

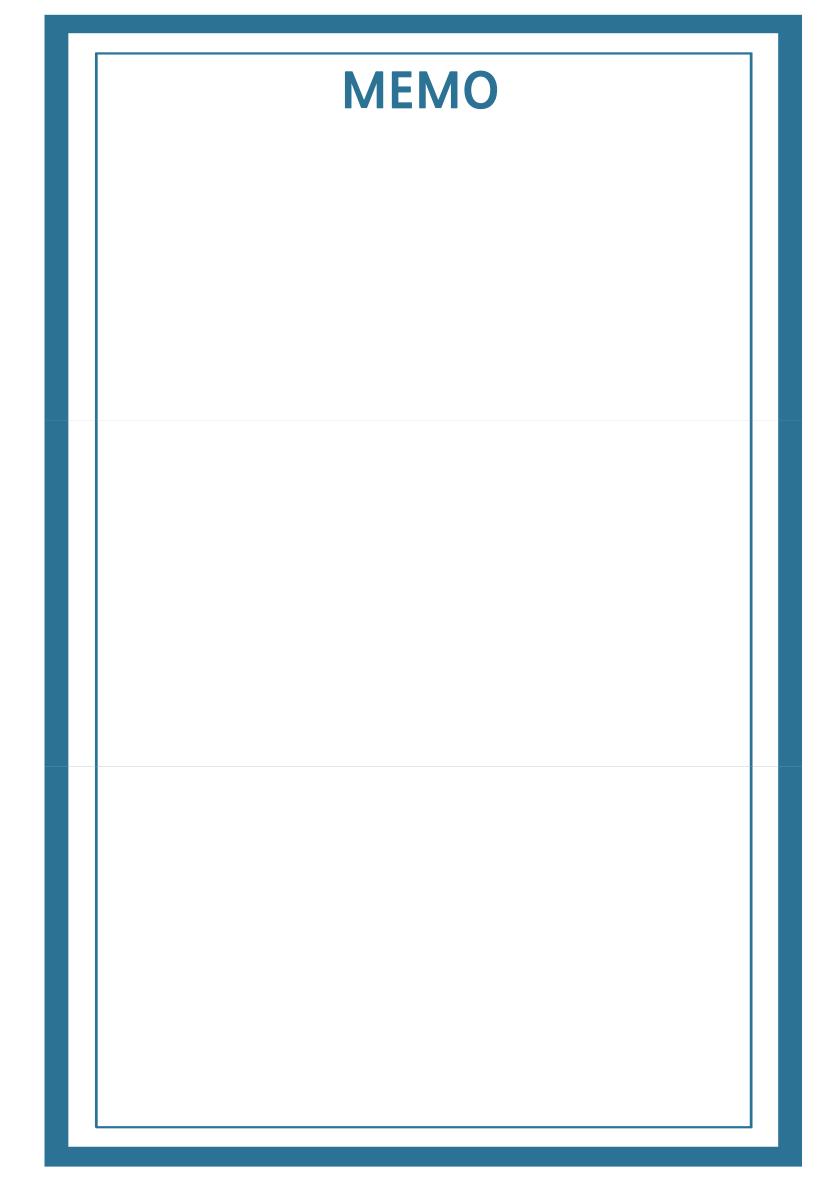


### **Best-First Search**

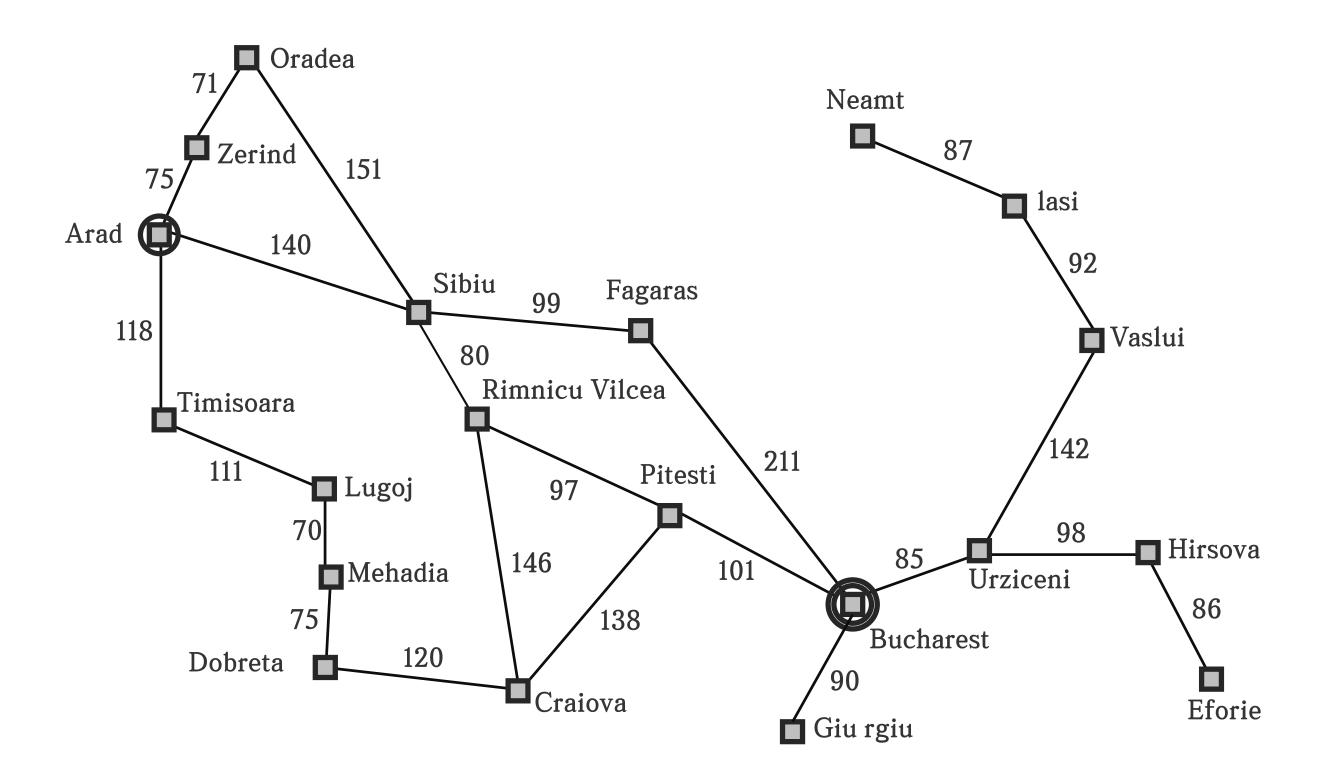
- Informed search strategy: uses problem-specific knowledge beyond the definition of the problem itself
  - ► Can find solutions more efficiently than an uninformed strategy
- $\otimes$  Idea: use an evaluation function f(n) at each node
  - ► Estimate of desirability
  - ► Expand the most desirable unexpanded node
  - ► Implementation: order the nodes in fringe in a decreasing order of desirability

### Special cases:

- ► Greedy best-first search
- ► A\* search



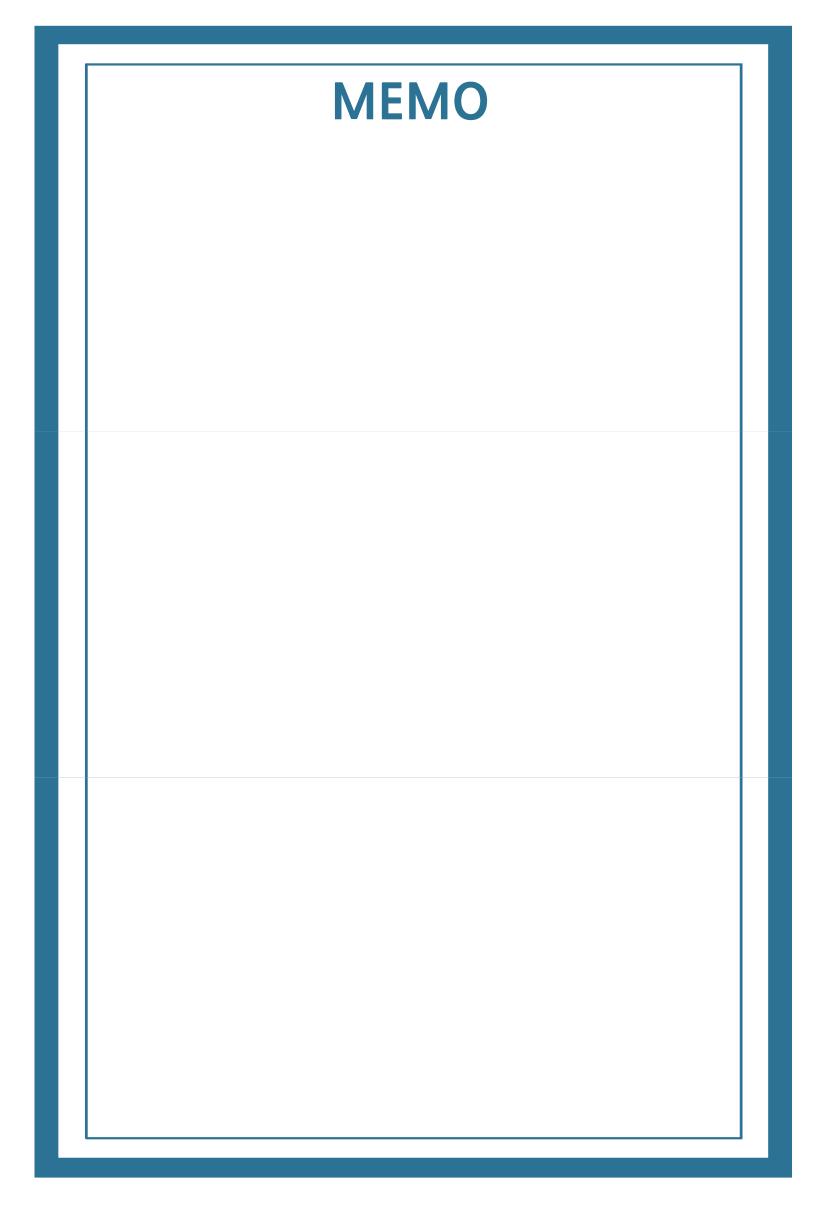




### Straight-line distance to Bucharest

Arad	366	Fagaras	176	Mehadia	241	Sibiu	253
Bucharest	0	Giurgiu	77	Neamt	234	Timisoara	329
Craiova	160	Hirsova	151	Oradea	380	Urziceni	80
Dobreta	242	Iasi	226	Pitesti	10	Vaslui	199
Eforie	161	Lugoj	244	Rimnicu Vilcea	193	Zerind	374

