PH2102 Problem Set

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Q 1) Let the velocity and the acceleration of a moving particle in a given frame be $\vec{u}$ and $\vec{a}$, respectively. If the acceleration of the particle in an instantaneously co-moving frame is $\vec{a}_0$, then show that its magnitude is given by

$$a_0 = \gamma_u^3 \sqrt{1 - \beta_u^2 \sin^2\theta}\, a$$

where θ is the angle between the vectors $\vec{u}$ and $\vec{a}$.

Q 2) Observer S measures the components of 4-covector A_μ to be $(5, 1, -2, 0)$ in some units. Another observer S' measures space-time coordinates that are related to those measured by S by the Lorentz transformation

$$L = \frac{1}{64}\begin{pmatrix} 125 & -48 & -60 & 75 \\ -75 & 80 & 36 & -45 \\ -60 & 0 & 80 & -36 \\ 48 & 0 & 0 & 80 \end{pmatrix}$$

Determine the components $A'_\mu$of the 4-covector measured by S'.

Q 3) A Lorentz transformation matrix L obeys $L^T \eta L = \eta$.

a) If two Lorentz transformations are carried out in succession, there corresponding matrices multiply. Show that if L_1 and L_2 are Lorentz transformation matrices, their product $L_1 L_2$ is a Lorentz transformation matrix.

b) Show that if both L_1 and L_2 have determinant $+1$, their product has determinant $+1$. *(Easy)*

c) If both L_1 and L_2 have 0-0 components that are bigger than 1, show that $L_1 L_2$ also has a 0-0 component that is bigger then 1. *(A bit more difficult!)*