

Practical 1

Table: Random Variable

Find the mean & variance for the following:

a)	X	-1	0	1	2
	P(X)	0.1	0.2	0.3	0.4

Solution

X	P(X)	X · P(X)	$E[X^2]$	$[E(X)]^2$
-1	0.1	-0.1	0.1	0.01
0	0.2	0	0	0
1	0.3	0.3	0.3	0.09
2	0.4	0.8	0.816	0.64
TOTAL	$\Sigma=1$	$\Sigma=1$	$\Sigma=2$	$\Sigma=0.74$

$\therefore \text{Mean } E(X) = \sum x_i \cdot p(x_i) = 1$

$$\text{Variance } V(X) = \sum E(X)^2 - [E(X)]^2$$

$$= 2 - 0.74$$

$$= 1.24$$

mean $E(X) = 1$ & variance $V(X) = 1.24$

$\{x\}$	x	-1	0	1	2
$\{f(x)\}$	$f(x)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

density

x	$f(x)$	$x \cdot f(x)$	$E(x^2)$	$[E(x)]^2$
-1	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
0	$\frac{1}{4}$	0	0	0
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{16}$
2	$\frac{1}{2}$	1	2	1
Total	$\Sigma = \frac{1}{2}$	$\Sigma = \frac{9}{8}$	$\Sigma = \frac{10}{8}$	$\Sigma = \frac{9}{64}$

$$\text{Mean } E(x) = \sum x \cdot f(x) = \frac{9}{8}$$

$$\begin{aligned} \text{Variance } V(x) &= \sum E(x)^2 - [E(x)]^2 \\ &= \frac{10}{8} - \frac{81}{64} \\ &= \frac{152 - 81}{64} \\ &= \frac{83}{64} \end{aligned}$$

~~$$\text{Mean } E(x) = \frac{9}{8} \text{ & Variance } V(x) = \frac{83}{64}$$~~

x	-3	10	15
p(x)	0.4	0.35	0.25

Solution

x	p(x)	x-p(x)	$E(x)^2$	$[E(x)]^2$
-3	0.4	-1.2	3.0	1.44
10	0.35	3.5	3.5	12.25
15	0.25	3.75	56.25	14.0625
TOTAL	$\Sigma = 1$	$\Sigma = 6.05$	$\Sigma = 94.85$	$\Sigma = 27 - 7.525$

$$\text{Mean} = E(x) = \sum x \cdot p(x) = 6.05$$

$$\text{Variance} = V(x) = \sum E(x)^2 - [E(x)]^2$$

$$= 94.85 - 27 - 7.525$$

$$= 67.025$$

$$\therefore \text{Mean } E(x) = 6.05 \text{ & variance } V(x) = 67.025$$

- Q] If $P(x)$ is pmf of a random variable X if $P(x)$ represents pmf for random variable X . Find value of k . Then evaluate mean & variance.

Solution As $P(x)$ is a pmf it should satisfy the properties of pmf which are

- a) $P(x_i) \geq 0$ for all sample space
- b) $\sum P(x_i) = 1$

No.	-1	0	1	2
P(x)	$\frac{6}{13}$	$\frac{5}{13}$	$\frac{1}{13}$	$\frac{1}{13}$

$$\therefore \sum P(x) = 1 = \frac{6+5+1+1}{13} = \frac{13}{13} = 1$$

$$1 = \frac{K+1+K+1+K-4}{13}$$

$$13 = 3K - 2$$

$$15 = 3K$$

$$K = 5$$

X	P(x)	XP(x)	E(X) ²	[E(X)] ²
-1	$\frac{6}{13}$	$-\frac{6}{13}$	$\frac{6}{13}$	$\frac{36}{169} = \frac{36}{169}$
0	$\frac{5}{13}$	0	0	0
1	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{169} = \frac{1}{169}$
2	$\frac{1}{13}$	$\frac{2}{13}$	$\frac{4}{13}$	$\frac{4}{169} = \frac{4}{169}$
Total	$\Sigma = 1$	$\Sigma = \frac{3}{13}$	$\Sigma = \frac{11}{13}$	$\Sigma = \frac{41}{169} = \frac{41}{169}$

$$\therefore \text{Mean} = E(x) = \Sigma xP(x) = -\frac{3}{13}$$

$$\begin{aligned} \text{Variance} &= V(x) = \Sigma E(x)^2 - [E(x)]^2 \\ &= \frac{11}{169} - \frac{9}{169} \\ &= \frac{2}{169} = \frac{10.2}{169} \end{aligned}$$

$$\therefore \text{Mean} = -\frac{3}{13} \text{ & Variance} = \frac{10.2}{169}$$

The pmf of random variable X is given by

x	-3	-1	0	1	2	3	5	8
$p(x)$	0.1	0.2	0.15	0.2	0.1	0.15	0.05	0.05
cumulative dist. func.			$P(-1 \leq X \leq 2)$	$P(1 \leq X \leq 5)$	$P(X > 0)$			
① $P(X \leq 2)$	0.1	0.3	0.45	0.65	0.75	0.90	0.95	1.0
② $P(X > 0)$								

Solutions

x	-3	-1	0	1	2	3	5	8
$P(x)$	0.1	0.2	0.15	0.2	0.1	0.15	0.05	0.05
$F(x)$	0.1	0.3	0.45	0.65	0.75	0.90	0.95	1.0

$$\begin{aligned} \text{① } P(-1 \leq X \leq 2) &= P(X \leq 2) - P(X \leq -1) + P(X = -1) \\ &= F(2) - F(-1) + P(-1) \\ &= 0.75 - 0.3 + 0.2 \\ &= 0.25 \end{aligned}$$

$$\begin{aligned} \text{② } P(1 \leq X \leq 5) &= F(5) - F(1) + P(1) \\ &= F(5) - F(1) + P(1) \\ &= 0.95 - 0.65 + 0.2 \\ &= 0.5 \end{aligned}$$

$$\begin{aligned} \text{③ } P(X \leq 2) &= P(X = -3) + P(X = -1) + P(X = 0) + P(X = 1) + P(X = 2) \\ &= 0.1 + 0.2 + 0.15 + 0.2 + 0.1 \\ &= 0.7 \end{aligned}$$

$$\begin{aligned} \text{④ } P(X > 0) &= 1 - F(0) + P(0) \\ &= 1 - 0.45 + 0.15 \\ &= 0.40 \end{aligned}$$

