3(x) =$$

$$= \epsilon_1 E_0 + \epsilon_1 E_1 e^{-\frac{h}{\xi}} \operatorname{ch}(\frac{x}{\xi}) + (\epsilon_0 - \epsilon_1)E_0 \left(1 - e^{-\beta\frac{h}{\xi}} \operatorname{ch}\left(\beta\frac{x}{\xi}\right) \right) +$$

$$+ \frac{\beta}{1-\beta^2}(\epsilon_0 - \epsilon_1)E_1 e^{-\frac{h}{\xi}} \left[-\beta\operatorname{ch}(\frac{x}{\xi}) + \operatorname{ch}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}} \left(\operatorname{sh}(\frac{h}{\xi}) + \beta\operatorname{ch}(\frac{h}{\xi}) \right) \right]$$

3 $\frac{d}{dx}D(x) = 0$

$$0 = \epsilon_1 E_1 e^{-\frac{h}{\xi}} \operatorname{sh}(\frac{x}{\xi}) + (\epsilon_0 - \epsilon_1) E_0 \beta e^{-\beta \frac{h}{\xi}} \operatorname{sh}\left(\beta \frac{x}{\xi}\right) + \frac{\beta}{1-\beta^2}(\epsilon_0 - \epsilon_1) E_1 e^{-\frac{h}{\xi}}\left[-\beta \operatorname{sh}(\frac{x}{\xi}) + \beta \operatorname{sh}(\beta\frac{x}{\xi}) e^{-\beta \frac{h}{\xi}}\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)\right]$$

$$\frac{\beta^2}{1-\beta^2}(\epsilon_0 - \epsilon_1) = \frac{\frac{\epsilon_1}{\epsilon_0}}{1-\frac{\epsilon_1}{\epsilon_0}}(\epsilon_0 - \epsilon_1) = \epsilon_1$$

$$0 = \left[(\epsilon_0 - \epsilon_1)E_0 \beta + \epsilon_1 E_1\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)\right]\operatorname{sh}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}}$$

$$E_1 = -\frac{(\epsilon_0 - \epsilon_1)\beta}{\epsilon_1\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)}E_0$$

$$D(x) = E_0\left\{\epsilon_1 - \frac{(\epsilon_0 - \epsilon_1)\beta}{\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)}e^{-\frac{h}{\xi}}\operatorname{ch}\left(\frac{x}{\xi}\right) + (\epsilon_0 - \epsilon_1)\left(1 - e^{-\beta\frac{h}{\xi}}\operatorname{ch}\left(\beta\frac{x}{\xi}\right)\right) - \right.$$

$$\left. - \frac{(\epsilon_0 - \epsilon_1)\beta}{\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)}e^{-\frac{h}{\xi}}\left[-\beta\operatorname{ch}\left(\frac{x}{\xi}\right) + \operatorname{ch}(\beta\frac{x}{\xi})e^{-\beta\frac{h}{\xi}}\left(\operatorname{sh}\frac{h}{\xi} + \beta \operatorname{ch}(\frac{h}{\xi})\right)\right]\right\} =$$

$$= E_0\left\{\epsilon_1 - \frac{(\epsilon_0 - \epsilon_1)\beta}{\left(\operatorname{sh}(\frac{h}{\xi}) + \beta \operatorname{ch}(\frac{h}{\xi})\right)}e^{-\frac{h}{\xi}}\operatorname{ch}\left(\frac{x}{\xi}\right)[1-\beta] + (\epsilon_0 - \epsilon_1)\left\{1 - \left[1 + \beta e^{-\frac{h}{\xi}}\right]e^{-\beta\frac{h}{\xi}}\operatorname{ch}\left(\beta\frac{x}{\xi}\right)\right\}\right\}$$

Итого мы имеем

$$D(x) = E_0\left\{\epsilon_1 - \frac{(\epsilon_0-\epsilon_1)\beta}{(\operatorname{sh}(\frac{h}{\xi})+\beta\operatorname{ch}(\frac{h}{\xi}))}e^{-\frac{h}{\xi}}\operatorname{ch}\left(\frac{x}{\xi}\right)[1-\beta] + (\epsilon_0 - \epsilon_1)\left\{1 - \left[1 + \beta e^{-\frac{h}{\xi}}\right]e^{-\beta\frac{h}{\xi}}\operatorname{ch}\left(\beta\frac{x}{\xi}\right)\right\}\right\}$$

$$E(x) = E_0\left(1 - \frac{2(\epsilon_0-\epsilon_1)\beta}{\epsilon_1(\operatorname{sh}(\frac{h}{\xi})+\beta\operatorname{ch}(\frac{h}{\xi}))}e^{-\frac{h}{\xi}}\operatorname{ch}(\frac{x}{\xi})\right)$$

Которые все равно различаются как минимум по порядку вхождения ϵ.

А значит $D(h) \neq E(\mathrm{h}) \Rightarrow P(h) = \frac{1}{4\pi}\left(D(h) - E(h)\right) \neq 0$

Рассмотрим предел при $\beta \ll 1, \frac{h}{\xi} \gg 1$

$$\frac{(\epsilon_0 - \epsilon_1)\beta}{\left(\operatorname{sh}(\frac{h}{\xi}) + \beta\operatorname{ch}(\frac{h}{\xi})\right)}e^{-\frac{h}{\xi}}\operatorname{ch}\left(\frac{h}{\xi}\right)[1-\beta] \ltimes \quad \# e^{-\frac{h}{\xi}} \to 0$$

$$\left[1 + \beta e^{-\frac{h}{\xi}}\right]e^{-\beta\frac{h}{\xi}}\operatorname{ch}\left(\beta\frac{x}{\xi}\right) \approx \left[1 + \beta e^{-\frac{h}{\xi}}\right]\frac{1}{2} \approx \frac{1}{2}$$

$$(\epsilon_0 - \epsilon_1)\left\{1 - \left[1 + \beta e^{-\frac{h}{\xi}}\right]e^{-\beta\frac{h}{\xi}}\operatorname{ch}\left(\beta\frac{x}{\xi}\right)\right\} \approx (\epsilon_0 - \epsilon_1)\frac{1}{2}$$

$$D(h) = E_0\left(\epsilon_1 + \frac{\epsilon_0 - \epsilon_1}{2}\right) = E_0\frac{\epsilon_0}{2}$$

Скорее всего мы потеряли 2 где-то перед $(\epsilon_0 - \epsilon_1)\left\{1 - \left[1 + \beta e^{-\frac{h}{\xi}}\right]e^{-\beta\frac{h}{\xi}}\operatorname{ch}\left(\beta\frac{x}{\xi}\right)\right\}$