

Machine Learning

Bayes Classifier Implementation from Scratch

(i.e. No library function for core implementation)

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```
## Objective : Using Naive Bayes classifier to classify data points
#             into classes ("Yes/No") for bank default based on
#             specific client attributes.
#             N.B. Dependent variable is categorical.
#Dataset found here -> https://archive.ics.uci.edu/ml/datasets/bank+marketing

#####
#####          BANK DATA          #####
#####

#Bank Info dataset most interesting features are :
# (1) Age
# (2) nr.employed: number of employees - quarterly indicator (numeric)
# (3) previous: number of contacts performed before this campaign and for this client (numeric)
# (4) emp.var.rate: employment variation rate - quarterly indicator (numeric)

# (5) Subscription status "y": "Has subscribed to term deposit?"
setwd("C:/Users/Kisha/Documents")
Bkdata <- read.csv("bank-additional.csv",header=TRUE,sep=";")

head(Bkdata)
```

```
##   age      job marital      education default housing   loan
## 1  30 blue-collar married      basic.9y      no    yes    no
## 2  39  services single      high.school      no    no    no
## 3  25  services married      high.school      no    yes    no
## 4  38  services married      basic.9y      no unknown unknown
## 5  47   admin. married university.degree      no    yes    no
## 6  32  services single university.degree      no    no    no
##   contact month day_of_week duration campaign pdays previous  poutcome
## 1  cellular   may         fri      487        2    999        0 nonexistent
## 2  telephone  may         fri      346        4    999        0 nonexistent
## 3  telephone  jun         wed      227        1    999        0 nonexistent
## 4  telephone  jun         fri       17        3    999        0 nonexistent
## 5  cellular   nov         mon       58        1    999        0 nonexistent
## 6  cellular   sep         thu      128        3    999        2 failure
##   emp.var.rate cons.price.idx cons.conf.idx euribor3m nr.employed y
## 1          -1.8      92.893      -46.2      1.313      5099.1 no
## 2           1.1      93.994      -36.4      4.855      5191.0 no
## 3           1.4      94.465      -41.8      4.962      5228.1 no
## 4           1.4      94.465      -41.8      4.959      5228.1 no
## 5          -0.1      93.200      -42.0      4.191      5195.8 no
## 6          -1.1      94.199      -37.5      0.884      4963.6 no
```

```
summary(Bkdata)
```

```
##      age      job      marital
## Min.   :18.00  admin.   :1012  divorced: 446
## 1st Qu.:32.00  blue-collar: 884  married :2509
## Median :38.00  technician : 691  single  :1153
## Mean   :40.11  services   : 393  unknown : 11
## 3rd Qu.:47.00  management : 324
## Max.   :88.00  retired    : 166
##      (Other)   : 649
##      education  default      housing      loan
## university.degree :1264  no :3315  no :1839  no :3349
## high.school       : 921  unknown: 803  unknown: 105  unknown: 105
## basic.9y          : 574  yes  : 1  yes :2175  yes : 665
## professional.course: 535
## basic.4y          : 429
## basic.6y          : 228
## (Other)           : 168
##      contact      month      day_of_week      duration
## cellular :2652  may :1378  fri:768  Min. : 0.0
## telephone:1467  jul : 711  mon:855  1st Qu.: 103.0
##      aug : 636  thu:860  Median : 181.0
##      jun : 530  tue:841  Mean : 256.8
##      nov : 446  wed:795  3rd Qu.: 317.0
##      apr : 215  Max. :3643.0
##      (Other): 203
##      campaign      pdays      previous      poutcome
## Min. : 1.000  Min. : 0.0  Min. :0.0000  failure : 454
## 1st Qu.: 1.000  1st Qu.:999.0  1st Qu.:0.0000  nonexistent:3523
## Median : 2.000  Median :999.0  Median :0.0000  success : 142
## Mean : 2.537  Mean :960.4  Mean :0.1903
## 3rd Qu.: 3.000  3rd Qu.:999.0  3rd Qu.:0.0000
## Max. :35.000  Max. :999.0  Max. :6.0000
##
## emp.var.rate  cons.price.idx  cons.conf.idx  euribor3m
## Min. :-3.40000  Min. :92.20  Min. : -50.8  Min. :0.635
## 1st Qu.: -1.80000  1st Qu.:93.08  1st Qu.: -42.7  1st Qu.:1.334
## Median : 1.10000  Median :93.75  Median : -41.8  Median :4.857
## Mean : 0.08497  Mean :93.58  Mean : -40.5  Mean :3.621
## 3rd Qu.: 1.40000  3rd Qu.:93.99  3rd Qu.: -36.4  3rd Qu.:4.961
## Max. : 1.40000  Max. :94.77  Max. : -26.9  Max. :5.045
##
## nr.employed  y
## Min. :4964  no :3668
## 1st Qu.:5099  yes: 451
## Median :5191
## Mean :5166
## 3rd Qu.:5228
## Max. :5228
##
```