Ques Let $f(x)$ continuous random variable with PMF
 $f(x) = \frac{2x}{2} \quad -1 \leq x \leq 1$

$\rightarrow 0$ elsewhere
 Obtain cdf of x . Find mean

Solution By definition of cdf we have

$$\begin{aligned} F(x) &= \int_{-\infty}^x f(t) dt \\ &= \int_{-1}^x \frac{2t}{2} dt \\ &= \frac{1}{2} \left(\frac{1}{2} x^2 + x \right) \text{ for } -1 \leq x \leq 1 \end{aligned}$$

Hence the cdf is

$$\begin{aligned} F(x) &= 0 \quad \text{for } x \leq -1 \\ &= \frac{1}{2} x^2 + \frac{1}{2} x \quad \text{for } -1 \leq x \leq 1 \\ &= 1 \quad \text{for } x \geq 1 \end{aligned}$$

Ans ✓

Binomial

Def: Binomial Distribution

- 1) An unbiased coin is tossed 4 times. Calculate the probability of obtaining no head, atleast one head & more than one tail.

No HEAD:

$$\text{Binom}(0, 4, 0.5)$$

$$[1] 0.0625$$

ATLEAST ONE HEAD:

$$\text{1 - Binom}(0, 4, 0.5)$$

$$[1] 0.9375$$

MORE THAN ONE TAIL:

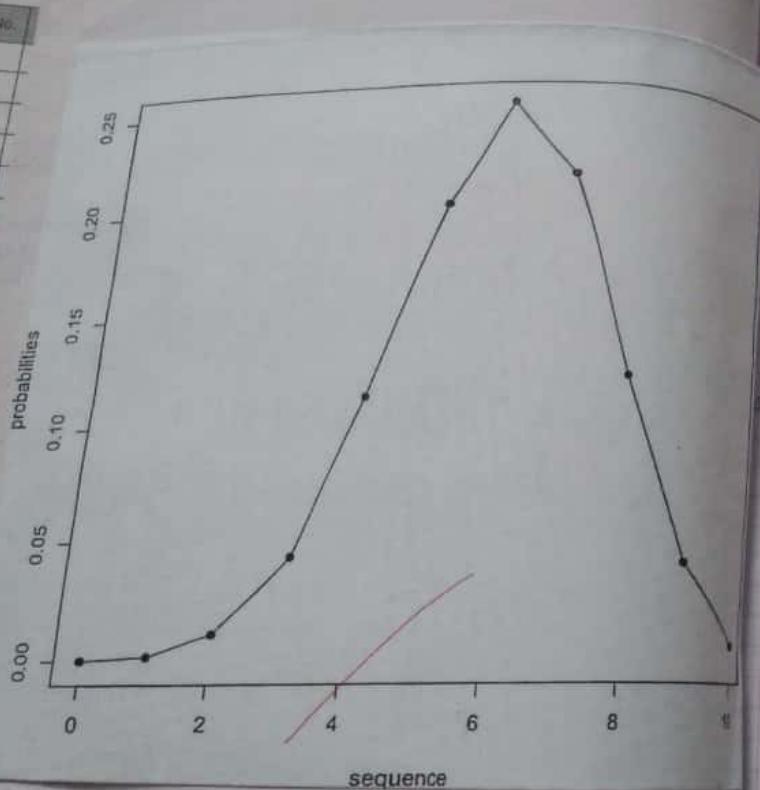
$$\text{Binom}(1, 4, 0.5, \text{lower tail} = \text{F})$$

$$[1] 0.9375$$

- 2) The probability that student is accepted to a prestigious college is 0.3. If 5 students apply. What is the probability of atmost 2 are accepted?

$$\text{Binom}(2, 5, 0.3)$$

$$[1] 0.83697$$



In unbiased case it is known 6 times the probabilities of read at any time = 0.3. Let x be no. of heads that comes up. Calculate $P(x=2), P(x=3), P(1 \leq x \leq 5)$

> $dbinom(2, 6, 0.3)$

[1] 0.324185

> $dbinom(3, 6, 0.3)$

[1] 0.18522

> $dbinom(2, 6, 0.3) + dbinom(3, 6, 0.3) + dbinom(4, 6, 0.3)$

[1] 0.74773

] For $n=10, p=0.6$, evaluate binomial probabilities and plot the graphs of pmf of idf.

> $x = seq(0, 10)$

> $y = dbinom(x, 10, 0.6)$

> y

[1]	0.0001048576	0.0015728640	0.0106162325
	0.0424673380	0.1114767360	0.2006581248
	0.2507226560	0.2147708480	0.170932367
	0.0403107840	0.0060466196	

> $plot(x, y, xlab = "sequence", ylab = "probability", 0, 10)$

8) Bits are sent for transmission channel in packet of 12. If the probability of bit error corrupted is 0.1 what is the probability of no more than 2 bits are corrupted in a packet?

$$\text{Required } (2, 12, 0.1), \text{ lower tail} = P(X \leq 2)$$

$$= 0.3400077$$

Practical 3

Left normal distribution

- a) A normal distribution of 100 students with mean 50 ± 10 .
 Find no. of students whose marks are (i) $P(X < 35)$ (ii) $P(45 \leq X \leq 70)$ (iii) $P(25 \leq X \leq 35)$ (iv) $P(X > 60)$.

