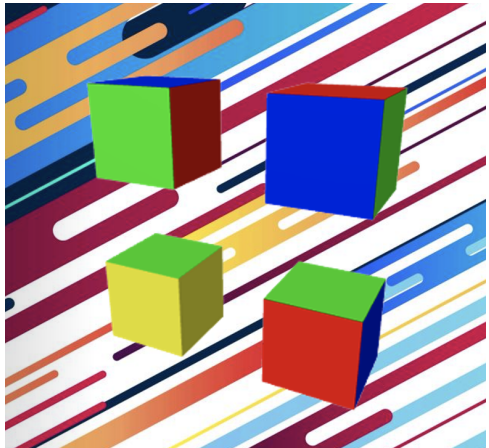


# Instant Insanity: The Four Cubes Problem



# Instant Insanity!

**Instant Insanity** is a puzzle consisting of four cubes with faces colored red, blue, green, and yellow.

# Instant Insanity!

**Instant Insanity** is a puzzle consisting of four cubes with faces colored red, blue, green, and yellow.

## Goal

Stack the cubes in such a way that each side (front, back, left, and right) of the stack shows each of the four colors.

Try to solve the puzzle!

Remember, stack the 4 cubes on top of each other so that each side (front, back, left, and right) of the stack shows each of the four colors.

You have 3 minutes!

This puzzle was created by **Franz Owen Armbruster** and published by Parker Brothers in 1967. Over 20 million puzzles were sold!



## Solving by Trial and Error?

Each cube has 24 different ways of being placed:

- 6 choices for the top face
- rotate the cube 4 times to bring any of the four sides to the front

$$6 \times 4 = 24$$

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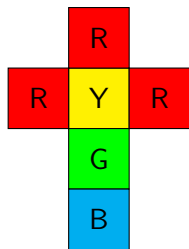
Once the 4 cubes are stacked, there are  $4! \times 8 = 192$  symmetries which give the same solution. So, the total number of possible stacks is

$$\frac{24^4 \times 4!}{192} = 41,472$$

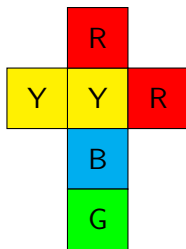


## A different approach

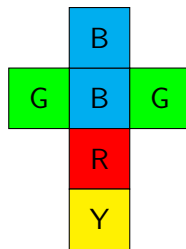
Lets visualize the cubes in a flattened-out form



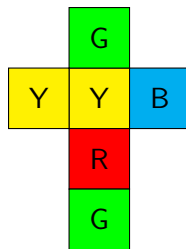
Cube 1



Cube 2

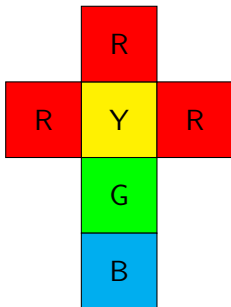


Cube 3



Cube 4

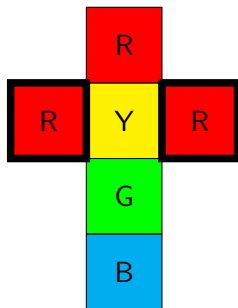
## Opposite Faces



Cube 1

What are the opposite faces in cube 1?

## Opposite Faces

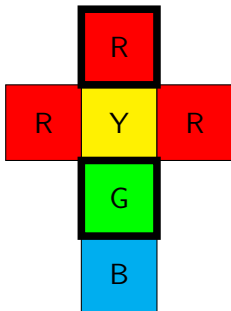


Cube 1

What are the opposite faces in cube 1?

- 1 Red is opposite to Red

## Opposite Faces

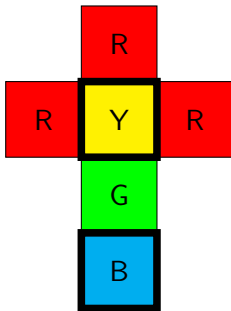


Cube 1

What are the opposite faces in cube 1?