```
dim(Bkdata)
```

```
## [1] 4119 21
```

```
str(Bkdata) # gives a nice decription of each column and its factors
```

```
## 'data.frame': 4119 obs. of 21 variables:
## $ age : int 30 39 25 38 47 32 32 41 31 35 ...
## $ job : Factor w/ 12 levels "admin.", "blue-collar",...: 2 8 8 8 1 8 1 3 8 2 ...
## $ marital : Factor w/ 4 levels "divorced", "married",...: 2 3 2 2 2 3 3 2 1 2 ...
## $ education : Factor w/ 8 levels "basic.4y", "basic.6y",...: 3 4 4 3 7 7 7 6 3 ...
## $ default : Factor w/ 3 levels "no", "unknown",...: 1 1 1 1 1 1 1 2 1 2 ...
## $ housing : Factor w/ 3 levels "no", "unknown",...: 3 1 3 2 3 1 3 3 1 1 ...
## $ loan : Factor w/ 3 levels "no", "unknown",...: 1 1 1 2 1 1 1 1 1 1 ...
## $ contact : Factor w/ 2 levels "cellular", "telephone": 1 2 2 2 1 1 1 1 1 2 ...
## $ month : Factor w/ 10 levels "apr", "aug", "dec",...: 7 7 5 5 8 10 10 8 8 7 ...
## $ day_of_week : Factor w/ 5 levels "fri", "mon", "thu",...: 1 1 5 1 2 3 2 2 4 3 ...
## $ duration : int 487 346 227 17 58 128 290 44 68 170 ...
## $ campaign : int 2 4 1 3 1 3 4 2 1 1 ...
## $ pdays : int 999 999 999 999 999 999 999 999 999 ...
## $ previous : int 0 0 0 0 0 2 0 0 1 0 ...
## $ poutcome : Factor w/ 3 levels "failure", "nonexistent",...: 2 2 2 2 2 1 2 2 1 2 ...
## $ emp.var.rate : num -1.8 1.1 1.4 1.4 -0.1 -1.1 -1.1 -0.1 -0.1 1.1 ...
## $ cons.price.idx: num 92.9 94 94.5 94.5 93.2 ...
## $ cons.conf.idx : num -46.2 -36.4 -41.8 -41.8 -42 -37.5 -37.5 -42 -42 -36.4 ...
## $ euribor3m : num 1.31 4.86 4.96 4.96 4.19 ...
## $ nr.employed : num 5099 5191 5228 5228 5196 ...
## $ y : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 1 1 1 1 ...
```

```
class(Bkdata)
```

```
## [1] "data.frame"
```

```
#####

# Finding correlation matrix for all attributes
# Then selecting the ones with moderate to high correlation to response variable i.e target of Subscription is "Yes"
#Repeating exercise to select attributes with fairly strong or moderate correlation coefficient

#####
```

```
#remove.packages("caret")
#install.packages('Rcpp', dependencies = TRUE)
#install.packages('caret', dependencies = TRUE)
library(caret)
```

```
## Loading required package: lattice
```

```
## Loading required package: ggplot2
```

```
#remove.packages("ggplot2")
#install.packages("ggplot2")
library(ggplot2)
```

```
##### EXPLORATION #####
int_factorsExplore <- dummyVars(" ~ .",data = Bkdata)
newBkdataExplore <- data.frame(predict(int_factorsExplore,Bkdata))
dim(newBkdataExplore)
```

```
## [1] 4119 65
```

```
explore_cormatBk <- cor(newBkdataExplore)
dim(explore_cormatBk)
```

```
## [1] 65 65
```

```
rownames(explore_cormatBk)[65] # Limiting correlation to all variables versus my response variable
```

```
## [1] "y.yes"
```

```
refined_corBk <- explore_cormatBk[,65]

refined_corBk2 <- refined_corBk[abs(refined_corBk[])>=0.20] # refining further to identify correlations above a specific value
# varied the min correlation amount several times to aid in selecting input variables for desired output variable
refined_corBk2 # reflects only values that have correlation above a min
```

```
##          duration          pdays          previous
##          0.4185654         -0.3320115          0.2556966
## poutcome.nonexistent poutcome.success emp.var.rate
##          -0.2071789          0.3258037         -0.2832157
##          euribor3m        nr.employed           y.no
##          -0.2985650         -0.3492412         -1.0000000
##          y.yes
##          1.0000000
```

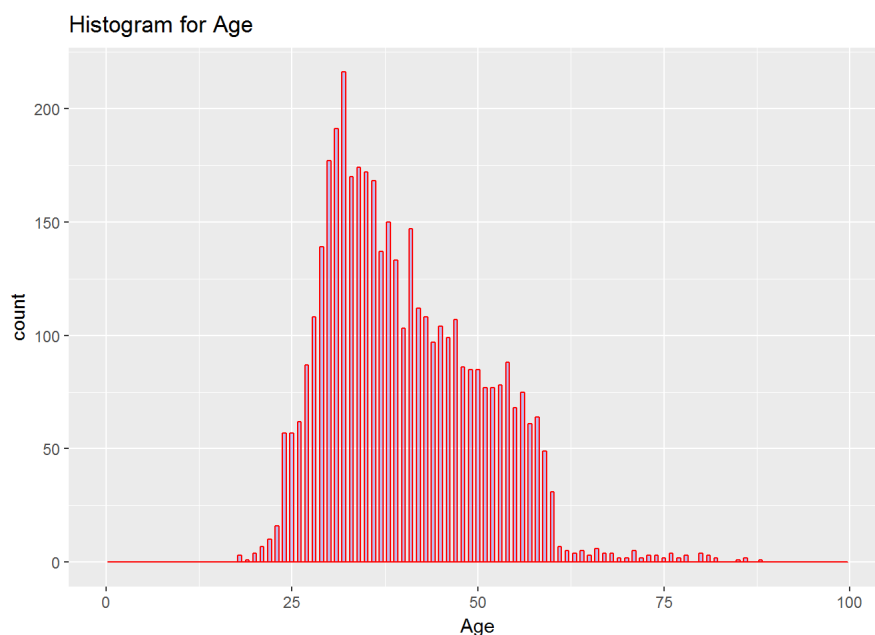
```
#####
# Attributes were finally selected based on the strength of the correlaton to response variable y.

#Attribute : Age

mean_age <- mean(Bkdata$age) # mean age of client
median_age <- median(Bkdata$age) # mean age of client
var_age <- var(Bkdata$age) # variance in age of client

#graphics.off()
#par("mar")
#par(mar=c(1,1,1,1))

qplot(Bkdata$age,
      geom="histogram",
      binwidth = 0.5,
      main = "Histogram for Age",
      xlab = "Age",
      fill=I("blue"),
      col=I("red"),
      alpha=I(.2),
      xlim=c(0,100))
```