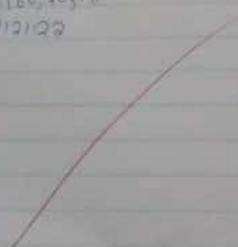
> $\text{pnorm}(35, 50, 10)$
 [i] 0.2524985

> $\text{pnorm}(70, 50, 10) - \text{pnorm}(45, 50, 10)$
 [ii] 0.4772400

> $\text{pnorm}(35, 50, 10) - \text{pnorm}(25, 50, 10)$
 [iii] 0.2107161

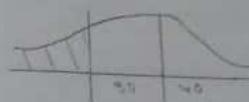
> $1 - \text{pnorm}(60, 50, 10)$
 [iv] 0.0912122

#2



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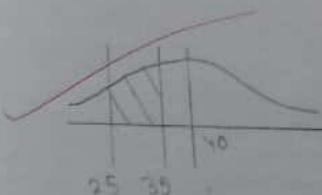
b)]



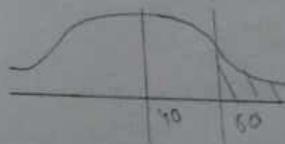
c)]



d)]



e)]



Q) If X is a random variable & follows the normal distribution with $\mu=50, \sigma^2=100$. Find
 Q1) $P(X>65)$ Q2) $P(X<32)$ Q3) $P(55 < X < 60)$ Q4) $P(20 < X < 30)$

$\rightarrow \text{pnorm}(70, 50, 10)$
 Q1) 0.0772499

$\rightarrow \text{pnorm}(68, 50, 10)$
 Q1) 0.0663072

$\rightarrow \text{pnorm}(32, 50, 10)$
 Q1) 0.03593032

$\rightarrow \text{pnorm}(60, 50, 10) - \text{pnorm}(35, 50, 10)$
 Q1) 0.7745375

$\rightarrow \text{pnorm}(30, 50, 10) - \text{pnorm}(20, 50, 10)$
 Q1) 0.02140023

Q) Let $X \sim N(160, 400)$ find $K_1 & K_2$ such that $P(X < K_1) = 0.2$
 $P(X > K_2) = 0.8$

$\rightarrow \text{qnorm}(0.8, 160, 20)$
 Q1) 165.0619

$\rightarrow \text{qnorm}(0.2, 160, 20)$
 Q1) 135.8374

- Q4] A random variable X follows normal distribution with $\mu=0, \sigma^2=2$. Generates 100 observation and evaluates its mean, median & variance.

```
> x=rnorm(100,10,2)
> summary(x)
[1]   nrs 1st 0 Median Mean 3rd 0    n
[1] 5713 8.444 9.723 9.914 11.325 14.22
> var(x)
[1] 3.648924
```

- Q5] Use a command to generate 100 random numbers for normally distribution with $\mu=50, \sigma^2=4$. Then the sample mean & median.

```
> x=rnorm(100,50,4)
> summary(x)
[1]   nrs 1st 0 Median Mean 3rd 0    n
[1] 44.73 50.46 52.01 52.35 54.34 58.65
```

Ans

Q3] A random variable Z follows normal distribution with $\mu=10, \sigma^2=2$. Generate 100 observation and calculate the mean, median & variance.

> `x=rnorm(100,10,2)`

> `summary(x)`

no	1st Q	Median	Mean	3rd Q	Max
5.713	8.664	9.723	9.914	11.378	16.73

> `var(x)`

[1] 3.64904

Q4] Write a command to generate 100 random numbers from uniformly distributed with $\mu=50, \sigma=1$. Find the sample mean & median.

7> `x=rnorm(100,50,1)`

7> `summary(x)`

no	1st Q	Median	Mean	3rd Q	Max
64.73	50.46	51.01	52.35	54.39	58.88

Now

sample mean & deviation give single population

Suppose the food level on the cookie is 2g. that it has almost 2 gms of saturated salt in a single cookie. If a sample of 5, 5 cookies, it was found that mean and of saturated salt per cookie is 2.1 gm. Assume that sample standard deviation is 0.3 at 5% level of significance can be rejected the claim no food salt added.

$$\sigma = 0.3$$

$$n = 35$$

$$\bar{x} = 2.1$$

$$x = 2$$

$$H_0 \text{ (null hypothesis)} = \mu \leq 2$$

$$H_1 \text{ (alt hypothesis)} = \mu > 2$$

$$Z = \frac{\bar{x} - \mu}{\sigma}$$

$$\frac{6}{\sqrt{35}}$$

$$Z = \frac{2.1 - 2}{\frac{0.3}{\sqrt{35}}} = 1.972021$$

$$\text{p-value} = 1 - \text{pnorm}(Z)$$

$$= 0.0243$$

Reject the null hypothesis p-value < 0.05

Accepted alternate hypothesis

32] A sample of 100 customers was randomly selected. It was found that average was 275. The standard deviation was 28. At the 0.05 level of significance, would you conclude that the claim by the customer is more than 250? Answer the question based on the statement that it is not 250.

$$\Rightarrow \bar{x} = 275, \mu = 250, \sigma = 30, n = 100.$$

$$z_0 = \frac{275 - 250}{30/\sqrt{100}} = 0.833$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\approx 0.833$$

$$\frac{30}{\sqrt{100}}$$

$\Rightarrow \text{nt}(z, 99, \text{lower tail}) = 0$

$$\text{p-value} = 2.3067 \cdot 10^{-13}$$

\Rightarrow Reject the null hypothesis - p-value < 0.05
Accept the alternative hypothesis ($\mu > 250$)

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3) A quality control engineer finds that sample of 100 have average life of 470 hours. Assuming population test whether the population mean is 480 hours. ($\alpha = 0.05$)

$$n = 100, \bar{x} = 470, \mu_0 = 480, \sigma = 25, \alpha = 0.05$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{470 - 480}{25/\sqrt{100}} = -4$$

 \sqrt{n}

$p(z \geq 4.09, \text{lower tail}) = P(z \leq -4)$
 $= 6.11257 \times 10^{-5}$

Reject the null hypothesis ($H_0: \mu = 480$)
 since the alternative hypothesis ($H_1: \mu < 480$)

4) A principal at school claims that the IQ is 100 of the students. A random sample of 30 students whose IQ was found to be 112. The SD of population is 15. Test the claim of principal.