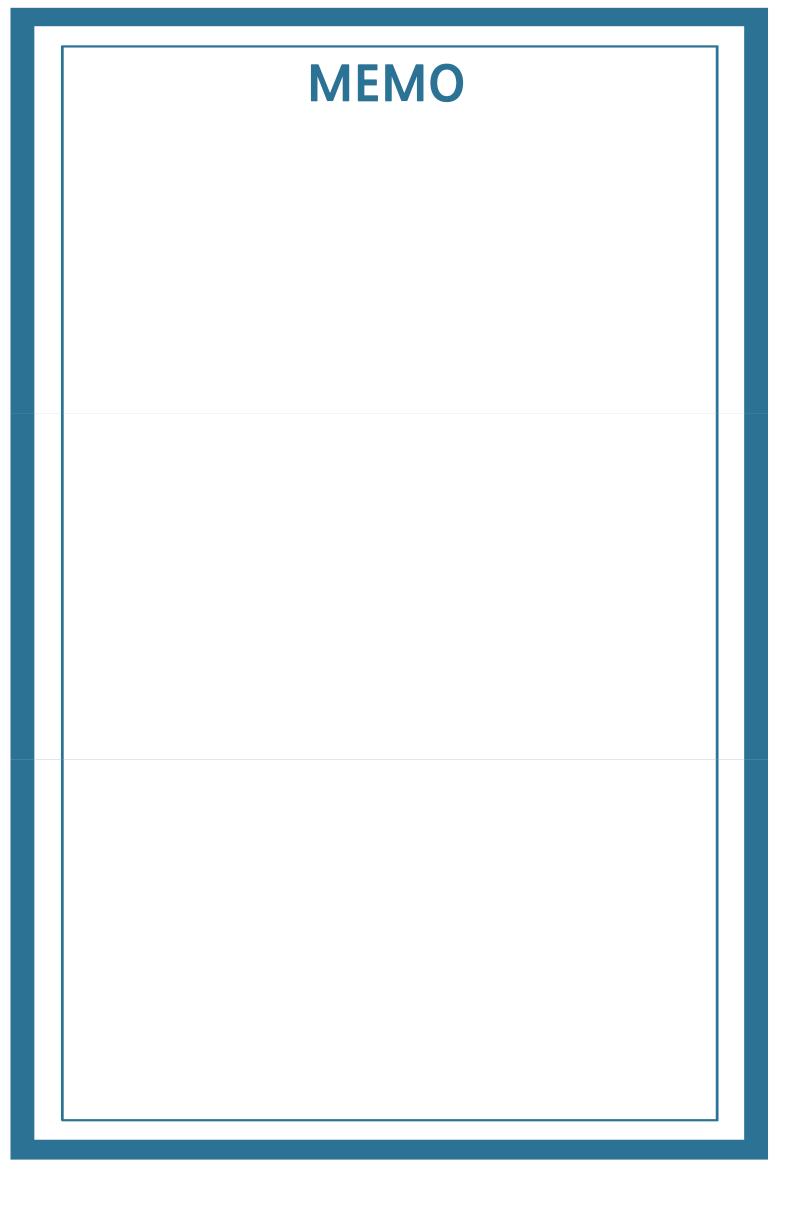






### **Greedy Best-First Search**

- $\otimes$  Evaluation function f(n) = h(n) (heuristic)
  - $\blacktriangleright h(n)$ : estimated cost of the cheapest path from node n to a goal
  - ▶ e.g.,  $h_{SLD}(n)$  = straight-line distance from n to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal



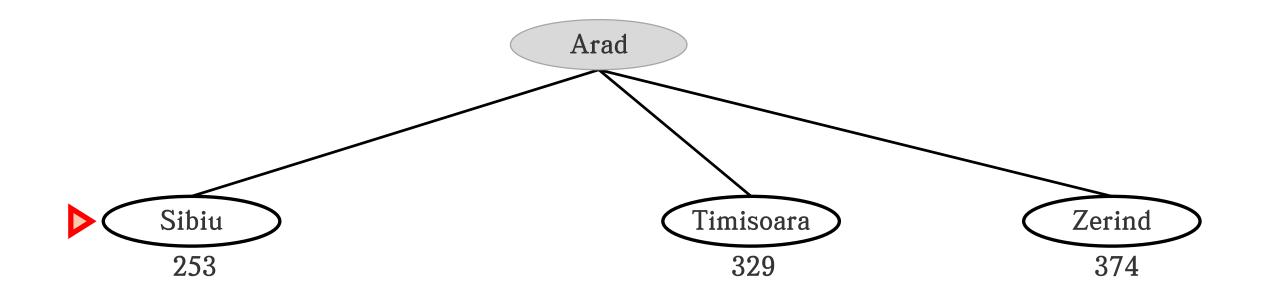




### Straight-line distance to Bucharest

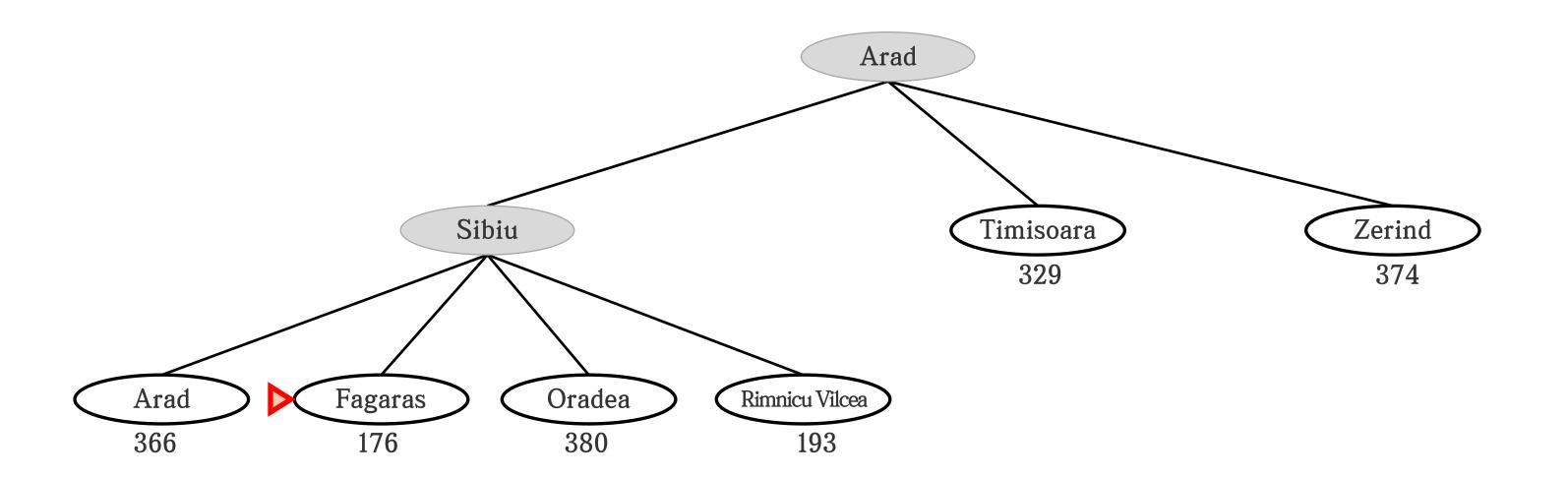
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# **MEMO**