```
#Code REf. for histogram:https://www.r-bloggers.com/how-to-make-a-histogram-with-ggplot2/
#####
# nr.employed: number of employees - quarterly indicator (numeric)

mean_numempl <- mean(Bkdata$nr.employed) # mean
median_numempl <- median(Bkdata$nr.employed) # median
var_numempl <- var(Bkdata$nr.employed) # variance

mean_numempl
```

```
## [1] 5166.482
```

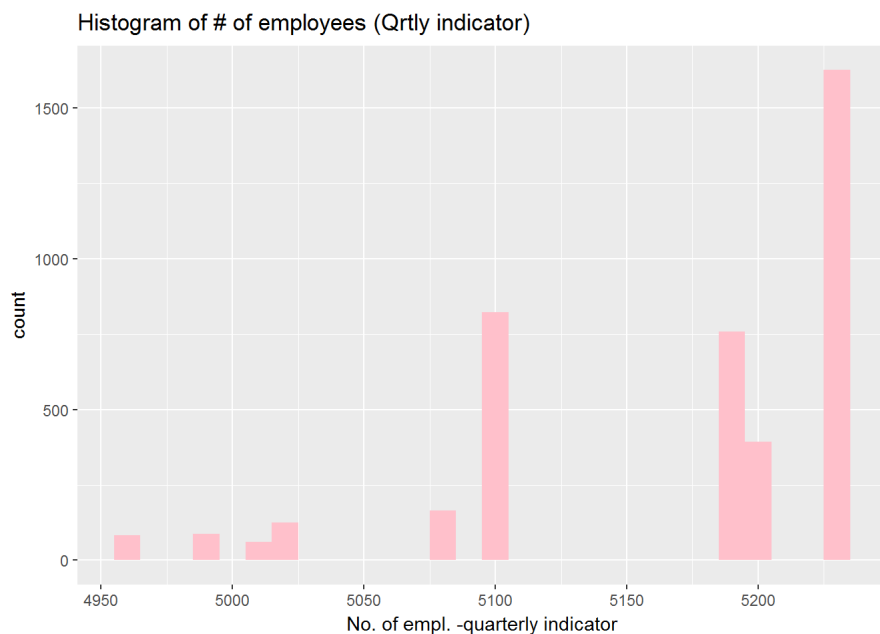
```
median_numempl
```

```
## [1] 5191
```

```
var_numempl
```

```
## [1] 5426.96
```

```
##Histogram of nr.employed: number of employees - quarterly indicator (numeric)
ggplot(Bkdata, aes(Bkdata$nr.employed)) +labs(title="Histogram of # of employees (Qrtly indicator)") + xlab("No. of empl. -q
uarterly indicator") +
  geom_histogram(binwidth=10,fill=I("pink"))
```



```
#####
```

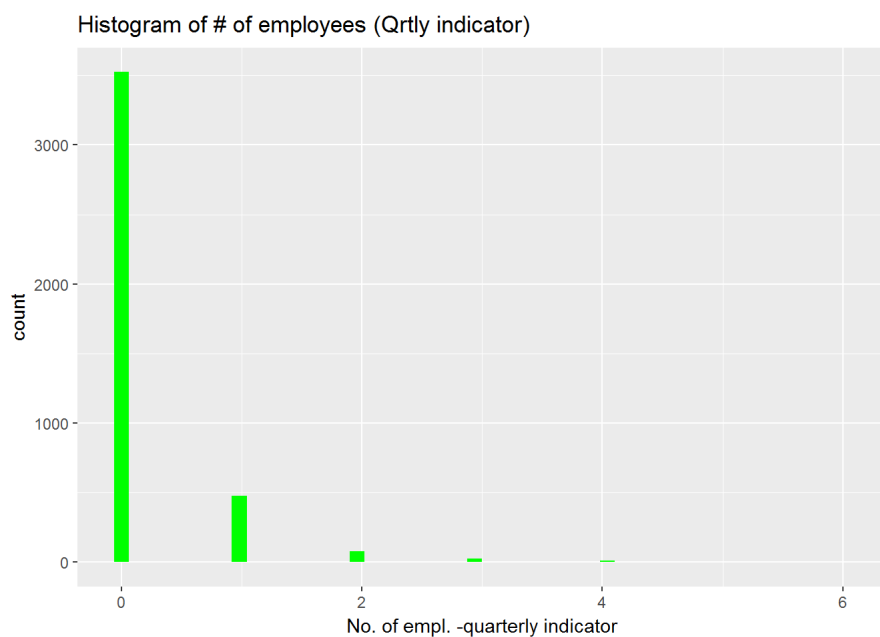
```
#Attribute : previous: number of contacts performed before this campaign and for this client (numeric)
```

```
mean_prev <- mean(Bkdata$previous) # mean No. of prev client contacts
```

```
median_prev <- median(Bkdata$previous)
```

```
var_prev <- var(Bkdata$previous) # variance in No. of prev client contacts
```

```
ggplot(Bkdata, aes(Bkdata$previous)) + labs(title="Histogram of # of employees (Qrtly indicator)") +
  xlab("No. of empl. -quarterly indicator") +
  geom_histogram(bins=50,fill=I("green"))
```



```
#####
# emp.var.rate: employment variation rate - quarterly indicator (numeric)

mean_empvarrt <- mean(Bkdata$emp.var.rate) # mean age
median_empvarrt <- median(Bkdata$emp.var.rate) # median age
var_empvarrt <- var(Bkdata$emp.var.rate) # variance in age

mean_empvarrt
```

```
## [1] 0.08497208
```

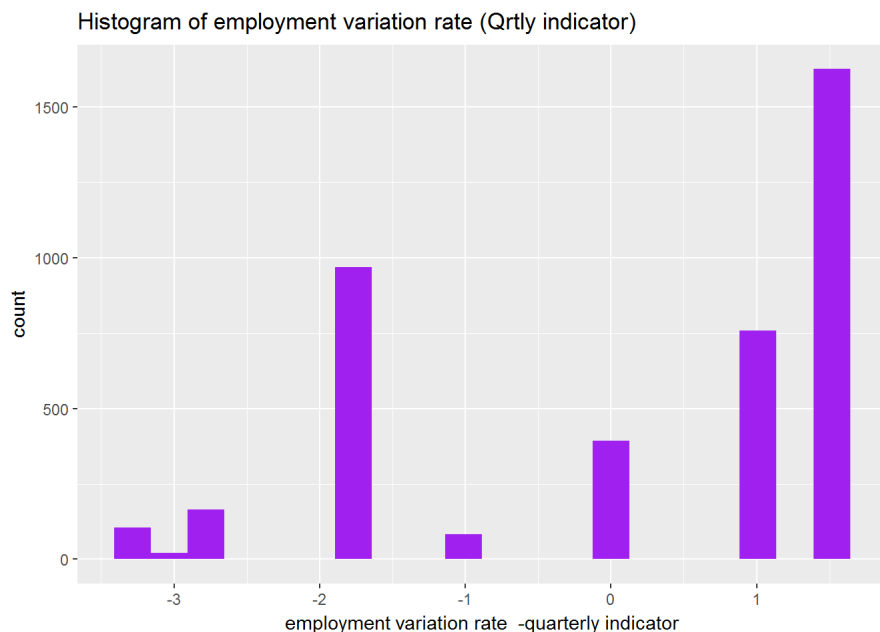
```
median_empvarrt
```