Mistake: 1 tail test

~~$x_0 = \mu = 100$~~

~~$x_1 = \mu = 100$~~

~~$\bar{x} = 112, S_D = 15, \mu = 100, n = 30$~~

~~$$z = \frac{\bar{x} - \mu}{\frac{S_D}{\sqrt{n}}} = \frac{112 - 100}{\frac{15}{\sqrt{30}}}$$~~

~~$= \frac{112 - 100}{\frac{15}{\sqrt{30}}} = 4.38118$~~

$p\text{value} = 6.19567 \times 10^{-6}$
 $\Rightarrow \text{Reject the null hypothesis - claim of number of runs}(u=100)$

Method 2: 2 tail test

$$\mu_0 = \mu = 100$$

$$\mu_1 = \mu = 100$$

$$p\text{value} = 2 \times [1 - \text{norm}(\text{abs}(z))] = 1.77 \times 10^{-6}$$

$\Rightarrow \text{Reject the null hypothesis - pvalue} = 0.05$

④ Single population proportion

- c) It is believed that coin is fair. The coin is tossed 90 times; 25 time-head occurs. Indicate whether the coin is fair or not at 95% LOC.

$$\therefore z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$\therefore p_0 = 0.5$$

$$p_0 = 1 - p_0 = 0.5$$

$$p = 25 = 0.2777777777777778$$

$$n = 90$$

$$\therefore z = \frac{0.2777777777777778 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{90}}}$$

$$\mu_0 - \mu = 0.5$$

$$\sigma_{\bar{X}_1} = \sigma / \sqrt{n}$$

$$p\text{-value} = 2 \times [1 - \text{pnorm}(\text{abs}(z))]$$

$$p\text{-value} = 0.01141209$$

Reject the null hypothesis - $p < 0.05$
 Accept the alternate hypothesis

In a hospital 480 females & 520 males are born in a day
 To confirm that male and female are equal in

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \quad \hat{p} = \frac{520}{1000} = 0.52, p_0 = 0.5, q_0 = 0.5, n = 1000$$

$$H_0 = [\hat{p} \leq p_0]$$

$$H_1 = [\hat{p} \neq p_0]$$

$$\gg z = (\hat{p} - p_0) / \sqrt{q_0 + (p_0 q_0 / n)}$$

$$\gg z = 1.2645$$

$$p\text{-value} \rightarrow 2 \times [1 - \text{pnorm}(\text{abs}(z))]$$

$$p\text{-value} = 0.2060508$$

Reject the null hypothesis - $p\text{-value} < 0.5$
 Accept the alternate hypothesis i.e. $\hat{p} \neq p_0$

c) In a big city, 325 men out of 600 men are found to be self-employed. Test whether the maximum men in city are self-employed.

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$H_0: [p = p_0]$$

$$H_1: [p \neq p_0]$$

$$\gg p = 325/600 = 0.541667, p_0 = 0.5, q_0 = 0.5, n = 600$$

$$\gg p_z = [(0.541667 - 0.5) / (\sqrt{0.5 * 0.5 / 600})]$$

$$\gg z = 2.037975$$

$$\gg p\text{value} = 2 \times (1 - \text{norm}(abs(z)))$$

$$\gg p\text{value} = 0.04155239$$

Reject the null hypothesis : p-value < 0.05
Accept the null hypothesis if p >= 0.05

c) Experience shows that 20% of manufactured products are of top quality. In 1 day production of 400 articles, 50 are of top quality. Test hypothesis that experience of 20% of products is wrong?

$$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}, p = 0.25 (50/400), p_0 = 0.2, q_0 = 0.8, n = 400$$

$$H_0: [p_1 = 0.2] \\ H_A: [p_1 < 0.2]$$

$$z = \frac{(0.125 - 0.2)}{\sqrt{0.484(0.2 + 0.8)/1000}} \\ z = -3.75$$

$$\Rightarrow p\text{-value} = 2 \times (1 - \text{norm}(z_{\text{obs}})) \\ p\text{-value} = 0.0001768348$$

Reject the null hypothesis
and the null hypothesis is $p_1 < 0.2$

Formula:

$$z = \sqrt{pq} \left(\frac{n}{n} + \frac{m}{m} \right) \quad \text{where } p = \frac{p_1 + p_2}{n+m}$$

In an election campaign, a telephone poll of 800 registered voters shows favour 460-second web opinion. 520 of 1000 registered voters favoured the candidate of 0.5%. LOC (Level of confidence), is there sufficient evidence that popularity has decreased?

$$n=800, p_1 = 460/800 = 0.575, m=1000, p_2 = 520/1000 = 0.52 \\ p = [(0.575 * 800) + (0.52 * 1000)] / (1800) \\ p = 0.54444$$

$$z = \sqrt{0.54444(0.45556 + 0.49999)} + 1/2 \\ z = 0.001121394.$$

$H_0: p_1 = p_2$
 $H_1: p_1 \neq p_2$

value = (21/21) - normal(0.5) = 0.489053

As per the null hypothesis : value > 0.5
 Accept $p < 0.5$ i.e. H_1

- Q27 From two consignment A, 200 articles are drawn & it were found defective from consignment B, 200 samples are drawn out of which 30 are defective. Test whether the proportion of defective items in 2 consignments are significantly different.