- ① Red is opposite to Red
- ② Red is opposite to Green

## Opposite Faces



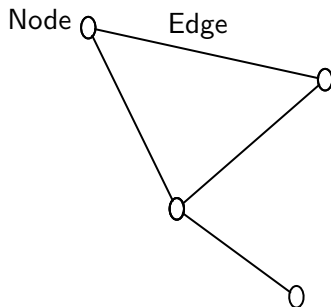
Cube 1

What are the opposite faces in cube 1?

- ① Red is opposite to Red
- ② Red is opposite to Green
- ③ Yellow is opposite to Blue

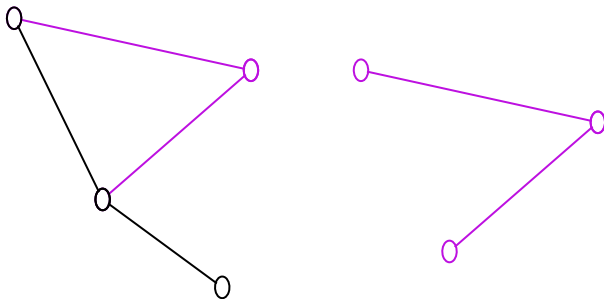
# An Introduction to Graph Theory

A **graph** is a mathematical structure used to model pairwise relations between objects.



# An Introduction to Graph Theory

A **subgraph** is a subset of nodes and edges.



# Solution using Graph Theory

Step 1 Draw a graph for each cube, connecting **opposite faces**

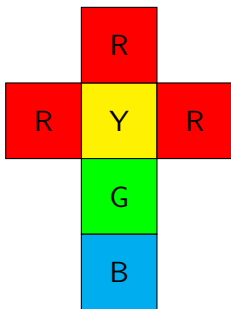
Step 2 Put all 4 graphs from Step 1 into a **single graph**

Step 3 Find 2 subgraphs using specific instructions



## Step 1: Turning Cubes into Graphs

Every cube will be turned into **one graph**. Every color will be a **node**. There is an **edge** between two colors if they are **opposite to each other**.

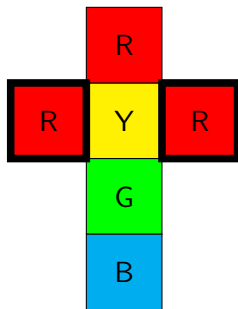


Cube 1



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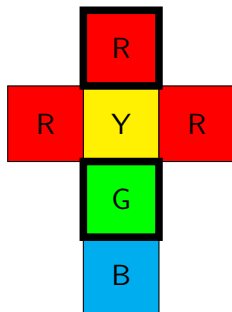


Cube 1

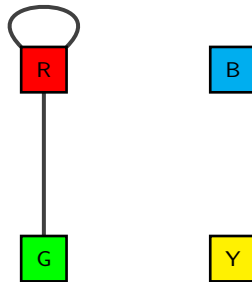


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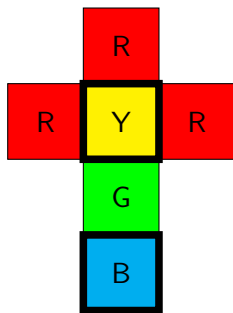


Cube 1



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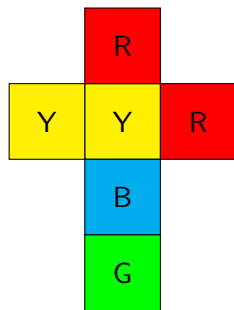


Cube 1

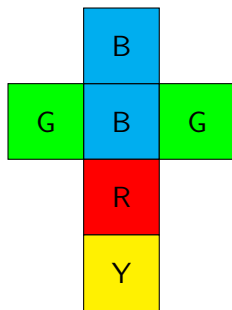


## Step 1: Turning Cubes into Graphs

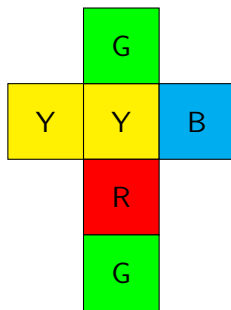
On your worksheet, complete the graphs for the other 3 cubes.