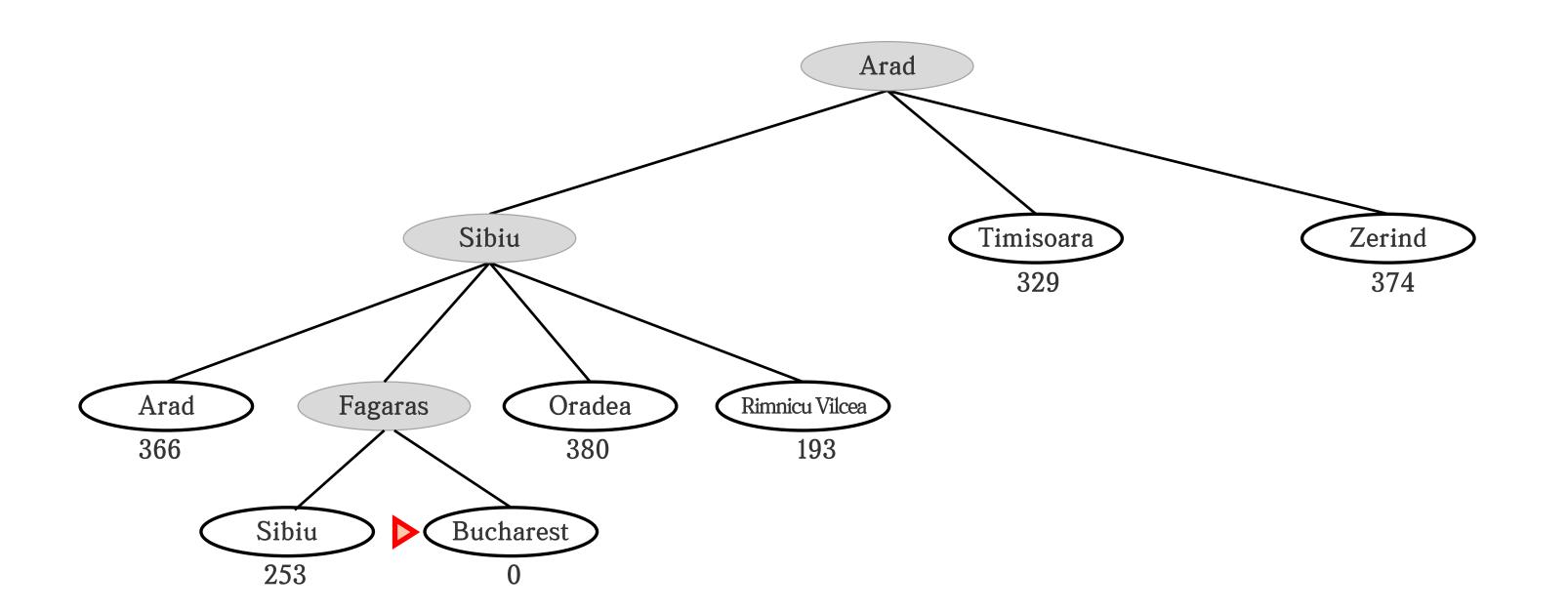


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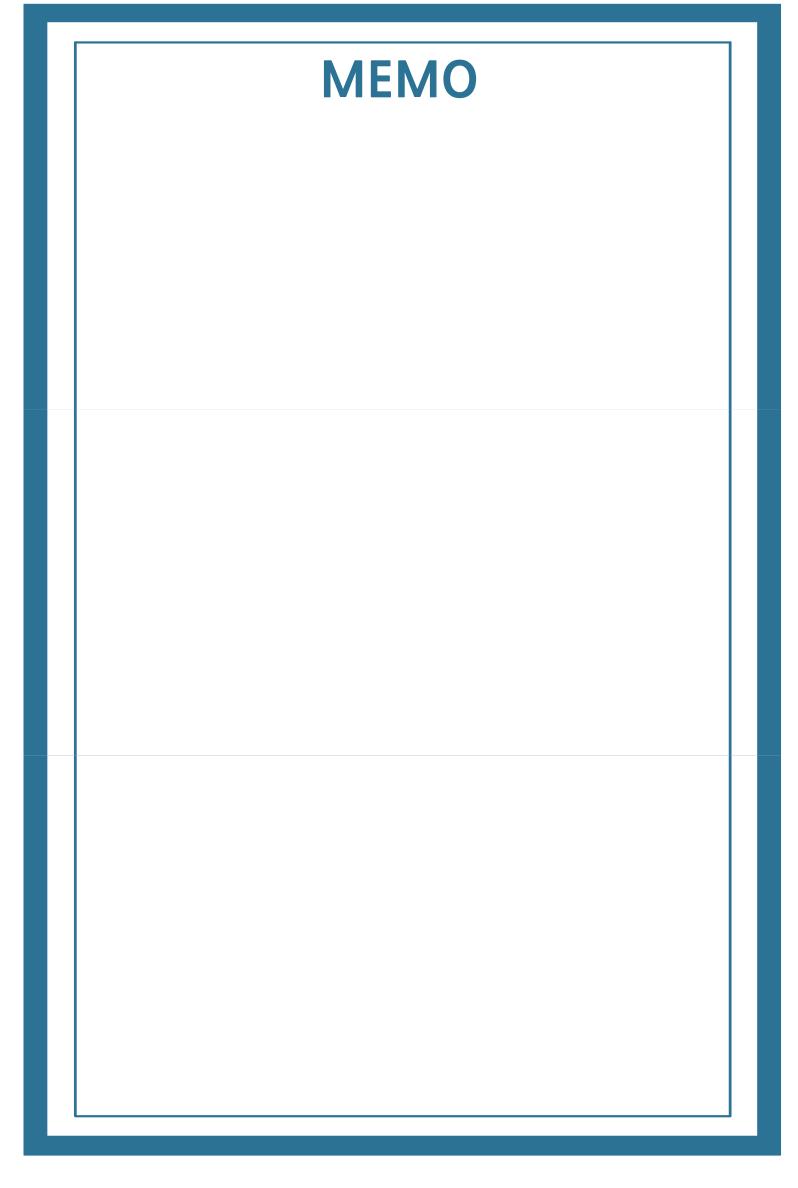


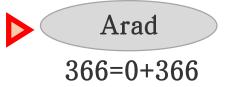
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### A\* Search (A-star)

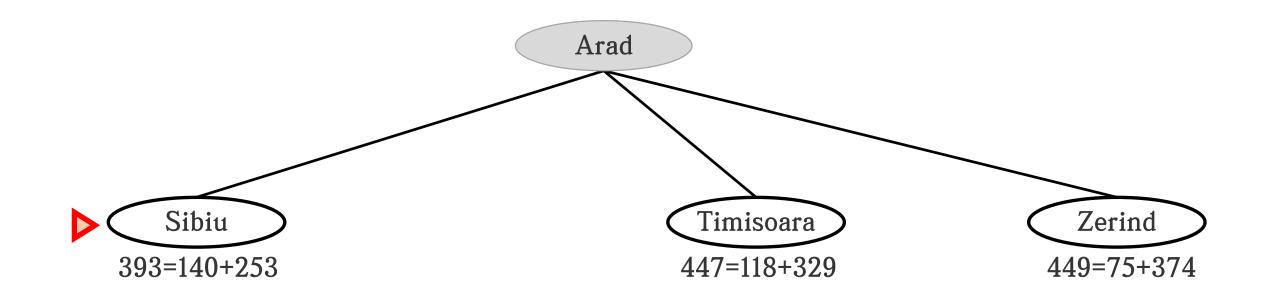
- $\otimes$  Evaluation function f(n) = g(n) + h(n)
  - ightharpoonup g(n): cost so far to reach n
  - $\blacktriangleright h(n)$ : estimated cost from n to goal
  - ightharpoonup f(n): estimated total cost of path through n to goal
- Ø Idea: avoid expanding paths that are already expensive
- Meuristic function
  - ► A way to inform the search about the direction to a goal
  - ► May be a solution to a simplified problem
  - ► A trade-off between (i) the amount of work it takes to derive a heuristic value for a node and (ii) how accurately the heuristic value of a node measures the actual path cost to a goal