```
## [1] 1.1
```

```
var_empvarrt
```

```
## [1] 2.443327
```

```
##Histogram of employment variation rate- quarterly indicator (numeric)
ggplot(Bkdata, aes(Bkdata$emp.var.rate)) + labs(title="Histogram of employment variation rate (Qrtly indicator)") +
  xlab("employment variation rate -quarterly indicator") +
  geom_histogram(bins=20,fill=I("purple"))
```



```
#####
#Attribute : Subscription status

Bk_subscript <- Bkdata$y
head(Bk_subscript)
```

```
## [1] no no no no no no
## Levels: no yes
```

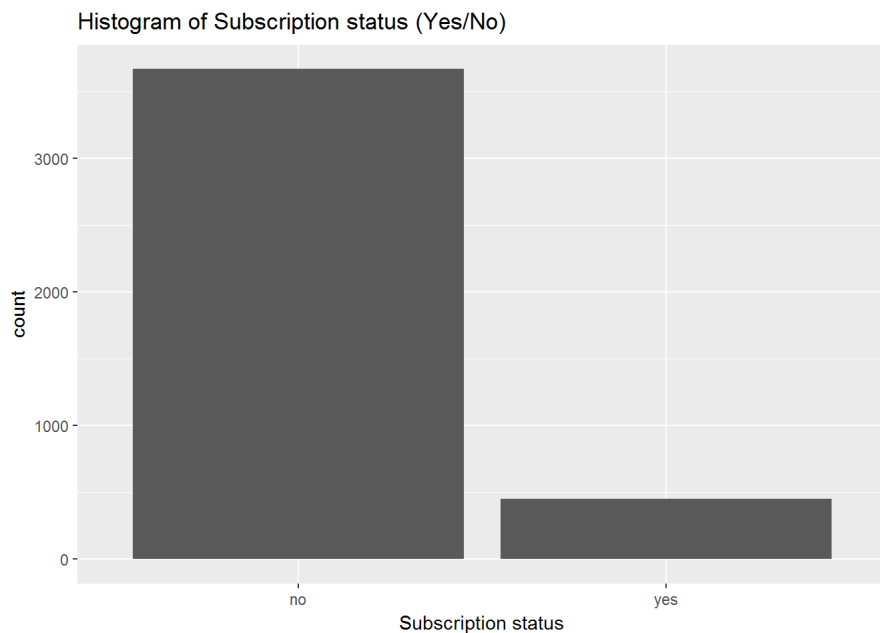
```
colnames(Bkdata)
```

```
## [1] "age"      "job"      "marital"  "education"
## [5] "default"  "housing"  "loan"     "contact"
## [9] "month"    "day_of_week" "duration" "campaign"
## [13] "pdays"   "previous"  "poutcome" "emp.var.rate"
## [17] "cons.price.idx" "cons.conf.idx" "euribor3m" "nr.employed"
## [21] "y"
```

```
target <- as.character(Bkdata$y) # Deposit subscription is referred to as target

targetplot <- ggplot(data.frame(target), aes(x=Bk_subscript)) +
  geom_bar() + ggtitle(label="Histogram of Subscription status (Yes/No)") + xlab(label="Subscription status")

targetplot
```



```
str(Bkdata) # gives a nice description of each column and its factors
```

```
## 'data.frame': 4119 obs. of 21 variables:
## $ age : int 30 39 25 38 47 32 32 41 31 35 ...
## $ job : Factor w/ 12 levels "admin.", "blue-collar", ...: 2 8 8 8 1 8 1 3 8 2 ...
## $ marital : Factor w/ 4 levels "divorced", "married", ...: 2 3 2 2 2 3 3 2 1 2 ...
## $ education : Factor w/ 8 levels "basic.4y", "basic.6y", ...: 3 4 4 3 7 7 7 6 3 ...
## $ default : Factor w/ 3 levels "no", "unknown", ...: 1 1 1 1 1 1 1 2 1 2 ...
## $ housing : Factor w/ 3 levels "no", "unknown", ...: 3 1 3 2 3 1 3 3 1 1 ...
## $ loan : Factor w/ 3 levels "no", "unknown", ...: 1 1 1 2 1 1 1 1 1 1 ...
## $ contact : Factor w/ 2 levels "cellular", "telephone": 1 2 2 2 1 1 1 1 1 2 ...
## $ month : Factor w/ 10 levels "apr", "aug", "dec", ...: 7 7 5 5 8 10 10 8 8 7 ...
## $ day_of_week : Factor w/ 5 levels "fri", "mon", "thu", ...: 1 1 5 1 2 3 2 2 4 3 ...
## $ duration : int 487 346 227 17 58 128 290 44 68 170 ...
## $ campaign : int 2 4 1 3 1 3 4 2 1 1 ...
## $ pdays : int 999 999 999 999 999 999 999 999 999 999 ...
## $ previous : int 0 0 0 0 0 2 0 0 1 0 ...
## $ poutcome : Factor w/ 3 levels "failure", "nonexistent", ...: 2 2 2 2 2 1 2 2 1 2 ...
## $ emp.var.rate : num -1.8 1.1 1.4 1.4 -0.1 -1.1 -1.1 -0.1 -0.1 1.1 ...
## $ cons.price.idx : num 92.9 94 94.5 94.5 93.2 ...
## $ cons.conf.idx : num -46.2 -36.4 -41.8 -41.8 -42 -37.5 -37.5 -42 -42 -36.4 ...
## $ euribor3m : num 1.31 4.86 4.96 4.96 4.19 ...
## $ nr.employed : num 5099 5191 5228 5228 5196 ...
## $ y : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 1 1 1 1 ...
```

