PH2102 Problem Set 8

Q 1) For some problems, it is more convenient to use the so called "light cone coordinates" instead of our usual quartet of $x^0, x^1, x^2$and x^3. These are defined by the following

$$x^+ = x^0 + x^1, \quad x^- = x^0 - x^1,$$

the other coordinates staying unchanged.

a) Try to explain why these are called "light cone coordinates" .

b) Write down the space-time invariant in terms of these coordinates $\left(x^+, x^-, x^2, x^3\right)$. Hence write down the matrix form for η in these coordinates.

c) What does the special Lorentz transformations look like in these coordinates?

d) A general 4-vector in light cone coordinates will be of the form $A^\mu = \left(A^+, A^-, A^2, A^3\right)$. How will the components of its corresponding covector be related to A^+, A^-, A^2, A^3 in these co-ordinates?

Q 2) In classical (i.e. Newtonian) physics, the kinetic energy T of a point particle is defined to be the work that needs to be done on it to accelerate it to its current velocity $\vec{u}$ starting from rest. The calculation goes somewhat like this :

$$\begin{aligned} T &= \int \vec{F} \cdot d\vec{r} = \int_{\vec{v}=\vec{0}}^{\vec{v}=\vec{u}} \frac{d}{dt}\left(m\vec{v}\right) \cdot d\vec{r} = \int d\left(m\vec{v}\right) \cdot \frac{d\vec{r}}{dt} \\ &= \int m\vec{v} \cdot d\vec{v} = \int md\left(\frac{v^2}{2}\right) = \frac{1}{2}mv^2\bigg|_0^{\vec{u}} = \frac{1}{2}mu^2 \end{aligned}$$

In relativistic physics we can use something similar to define kinetic energy. However, instead of $T = \int_{\vec{v}=\vec{0}}^{\vec{v}=\vec{u}} \frac{d}{dt}\left(m\vec{v}\right) \cdot d\vec{r}$ we have to use

$$T = \int_{\vec{v}=\vec{0}}^{\vec{v}=\vec{u}} \frac{d}{dt}\left(m\gamma\left(v\right)\vec{v}\right) \cdot d\vec{r}$$

a) By carrying out this integral, show that the relativistic expression for the kinetic energy of a point particle is

$$T = m\left(\gamma\left(u\right) - 1\right)c^2$$

b) Show that when $u \ll c$ this reduces to the familiar Newtonian formula. What is next order correction to the classical formula? Estimate what is the percentage of error that will be incurred if we were to use the Newtonian formula instead of the classical formula for $u = 0.01c$, $u = 0.1c$,and $u = 0.5c$, respectively.