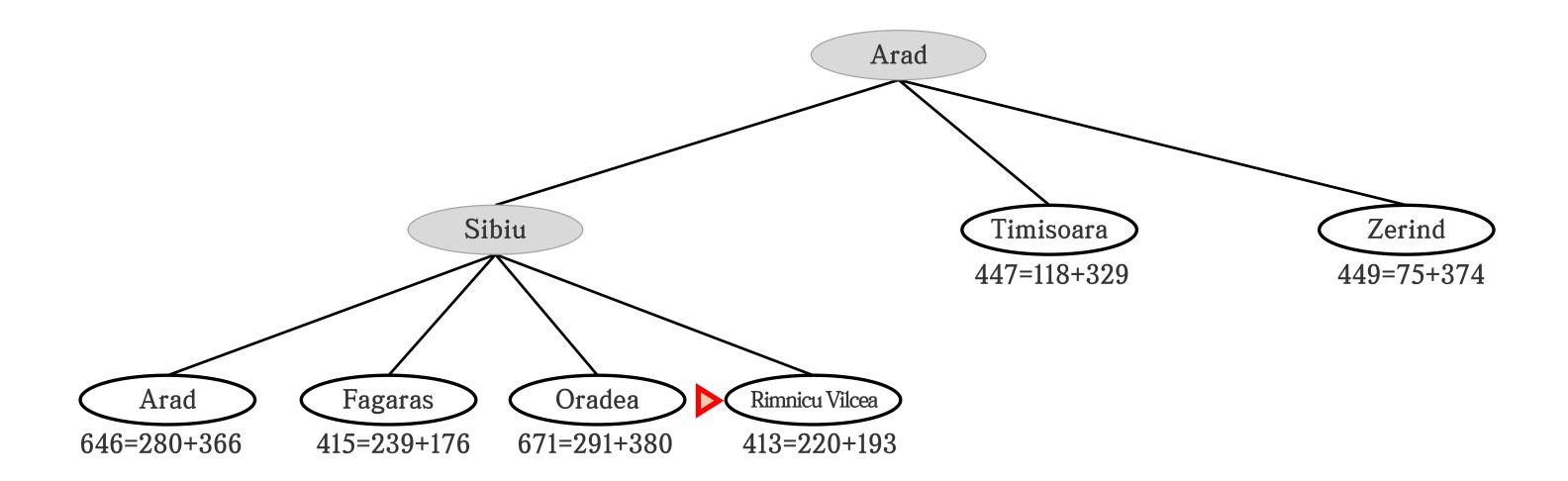


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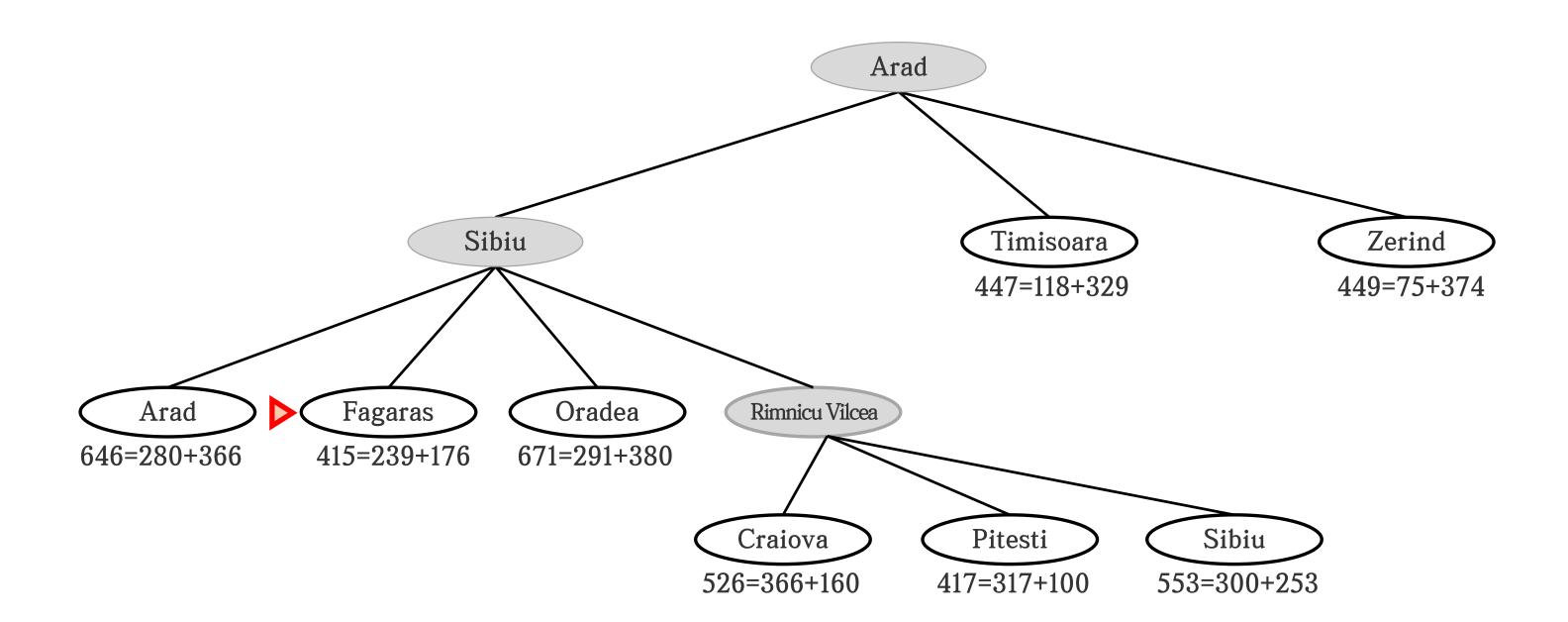


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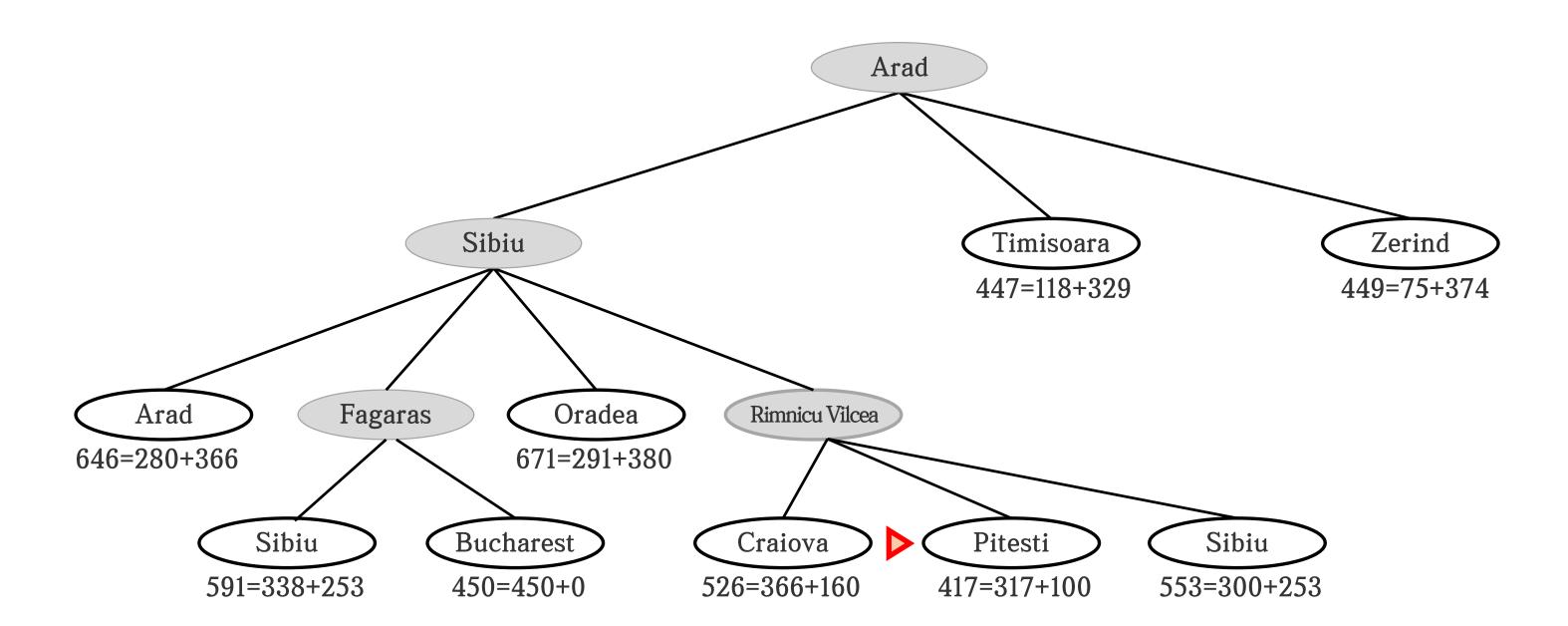


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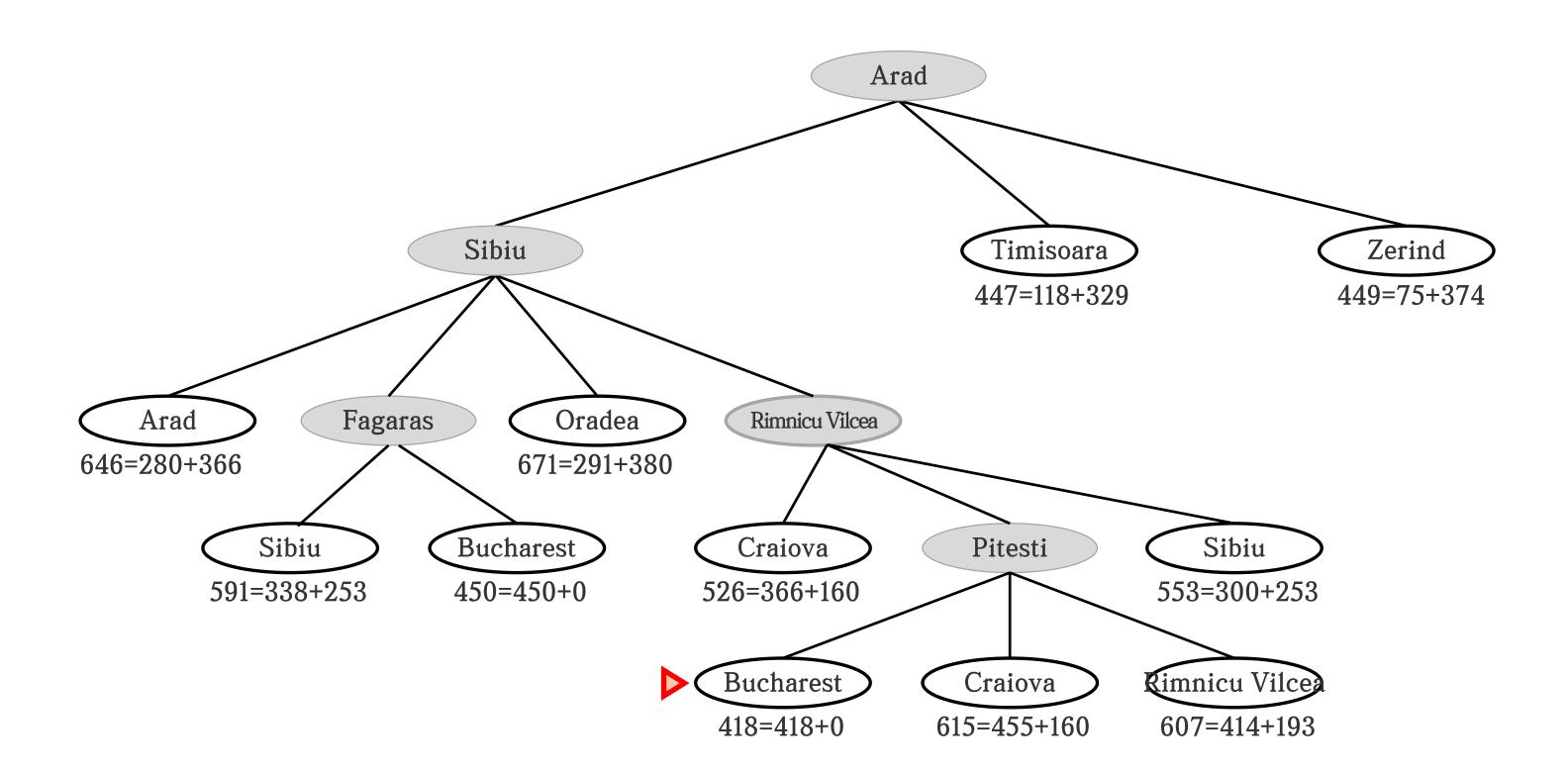


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### Admissible Heuristics

- $\otimes$  A heuristic h(n) is <u>admissible</u> if for every node n,  $h(n) \leq h^*(n)$ , where  $h^*(n)$  is the true cost to reach the goal state from n
  - ► An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
  - ► Example:  $h_{SLD}(n)$  (never overestimates the actual distance)