$$H_0: p_1 = p_2$$

$$H_1: p_1 \neq p_2$$

$$p_1 = 44/200 = 0.22$$

$$n = 200 = m$$

$$p_2 = 30/200 = 0.15$$

$$\Rightarrow p = \frac{p_1 + p_2}{n+m}$$

$$n+m$$

$$\Rightarrow p = (0.22 \times 200 + 0.15 \times 200)/400$$

$$\Rightarrow p = 0.185$$

$$\therefore \text{Z} = \text{Z}_{\text{sig}}(0.489053 + 0.185) = (21/200)$$

$$= 0.003882976$$

$$\text{pvalue} = 2 * (1 - \text{pnorm}(\text{tstat}(x)))$$

$$\begin{aligned}\text{pvalue} &= 0.9969012 \\ \text{pvalue} &> p\end{aligned}$$

- Accord the null hypothesis i.e. $p_1 = p_2$

Null

Ex 4
Chi-Square Test

- a) Use the following data to test whether the aforesaid conditions of horse & child are independent.
- | | | Condition of Horses | |
|--------------------|-------|---------------------|-------|
| | | Clear | Dusty |
| Condition of Child | Clear | 70 | 50 |
| | Dusty | 80 | 20 |
| | | 35 | 45 |

H_0 = Both are independent, H_1 = Both are dependent

> x = c(70, 80, 35)
 > y = c(50, 20, 45)
 > z = data.frame(x, y)

> z

	x	y
1	70	50
2	80	20
3	35	45

> chisq.test(z)

Pearson's chi-squared test

data: z

χ^2 -square = 25.646, df = 2, p-value = 2.698e-06

: Reject the null hypothesis
 : Both are dependent.

Q. A dice is thrown 120 times & following results are obtained.

No. of turns	Frequency
1	30
2	25
3	18
4	10
5	22
6	15

Test the hypothesis that dice is unbiased.

H_0 = dice is unbiased, H_1 = dice is biased.

$$\text{obs} = [30, 25, 18, 10, 22, 15]$$

$$\chi^2_{\text{exp}} = \sum (\text{obs} - \text{exp})^2 / \text{exp}$$

$$\chi^2_{\text{exp}} \\ [0] 2.0$$

$$\chi^2 = \sum ((\text{obs} - \text{exp})^2 / \text{exp})$$

$$\chi^2_{\text{crit}} (\chi^2, df = \text{length}(\text{obs}) - 1)$$

$$[1] 0.956659$$

Accept the null hypothesis.

Dice is unbiased.

Q3] An IQ test was conducted & the students were assessed again after training. The results are following:

Before	After
110	120
120	118
123	126
127	136
132	121
125	125

Test whether there is change in the IQ after the training.

$$H_1 = \text{no change in IQ}$$

$$H_0 = \text{IQ increases after training}$$

$$\gamma_a = \{120, 118, 125, 136, 121\}$$

$$\gamma_b = \{110, 120, 123, 132, 125\}$$

$$\gamma_{Z} = \text{sum}((b - a)^2) / a$$

$$\gamma_{\text{reduced}} [z_j, df = \text{length}(b) - 1]$$

$$[0.11, 35.954]$$

Accept the null hypothesis

There is change in IQ after training.

	grades	Undergraduates
below few to fail	20 47	25 9

Is there any association between student's number
for type of education & method

H0: Independent, H1: Dependent

chi-squared (20, 47, 25, 9)

math (2, chisq = 2)

using test (2)

Pearson chi-squared test with Yate's continuity
corrected

data = 2

chi-squared = $\frac{18.03}{25-4-2} = 3.75$, df = 3, p-value = 0.00059215 + 10^{-13}

reject null Hypothesis

Both are dependent

Q5) A die is tossed 180 times.	
No. of turns	frequency
1	20
2	30
3	35
4	40
5	12
6	43

Test the hypothesis that die is unbiased.

$$H_0 = \text{die is biased}$$

$$H_1 = \text{die is unbiased}$$

$$> x=c(20,30,35,40,12,43)$$

Chi-squared test (χ²)

Chi-squared test for given probabilities

$$\text{Data: } x$$

$$\chi^2\text{-squared} = 23.973, df = 5, p\text{-value} = 0.000227$$

Reject null hypothesis
Die is unbiased.

Mark

Titled T Test

Let $x = [3366, 3337, 3361, 3318, 3316, 3359, 3348, 3356, 3376, 3322, 3377, 3355, 3408, 3401, 3377, 3324, 3383]$

With the R command for following to test hypothesis

$$\text{① } H_0: \mu = 3400, H_1: \mu \neq 3400$$

$$\text{② } H_0: \mu < 3400, H_1: \mu > 3400$$

$$\text{③ } H_0: \mu = 3400, H_1: \mu < 3400$$

at 95% level of confidence. At 5% level at 77%

level of confidence

$$\text{④ } H_0: \mu = 3400$$

$$H_1: \mu \neq 3400$$

$x = [3366, 3337, 3361, 3318, 3316, 3359, 3348, 3356, 3376, 3322, 3377, 3355, 3408, 3401, 3398, 3404, 3373, 3374, 3314, 3379]$

\Rightarrow test [x , mu=3400, alter='two.sided', conf.level=0.95]

One sample test

data: x

$$t = -4.4765, df = 19, p-value = 0.0002528$$

alternative hypothesis: true mean is not equal to 3400

95% current confidence level:
~~3361.77 ± 3376.103~~

Sample estimate:

mean of x :

$$3373.95$$

④ Rejected H_0
 $t = -3.4815, df = 9, p\text{-value} = 0.000752$ alternative hypothesis: true mean is not equal to 3400
 $\bar{x} = 3387.57$
 $s = 310.73$