Cube 2



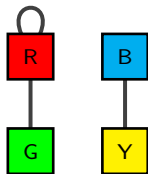
Cube 3



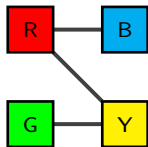
Cube 4

Remember, each graph should have exactly **3 edges** connecting the **opposite faces**.

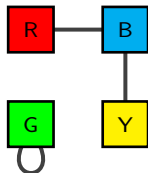
## Solution to Step 1



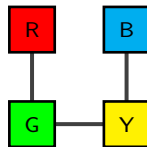
Cube 1  
Graph



Cube 2  
Graph



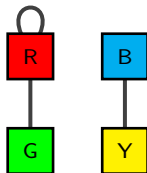
Cube 3  
Graph



Cube 4  
Graph

## Step 2: Create 1 big Graph

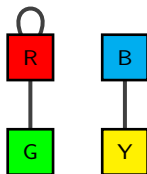
Put all 4 graphs together into a single graph. Put a number next to each edge that represents the **cube number**.



Cube 1 Graph

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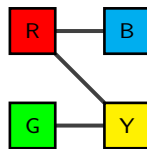


Cube 1 Graph



## Step 2: Create 1 big Graph

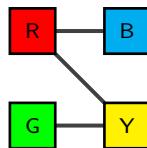
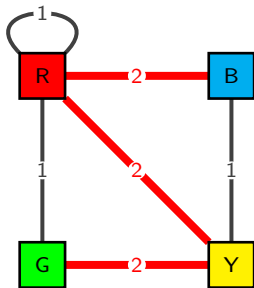
Put all 4 graphs together into a single graph. Put a number next to each edge that represents the **cube number**.



Cube 2 Graph

## Step 2: Create 1 big Graph

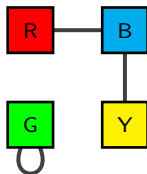
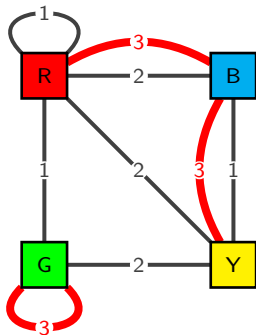
Put all 4 graphs together into a single graph. Put a number next to each edge that represents the **cube number**.



Cube 2 Graph

## Step 2: Create 1 big Graph

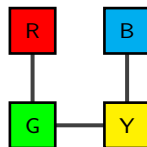
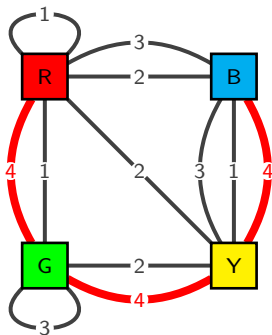
Put all 4 graphs together into a single graph. Put a number next to each edge that represents the **cube number**.



Cube 3 Graph

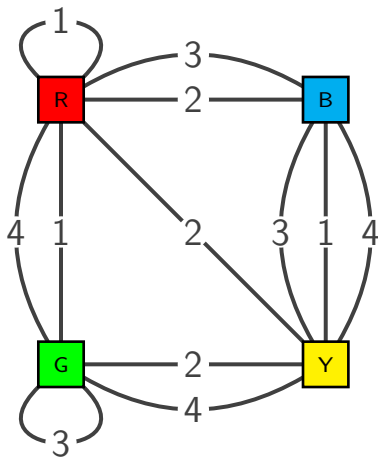
## Step 2: Create 1 big Graph

Put all 4 graphs together into a single graph. Put a number next to each edge that represents the **cube number**.



Cube 4 Graph

## Step 2: Create 1 big graph



## Step 3: Find 2 subgraphs

Create **2 subgraphs** which satisfy:

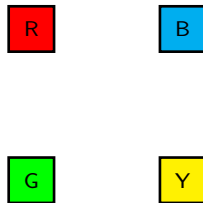
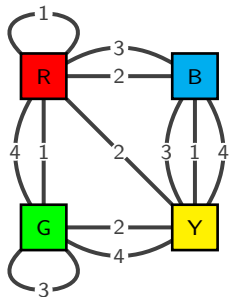
- ① The 2 subgraphs have no edges in common
- ② Each subgraph contains exactly one edge from each cube
- ③ Each node only has 2 edges connected to it

One subgraph corresponds to the colors that will be on the **left and right faces**, the other subgraph will correspond to the colors that will be on the **front and back faces** of the stack.