```
class(Bkdata)
```

```
## [1] "data.frame"
```

```
#install.packages("ggplot2")
#library(ggplot2)

#remove.packages("caret")
#install.packages('Rcpp', dependencies = TRUE)
#install.packages('caret', dependencies = TRUE)
library(caret)

#####

myBkdata <- data.frame(Bkdata$nr.employed,Bkdata$previous,Bkdata$emp.var.rate,Bkdata$age,Bkdata$y)
colnames(myBkdata)
```

```
## [1] "Bkdata.nr.employed" "Bkdata.previous" "Bkdata.emp.var.rate"
## [4] "Bkdata.age" "Bkdata.y"
```

```
library(caret)

myint_factors <- dummyVars(~ ., data = myBkdata)
newBkdata <- data.frame(predict(myint_factors, myBkdata))
class(newBkdata)
```

```
## [1] "data.frame"
```

```
colnames(newBkdata)
```

```
## [1] "Bkdata.nr.employed" "Bkdata.previous" "Bkdata.emp.var.rate"
## [4] "Bkdata.age" "Bkdata.y.no" "Bkdata.y.yes"
```

```
dfBk <- newBkdata
class(dfBk)
```

```
## [1] "data.frame"
```

```
head(dfBk)
```

```
## Bkdata.nr.employed Bkdata.previous Bkdata.emp.var.rate Bkdata.age
## 1 5099.1 0 -1.8 30
## 2 5191.0 0 1.1 39
## 3 5228.1 0 1.4 25
## 4 5228.1 0 1.4 38
## 5 5195.8 0 -0.1 47
## 6 4963.6 2 -1.1 32
## Bkdata.y.no Bkdata.y.yes
## 1 1 0
## 2 1 0
## 3 1 0
## 4 1 0
## 5 1 0
## 6 1 0
```

```
colnames(dfBk)
```

```
## [1] "Bkdata.nr.employed" "Bkdata.previous" "Bkdata.emp.var.rate"
## [4] "Bkdata.age" "Bkdata.y.no" "Bkdata.y.yes"
```

```
dim(dfBk)
```

```
## [1] 4119 6
```



```
#####
```

```
##Visualization of Correlation matrix
```

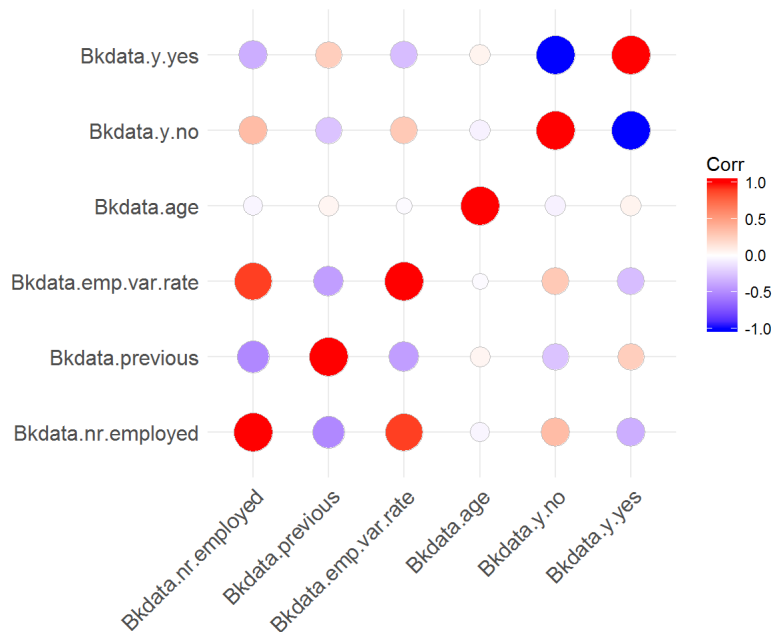
```
#install.packages("ggcorrplot")
```

```
library(ggcorrplot)
```

```
mycorBank <- cor(dfBk)
```

```
# method = "circle"
```

```
ggcorrplot(mycorBank, method = "circle")
```



```
#####
```

```
##Dependencies observed
```

```
# The graph clearly depicts the strength of the correlation  
#by the size and colour coding of diagram.
```

```
# The strongest positive dependencies were between:
```

```
# (i) emp.variation rate & Number employed (Qtrly indicators)  
# - reflecting pos. cor of 0.90
```

```
# The strongest negative correlation were :
```

```
# (i) between the previous: number of contacts performed before this campaign  
# and for this client and the number employed Qtrly indicator.  
# - reflecting neg. cor of -0.51
```

```
# (i) between the previous: number of contacts performed before this campaign  
# and emp.var.rate.
```

```
# - reflecting neg. cor of -0.42
```

```
# Low to moderate dependencies were between deposit yes:
```

```
# (i) nr.employed (# employed) : neg. cor of -0.35
```

```
# (ii) previous : pos cor of 0.26
```

```
# (iii) emp.var.rate : neg. cor of -0.28
```

```
mycorBank[, "Bkdata.nr.employed"]
```

```
## Bkdata.nr.employed Bkdata.previous Bkdata.emp.var.rate  
## 1.00000000 -0.51485341 0.89717322  
## Bkdata.age Bkdata.y.no Bkdata.y.yes  
## -0.04193615 0.34924123 -0.34924123
```

```
mycorBank["Bkdata.y.yes", ]
```

```
## Bkdata.nr.employed Bkdata.previous Bkdata.emp.var.rate  
## -0.34924123 0.25569663 -0.28321568  
## Bkdata.age Bkdata.y.no Bkdata.y.yes  
## 0.06037408 -1.00000000 1.00000000
```

```
mycorBank[, "Bkdata.previous"]
```

```
## Bkdata.nr.employed Bkdata.previous Bkdata.emp.var.rate
## -0.51485341 1.00000000 -0.41523813
## Bkdata.age Bkdata.y.no Bkdata.y.yes
## 0.05093104 -0.25569663 0.25569663
```