Sample estimates:

mean of x :
 3387.57

Rejected H_0
 accept H_1

⑤ $H_0: \mu = 3400$
 $H_1: \mu > 3400$
 $t + \text{t.test}(T, mu=3400, alt="greater", conf.level=0.99)$
 one sample t-test

data: T
 $t = -4.468, df = 9, p\text{-value} = 0.9999$

alternative hypothesis: true mean is greater than 3400

$\bar{x} = 33.6341$ $S_{\bar{x}}$

Sample estimates:

mean of x :

33.6341 accept H_1

$t + \text{t.test}(T, mu=3400, alt="greater", conf.level=0.97)$
 one sample t-test

data: x
 $t = -4.4865$, df = 19, p-value = 0.9999
 alternative hypothesis: true mean is greater than 3100
 3357.337 Inf
 sample estimate:
 mean of x
 3373.95
 assert H0

Below are the data of gain in weight on 2 different diets A & B

Diet A: 25, 32, 30, 43, 24, 14, 32, 24, 31, 31, 35, 25

Diet B: 44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18, 21

$$\begin{aligned} \therefore H_0 &= a - b = 0 \\ H_1 &= a - b \neq 0 \end{aligned}$$

$\gt a = c(25, 32, 30, 43, 24, 14, 32, 24, 31, 31, 35, 25)$
 $\gt b = c(44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18, 21)$

$t.test(a, b, paired = \text{TRUE}, alt = \text{"two.sided"}, conf.level = 0.95)$

Paired t-test

data: a and b

$t = -0.62757$, df = 11, p-value = 0.5429

alternative hypothesis: true difference in means is not equal to 0

- 95 percent confidence interval:
 -14.26 ± 3.30 $t = -0.33397$
- Sample estimates:
 mean of the differences
 -3.16667
- Accept H_0
 There are no differences in weight
- (2) 11 students gave the test after 1 month they again gave the test after the tuition, so the marks give evidence that students have benefited by coaching
- E1: 23, 20, 19, 21, 18, 20, 16, 17, 23, 16, 19
 E2: 24, 19, 22, 18, 20, 22, 20, 20, 23, 20, 17
 Test at 99 level of confidence
- $E1 = 23, 20, 19, 21, 18, 20, 16, 17, 23, 16, 19$
 $E2 = 24, 19, 22, 18, 20, 22, 20, 20, 23, 20, 17$
- $H_0: \mu_1 = \mu_2$
 • $H_A: \mu_1 < \mu_2$
- $t = t_{\text{test}}(e_1, e_2, \text{paired} = \text{t}, \text{alter} = \text{"less"}, \text{conf.level} = 0.99)$

Paired t-test

data : e1 and e2

$t = -1.432$, $df = 10$, $p\text{-value} = 0.08441$

alternative hypothesis: true difference in mean is less than 0

99 percent confidence interval:

-2.07 ± 0.867383

sample estimates:
mean of the differences:
-1

Accept H_0

Two drugs for BP were given & data was collected
 $D_1 = \{0.7, -1.8, -0.2, -1.2, -0.1, 3.4, 3.7, 0.8, 0.2\}$

$D_2 = \{1.9, 0.8, 1.1, 0.1, -0.1, 4.6, 5.6, 1.6, 4.6, 3.4\}$

To two drugs have same effect, check whether two drugs have same effect on patient or not.

$$H_0: D_1 = D_2$$

$$H_1: D_1 \neq D_2$$

$$\bar{D}_1 = \{0.7, -1.8, -0.2, -1.2, -0.1, 3.4, 3.7, 0.8, 0.2\}$$

$$\bar{D}_2 = \{1.9, 0.8, 1.1, 0.1, -0.1, 4.6, 5.6, 1.6, 4.6, 3.4\}$$

$$t \text{ test } [D_1, D_2, \text{ alter: "two-sided", paired-T, conf level} = 0.95]$$

Paired t test

data: D_1 and D_2

$t = -4.0621$, $df = 9$, $p\text{value} = 0.002833$

Alternative hypothesis: true difference in mean is not equal to 0.

95 percent confidence interval:

Mean of the differences:

-1.51

Reject H_0

Accept H_1

a) If there is difference in relation for the same μ_1
 2 different methods:
 $\text{CB} = \{53000, 40051, 41974, 44031, 40470, 36963\}$
 $\text{DCB} = \{62490, 58150, 48495, 52313, 47574, 43552\}$

$$\Rightarrow H_0: \mu_1 = \mu_2$$

$$\begin{aligned} H_0 &= \mu_1 = \mu_2 \\ \text{PRL} &= \{53000, 40051, 41974, 44031, 40470, 36963\} \\ \text{DCB} &= \{62490, 58150, 48495, 52313, 47574, 43552\} \end{aligned}$$

$\Rightarrow \text{tstat}(c_0, c_1, \text{value}) = T_{\text{value}} = \text{"true value"}$, conf.level = 0.95

Paired t-test

data: cb and dc

t = -4.6568, df = 5, p-value = 0.00666

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

$$-10404.821 \quad -2792.146$$

Sample estimates:

mean of the differences:

$$-6691.833$$

Reject H_0

conf. 95%

new

Practical - 7

F Test

65

7 life expectancy in 10 regions of India in 1970 & 2000 are given below. Test whether the variances of 2000 & 1970 are same.