Theorem: If h(n) is admissible, A\* using TREE-SEARCH is optimal

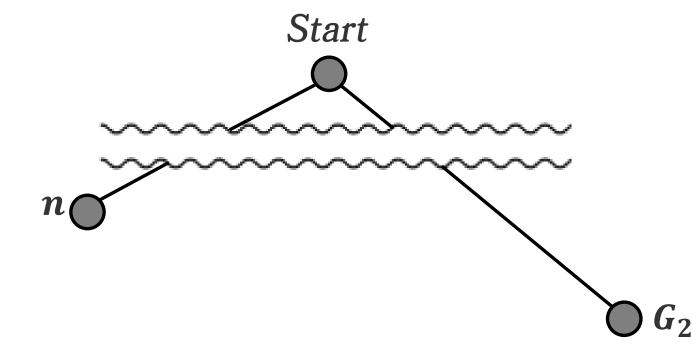
# **MEMO**



### Optimality of A\*

### Proof

▶ Suppose some suboptimal goal  $G_2$  has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



### Then

$$\blacktriangleright f(G_2) = g(G_2)$$

 $G \bigcirc$ 

$$ightharpoonup f(G) = g(G)$$

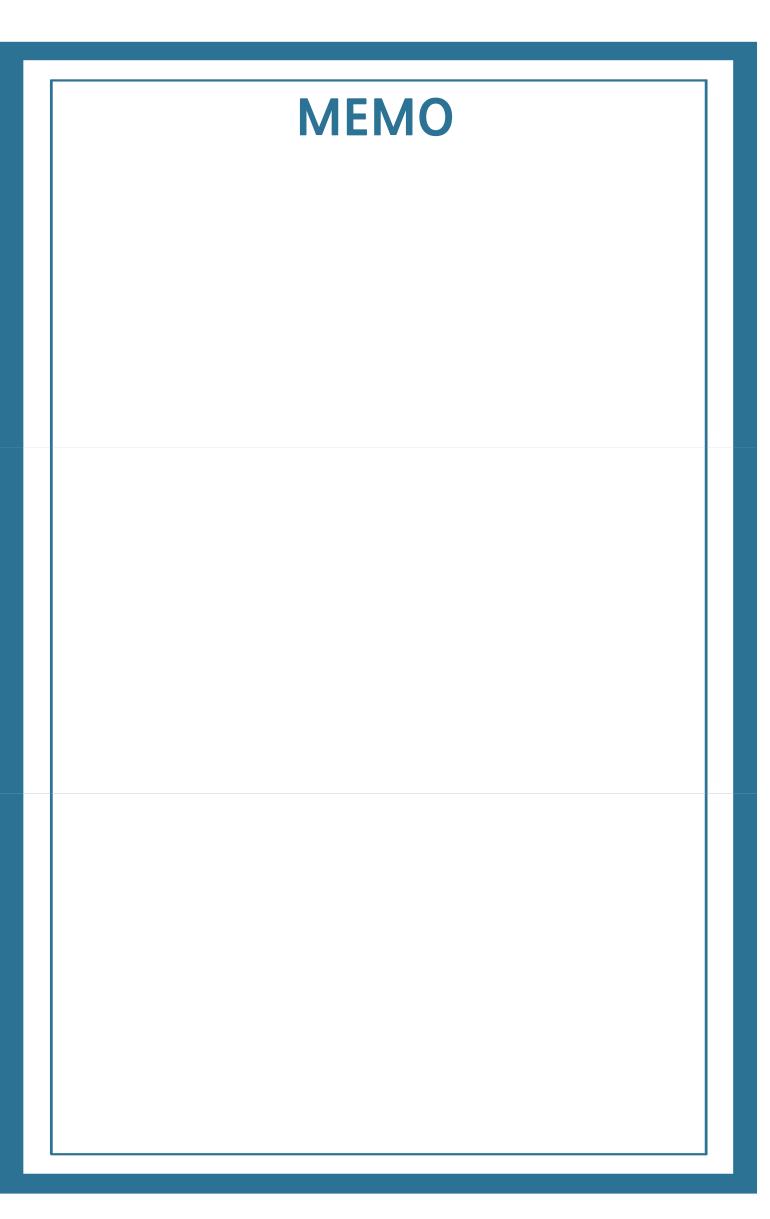
$$\blacktriangleright f(G_2) > f(G)$$

since 
$$h(G_2) = 0$$

since  $G_2$  is suboptimal

since 
$$h(G) = 0$$

from above

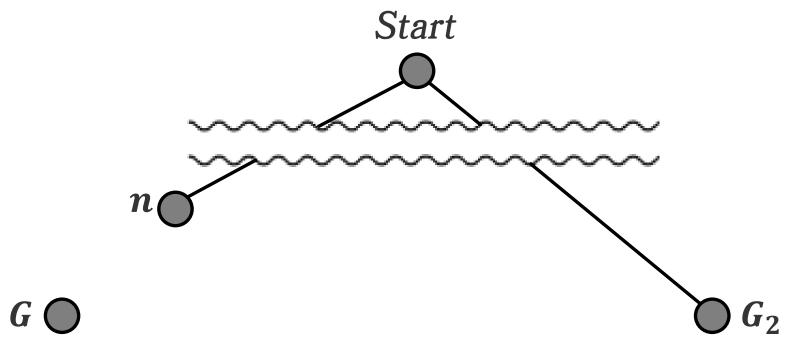




# Optimality of A\*

### Proof

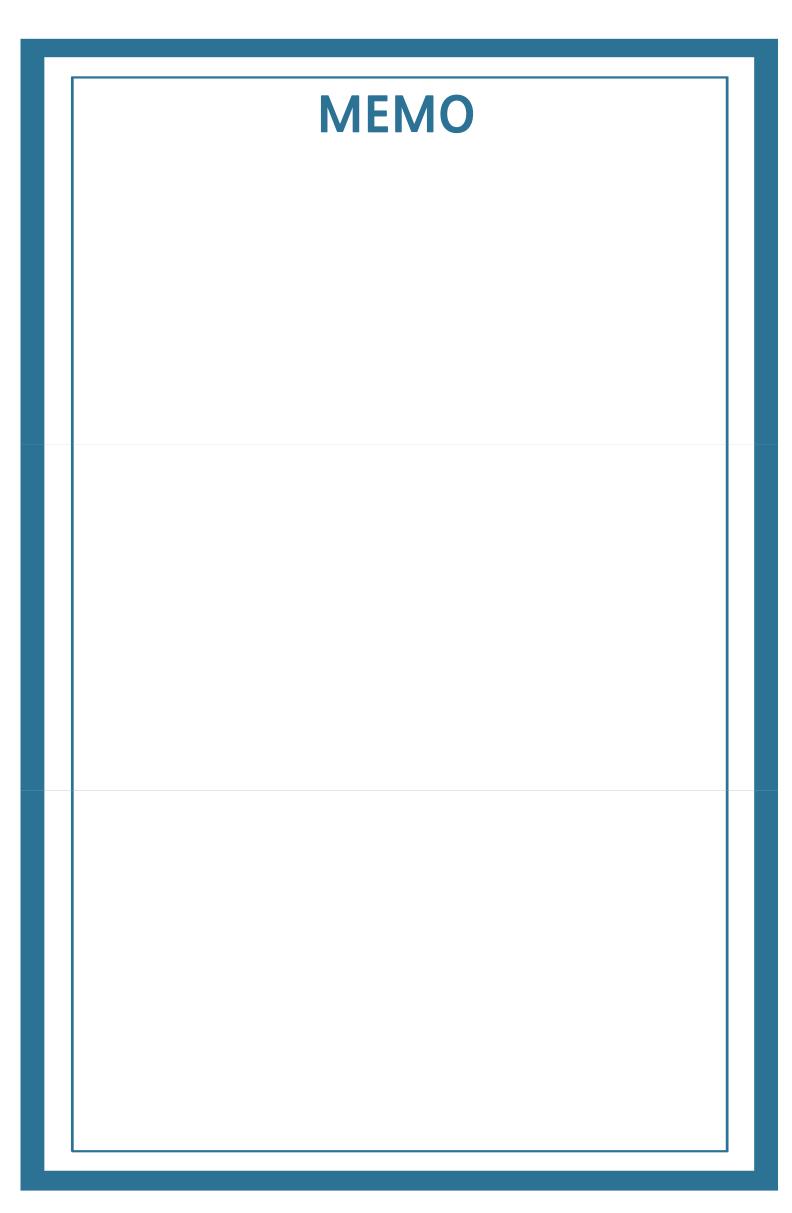
▶ Suppose some suboptimal goal  $G_2$  has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



### Then

- $ightharpoonup f(G_2) > f(G)$  from above
- ►  $h(n) \le h^*(n)$  since h is admissible
- $h(n) + g(n) \le h^*(n) + g(n)$
- ►  $f(n) \le f(G)$  since f is increasing

