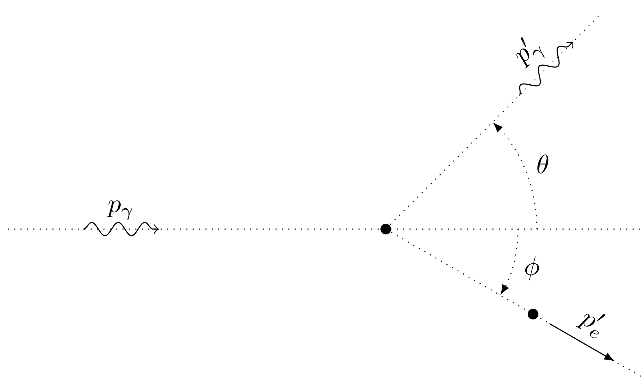
PH2102 Problem Set 9

Q 1) Consider a point particle of rest mass m which starts from rest under a uniform constant force of magnitude F. Show that when its displacement from the initial position is x, its velocity is given by

$$u = \frac{\sqrt{x\,(x+2\xi)}}{x+\xi}\,c$$

where $\xi = \frac{mc^2}{F}$. Solve this to find how x depends on time t

Q 2) We have analyzed the Compton effect by using 4-vector methods. In this problem we will repeat the same - but using energy and momentum conservation directly. The figure below depicts the scattering of a photon off a stationary electron. After the collision the photon scatters through an angle θ as shown in the figure. The electron recoils in a direction making an angle ϕ with that of the initial direction of motion of the photon.



Using the fact that the energy of a photon is related to its momentum by $E_\gamma = p_\gamma c$ we can write energy conservation as

$$m_e c^2 + p_\gamma c = p'_\gamma c + E'_e \tag{1}$$

Momentum conservation leads to two equations

$$p_\gamma = p'_\gamma \cos\theta + p'_e \cos\phi \tag{2}$$

$$0 = p'_\gamma \sin\theta - p'_e \sin\phi \tag{3}$$

Carry out the following steps to derive the Compton shift:

- eliminate ϕ between (2) and (3).
- Now eliminate p'_e and E'_e between this result and (1) by using the relativistic relation between energy and momentum.

- Use the relation $\frac{h}{p_\gamma} = \lambda$ to determine the shift $\lambda' - \lambda$ as a function of θ.

Q 3) In Compton scattering a high energy photon scatters off an electron. The photon is not absorbed by the electron - just moves away in a different direction. Show that the process of a free electron absorbing a photon $\gamma + e \to e$ is not allowed due to energy-momentum conservation.

Q 4) The isotope ^{220}Fr of Francium can decay by either emitting α and β particles. By looking up a suitable tables of atomic masses, calculate the kinetic energy of the emitted α particles and the maximum kinetic energy of the emitted β particles.

One possible source of atomic masses could be

https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html.

Note that the atomic mass is the sum of the nuclear and electronic masses (the actual value is very slightly smaller - because of the binding energy of the electrons - but this is too tiny to make a significant difference).

Q 5) Alice observes two particles, of rest masses m_1 and m_2, respectively, to have velocities $\vec{u}_1$ and $\vec{u}_2$, respectively. Bob has a velocity of $\vec{u}_2$ with respect to Alice (the same as the second particle's) . What is the energy of the first particle that is observed by Bob? *Hint : Use that fact that the scalar product of the 4-momenta, P_1 and P_2, of the two particles is an invariant.*

Q 6) a) Show that for two particles, of rest masses m_1 and m_2, and 4-momenta P_1 and P_2, the invariant

$$P_1^\mu P_{2\,\mu} \geq m_1 m_2.$$

b) Show that if there are n particles with masses $m_1, m_2, \ldots, m_n$ and 4-momenta $P_1,\, P_2, \ldots, P_n$ we have

$$P^\mu P_\mu \geq M^2 c^2$$

where $P = P_1 + P_2 + \ldots + P_n$ and $M = m_1 + m_2 + \ldots + m_n$. Under what condition will the above inequality be satisfied as an equality?

c) Consider a reaction where a projectile of rest mass m_1 hits a stationary target of rest mass m_2. Let the reaction lead to the creation of n particles, whose net mass M is *more* than $m_1 + m_2$. What is the minimum kinetic energy that the projectile must have in order that this reaction takes place?