

Assignment 4 (Due December 10 2023)

Submit clear and well explained answers. You can submit one answer per group. You can use the internet (but not pay wall sites like Chegg) to look for answers, but the final answer must be in your own words. You can also use code you found online for the experiments. Please cite all resources used for answers as well as codes.

1. Prove the following statements. Please write out the proof in detail, without skipping any vital step. You can use figures to illustrate your work if necessary (6*10=60 points)
 - (a) The centre of a graph is the set of vertices with minimum eccentricity. Show that a tree can have either (i) exactly one vertex as a centre or (ii) two vertices as centres, and these vertices will be connected by an edge
 - (b) Any tree with at least two vertices has at least two vertices of degree 1
 - (c) Two non-isomorphic trees can have the same degree distribution
 - (d) The upper bound on the number of articulation points in a graph is the number of biconnected components minus 1
 - (e) G is a simple graph with n vertices $n \geq 2$; and at least $\lfloor (n-1)(n-2)/2 \rfloor + 1$ edges. Prove that G is connected
 - (f) Show that if v is an articulation point in a graph G , then it cannot be an articulation point in the complement graph of G .
2. Chain of numbers (20) Given the set of positive integers from 1 to 15, arrange all of them in a sequence such that the sum of consecutive numbers in the sequence is a perfect square. For example if part of the sequence is (9,7,2,14); $9+7=16$; $7+2=9$; $2+14=16$, etc.

Show that solving this problem from any number n is equivalent to solving the Hamiltonian path problem. Here solving means either finding the sequence or identifying that such a sequence, including all numbers cannot be formed. (10)

Explain your method and show step by step how you obtained your sequence for (2*5=10)

- (a) 1 to 15
- (b) 1 to 16

3. Water Jug Problem (20)
You are given three jugs J_1 , J_2 and J_3 with capacity a , b , and c gallons

respectively. Let $a \leq b \leq c$. There are no markings on the jugs, so they can be used to measure only their respective capacities. You can measure different amounts of liquid by pouring from one jar to another. Thus at each step the following things will happen

- (i) One of jars will remain unchanged
- (ii) one jar that was not completely filled will become full
- (iii) one jar which was not empty becomes empty.

Given that jug J1 is filled, develop an algorithm to measure exactly a given amount, x , of liquid using the three jars. [HINT: Express the different options of pouring the liquid as vertices in a graph and then find the shortest path from the start state to the end state].

(a) Clearly explain your algorithm with clearly steps. You do not have to submit the code. (10)

(b) Show how it works for the example given above; 8(J1), 5(J2), and 3(J3) gallons and $x=4$. You should not just write the steps, but also explain how your algorithm is used to arrive at the steps. (10)