```
mycorBank[, "Bkdata.emp.var.rate"]
```

```
## Bkdata.nr.employed Bkdata.previous Bkdata.emp.var.rate
## 0.89717322 -0.41523813 1.00000000
## Bkdata.age Bkdata.y.no Bkdata.y.yes
## -0.01919176 0.28321568 -0.28321568
```

```
#####
##### ANALYSIS #####
#####
### TRAINING the model #####

### We are selecting the response variable as y - has the client subscribed a term deposit? (binary: 'yes', 'no')

#### Step 1

### Splitting the data set into two parts : 75% - training set & 25% - test set

bkTr_rnum <- 0.75*nrow(dfBk)

mydfBk_Tr <- dfBk[(1:bkTr_rnum),]
dim(mydfBk_Tr)
```

```
## [1] 3089 6
```

```
mydfBk_Test <- dfBk[((bkTr_rnum+1):nrow(dfBk)),]

##Step #2 : Training the model - we use training data set only
#### Find Posterior probability of each class  $P(C_i|x)$   $i = 1, 2$  only ( i.e. 2 classes)

#To find the posterior we need the Prior Probability  $P(x|C_i)$  i.e class Likelihood
# and Probability of each class  $P(C_i)$ 
# Note we do need the evidence since it will be common in both classes we can
#eliminate/ignore it as we will be comparing the posterior in both classes to
#choose a max for class assignment. Hence, a common denominator ( evidence :  $P(X)$ )
#can be ignored

# Let us proceed with deriving an estimate for Sigma (covariance matrix)
# since it will be needed to compute the Posterior Prob.
# We must separate training set into the different classes and find
#Posterior for each class

colnumBk_Tr <- dim(mydfBk_Tr)[2]

# using only input variables and ensuring data points belong to class 1
mydfBk_Tr_class1 <- mydfBk_Tr[(mydfBk_Tr[,colnumBk_Tr]==1),-c(colnumBk_Tr-1,colnumBk_Tr)]

mydfBk_Tr_class0 <- mydfBk_Tr[(mydfBk_Tr[,colnumBk_Tr]==0),-c(colnumBk_Tr-1,colnumBk_Tr)]

# Separating the input values from the response values for the test set as well
mydfBk_TestInput <- dfBk[((bkTr_rnum+1):nrow(dfBk)),-c(colnumBk_Tr-1,colnumBk_Tr)]

# response variable only for test set
r_test <- dfBk[((bkTr_rnum+1):nrow(dfBk)),colnumBk_Tr]

dim(mydfBk_Tr_class1) # confirming dim of class for training set
```

```
## [1] 342 4
```

```
dim(mydfBk_Tr_class0)
```

```
## [1] 2747 4
```

```
#Finding the estimate for the covarince matrix denoted by Si ( for class i).
# Starting with class i=1 ( representing success re deposit subscription)
```

```
m1 <- apply(mydfBk_Tr_class1,2,mean)
m0 <- apply(mydfBk_Tr_class0,2,mean)
```

```
x_C11 <-mydfBk_Tr_class1
x_C10 <-mydfBk_Tr_class0
```

```
nrows1 <- dim(x_C11)[1] ## instances (rows) for class1
nrows0 <- dim(x_C10)[1]
```

```
#install.packages("MASS")
#Library(MASS)
```

```
covariance_matrix1 <- function(d) {
  # calculating covriance matrix from the given sample d

  cn <- ncol(d)
  tn <- nrow(d)
  result <- matrix(c(rep(0,cn*cn)),cn,cn)
  ms <- apply(d,2,mean) #calculates actual mean of Sample per column
  i <- 1
  while (i <= cn)
  {
    j <- 1
    while (j <=cn) {
      if (j !=i) {
        diffi <- matrix(c(rep(0,tn)),tn,cn)
        diffj <- matrix(c(rep(0,tn)),tn,cn)
        prodt <- matrix(c(rep(0,tn)),tn,cn)
        for (t in 1:tn) {
          diffi[t,i] <- d[t,i] - ms[i]

          diffj[t,j] <- d[t,j] -ms[j]

          prodt[t] <- diffi[t,i] * diffj[t,j]

        }#end for
        result[i,j] <- (sum(prodt))/tn

      }# endif
      j <- j +1
    }#end nested while
    result[i,i] <- (sum((d[,i]-ms[i])^2))/tn # cal. of variance
    i <- i +1

  } #end 1st while
  return(result);
} #end function
```

```
S1 <- covariance_matrix1(x_C11)
S0 <- covariance_matrix1(x_C10)
```

```
#calculating class Likelihood p(x/Ci)
Class_likeihoodi <- function(di,mySi,mi){
```

```
  drn <- dim(di)[1]
  cn <- dim(di)[2]
  result_like <- rep(0,drn)
  for (t in 1:drn){
    a <- (t(di[t,]-mi))
    b <- solve(mySi)
    c <- as.matrix(di[t,]-mi)
    dd <- sqrt(det(mySi))
    e <- (2*pi)^(cn*0.5)

    result_like[t] <- (1/(e*dd))*exp(-0.5*c%*b%*%a)
  }
  return(result_like)
}
likelihood_c11<- Class_likeihoodi(mydfBk_TestInput,S1,m1)

likelihood_c10<- Class_likeihoodi(mydfBk_TestInput,S0,m0)
```

```

Pc1 <- (dim(x_C11)[1]) / ((dim(x_C11)[1])+(dim(x_C10)[1])) # Prob. of Class 1
Pc0 <- (dim(x_C10)[1]) / ((dim(x_C11)[1])+(dim(x_C10)[1])) # Prob. of class 0

PostProb1 <- likelihood_c11*Pc1 # Posterior Prob for class 1 for test dataset
PostProb0 <- likelihood_c10*Pc0 # Posterior Prob for class 0 for test dataset