We'll do this **one subgraph at a time**.

## Step 3 - Subgraph 1

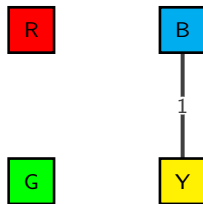
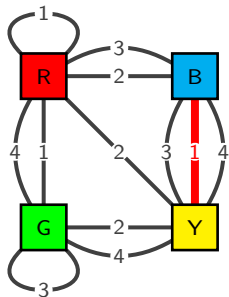
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Subgraph 1

## Step 3 - Subgraph 1

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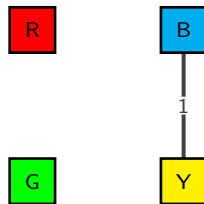
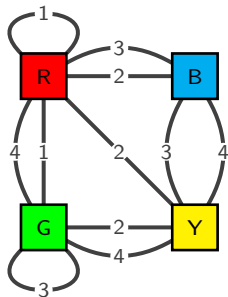


Subgraph 1



## Step 3 - Subgraph 1

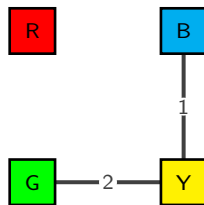
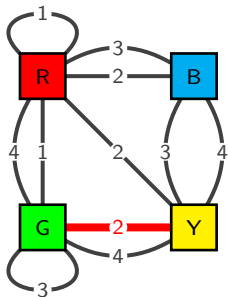
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Subgraph 1

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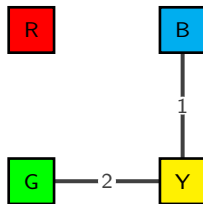
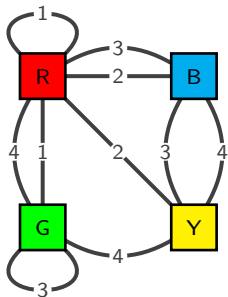
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Subgraph 1

## Step 3 - Subgraph 1

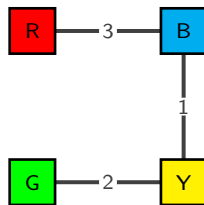
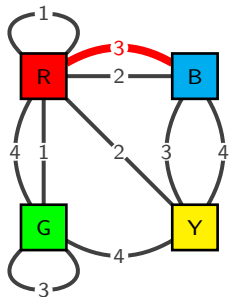
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Subgraph 1

## Step 3 - Subgraph 1

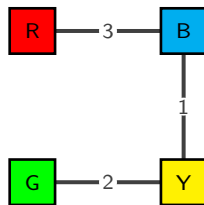
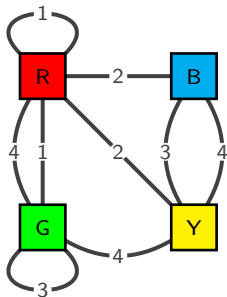
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Subgraph 1

## Step 3 - Subgraph 1

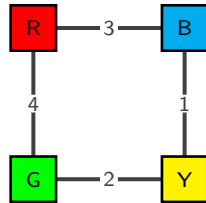
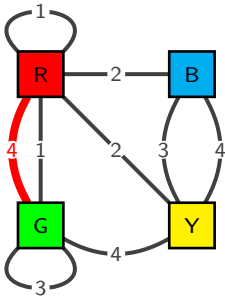
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Subgraph 1

### Step 3 - Subgraph 1

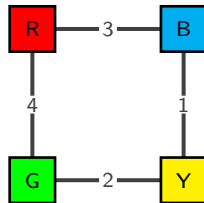
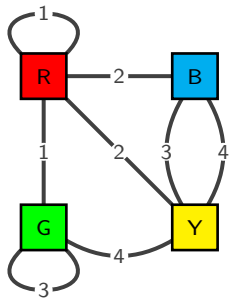
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### Subgraph 1