 $\otimes$  Hence  $f(G_2) > f(n)$ . A\* never selects  $G_2$  for expansion







### Local Search Algorithms

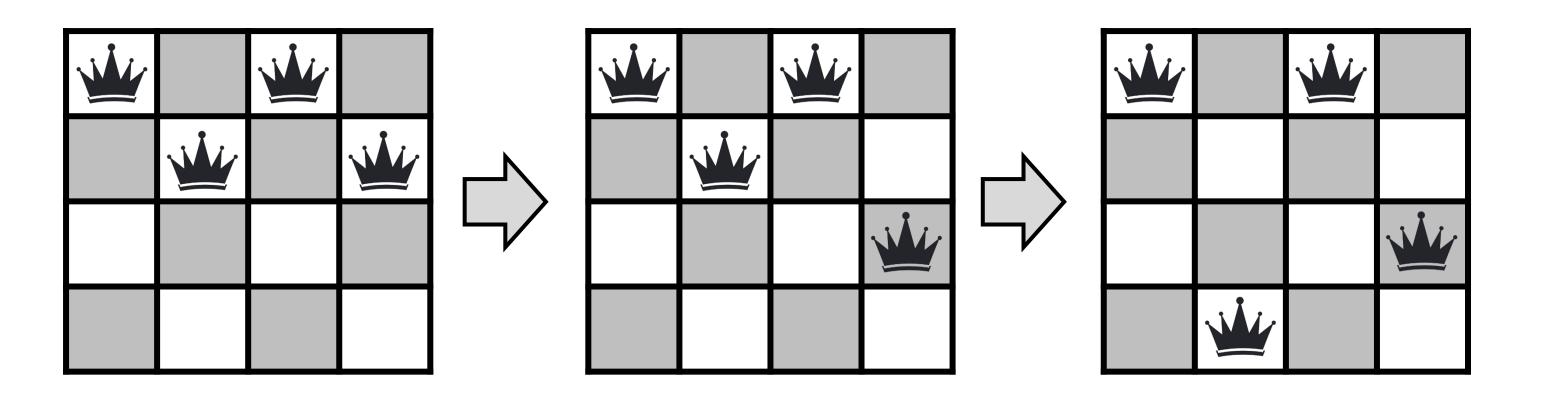
- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution
  - ► State space = a set of "complete" configurations
  - ► Find configuration satisfying constraints, e.g., n-queens
- @ In such cases, we can use <u>local search</u> algorithms
  - ► Keep a single "current" state, try to improve it (by moving only to neighbors)
- Two advantages
  - ► Use very little memory usually a constant amount
  - ► Can often find reasonable solutions in large or infinite (continuous) state spaces

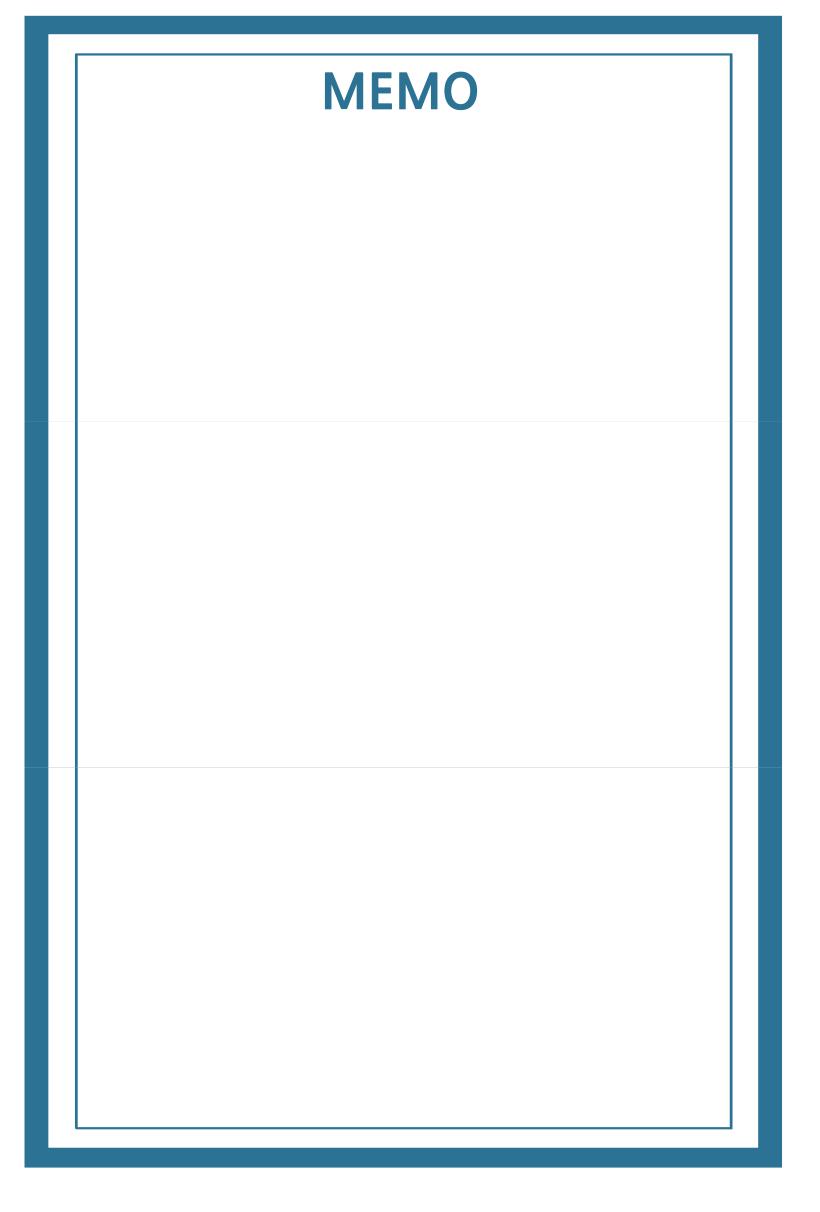
# **MEMO**



## Example: n-queens

 $\otimes$  Put n queens on an  $n \times n$  board with no two queens on the same row, column, or diagonal