$$1970 = \{37, 39, 35, 43, 45, 44, 46, 49, 50, 51\}$$

$$2000 = \{44, 45, 47, 43, 42, 47, 50, 41, 48, 52, 42, 57\}$$

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

$$S_1 = \{(37, 39, 35, 43, 45, 44, 46, 49, 50, 51)\}$$

$$S_2 = \{(44, 45, 47, 43, 42, 47, 50, 41, 48, 52, 42, 57)\}$$

$$\text{var. test } (S_1, S_2) \quad p\text{-value} = 0.9136$$

accept H_0

$$\begin{array}{|c|c|} \hline I & 25, 28, 21, 22, 22, 29, 31, 31, 26, 31 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline II & 30, 25, 31, 32, 23, 28, 35, 36, 31, 32, 27, 31, 30, 27 \\ \hline \end{array}$$

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

$$S_1 = \{(25, 28, 21, 22, 22, 29, 31, 31, 26, 31)\}$$

$$S_2 = \{(30, 25, 31, 32, 23, 28, 35, 36, 31, 32, 32, 27, 31, 30, 27)\}$$

$$p\text{-value} = 0.5341$$

accept H_0

For the following data test the hypothesis of equality of 2 population means

of equality of population variances

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

$x = c(175, 168, 145, 190, 181, 185, 175, 200)$
 $y = c(180, 170, 153, 180, 170, 183, 187, 205)$
 levene test (2,4)
 p value = 0.779

accept H_0
 t test (2,4)
 p value = 0.8216
 accept H_0

- Q6] The following are the prices of commodity in rupees of shirts selected at random from different cities.

city A: 74.10, 77.30, 75.35, 74, 73.00, 79.30, 75.30, 76.80, 77,

76.40

city B: 70.80, 74.90, 76.20, 72.80, 78.10, 74.70, 69.80, 81.20

$x = c(74.10, 77.30, 75.35, 74, 73.00, 79.30, 75.30, 76.80, 77,$
 76.40)

$y = c(70.80, 74.90, 76.20, 72.80, 78.10, 74.70, 69.80, 81.20)$

Shapiro test (7)
 p value = 0.6569
 data is normal

Shapiro test (21)
 p value = 0.9304.
 data is normal

$H_0: \sigma_1^2 = \sigma_2^2$
 $H_1: \sigma_1^2 \neq \sigma_2^2$
 F value = 0.04249
 2 variances are not equal
 Reject H_0
 accept H_1

$H_0: \mu_1 = \mu_2$
 $H_1: \mu_1 \neq \mu_2$
 t-test (var.equal=F)
 p-value = 3.488e-16
 t-test (var.equal=F)
 p-value = 1.46e-10
 accept H_0

Create a csv file in excel. Import the file in R & apply the test to check the equality of variance of 2 data.
 obs 1: 10, 15, 17, 11, 16, 20
 obs 2: 15, 14, 16, 11, 12, 19

$H_0: \sigma_1^2 = \sigma_2^2$
 $H_1: \sigma_1^2 \neq \sigma_2^2$

> Data = read.csv("file name [], header=T")
> Date

observation1	observation2
10	15
15	14
17	16
11	11
16	12
20	19

> attach(Data)
> mean(observation1)
14.83333

> t.test(observation1, observation2)
p value = 0.5717.
- do not H0.

Alert

Practical 1

Title: Non Parametric Test

In times of failures in hrs of 10 randomly selected
a vast battery of a certain company is as follows.

28.9, 15.2, 28.7, 72.5, 48.6, 52.4, 37.6, 49.5, 62.1, 54.5

Test the hypothesis that the population median is 63
against alternative is less than 63 at 5% of level of
significance

$$H_0: \text{median} = 63$$

$$H_1: \text{median} < 63$$

$$\gamma_x = c [28.9, 15.2, 28.7, 72.5, 48.6, 52.4, 37.6, 49.5, 62.1, 54.5]$$

$$\gamma_{sp} = \text{length}(\text{which}(x > 63))$$

$$\gamma_{sn} = \text{length}(\text{which}(x < 63))$$

$$\gamma_{sp}$$

$$4$$

$$\gamma_{sn}$$

$$1$$

$$\gamma_n = \gamma_{sp} + \gamma_{sn}$$

$$\gamma_{unifm}(0.05, n, 0.5)$$

$$2$$

$$\therefore \text{glum} \leq \text{sn}$$

$$\text{Accept } H_0$$

$$\therefore \text{median} = 63$$

e) The following data give the weight of 40 students in random sample
45, 48, 52, 64, 46, 67, 56, 41, 69, 61, 57, 54, 50, 48, 55, 63, 61, 51, 53, 56, 59, 53, 59, 63, 53, 56, 57, 49, 50, 52, 54.
Test the sign test to see whether the median
height of population is 50kg against alternative which

$$H_0: \text{median} = 50$$

$$H_1: \text{median} > 50$$

$x = [45, 48, 52, 64, 46, 67, 56, 41, 69, 61, 57, 54, 50, 48, 55, 63, 61, 51, 53, 56, 59, 53, 59, 63, 53, 56, 57, 49, 50, 52, 54]$

$$n = 40 \text{ (even)} (x > 50)$$

2.40

2.25

2.50: larger / which ($x > 50$)

2.50

1.2

2.00: after

? $\text{glirant}(0.03, n, 0.5)$

1.6

? $\text{glirant}(2.0)$

$\therefore \text{Reject } H_0$

70

Q) The median age of tourists visiting a certain place is claim to be 41 yrs. A random sample of 20 tourists have the ages
 $25, 29, 52, 48, 57, 30, 43, 36, 30, 49, 28, 39, 44, 63, 33, 65, 42$.
 Use the sign test to check the claim.

$H_0: \text{median} = 41$

$H_1: \text{median} \neq 41$

$\tau_n = \text{length}(\{\text{which}(x > 41)\})$

τ_{10}

9

$\tau_{10} = \text{length}(\{\text{which}(x > 41)\})$

τ_{10}

8

$\tau_{10} = 8 < 10$

$\tau_{10} \text{ binom}(0.05, n=10)$

$\therefore \text{sign test}$

$\text{Accept } H_0$

$\text{Median} = 41$.