## Step 3 - Subgraph 1

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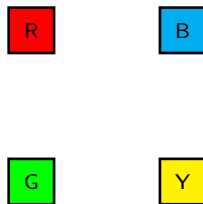
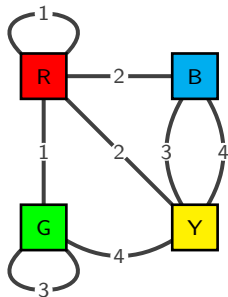


Subgraph 1

## Step 3 - Subgraph 2

**Now it's your turn!** Draw a second subgraph that satisfies these conditions:

- 1 The 2 subgraphs have no edges in common
- 2 Each subgraph contains exactly one edge from each cube
- 3 Each node only has 2 edges connected to it

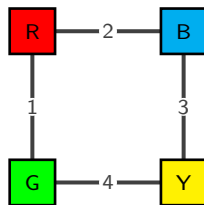
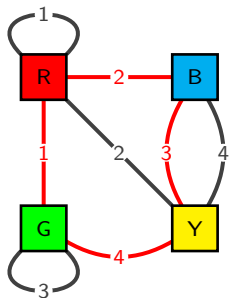


Subgraph 2



## Step 3 - Subgraph 2

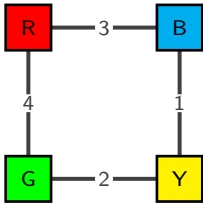
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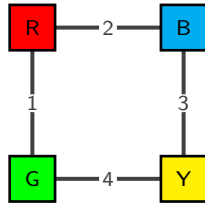
Subgraph 2

## Step 3 - Subgraphs

Subgraph 1

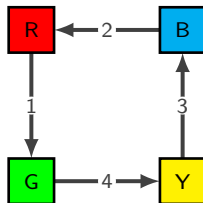
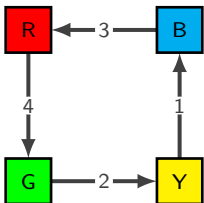


Subgraph 2



## Step 3

For each subgraph, change the edges from **lines** to **arrows** that all point in the same direction.

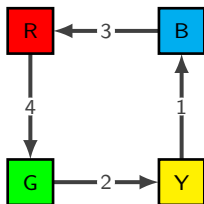


One subgraph will tell us which pair of colors appears on the **front and back** faces of each cube. The other will tell us which pair of colors appears of the **left and right** of each cube.

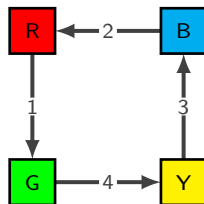
# Stack the Cubes!

Using the 2 subgraphs we found, stack the cubes to get the solution to the puzzle!

Front/Back

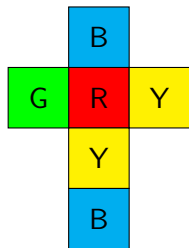


Left/Right

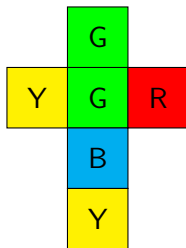


## Another Example

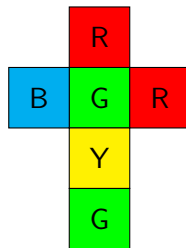
Find the solution to the following four cube problem:



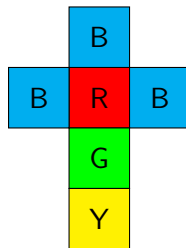
Cube 1



Cube 2



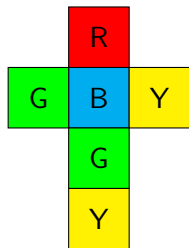
Cube 3



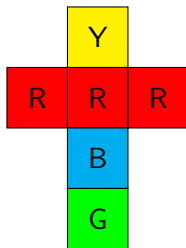
Cube 4

## Another Example

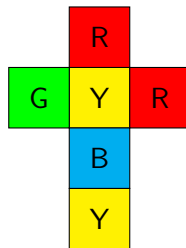
Find the solution to the following four cube problem:



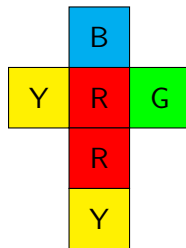
Cube 1



Cube 2



Cube 3



Cube 4