# Classifying based on the maximum post. prob. for each test data point
classification <- function(p1,p0){
  lenp <- length(p1)
  rclass <- rep(99,lenp) # 99 depicts the rejected datapoint
                        # value will be preserved if p0 = p1

  for (cnt in 1:lenp){

    if (p1[cnt] > p0[cnt]){
      # if posterior prob. for class1 is greater than that of class0
      # then assign the data point to class 1
      rclass[cnt]<- 1}

    else if (p0[cnt] > p1[cnt]){
      rclass[cnt] <- 0}
  }# end for-Loop
  return(rclass);
}#end function

modelresultTest <- classification(PostProb1,PostProb0)
ErrorTest <- function(r_model,r_actual){
  # Calculating how accurate model was vs. actual "correct" values

  lenr <- length(r_actual)
  resultTest <- rep(0,lenr)

  for (a in 1:lenr){
    if (r_model[a] == r_actual[a]){
      resultTest[a] = 1

    } else if (r_model[a] != 99){
      resultTest[a] = 0

    }#end else if
  }#end for Loop

  return(resultTest);
}# end function

ErrorTestresults <- ErrorTest(modelresultTest,r_test)
PercError <- 100*sum(ErrorTestresults==0)/length(ErrorTestresults)
PercCorrect <- 100*sum(ErrorTestresults==1)/length(ErrorTestresults)
PercReject <- 100*sum(ErrorTestresults==99)/length(ErrorTestresults)

PercError

```

```
## [1] 10.30126
```

```
PercCorrect
```

```
## [1] 89.69874
```

```
PercReject
```

```
## [1] 0
```

```

# Model has an error rate of 10.30% or an Accuracy rate of 89.70%
#and zero rejection rate

# We will now test the accuracy of model with different parameters,
#specifically, we will vary the covariance matrix
### Using the equation below we will vary alpha ("A") and Beta ("B")
#Sinew rep new Si (covariance matrix after varying A and B)
# s2 rep. variance which is going to be an average of the vars in S
# where S is derived by using  $P(C1)*S1 + P(C0)*S0$ 
# and S1 rep covariance matrix for class 1 when i = 1 in Si
#and S0 rep covariance matrix for class 0 when i = 0 in Si

Sinew <- A*s2*I + B*S + (1-A-B)*Si

S <- Pc1*S1 + Pc0*S0 # Matrix for entire dataset; notice weighted based on resp. prob.

# Let us derive s2, the variance, which is going to be an average of the vars in S
avgs2 <- function(covS){
  dimS <- dim(covS)[2] # symmetrical so either row or column will denote data dimension

  getavg <- (1/dimS)*(sum(diag(covS))) # changes diagonals (variance values) to the avg.

  return(getavg)
}# end function

s2 <- avgs2(S) # returns avg. variance value of given covariance matrix
# in this case, it is the avg. using covariance matrix irrespective of class

# Case 1 : Let A ("alpha") = 1 & B ("Beta") = 0
# Equation reduces to: Sinew = s2*I (I rep Identity matrix)
# In this case, Sinew for each class will have the attributes that are independent
# since the covariance values will be zero and all the attributes will have
# the same variance value.
### Note the main equation in the function below Sinew which will be reduced
### as alpha and beta values vary or the various scenarios
### In case #2 and additional constraint is imposed further to a change of value
#for alpha and beta

Sinew <- function(s2,Si,S,A,B){
  I <- diag(dim(S)[1])

  resultSinew <- A*s2*I + B*S + (1-A-B)*Si
  return(resultSinew)
}

offdiagzero <- function(Scov){
  ## takes matrix and sets off-diagonal elements to zero

  I <- diag(dim(S)[1]) # Identity matrix created

  for (cnti in 1:dim(S)[1]){
    #setting diagonal elements in matrix
    #to the identity matrix
    I[cnti,cnti] <- Scov[cnti,cnti]
  }#end for Loop
  return(I)
}# end function

#s2 rep variance
# S rep. cov. matrix derived using both classes
# S1 & S0 rep. cov. matrix from class1 and class0 respectively

SinewCase1 <- Sinew(s2,S1,S,1,0) #alpha = 1 & Beta = 0
SinewCase2 <- offdiagzero(Sinew(s2,S1,S,0,1)) #alpha = 0 & Beta = 1 and offdiag.set to zero
SinewCase3 <- Sinew(s2,S1,S,0,1) #alpha = 0 & Beta = 1
SinewCase4 <- Sinew(s2,S1,S,0,0) #alpha = 0 & Beta = 0

S0newCase1 <- Sinew(s2,S0,S,1,0)
S0newCase2 <- offdiagzero(Sinew(s2,S0,S,0,1))
S0newCase3 <- Sinew(s2,S0,S,0,1)
S0newCase4 <- Sinew(s2,S0,S,0,0)

### Case1 cont'd. Deriving posterior and effecting classification based on
#### new cov matrix (where newSi = s2*I); recall, we let A ("alpha") = 1 & B ("Beta") = 0
#### as shown above SinewCase1, look at arguments passed to function on RHS

```

```

like_Newcase1_c11<- Class_likeihoodi(mydfBk_TestInput,S1newCase1,m1)
like_Newcase1_c10<- Class_likeihoodi(mydfBk_TestInput,S0newCase1,m0)

###These will not change as we vary S (covariance matrix from case to case)
Pc1 <- (dim(x_C11)[1]) / ((dim(x_C11)[1])+(dim(x_C10)[1])) # Prob. of Class 1
Pc0 <- (dim(x_C10)[1]) / ((dim(x_C11)[1])+(dim(x_C10)[1])) # Prob. of Class 0

PostProb1_newCase1 <- like_Newcase1_c11*Pc1 # Posterior Prob for class 1 for test dataset
PostProb0_newCase1 <- like_Newcase1_c10*Pc0 # Posterior Prob for class 0 for test dataset

##Classifying

#initializing (only necessary for case1)
PercError_newCase <- rep(55,4)
PercCorrect_newCase<- rep(55,4)
PercReject<- rep(55,4)

Tot <- length(r_test) # Will not change as case varies

modelresultTest_newCase1 <- classification(PostProb1_newCase1,PostProb0_newCase1)
ErrorTestresults_newCase1 <- ErrorTest(modelresultTest_newCase1,r_test)
PercError_newCase[1] <- 100*sum(ErrorTestresults_newCase1==0)/Tot
PercCorrect_newCase[1] <- 100*sum(ErrorTestresults_newCase1==1)/Tot
PercReject[1] <- 100*sum(ErrorTestresults_newCase1==99)/Tot