Q4) The time in minutes that a patient has to wait for consultation is recorded as follows.
 $15, 17, 24, 25, 20, 21, 22, 23, 17, 25, 24, 26$
 Use wilcoxon sign test to check whether the median weight is still is more than 20 at 5% level of significance.

$H_0: \text{median} \geq 20$
 $H_1: \text{median} < 20$
 $\tau = 5 (15, 17, 24, 25, 20, 21, 23, 28, 17, 25, 24, 26)$
 wilcoxon-test (τ , alternative="less")
 $p\text{-value} = 0.009$

Accept H_0 .

weight before	weight after
6.5	7.2
7.5	8.2
7.5	7.2
6.2	6.6
7.2	7.3

Use wilcoxon test to check whether the weight of person increases after stopping the smoking at 5% level of significance at 5% level of confidence.

H_0 : increase after the stopping smoking
 H_1 : does not increase after stopping smoking.

$x = \{65, 76, 75, 62, 72\}$
 $y = \{72, 52, 72, 66, 73\}$

$t = x - y$

Wilcoxon test (τ , $\mu = 0$)

p-value = 0.1936
do not H_0

reject

Special:

One-way Test

- a) The following data gives the effect of 3 treatments
- $$T_1 = 2, 3, 7, 2, 6$$
- $$T_2 = 10, 8, 7, 5, 10$$
- $$T_3 = 12, 13, 14, 13, 15$$

Test whether significant that all treatments are equally affected

```

> t1=c(2,3,7,2,6)
> t2=c(10,8,7,5,10)
> t3=c(12,13,14,13,15)
> z=data.frame(t1,t2,t3)

```

	t1	t2	t3
1	2	10	10
2	3	8	13
3	7	7	14
4	2	5	13
5	6	10	15

> stack(z)

	values	ind
--	--------	-----

1	2	t1
2	3	t1
3	7	t1
4	2	t1
5	6	t1
6	10	t2

1	8	t2
2	7	t2
3	5	t2
4	10	t2
10	10	t3
11	13	t3
12	14	t3
13	13	t3
14	15	t3
15	15	t3

lapply(z)
xx(values ~ ind, data = z)

all:
ovt(formula = values ~ ind, data = z)

Terms:

	1st	2nd
sum of Squares	203.3333	Products
Deg. of Freedom	2	12

Residual standard error: 2.12137
Estimated effects may be unreliable.
oneway-test(values ~ ind, data = z)

data: values and ind

F = 21.537, num df = 2,0000, denom df = 7.9316, p-value = 0.0001222

a) The following data gives life of tyres of four brands
 A: 20, 23, 18, 17, 23, 21
 B: 19, 15, 17, 20, 16, 17
 C: 21, 19, 22, 19, 20
 D: 16, 14, 18, 17, 16
 Test the hypothesis that average life of all tyres is same.

$\rightarrow a = \{20, 23, 18, 17, 23, 21\}$
 $\rightarrow b = \{19, 15, 17, 20, 16, 17\}$
 $\rightarrow c = \{21, 19, 22, 19, 20\}$
 $\rightarrow d = \{16, 14, 18, 17, 16\}$
 $\rightarrow z = \text{dstat}(a, b, c, d)$

\rightarrow^2
 [[1]]
 [1] 20 23 18 17 23 21

[2]
 [1] 19 15 17 20 16 17

[3]
 [1] 21 19 22 19 20

[4]
 [1] 16 14 18 17 16

$\rightarrow z = \text{dstat}(a, b, c, d)$
 \rightarrow^2

77

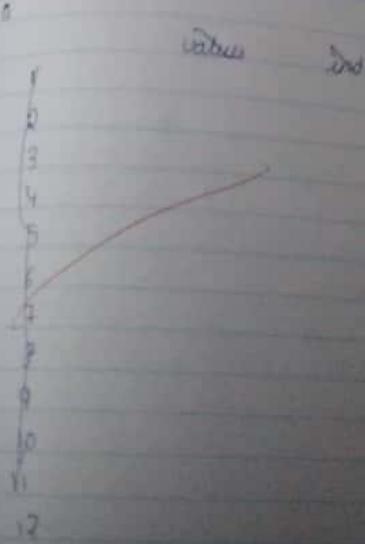
f1 [13] 20 23 18 17 23 23

f2 [13] 19 15 17 20 16 12

f3 [13] 21 19 22 17 20

f4 [13] 15 14 16 18 14 16

7. $t = \text{stack}(D)$



? anova (value ~ ind, data=c)
? one way test (value ~ ind, data=c),
 $p\text{-value} = 0.74442 \approx 0.74$

H₀ is rejected

Q] 3 types of mask are applied for the protection of eye
no. of days of restriction were noted test whether
these are equally effective

H_0 : The mask are equally effective
 H_1 : The mask are not equally effective

? a=c(26,45,46,47,48,49)
? b=c(40,42,51,52,53)
? c=c(50,53,58,59)

? m=stack(a,b,c)
? t=stack(m)
? one way test (value ~ ind, data=t)

p-value = 0.03822

Reject H₀

An experiment was conducted on 3 groups and the assumption were null-test whether the hypothesis that the group has equal means results in their null.

H_0 - The result are equal
 H_1 - The result are unequal

n = c(23, 26, 51, 18, 52, 37, 27, 44)
n20 = c(22, 27, 29, 37, 45, 48, 49, 55)
n60 = c(59, 66, 38, 43, 51, 60, 50, 52)

m = data(a = n, b = n20, c = n60)
t = stack(m)

anova.test(value ~ ind, data = t)

P-value 0.1412

H_0 is rejected

not