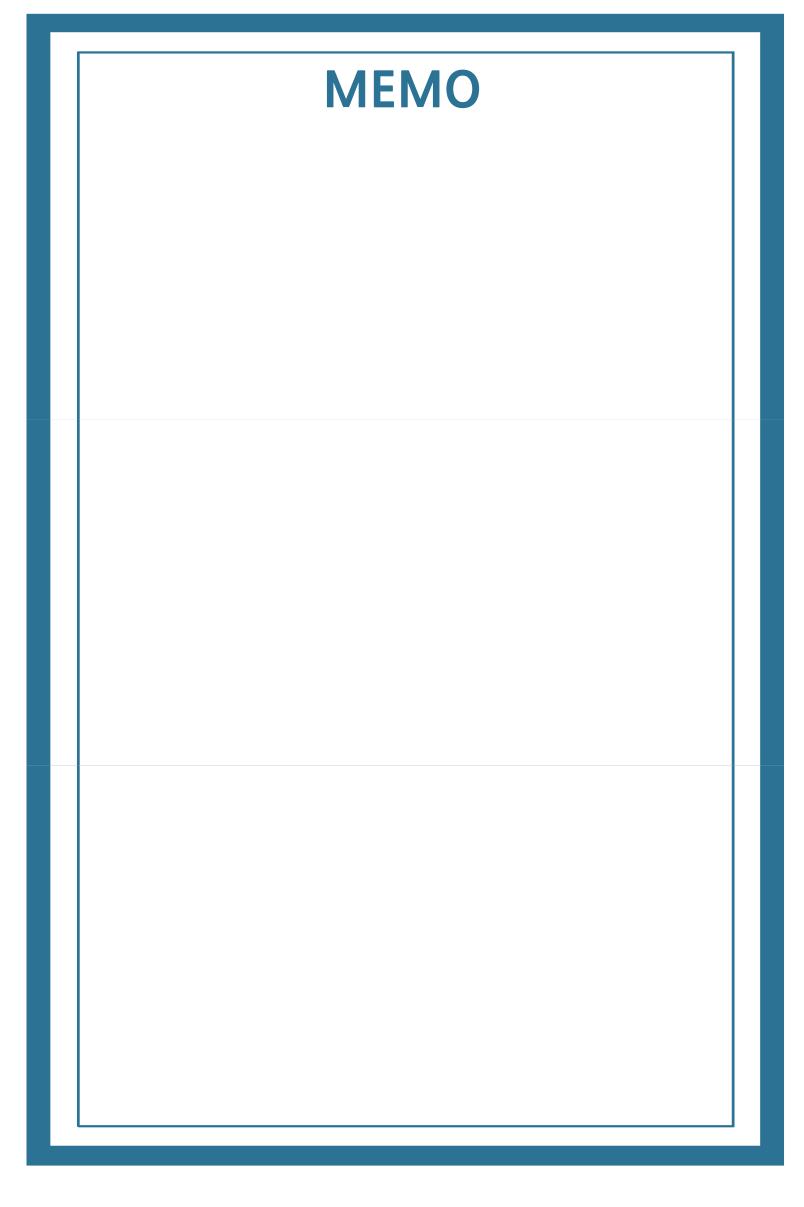






### Hill-Climbing Search

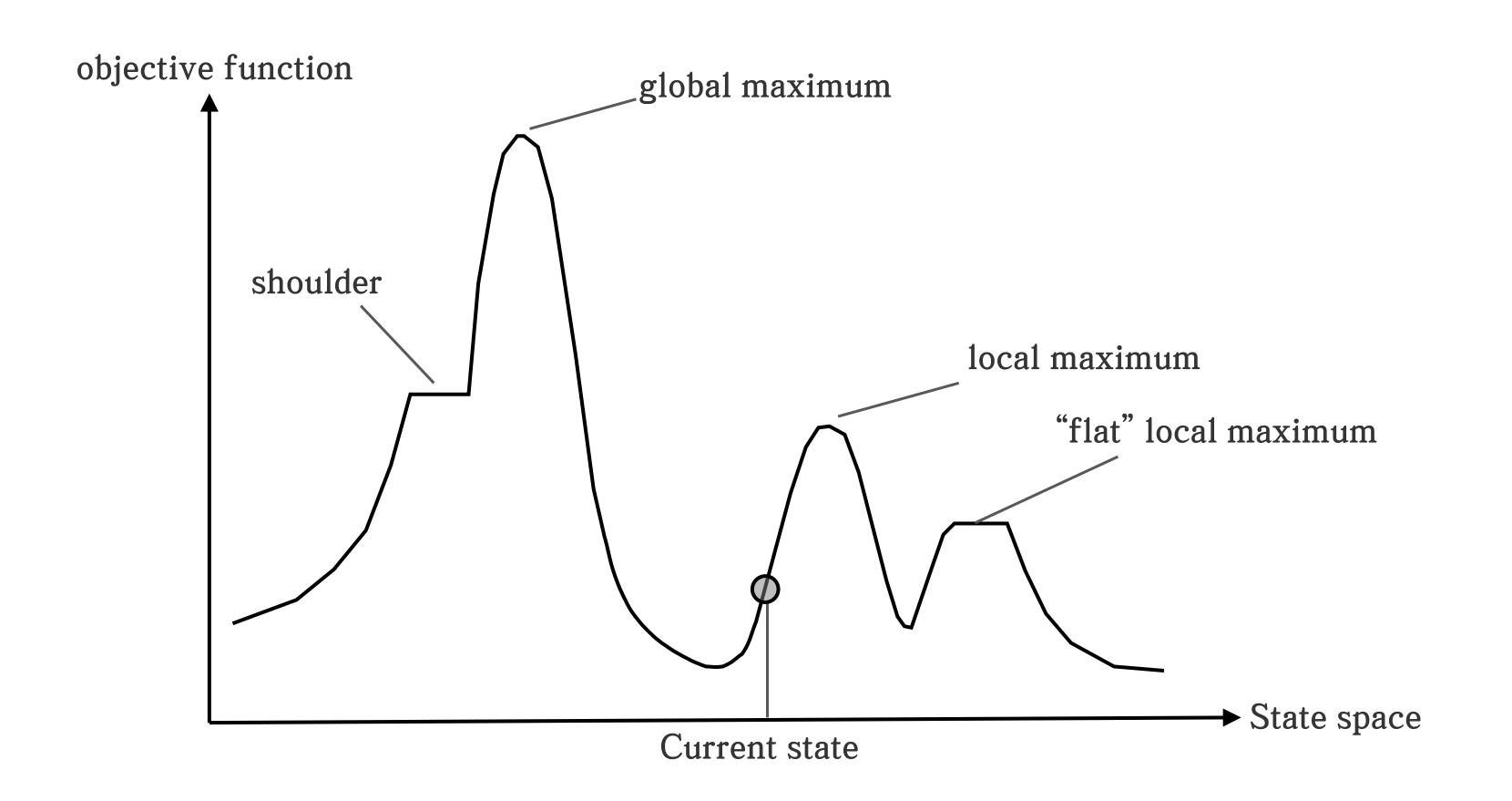
- Like climbing Everest in thick fog with amnesia
- The most basic local search technique
  - ► Steepest ascent: at each step the current node is replaced by the best neighbor
  - ► Greedy local search: grabs a good neighbor state without thinking ahead about where to go next

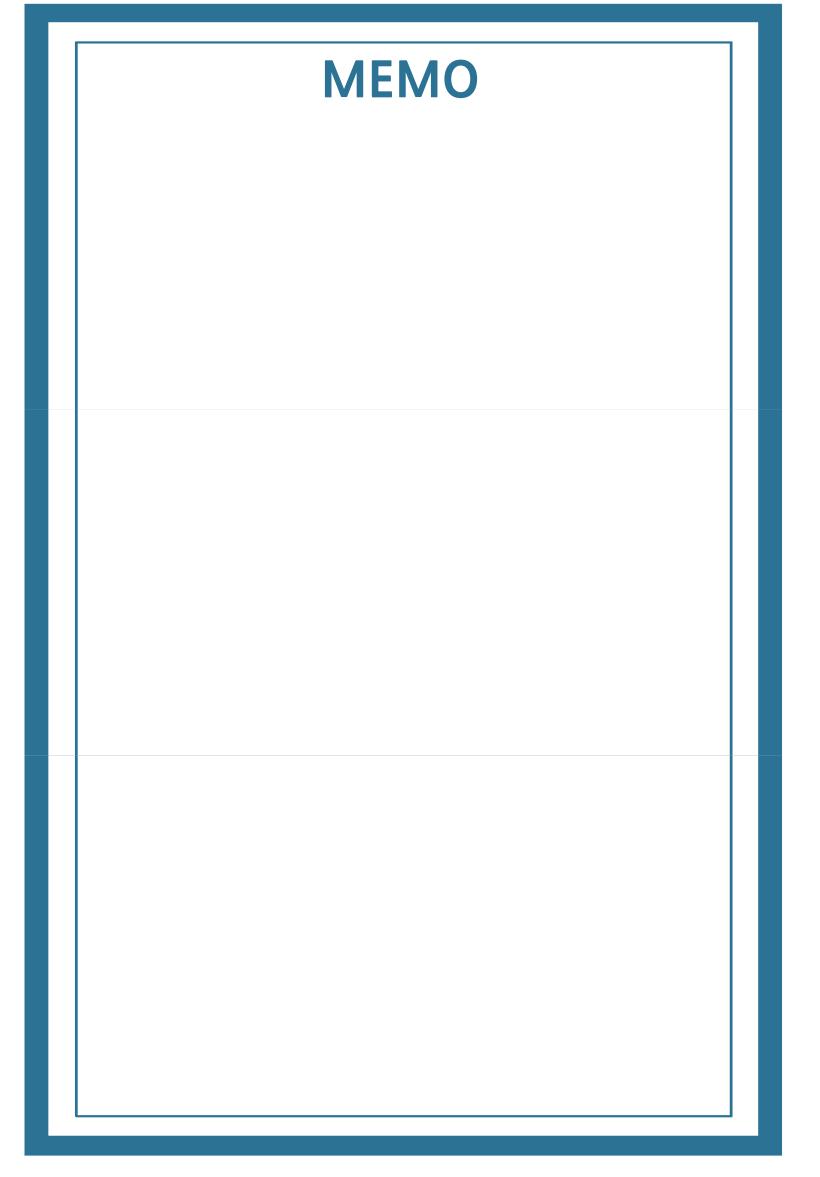




# Hill-Climbing Search

Problem: depending on initial state, can get stuck in local maxima



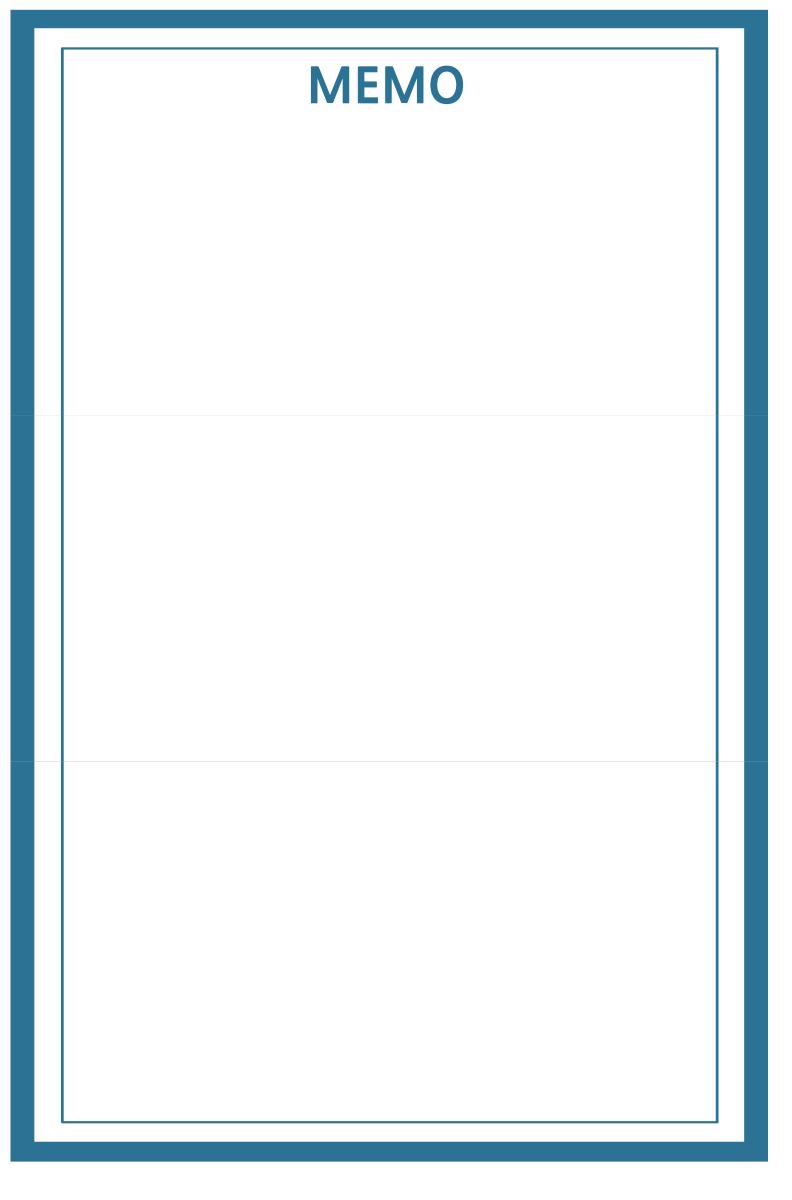




### Hill-Climbing Search: 8-queens Problem

- - $\blacktriangleright h = 0$  only at the global minimum (i.e. perfect solution)