### Case2 Deriving posterior and effecting classification based on
#### new cov matrix (where newSi = S where sij =0 i.e off-diag. are zero
#### in other words when the attributes are independent since covariances equal zero)

like_Newcase2_c11<- Class_likeihoodi(mydfBk_TestInput,S1newCase2,m1)
like_Newcase2_c10<- Class_likeihoodi(mydfBk_TestInput,S0newCase2,m0)

PostProb1_newCase2 <- like_Newcase2_c11*Pc1 # Posterior Prob for class 1 for test dataset
PostProb0_newCase2 <- like_Newcase2_c10*Pc0 # Posterior Prob for class 0 for test dataset

##Classifying

modelresultTest_newCase2 <- classification(PostProb1_newCase2,PostProb0_newCase2)
ErrorTestresults_newCase2 <- ErrorTest(modelresultTest_newCase2,r_test)

PercError_newCase[2] <- 100*sum(ErrorTestresults_newCase2==0)/Tot
PercCorrect_newCase[2] <- 100*sum(ErrorTestresults_newCase2==1)/Tot
PercReject[2] <- 100*sum(ErrorTestresults_newCase2==99)/Tot

### Case3 Deriving posterior and effecting classification based on
#### new cov matrix (where newSi = S so both classes share the same cov. matrix)
#### This occurs when "A" (alpha = 0) and "B" (Beta = 1)

like_Newcase3_c11<- Class_likeihoodi(mydfBk_TestInput,S1newCase3,m1)
like_Newcase3_c10<- Class_likeihoodi(mydfBk_TestInput,S0newCase3,m0)

PostProb1_newCase3 <- like_Newcase3_c11*Pc1 # Posterior Prob for class 1 for test dataset
PostProb0_newCase3 <- like_Newcase3_c10*Pc0 # Posterior Prob for class 0 for test dataset

##Classifying

modelresultTest_newCase3 <- classification(PostProb1_newCase3,PostProb0_newCase3)
ErrorTestresults_newCase3 <- ErrorTest(modelresultTest_newCase3,r_test)

PercError_newCase[3] <- 100*sum(ErrorTestresults_newCase3==0)/Tot
PercCorrect_newCase[3] <- 100*sum(ErrorTestresults_newCase3==1)/Tot
PercReject[3] <- 100*sum(ErrorTestresults_newCase3==99)/Tot

### Case4 - Keeping original values for Si
#### This occurs when "A" (alpha = 0) and "B" (Beta = 0)

```

```

like_Newcase4_c11<- Class_likeihoodi(mydfBk_TestInput,S1newCase4,m1)
like_Newcase4_c10<- Class_likeihoodi(mydfBk_TestInput,S0newCase4,m0)

PostProb1_newCase4 <- like_Newcase4_c11*Pc1 # Posterior Prob for class 1 for test dataset
PostProb0_newCase4 <- like_Newcase4_c10*Pc0 # Posterior Prob for class 0 for test dataset

##Classifying

modelresultTest_newCase4 <- classification(PostProb1_newCase4,PostProb0_newCase4)
ErrorTestresults_newCase4 <- ErrorTest(modelresultTest_newCase4,r_test)

PercError_newCase[4] <- 100*sum(ErrorTestresults_newCase4==0)/Tot
PercCorrect_newCase[4] <- 100*sum(ErrorTestresults_newCase4==1)/Tot
PercReject[4] <- 100*sum(ErrorTestresults_newCase4==99)/Tot

#####
## Case#5 - We will try one more case where alpha = 0.5 and beta = 0.5

S1newCase5 <- Sinew(s2,S1,S,0.5,0.5)
S0newCase5 <- Sinew(s2,S0,S,0.5,0.5)

like_Newcase5_c11<- Class_likeihoodi(mydfBk_TestInput,S1newCase5,m1)
like_Newcase5_c10<- Class_likeihoodi(mydfBk_TestInput,S0newCase5,m0)

PostProb1_newCase5 <- like_Newcase5_c11*Pc1 # Posterior Prob for class 1 for test dataset
PostProb0_newCase5 <- like_Newcase5_c10*Pc0 # Posterior Prob for class 0 for test dataset

##Classifying

modelresultTest_newCase5 <- classification(PostProb1_newCase5,PostProb0_newCase5)
ErrorTestresults_newCase5 <- ErrorTest(modelresultTest_newCase5,r_test)

PercError_newCase[5] <- 100*sum(ErrorTestresults_newCase5==0)/Tot
PercCorrect_newCase[5] <- 100*sum(ErrorTestresults_newCase5==1)/Tot
PercReject[5] <- 100*sum(ErrorTestresults_newCase5==99)/Tot

#####
## Comparing we get
ErrorComparisonCase1_2_3_4_5 <- data.frame(CaseNo=c(1,2,3,4,5),alpha=c(1,0,0,0,0.5),Beta=c(0,1,1,0,0.5),PercError=PercError_
newCase,PercCorrect=PercCorrect_newCase,PercReject)

ErrorComparisonCase1_2_3_4_5

## CaseNo alpha Beta PercError PercCorrect PercReject
## 1      1  1.0  0.0  25.07289    74.92711      0
## 2      2  0.0  1.0  15.16035    84.83965      0
## 3      3  0.0  1.0  10.59281    89.40719      0
## 4      4  0.0  0.0  10.30126    89.69874      0
## 5      5  0.5  0.5  10.68999    89.31001      0

# Case#4 which is our original case where we have two different covariance matrices
# is when the accuracy of our model is the highest.

#Recall that Case#2 has an additional constraint imposed to that of case #2 where
# off-diagonal terms are set to zero
library(caret)

ErrorComparisonCase1_2_3_4_5

```

```
## CaseNo alpha Beta PercError PercCorrect PercReject
## 1      1   1.0   0.0  25.07289    74.92711         0
## 2      2   0.0   1.0  15.16035    84.83965         0
## 3      3   0.0   1.0  10.59281    89.40719         0
## 4      4   0.0   0.0  10.30126    89.69874         0
## 5      5   0.5   0.5  10.68999    89.31001         0
```