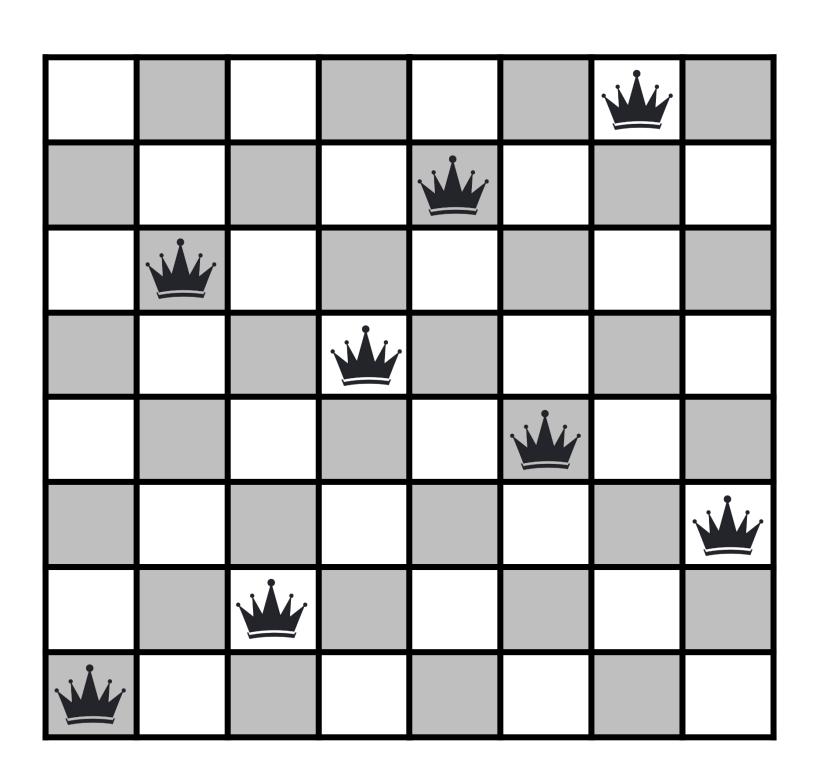
18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	<b>W</b>	13	16	13	16
<b>W</b>	14	17	15	4	14	16	16
17	1	16	18	15		15	
18	14	4	15	15	14	<u>w</u>	16
					14		

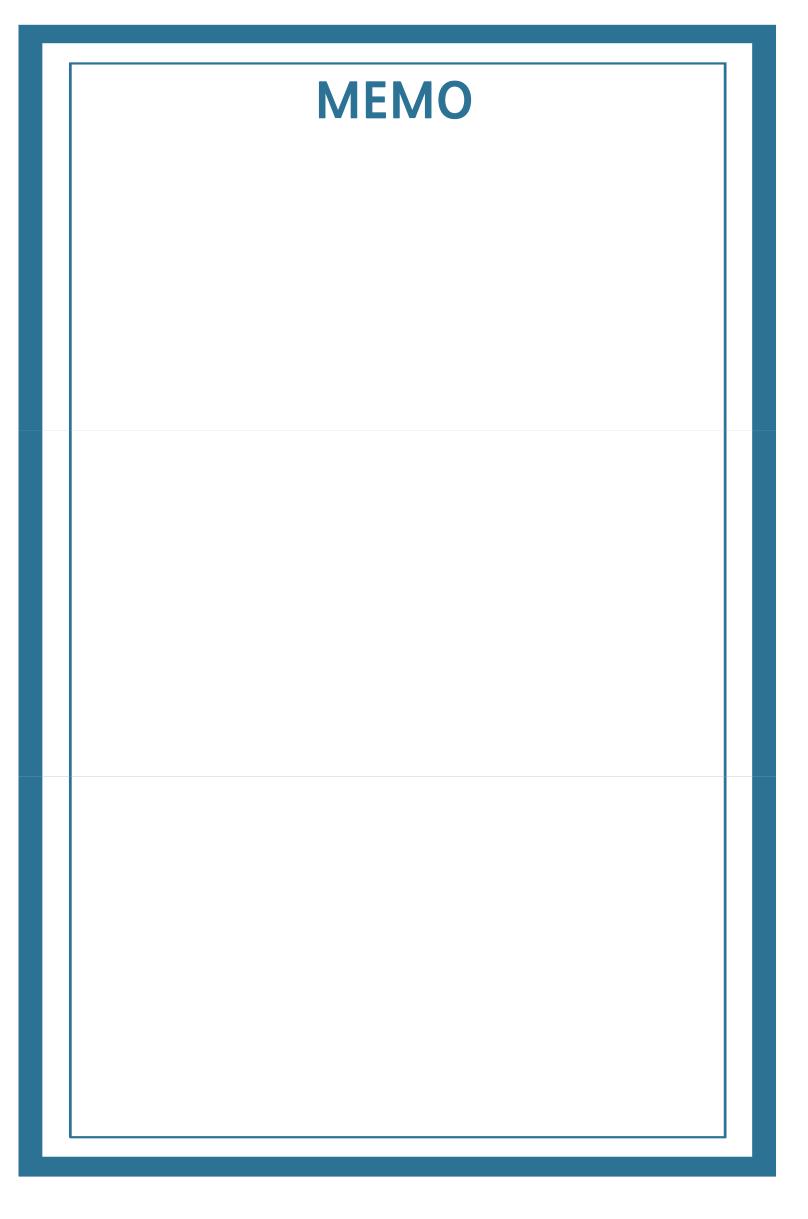




### Hill-Climbing Search: 8-queens Problem

- - ightharpoonup A local minimum with h=1 (every successors has a higher cost)







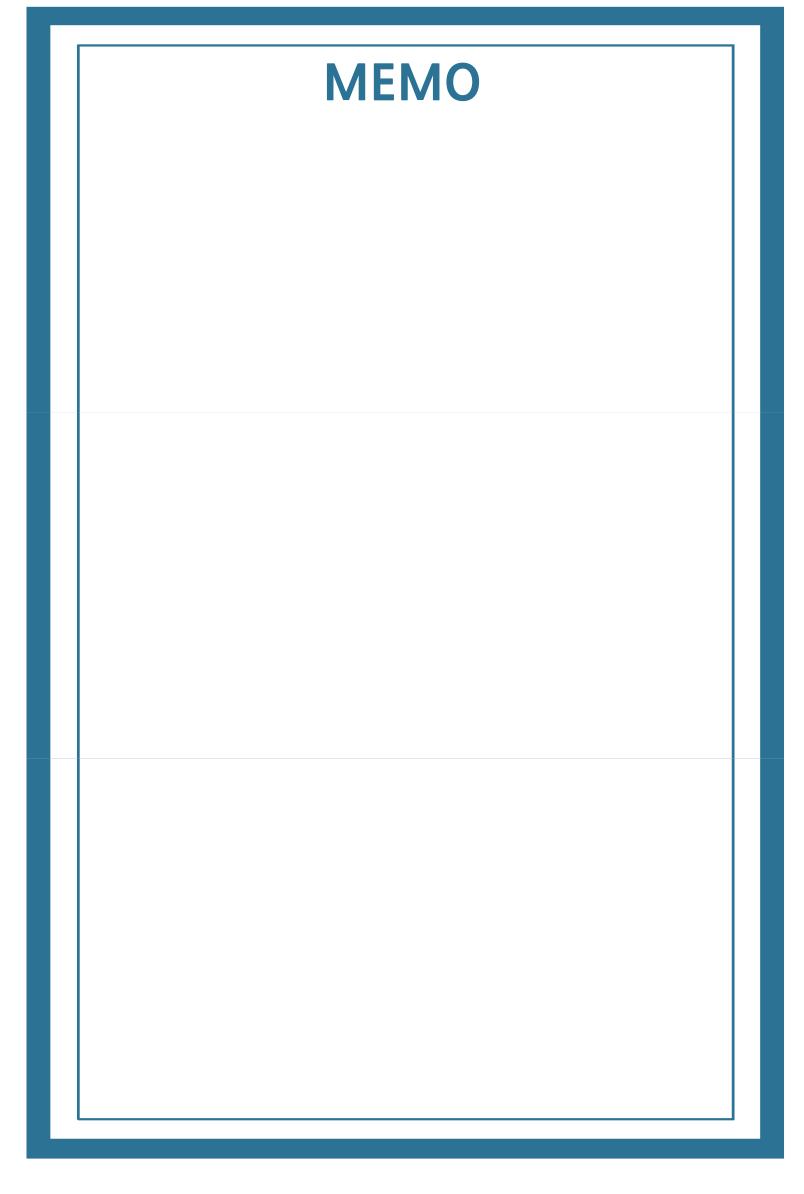
### Simulated Annealing Search

### Ø Idea: escape local maxima by allowing some bad moves but gradually decrease their frequency

- ► A hill-climbing algorithm never makes downhill
- ► Cannot escape from a local maximum
- ► Hill-climbing + random walk (for efficiency and completeness)

### Intuition with gradient descent

- ► Down a ball into a bumpy surface; then it comes to rest at a local minimum
- ► Shake the surface to bounce the ball out
- ➤ Start by shaking hard (at a high temperature), then gradually reduce the intensity of the shaking (lower the temperature)





### Algorithm

- ▶ Instead of picking the best move, it picks a random move
- ▶ If the move improves the situation, it is always accepted
- ► Otherwise, less likely to be accepted (i)  $|\Delta E|$  is high, (ii) temperature T is low

```
function Simulated-Annealing (problem, schedule) returns a solution state inputs: problem, a problem schedule, a mapping from time to "temperature" local variables: current, a node next, a node T, a "temperature" controlling prob. of downward steps  \begin{array}{c} current \leftarrow \text{Make-Node}(\text{Initial-State}[problem]) \\ \text{for } t \leftarrow 1 \text{ to} \infty \text{ do} \\ T \leftarrow schedule[t] \\ \text{if } T = 0 \text{ then return } current \\ next \leftarrow \text{a randomly selected successor of } current \\ \Delta E \leftarrow \text{Value}[next] - \text{Value}[current] \\ \text{if } \Delta E > 0 \text{ then } current \leftarrow next \\ \text{else } current \leftarrow next \text{ only with probability } e^{\Delta E/T} \end{array}
```



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